Simple model for the energy change for central slice

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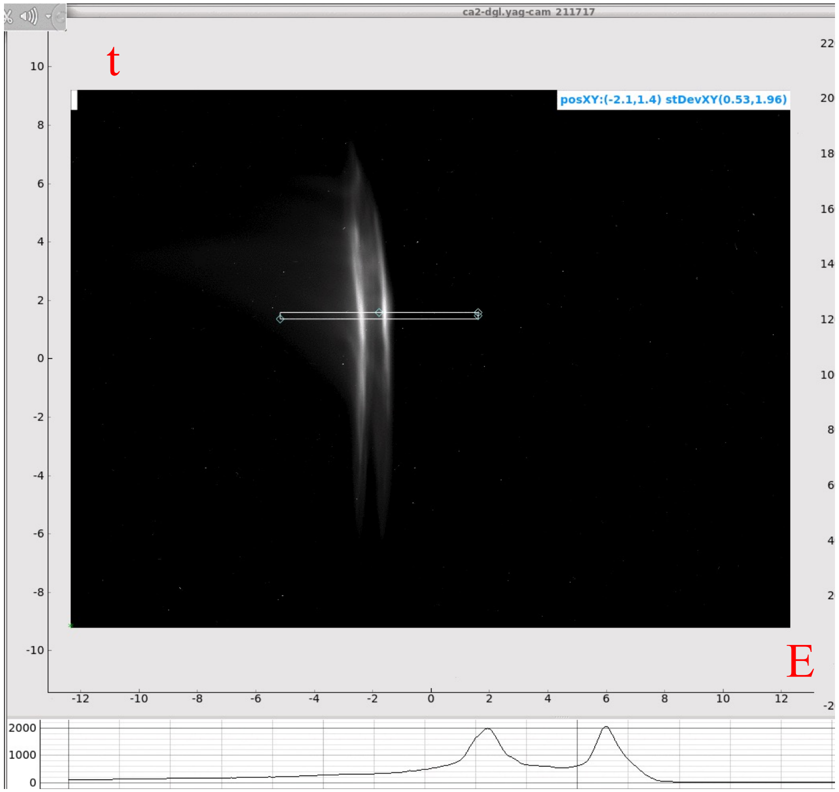
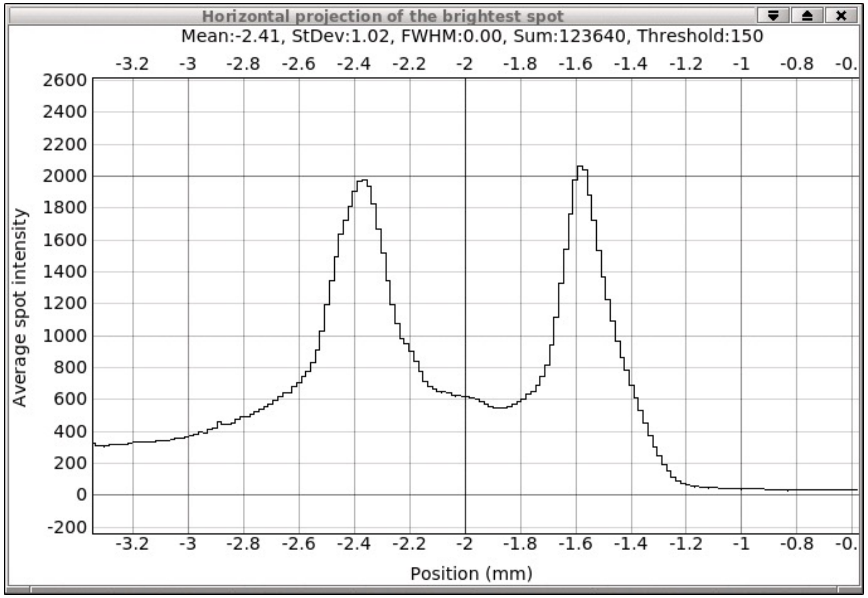
We observed significant variations in the energy of the central slices. In a simple model of CeC accelerator, with all three RF synchronized, the central slices are bunched (because of significant 520 kV/nsec chirp from the bunching cavity) from 4 A to 50 A peak current and de-chirped in the linac. But we observed up to 0.35% in the beam energy, i.e. $\Delta\gamma$=0.1. Working theory is that this jumps are result of the laser timing jitter. This note is to check this assumption.

