PHY 554
Fundamentals of Accelerator Physics
Lecture 11: RF and SRF accelerators

October 5, 2016

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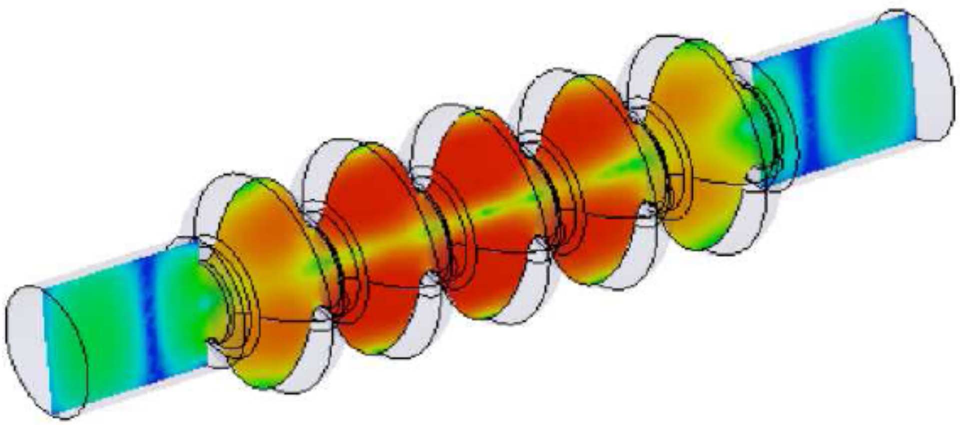

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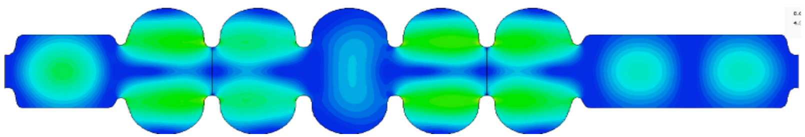
What we learned last time

- Resonant modes in a cavity resonator belong to two families: TE and TM. There is an infinite number of resonant modes. The lowest frequency TM mode is usually used for acceleration. All other modes (HOMs) are considered parasitic as they can harm the beam.
- Solution is given by Maxwell equations + boundary conditions

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_o(\vec{r})\cos(\omega t + \varphi(\vec{r})) = E_o \vec{u}_e(\vec{r})\cos(\omega t + \varphi(\vec{r}))$$

$$\vec{\mathbf{B}} = \vec{\mathbf{B}}_o(\vec{r})\sin(\omega t + \psi(\vec{r})) = E_o \vec{u}_b(\vec{r})\sin(\omega t + \psi(\vec{r}))$$

$$\int \vec{\mathbf{E}}_o^{\,2}\, dV = c^2 \int \vec{\mathbf{B}}_o^{\,2}\, dV \Leftrightarrow \int \vec{u}_e^{\,2}\, dV = c^2 \int \vec{u}_b^{\,2}\, dV$$



- A charged particle with a constant velocity in any RF system is described as

$$\Delta E = qV_{RF}\cos(\varphi);\ \varphi = \omega t;\ \lambda_{RF} = 2\pi c/\omega$$

$$V_{RF} = \sqrt{V_s^2 + V_c^2}; V_c = \int_{-\infty}^{\infty} \mathbf{E_o}(z)\cos\left(\omega_0 \frac{z}{\mathrm{v}}\right)dz;\ V_s = \int_{-\infty}^{\infty} \mathbf{E_o}(z)\sin\left(\omega_0 \frac{z}{\mathrm{v}}\right)dz$$

- Maximum RF voltage of a pillbox cavity (cell) is limited to $\boxed{V_{RF} = \frac{\mathbf{E_o}\lambda_{RF}}{\pi}\cdot\frac{\mathrm{v}}{c}}$ but in multi-cell a cavities RF from each cell voltage adds
- Several figures of merits are used to characterize accelerating cavities: main are accelerating voltage, transit time and Q-factor.

