CeC: eCooling rate estimate

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CeC team meeting

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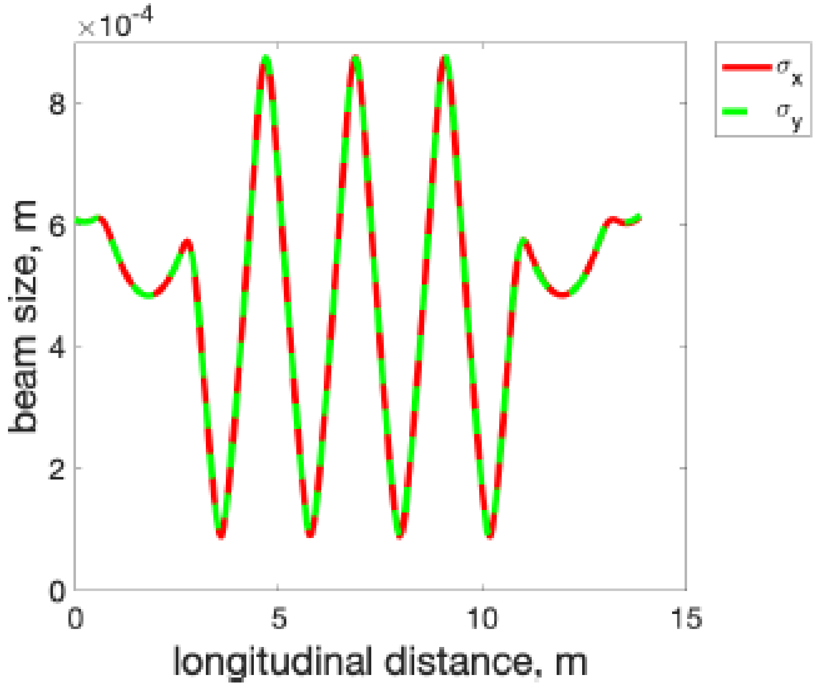
Parameters for CeC PoP experiment

Electron beam parameters		Ion beam parameters	
Energy, γ	28.5	Energy, γ	28.5
Bunch charge, nC	1.5	Bunch intensity	0.86e9
RMS bunch length, ps	12	Bunch length (FWHM), RMS, ns	(8.4), 3.6
RMS relative energy spread	2e-4 (slice) <5e-4 (projected)	Relative energy spread (FWHM), RMS	(3e-3), 1.3e-3
Common section length, m	14	RF voltage (28MHz), KV	400
Normalized emittance, RMS, mm.mrad	3e-6	Normalized emittance, RMS, mm.mrad	2.5
Beam width at modulator/kicker, RMS, mm	0.5	β^* at cooling section, m	5
Minimal beam width at amplifier, RMS, mm	0.1	Average β function at cooling section, m	10???
Angular spread min /max , mrad	0.2/0.8	Angular spread, mrad	0.13
vx_min/c	5.7e-3	vx/c	3.7e-3

Electron transverse beam size (rms)



(a) Beam size

Ion vx/vz ~3>>1

CeC_vx/ RHIC_vz=5.7/1.3= 4.4 >>1

Some formulas from “THEORY OF ELECTRON COOLING” paper *

$$\vec{F} = -\frac{4\pi z^2 e^4}{m} \int L(u) \frac{\vec{u}}{u^3} f(\vec{v}_e, \vec{r}) d^3 v_e \,, \quad \text{where:} \quad \int f(\vec{v}_e, \vec{r}) d^3 v_e = n'_e(\vec{r})$$

For example, for a Maxwell distribution with a temperature $T_e = m v^2_{eT}$,

$$\vec{F}(\vec{v}) = -\frac{4\sqrt{2\pi}}{3} \frac{z^2 n'_e e^4 L}{m v_{eT}^3} \vec{v} \quad \text{for} \quad v \ll v_{eT}$$

$$\vec{F}(\vec{v}) = -\frac{4\pi z^2 n'_e e^4 L}{m v^3} \vec{v} \quad \text{for} \quad v \gg v_{eT}$$

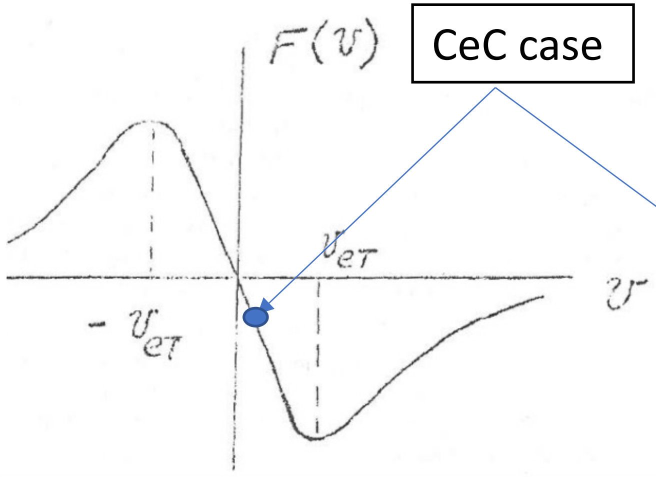


Fig. 1.