Observed bunch-by-bunch energy jitter





0.8 mm separation, which is 0.3% energy difference between consecutive bunches

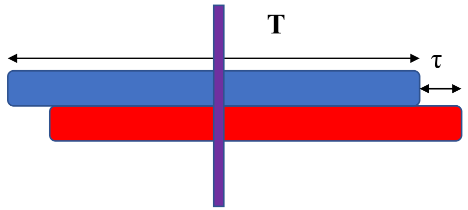
Bunches are 12 μsec apart

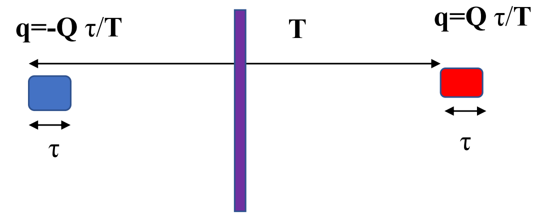
Simulations by Y. Jing:

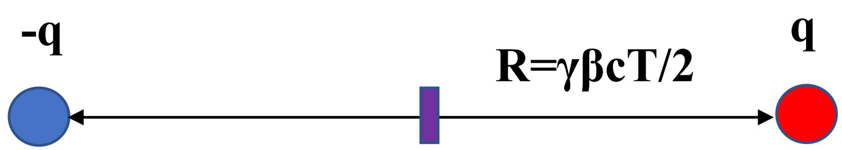
➢Laser jitter affects the average slice energy significantly.

➢A jitter in Laser time (+/- 40 ps) changes slice energy from +0.14%/-0.16% w.r.t. the designed value.

Simple model







$$E_z(z) = -\frac{2q}{R(z)^2};\ q = Q\frac{\tau_o}{T_o} = const;$$

$$E_z(z) = \frac{2Q}{R(z)^2}\frac{\tau_o}{T_o} = -\frac{8Q}{\gamma^2\beta^2c^2T(z)^2}\frac{\tau_o}{T_o} = \frac{8N_e e}{\gamma^2\beta^2c^2T(z)^2}\frac{\tau_o}{T_o};$$

$$\delta\gamma = \frac{e}{mc^2}\int_o^L E_z(z)dz = \frac{8N_e e^2}{\gamma^2\beta^2mc^2}\frac{\tau_o}{T_o}\int_o^L\frac{dz}{c^2T(z)^2}$$

In a simple model, laser time jitter (delay) result in a portion of τ /T of electrons appearing in front of the bunch and the same portion of τ /T of electrons disappearing from its tail. The central slice is puled backwards by this force.

Energy change

- Let's assume linear compression of the bunch length

$$T(z) = T_o\left(1-\alpha\frac{z}{L}\right); x = \alpha\frac{z}{L}$$

$$\int_o^L \frac{dz}{c^2 T(z)^2} = \frac{L}{\alpha(cT_o)^2}\int_o^\alpha \frac{dx}{(1-x)^2} = \frac{L}{\alpha(cT_o)^2}\left(\frac{1}{1-\alpha}-1\right) = \frac{L}{(1-\alpha)(cT_o)^2}$$

- Increased by the bunch compression factor with resulting change of the central slice energy is

$$\delta\gamma = \frac{8N_e}{(1-\alpha)}\frac{r_e L}{(\gamma\beta c T_o)^2}\frac{\tau_o}{T_o}.$$

- Putting numbers together gives

$$\delta\gamma = 0.23\frac{\tau_o}{T_o}.$$

Summary

- In this model, $\delta\gamma=0.1$ we will need peak-to-peak time jitter of 160 psec, or ±80 psec
- This simple model predicts energy slip of 2.2%/nsec of the laser jitter
- Simulations by Yichao Jing using Impact T show that ±40 psec gives +0.14%/-0.16% change for the central slice energy, i.e. 0.3%/80 psec=3.75%/nsec, i.e. 70% larger than this simple estimate
- Most likely explanations for this difference is a change of the arrival time to the linac