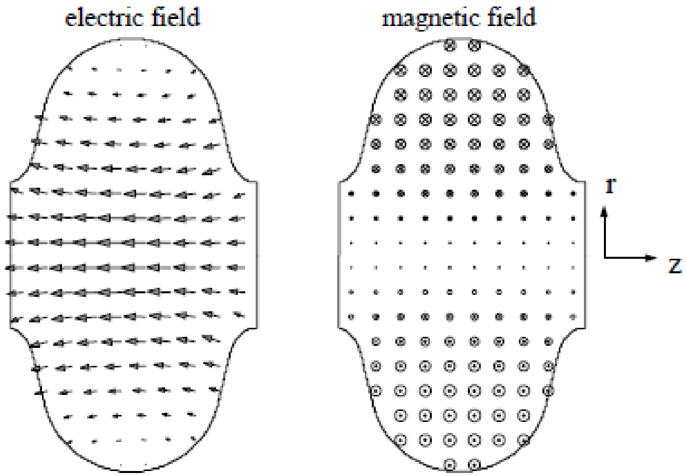
Realistic RF cavity (linac)
Figures of merit

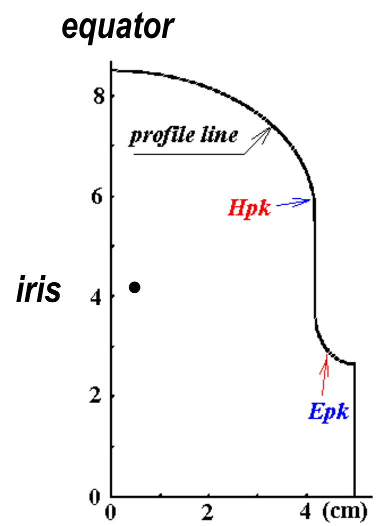
- Final conductivity of the surfaces
    - Approximation of the boundary conditions
    - Surface impedance, losses in the surface
- Main RF cavity characteristics
    - Accelerating voltage, peak electric and magnetic field
        - Q factor: internal, external, total
        - Geometrical factor, G
        - Shunt impedance R_{sh}, R_{sh}/Q
        - Coupling coefficient, *ONE MORE β!*

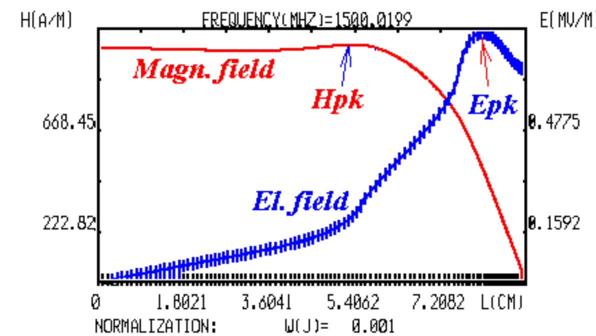
This part is usually related to more "engineering" factors measured in ohms, watts, etc.… – hence, for a change, we are using SI system…

Again, the main idea of this course: you are learning accelerator lingo and basis behind it

Typical SRF Cell fields (simulated using an EM code)