Then cooling time τ can be expressed in the lab. Frame using angular spread

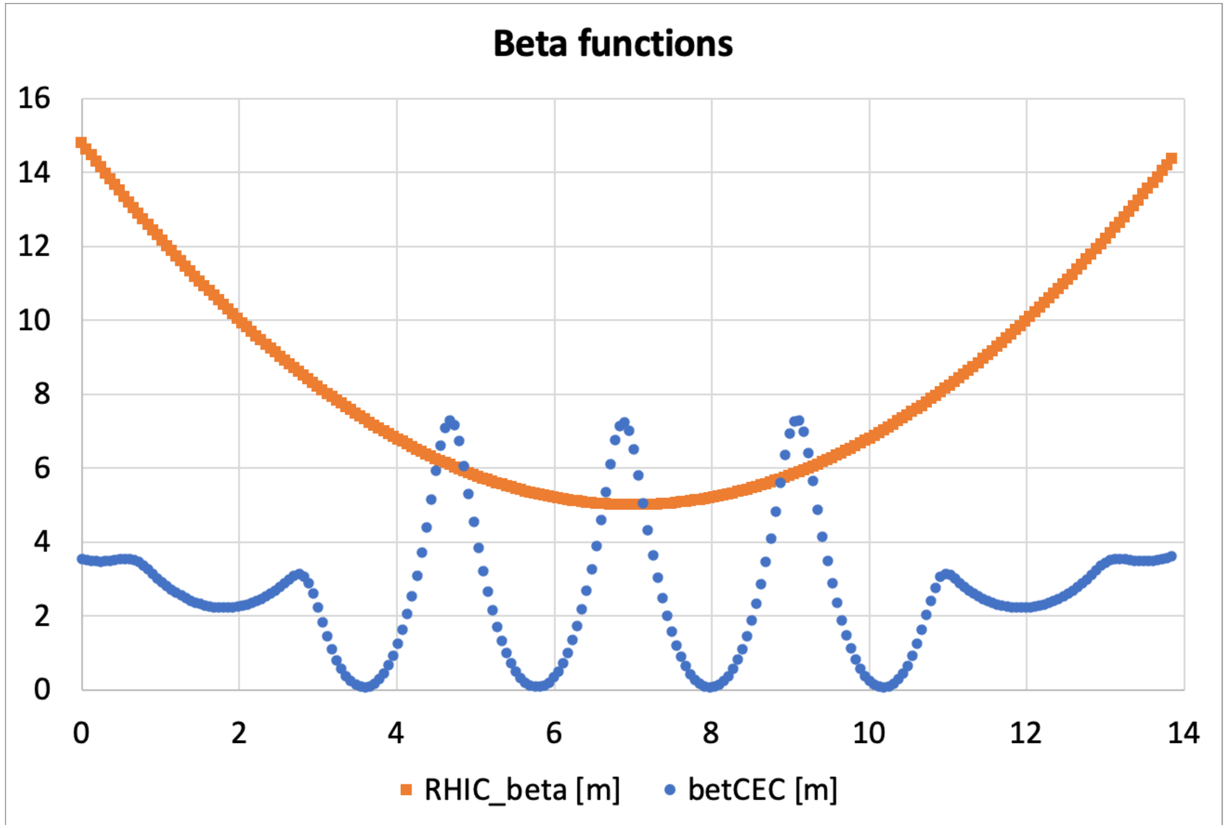
$$\tau_1 \approx \frac{3}{32\sqrt{\pi}} \frac{\gamma^5 (\beta\theta_e)^3}{\eta n_e z^2 r_e^2 c L} \frac{M}{m} \quad \text{when} \quad \theta < \theta_e \,,$$

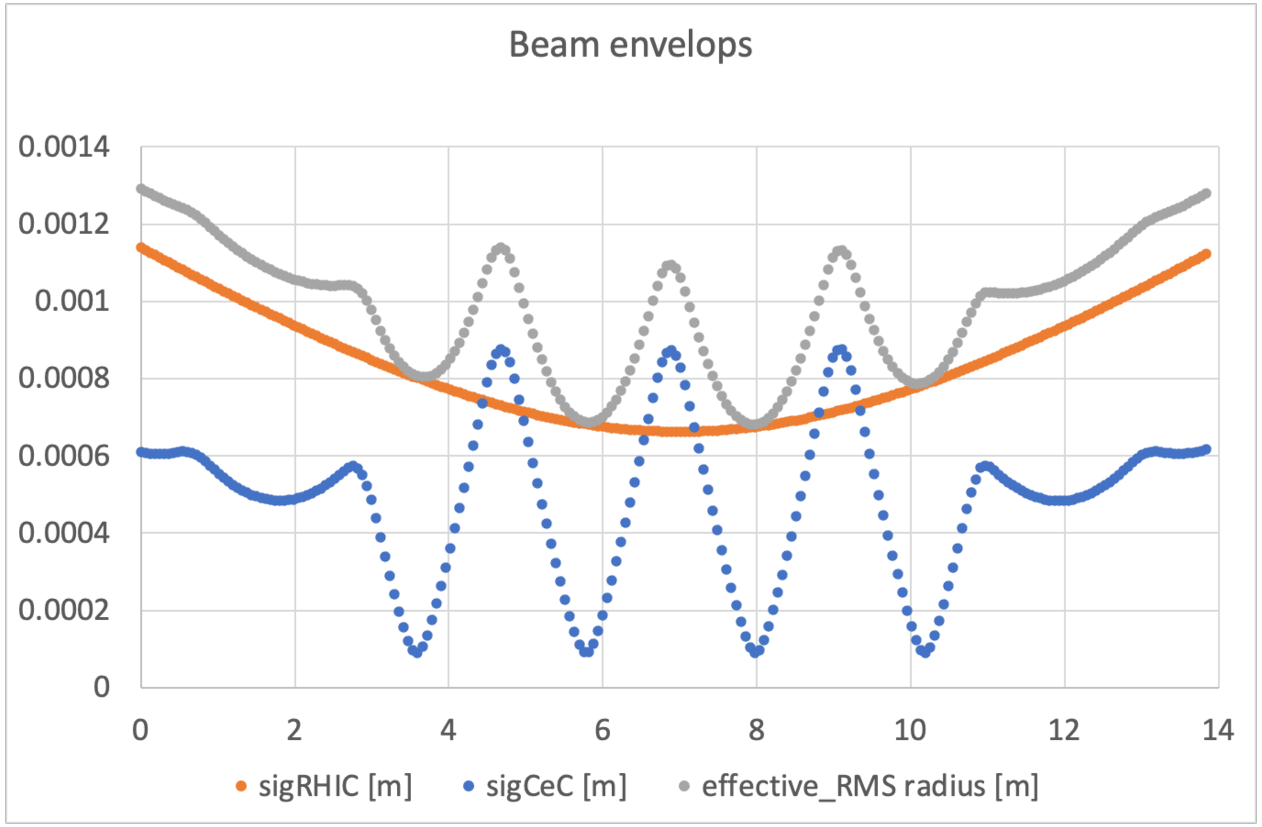
$$\tau_2 \approx \frac{1}{16\sqrt{\pi}} \frac{\gamma^5 (\beta\theta)^3}{\eta n_e z^2 r_e^2 c L} \quad \text{when} \quad \theta > \theta_e \,.$$

