HW III.1 (3 points): Comparison of SRF and NC RF cavities for a storage ring

An electron storage ring with a beam current of 300 mA can use either one superconducting or several normal conducting cavities. A single cell Nb cavity would operate at an accelerating gradient of 7.5 MV/m. The cavity parameters are: R/Q = 90 Ohm, $Q_0 = 2.0\cdot 10^9$. Normal conducting cavities would operate at 2.5 MV/m. Parameters of a single cell copper cavity are: R/Q = 220 Ohm, $Q_0 = 3.0\cdot 10^4$.

(a) Calculate and compare accelerating voltages of the SRF and NC RF cavities. How many NC cavities one would need to match the accelerating voltage of the SRF cavity? Calculate RF power loss in the cavity walls for both cases.

Assume an RF frequency of 500 MHz (as the frequency is not specified, we choose one), then $g = \lambda/2 \approx 0.3$ m. $V_c = E_{acc}\cdot g$ = 2.25 MV for the SRF cavity and 0.75 MV for the NC cavity. One would need 3 NC cavities to match the accelerating voltage of one SRF cavity.

$$P_c = \frac{V_c^2}{R/Q\cdot Q_0} =$$

= 28.1 W for one SRF cavity and

= 85.2 kW for one NC cavity or 225.7 kW for all three NC cavities.

(b) Assuming the beam phase of 80 degrees, calculate beam power delivered by the RF system. What is the total RF power required for two cases? Assume that there the cavities are ideally matched and there is no reflected power.

For the matched condition: i) the cavity has to be appropriately detuned to cancel the reactive part of beam loading and ii) the following condition has to be satisfied

$$1-\frac{1}{\beta}=\frac{I_b\, R/Q\cdot Q_{ext}}{V_c}\cos\phi_0 \quad \text{or} \quad \beta-1=\frac{I_b\, R/Q\cdot Q_0}{V_c}\cos\phi_0 .$$

From here we can calculate the coupling parameter and the external quality factor

$$Q_{ext}=\frac{Q_0}{\beta}=\frac{Q_0}{1+\dfrac{I_b\cdot R/Q\cdot Q_0\cdot\cos\phi_0}{V_c}},$$

which is $4.8\cdot 10^5$ for the SRF cavity and $2.06\cdot 10^4$ for the NC cavities.

Then the total power is

$$P_{\text{forw}} = \frac{V_c^2}{R/Q\cdot Q_{ext}} =$$

= 117 kW for the SRF cavity and
= 124 kW per NC cavity or 372 kW total.

(c) Assuming that the SRF cavity operates at 4.5 K, calculate the wall-plug power of the cryogenic system with COP of 300. Assuming an RF power generator efficiency of 50%, calculate the wall-plug power for two cases. Compare the total AC power needed for the SRF and NC RF accelerating systems.

The total AC power for the SRF cavity is a sum of the power needed for cryogenics and the power needed for RF:

$$P_{AC} = COP \cdot P_c + 2 \cdot P_{\mathrm{forw}} = 300 \cdot 28.1 + 2 \cdot 117 \cdot 10^3 = 8.43 \cdot 10^3 + 234 \cdot 10^3 = 242 \text{ kW}$$

For three NC cavities the AC power is needed only for RF:

$$P_{AC} = 3 \cdot 2 \cdot P_{\mathrm{forw}} = 6 \cdot 124 \cdot 10^3 = 744 \cdot 10^3 = 744 \text{ kW}$$

As we can see, the normal conducting accelerating system requires much more wall-plug power than the superconducting one.

HW III.2 (1 point): Pillbox cavity

For a 1300-MHz pillbox cavity operating at an accelerating gradient of 10 MV/m, calculate the cavity dimensions, and accelerating voltage. Calculate an energy gain for an ultra-relativistic electron traveling: a) along the cavity axis of symmetry; b) parallel to the axis with a radial offset of 2 cm.

$\lambda_{010} = c/f \approx 3 \cdot 10^8 / 1300 \cdot 10^6 = 0.23$ m = 23 cm,

$g = \lambda/2 = 0.115$ m = 11.5 cm,

$V_c = E_{acc} \cdot g = 10 \cdot 0.115 = 1.15$ MV, the energy gain is 1.15 MeV on axis.

At 2 cm offset $V_c(r) = V_c(0) \cdot J_0(2\pi r/\lambda_{010}) = 1.15 \cdot 0.927 = 1.07$ MV

HW III.3 (2 points): RF power and beam loading

Consider a 350-MHz SRF cavity operating at 2 MV in a storage ring. The cavity parameters: R/Q = 100 Ohm, $Q_0 = 2.0 \cdot 10^9$. The beam current is 500 mA and the beam phase is 85 degrees.

(a) What should be loaded quality factor of the cavity so that there is no RF power reflected back to the RF power generator? What is the coupling parameter?

From problem III.1b we know that

$$\beta - 1 = \frac{I_b R/Q \cdot Q_0}{V_c} \cos\phi_0$$

and can calculate $\beta = 4359$ and $Q_L = Q_0/(\beta + 1) = 4.59 \cdot 10^5$

(b) Calculate the cavity detuning needed to compensate the reactive part of the beam loading.

$$\Delta f = -\frac{I_b R/Q \cdot f \cdot \sin\phi_0}{2V_c} = -\frac{0.5 \cdot 100 \cdot 350 \cdot 10^6 \cdot \sin(85°)}{2 \cdot 2 \cdot 10^6} = 4.36 \cdot 10^3 \text{ Hz} = 4.36 \text{ kHz}$$