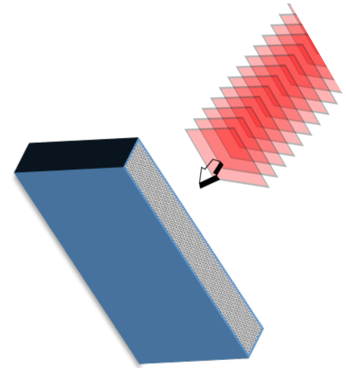
Fields along the profile line

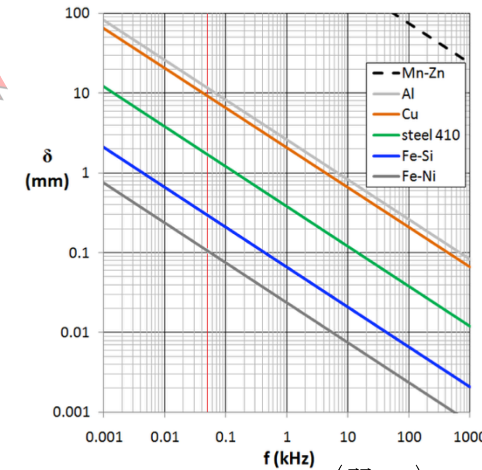
- Important for the cavity performance are the ratios of the peak surface fields to the accelerating field. Peak surface electric field is responsible for field emission; typically for real cavities E_{pk}/E_{acc} = **2…2.6**, as compared to **1.6** for a pillbox cavity.
- Peak surface magnetic field has fundamental limit (critical field for SRF cavities – will discuss at next lecture); surface magnetic field is also responsible for wall current losses; typical values for real cavities H_{pk}/E_{acc} = **40…50 Oe/MV/m**, compare this to **30.5** for the pillbox
- In SGS system 1**Oe -> 1 Gs;** /MV/m is 33.3 Gs, hence ratio H_{pk}/E_{acc} is dimensionless and is close to unity: 0.92 for a pillbox cavity, 1.2 – 1.5 for elliptical cavities.
- Tangential magnetic field on the surface induces Ohmic losses and affect Q-factor

Real: the conducting surface

- As input we have
- Inside the conductor the EM decays with typical length called skin depth

$$\delta = \sqrt{\frac{2}{\mu\sigma\omega}}$$





$$\vec{\mathbf{E}}_{//} \cong \mathrm{Re}\,\mathbf{E}_o\, e^{-\frac{\vec{n}\vec{r}}{\delta}} e^{i\left(\frac{\vec{n}\vec{r}}{\delta}-\omega t\right)}; \mathbf{H}_{//} \cong \sigma\delta\,\mathrm{Re}\frac{\mathbf{E}_o}{1+i} e^{-\frac{\vec{n}\vec{r}}{\delta}} e^{i\left(\frac{\vec{n}\vec{r}}{\delta}-\omega t\right)}; \vec{n}\cdot\vec{\mathbf{E}} = 0;$$

$$\vec{\mathbf{H}}_{//} = e^{-x/\delta}\,\mathrm{Re}\,\mathbf{H}_0\, e^{-\frac{\vec{n}\vec{r}}{\delta}} e^{i\left(\frac{\vec{n}\vec{r}}{\delta}-\omega t\right)}$$

- The current density is

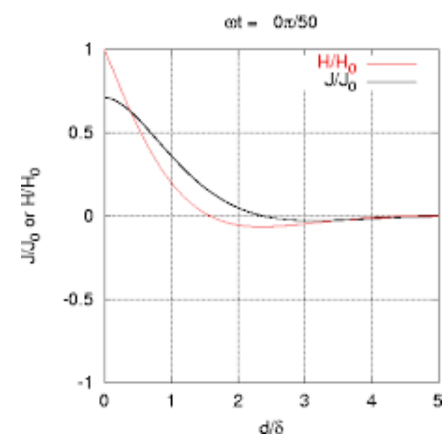
$$J \cong \frac{\sqrt{2}}{\delta}\left|\vec{\mathbf{H}}_{//}\right| e^{-\frac{\vec{n}\vec{r}}{\delta}} \cos\left(\frac{\vec{n}\vec{r}}{\delta} - \omega t\right)$$

$$K = \int J\, d\zeta \cong \vec{\mathbf{H}}_{//}$$

$$\left|\vec{\mathbf{E}}_{//}\right| = \frac{\sqrt{2}}{\sigma\delta}\left|\vec{\mathbf{H}}_{//}\right|$$

- And Ohmic losses per unit area

$$\frac{P_{loss}}{A} = \int \frac{\left\langle J^2(\zeta)\right\rangle_t}{\sigma} d\zeta \cong \frac{1}{2\delta\sigma}\left|\vec{\mathbf{H}}_{//}\right|^2$$