- The cooling time is proportional to the cube of the maximum of the velocity spreads of the two beams

*) Yaroslav Derbenev, THEORY OF ELECTRON COOLING, https://arxiv.org/pdf/1703.09735.pdf

Electron/ions transversely overlapping effect





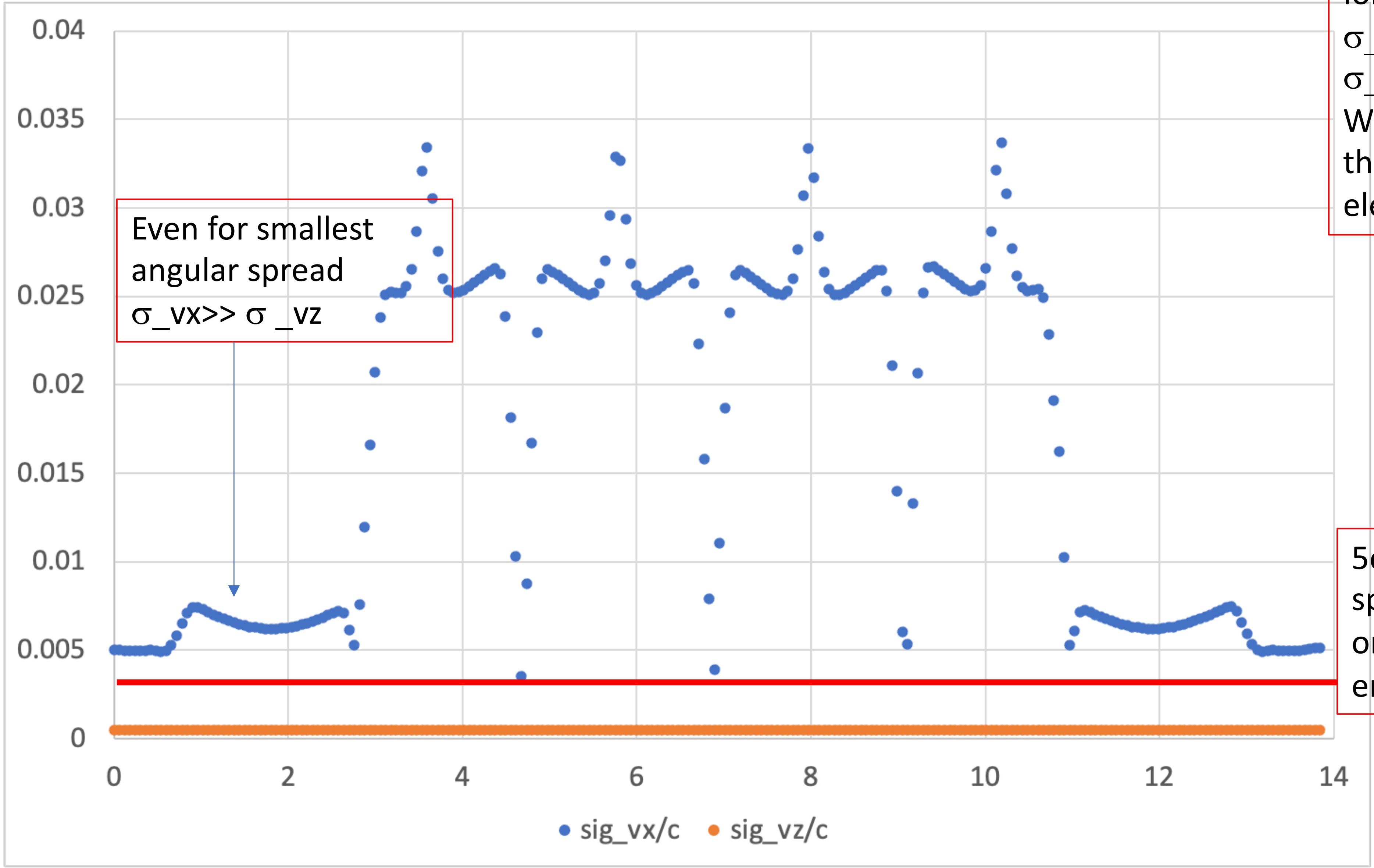
The cooling rate equations assumed that electron beam density is the constant constant across the interaction area. Here it's not the case as shown.

- Most of the time electron bunch transvers size is smaller than ion bunch. Overlapping is not 100%
- For effective density estimation I would use effective rms size: $\sigma_{eff}=\text{sqrt}(\sigma_{RHIC}^2+\sigma_{CeC}^2)$
- The electron bunch length is significantly shorter than ion bunch