Surface impedance

$$Z_s \equiv \frac{\mathbf{E}_o}{K} = \frac{\mathbf{E}_o}{\mathbf{H}_0} = \frac{1+i}{\sigma\delta} = R_s + iX_s$$

At 1 GHz

Conductor	**Skin depth (μm)**
Aluminum	2.52
Copper	2.06
Gold	2,50
Silver	2.02

$$\frac{P_{loss}}{A} = \frac{1}{2} R_s \left|\vec{\mathbf{H}}_{//}\right|^2$$

$$R_s = \sqrt{\frac{\omega\mu}{\sigma}}\,[\Omega]$$

Beware of factors 2!

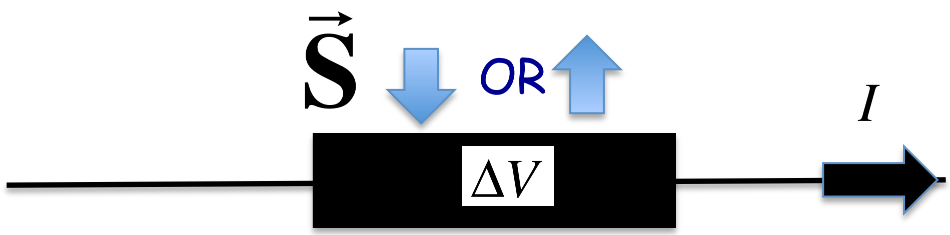
Question

- You should from E&M expression for Pointing vector

$$\vec{\mathbf{S}} = \vec{\mathbf{E}} \times \vec{\mathbf{H}}$$

indicating the flow of EM energy: direction and power density. Depending on you memory to remember "left hand" or "right hand" screw rule, you may get the direction either right or wrong… I have 50% success.

Based on the energy conservation law, please find direction of the EM energy flow in the case of a simple resistor with a current flowing through it. Is it pointed inside the surface of the resistor or outside? Does the result depends on the direction of the current?



$$\Delta V = -\mathbf{E}_z \cdot L = RI; \quad \oint \vec{\mathbf{H}} d\vec{l} = 2\pi r \vec{\mathbf{H}}_\varphi = I$$

Quality factor (SI)

- Let's consider a stand-alone cavity without any external couplers

$$\vec{\mathbf{H}}_0(\vec{r}) = H_o \vec{u}(\vec{r});\ \int |\vec{u}(\vec{r})|^2 dV = 1 \Rightarrow H_o = \sqrt{\frac{\int |\vec{\mathbf{H}}_0(\vec{r})|^2 dV}{\int |\vec{u}(\vec{r})|^2 dV}}$$

- Energy stored in the cavity

$$\boxed{W = \int\left(\varepsilon_o \frac{\vec{\mathbf{E}}^2}{2} + \mu_o \frac{\vec{\mathbf{H}}^2}{2}\right) dV = \frac{\mu_o}{2}\int \vec{\mathbf{H}}_o^2\, dV}$$

- Losses in the walls

$$\boxed{P_{loss} = \oint\!\!\oint \frac{1}{2} R_s \left|\vec{\mathbf{H}}_0\right|^2 dA = \frac{dW}{dt}}$$

- Quality factor (definition)

$$Q_0 \equiv \frac{\omega_0 \cdot (\text{stored energy})}{\text{average power loss}} = \frac{\omega_0 U}{P_c} = 2\pi \frac{1}{T_0}\frac{U}{P_c} = \omega_0 \tau_0 = \frac{\omega_0}{\Delta\omega_0}$$

- It is number of RF oscillation times 2π required for energy inside the cavity to reduce e-fold.