Resulted effective density $<1/2\pi/\sigma_{eff}^2>$ where

σ_{eff}=0.9mm

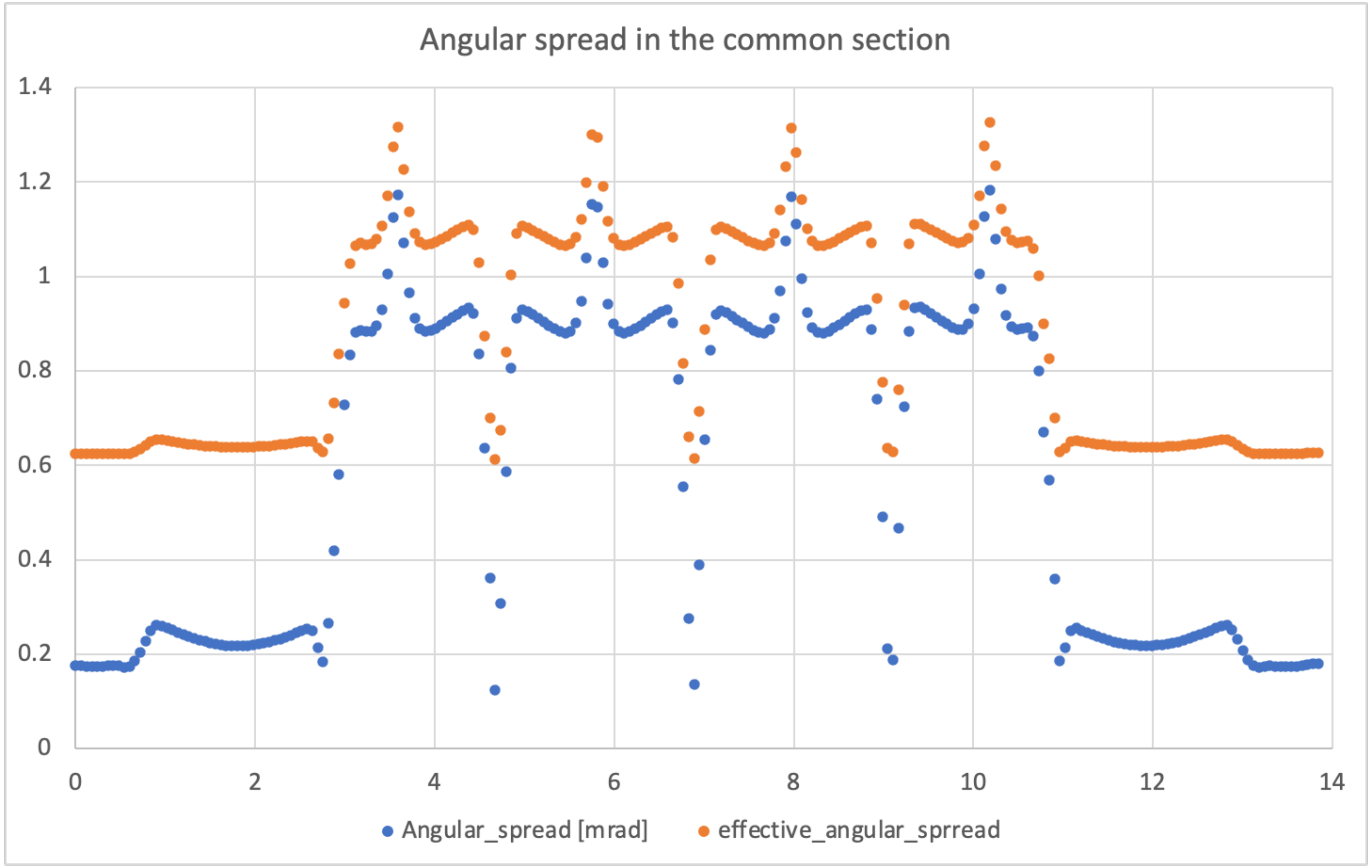
Longitudinal velocity vs transverse velocity in COF



Ions:
$\sigma\_vz/c=1.3e-3$,
$\sigma\_vx/c=3.7e-3$
Which is smaller than transverse electrons velocity

5e-4 projected energy spread
or 3e-3 if we include energy jitter

CeC: Angular spread in the common section with/without misalignment



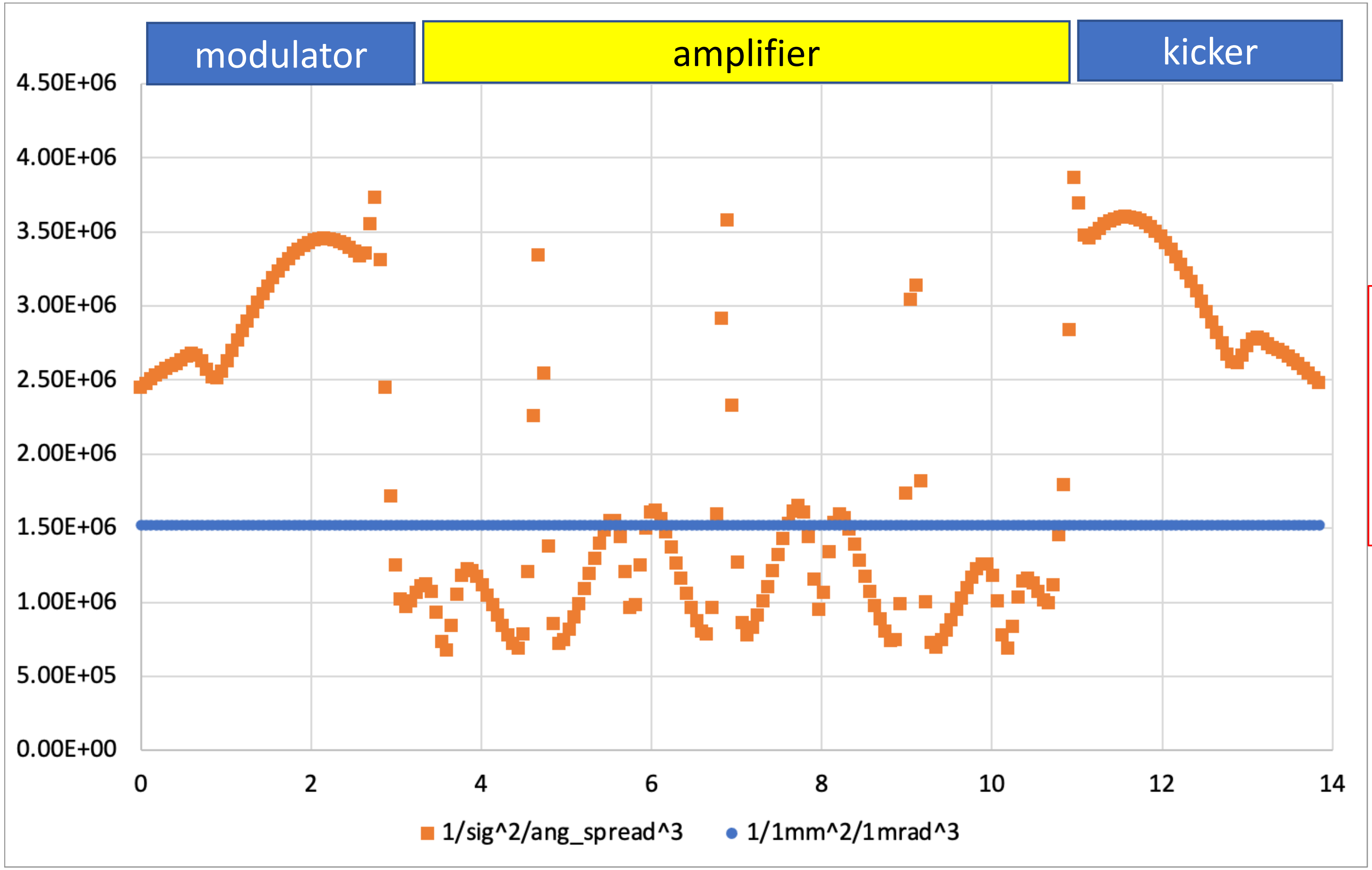
Without misalignment average angular spread 0.3 mrad

With misalignment resulted effective average angular spread ~0.8mrad

From Vladimir's email on Aug. 5:
"There are additional effects coming from the Earth magnetic field -
Estimation for RMS angle introduce by it is ~ 0.6 mrad. There are also solenoids misalignments ~ 0.25 mrad."

The resulted effective angular spread : emittance relative angular spread and some errors adds in quadrature

Which part of the common section provides cooling?



Here we plot:

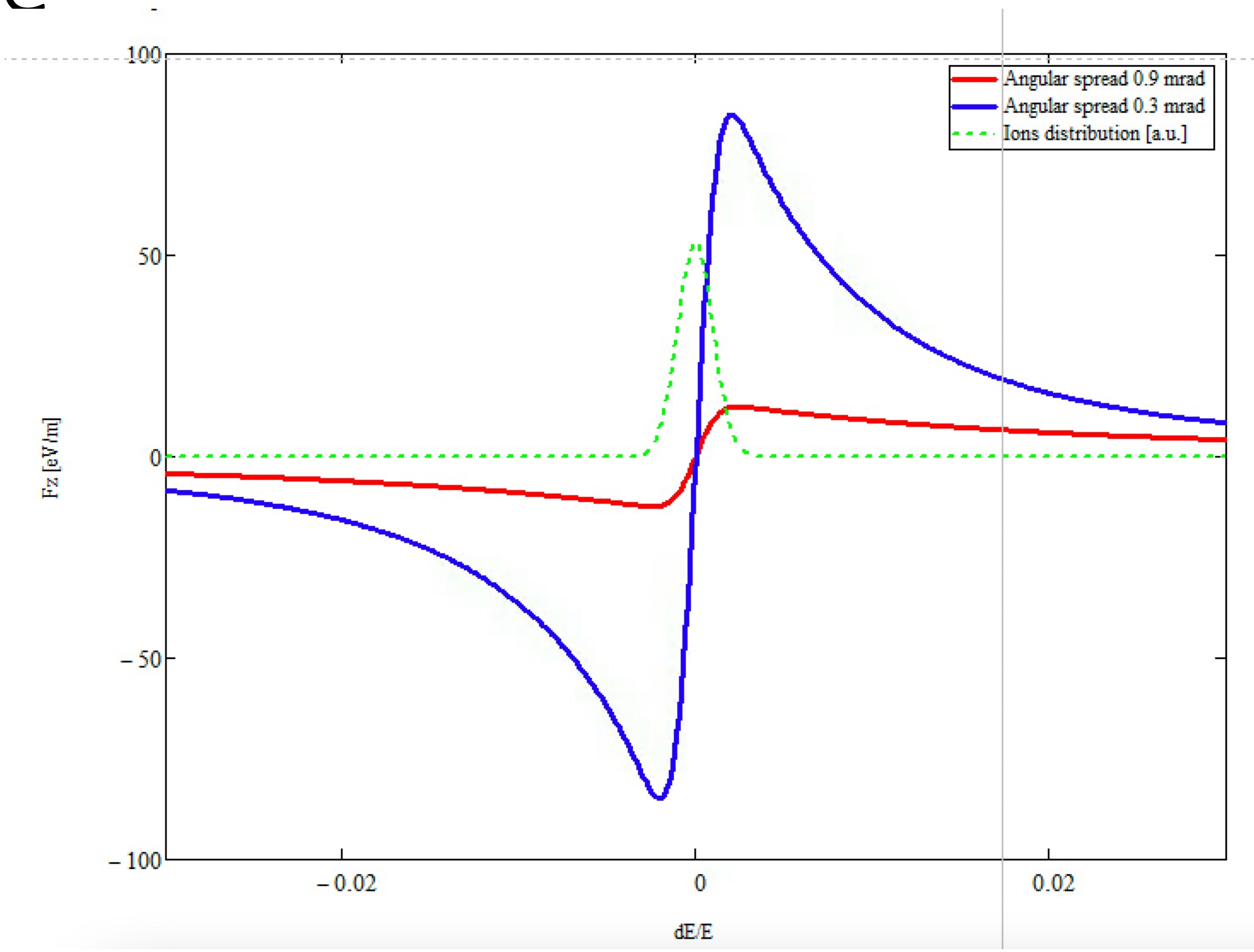
1/sig^2/ang_spread^3

Modulator and kicker together provide about 3 times more cooling (or more if can optimize angular alignment)
than amplifier section.