$$Q_0 = \frac{\omega_0 \mu_0 \int_V \left|\vec{\mathbf{H}}_0\right|^2 dV}{\boxed{R_s} \oint\!\!\oint \left|\vec{\mathbf{H}}_0\right|^2 dA} = \frac{\omega_0 \mu_0 \int_V |\vec{u}|^2 dV}{\boxed{R_s} \oint\!\!\oint |\vec{u}|^2 dA} \qquad \int_V |\vec{u}|^2 dV = 1$$

$$\boxed{R_s = \sqrt{\frac{\omega\mu}{\sigma}}\,[\Omega]}$$

Geometry factor: definition

- The ratio of two integrals determining Q-factor depends only on the cavity geometry: geometry defines eigen mode

$$G = \frac{\omega_0 \mu_0 \int_V \left|\vec{\mathbf{H}}_0\right|^2 dV}{\oint\!\!\oint \left|\vec{\mathbf{H}}_o\right|^2 dA} \equiv \frac{\omega_0 \mu_0 \int_V |\vec{u}|^2 dV}{\oint\!\!\oint |\vec{u}|^2 dA} \equiv F(geometry); \qquad \vec{r} \to \alpha\vec{r} \to \left\{ \begin{array}{c} \omega_0 \to \omega_0/\alpha \\ dV \to \alpha^3 dV \\ dA \to \alpha^2 dA \end{array} \right\}$$

The parameter G is the geometry factor (also known as geometry constant)

Obviously

$$Q_0 = \frac{G}{R_s} \qquad \boxed{R_s = \sqrt{\frac{\omega\mu}{\sigma}}\,[\Omega]}$$

- The geometry factor depends only on the cavity shape and electromagnetic mode, but not its size: Scaling the cavity size x-fold, increases volume as x^3, reduces frequency as $1/x$ and increasing surface as x^2. Hence, G does not change.
- **It is very useful for comparing different cavity shapes. TEM_{010} mode in a pillbox cavity had G = 257 Ohm independent the pillbox cavity length (d):**

  G_{TEM010}= 257 Ohm for any ratio of the length to the radius.
- At f=1.5 GHz for a normal conducting copper (σ= 5.8×10^7 S/m) cavity we get δ = 1.7 μm, R_s = 10 mOhm, and $Q_0 = G/R_s$ = 25,700.

Example: a pillbox cavity

- For a 1.5 GHz RF cavity
- normal conducting copper (σ= 5.8×10^7 S/m)

$$\sigma = 5.8 \cdot 10^7 S/m; \quad \delta = 1.7 \mu m \quad \Rightarrow R_s = 10 m\Omega$$

$$Q_{Cu} = \frac{G}{R_s} = 25{,}700$$

- for superconducting Nb at 1.8 K surface resistance can be as low as few nOhm, but typically is ~20 nOhm.

$$R_s = 20 n\Omega$$

$$Q_{SRF} = \frac{G}{R_s} \propto 1.2 \cdot 10^{10}$$

- Six orders of magnitude in heat losses making SRF cavities very attractive. Even with loss in cooling efficiency 500 to 1,000-fold, there is still three orders of magnitude in cooling.
- Hence, SRF cavity can operate at 30-fold higher accelerating gradient compared with room temperature Cu cavity using the same amount of cooling.

Shunt impedance and R/Q: definitions

- The shunt impedance determines how much acceleration a particle can get for a given power dissipation in a cavity

  $$R_{sh} = \frac{V_{RF}^2}{P_{loss}} \sim \frac{E_o^2}{H_o^2} \frac{FF_t^2}{\boxed{R_s} \oiint |\vec{u}|^2 dA}$$

  It characterized the cavity losses.