Longitudinal cooling force

Electron effective energy spread 1e-3
For example :angular spread 0.9 mrad and 0.3 mrad arte plotted

For CeC 2021 parameters we can use linear approximation for cooling force and for the rate estimate



Plotted for ion with transverse transverse velocity equal to transverse RMS spread

Cooling rate estimate:

- Effective rms transverse size for the average density calculation: 0.9 mm
- Effective ratio of longitudinal overlapping: 0.012/3.6=3.3e-3
- Cooling section fill factor: 14/3832= 3.6
- In my estimate the Coulomb logarithm (Log(rho_max/rho_min)) ~4.
- Effective angular spread: 0.8 mrad

Cooling rate tau1=340 hours

For smaller angular misalignment we accidently can get much faster cooling.

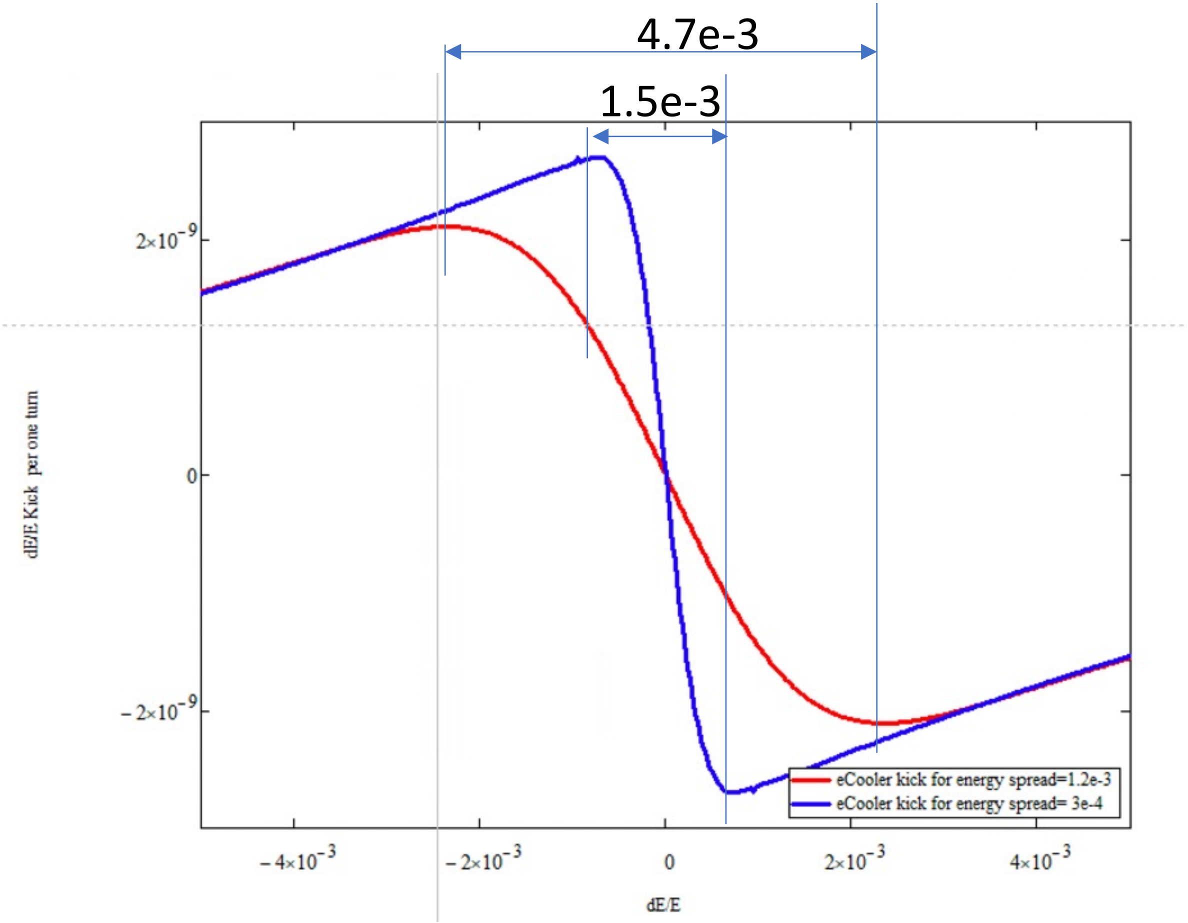
For example, if we assume no other contribution in the angular spread then relative average angular spread: 0.3 mrad

Cooling rate could as fast as tau1=20 hours

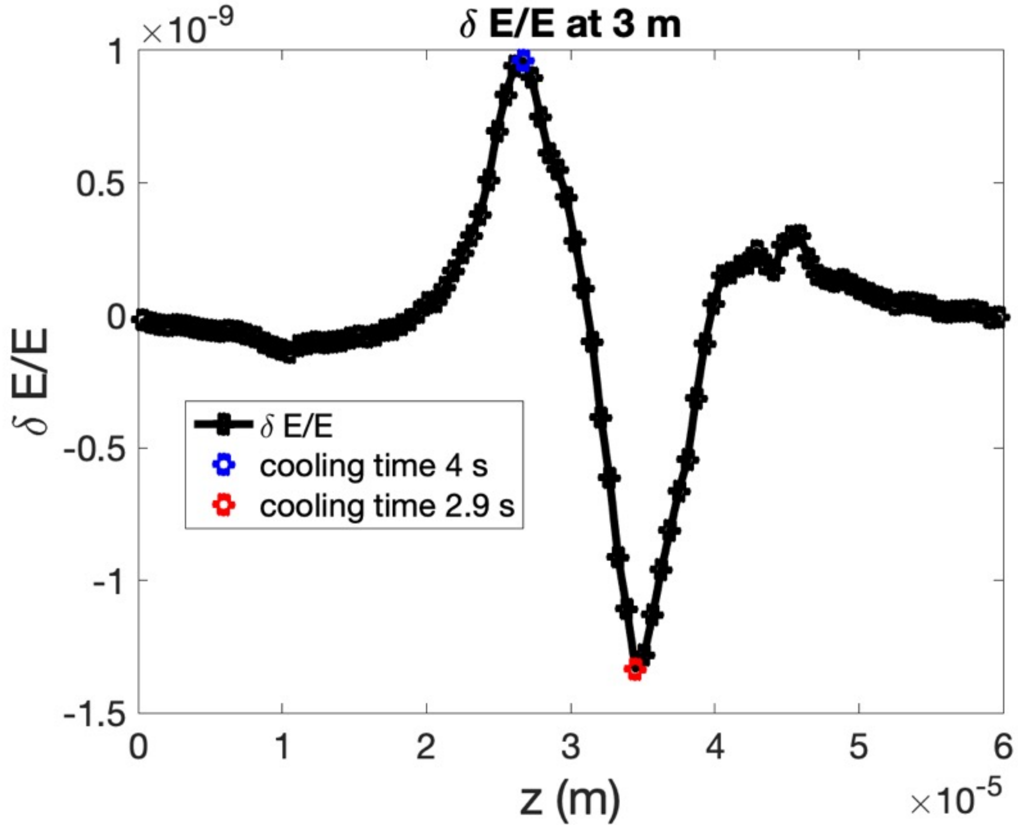
In reality we might observed cooling with rate somewhere between 20-340 hours.

For run 2022:
expected resulted energy jitter improvement from 1.2E-3 to 3E-4 (rms)