  Often the shunt impedance is defined as in the circuit theory $R_{sh} = \frac{V_{RF}^2}{2P_{loss}}$

  and, to add to the confusion, a common definition in linacs is $r_{sh} = \frac{E_{acc}^2}{P'_{loss}}$

  where P'_{loss} is the power dissipation per unit length and the shunt impedance is in Ohms per meter.
- A related quantity is the ratio of the shunt impedance to the quality factor, **which is independent of the surface resistivity and the cavity size**:

  $$\boxed{\frac{R_{sh}}{Q_0} = \frac{V_{RF}^2}{\omega_0 W}}$$

- This parameter is frequently used as a figure of merit and useful in determining the level of mode excitation by bunches of charged particles passing through the cavity. Sometimes it is called the geometric shunt impedance.
- **Pillbox cavity has *R/Q* = 196 Ohm.**

Dissipated power

- The power loss in the cavity walls is

$$P_{loss} = \frac{V_c^2}{R_{sh}} \equiv \frac{V_c^2}{Q_0 \cdot (R_{sh}/Q_0)} \equiv \frac{V_c^2}{(R_s \cdot Q_0)(R_{sh}/Q_0)/R_s} \equiv \frac{V_c^2 \cdot R_s}{G \cdot (R_{sh}/Q_0)}$$

- To minimize the losses one needs to maximize the denominator.
- The material-independent denominator is *G*R/Q*
- This parameter should be used during cavity shape optimization.

Consider now frequency dependence.

- For normal conductors $R_s \sim \omega^{1/2}$:

$$\boxed{\frac{P_{loss}}{L} \propto \frac{1}{G \cdot (R_{sh}/Q_0)} \cdot \frac{E_{acc}^2 R_s}{\omega} \propto \omega^{-1/2}} \qquad \boxed{\frac{P}{A} \propto \omega^{1/2}}$$

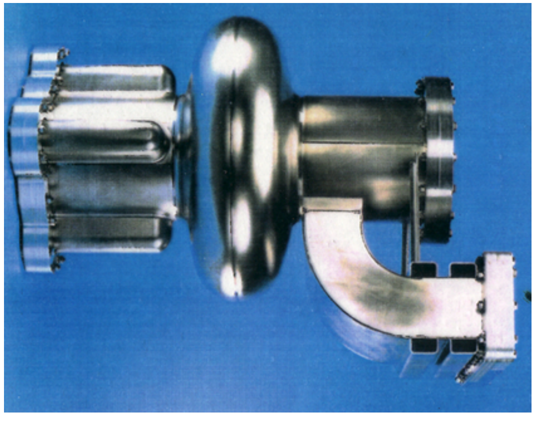
- For superconductors $R_s \sim \omega^2$

$$\boxed{\frac{P_{loss}}{L} \propto \omega} \qquad \boxed{\frac{P}{A} \propto \omega^2}$$

- NC cavities favor high frequencies, SC cavities favor low frequencies.

Pillbox *vs.* "real life" cavity

Quantity	Cornell SC 500 MHz	Pillbox
G	270 Ω	257 Ω
R_a/Q_0	88 Ω /cell	196 Ω/cell
E_{pk}/E_{acc}	2.5	1.6
H_{pk}/E_{acc}	52 Oe/(MV/m)	30.5 Oe/(MV/m)



- **In a high-current storage rings, it is necessary to damp Higher-Order Modes (HOMs) to avoid beam instabilities.**
- The beam pipes are made large to allow HOMs propagation toward microwave absorbers
- This enhances H_{pk} and E_{pk} and reduces R/Q.

Parameters of the 5-cell BNL3 cavity

Parameter	704 MHz BNL3 cavity
V_{acc} [MV]	20
No. of cells	5
Geometry Factor	283
R/Q [Ohm]	506.3
E_{pk}/E_{acc}	2.46
B_{pk}/E_{acc} [mT/MV/m]	4.26
Q_0	$> 2 \times 10^{10}$
Length [cm]	158
Beam pipe radius [mm]	**110**
Operating temperature [K]	1.9

- **It was designed for high current Energy Recovery Linacs. It is necessary to damp dipole Higher-Order Modes (HOMs) to avoid beam instabilities.**
- The beam pipes are made large to allow HOMs propagation toward HOM couplers to damp the modes
- This enhances B_{pk} and E_{pk} and reduces R/Q.