Regular electron cooler kick per one turn

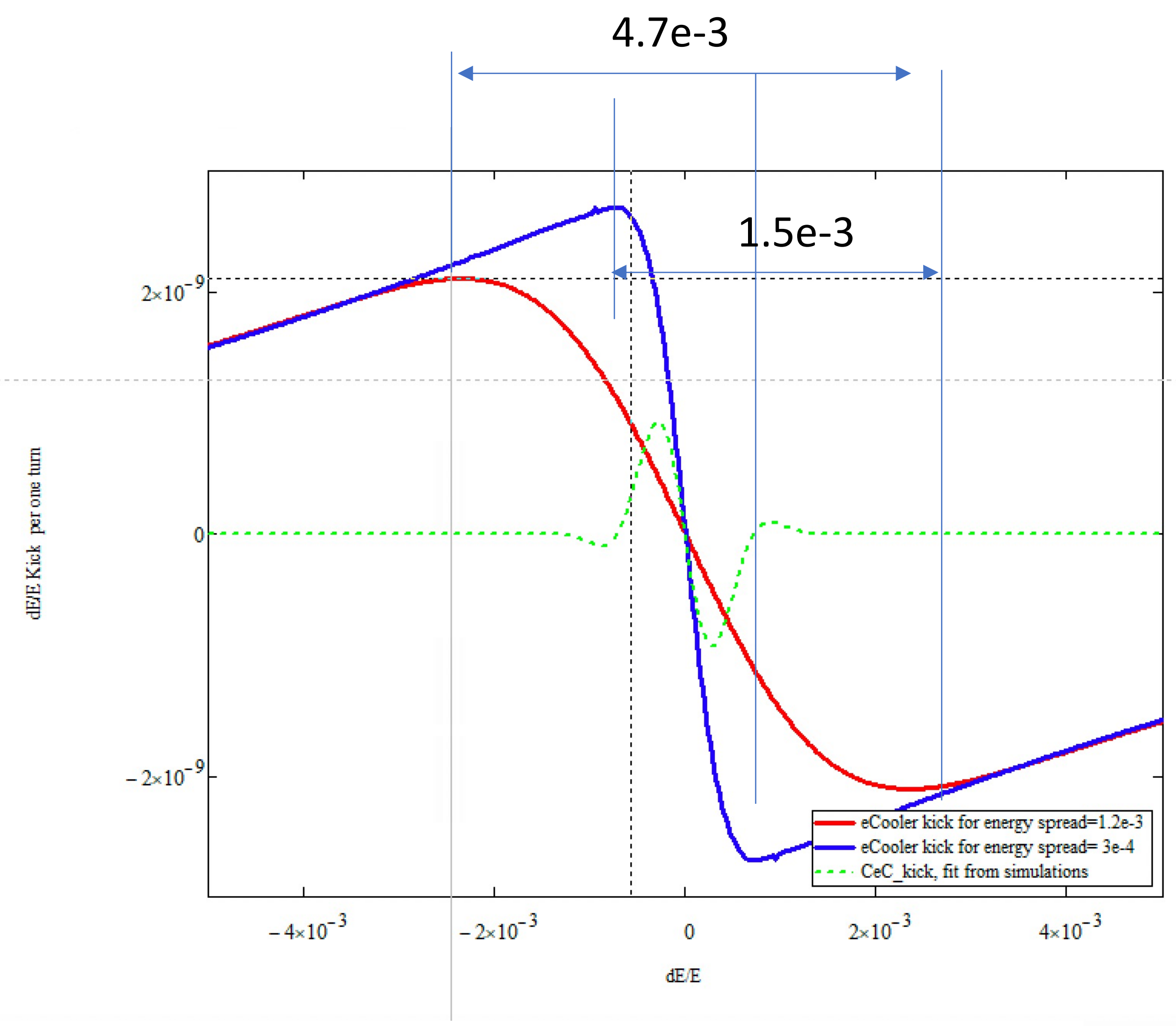


CEC kick per one turn

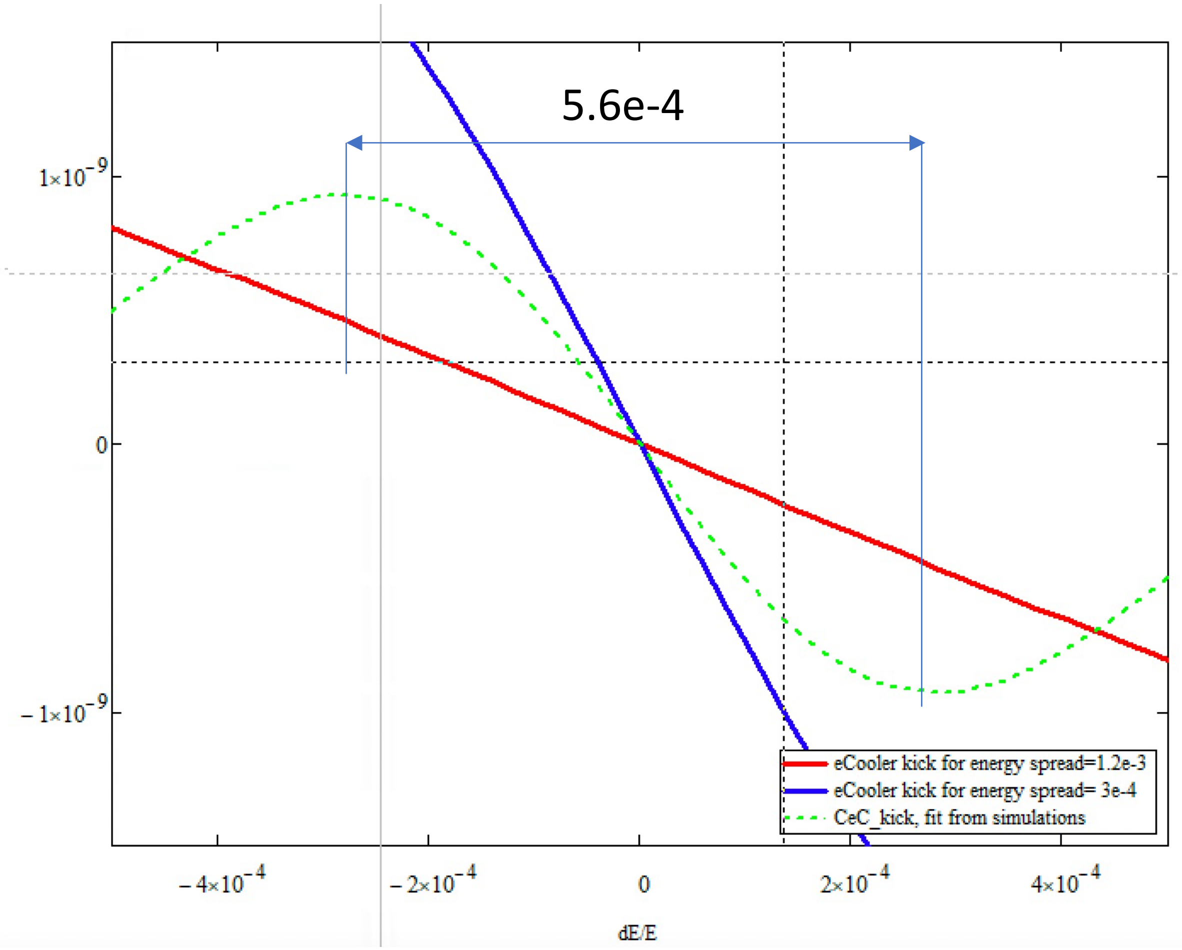


http://case.physics.stonybrook.edu/images/1/1b/Effect_of_energy_jitter.pdf

More materials



Zoom in



Electron cooling useful formulas:

LEReC: non-magnetized electron cooling

Non-magnetized friction force:

$$\vec{F} = -\frac{4\pi n_e e^4 Z^2}{m} \int \ln\left(\frac{\rho_{max}}{\rho_{min}}\right) \frac{\vec{V} - \vec{v}_e}{\left|\vec{V} - \vec{v}_e\right|^3} f(v_e) d^3 v_e$$

- Non-magnetized cooling:

  Very strong dependence on relative angles between electrons and ions.
- Requires strict control of both transverse angular spread and energy spread of electrons in the cooling section.
- LEReC: need to keep total contribution (including from emittances, energy spread, space charge, remnant magnetic fields, etc.) below 150 μrad!

asymptotic for $v_{ion} < \Delta_e$:

$$\vec{F} = -\frac{4\pi Z^2 e^4 n_e L}{m} \frac{\vec{v}_i}{\Delta_e^3}$$

$$\vec{F} = -\frac{4\pi Z^2 e^4 n_e L}{m} \frac{\vec{v}_i}{\beta^3 c^3 ((\gamma \vartheta)^2 + \sigma_p^2)^{3/2}}$$

Requirement on electron angles:
For γ=4.1: σ_p=5e-4; θ<150 μrad

U.S. DEPARTMENT OF ENERGY

BROOKHAVEN NATIONAL LABORATORY

A. Fedotov, MSU/FRIB APE Seminar, January 15, 2021

Cooling time estimates (based on friction force without magnetic field)

$$\frac{1}{\tau} = -\frac{1}{v_i}\frac{dv_i}{dt} = -\frac{F(v_i)}{p_i} \qquad \tau_{lab} = \gamma \frac{\tau}{L_{cool}/C} = \gamma \frac{\tau}{\eta}$$

Cooling time in lab frame:

$$\tau \propto \frac{A}{Z^2} \gamma^2 \frac{\theta^3 \gamma^3}{\eta n_e}$$

$v_i > \Delta_e$:

$$\vec{F} = -\frac{4\pi Z^2 e^4 n_e L}{m} \frac{\vec{v}_i}{v_i^3}$$

For dominant transverse ion velocities:

$$\tau \propto \frac{A}{Z^2} \frac{\gamma^2}{4\pi r_p r_e n_e c \eta \Lambda_c} \left(\frac{\gamma \varepsilon_n}{\beta_c}\right)^{3/2}$$

Here, instead of emittance angular spread we should use effective angular spread as a result of all contributions (earth field, solenoids misalignments etc.)