Parallel circuit model

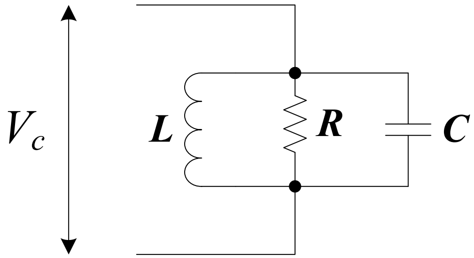
A resonant cavity can be modeled as a series of parallel *RLC* circuits representing the cavity eigen modes. For each mode:

dissipated power $$P_{loss} = \frac{V_c^2}{2R_{sh}}$$

shunt impedance $$R_{sh} = 2R$$

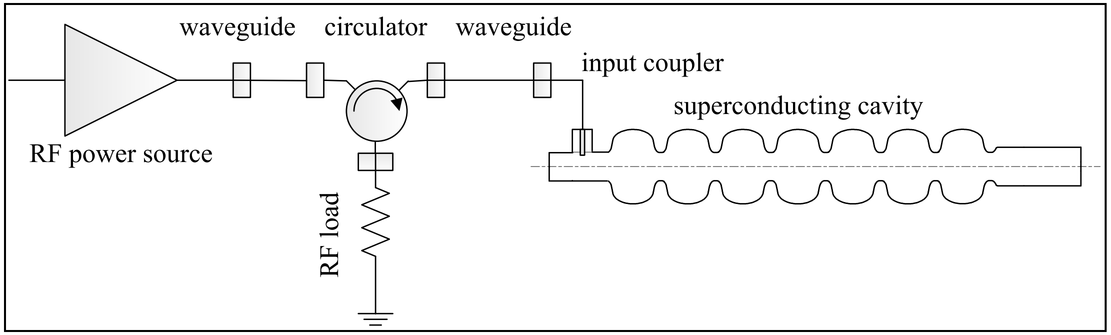
quality factor $$Q_0 = \omega_0 CR = \frac{R}{\omega_0 L} = R\sqrt{\frac{C}{L}}$$

impedance $$Z = \frac{R}{1 + iQ\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)} \approx \frac{R}{1 + 2iQ\left(\frac{\omega - \omega_0}{\omega_0}\right)}$$

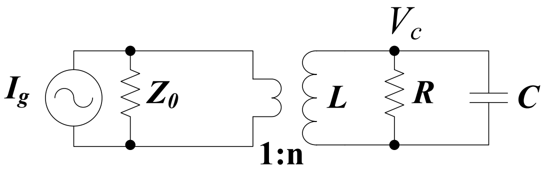


Connecting to a power source

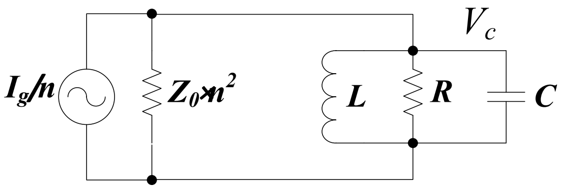
- Consider a cavity connected to an RF power source



- The input coupler can be modeled as an ideal transformer:



or



External & loaded Q factors

- If RF is turned off, stored energy will be dissipated now not only in R, but also in Z_0/n^2, thus

$$P_{tot} = P_o + P_{ext}$$

$$P_o = P_{loss} = \frac{V_c^2}{2R_{sh}} = \frac{V_c^2}{R_{sh}/Q \cdot Q_0} \qquad P_{ext} = \frac{V_c^2}{2Z_0 \cdot n^2} = \frac{V_c^2}{R_{sh}/Q \cdot Q_{ext}}$$

- This is definitions of an external quality factor associated with a coupler.
- Such Q factors can be identified with any external ports on the cavity: input coupler, RF probe, HOM couplers, beam pipes, etc.
- Then the total power loss can be associated with the loaded Q factor of

$$\boxed{\frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_{ext1}} + \frac{1}{Q_{ext2}} + \dots}$$

Coupling parameter β

- Coupling parameter is defined as

$$\beta \equiv \frac{Q_0}{Q_{ext}}$$

e.g.

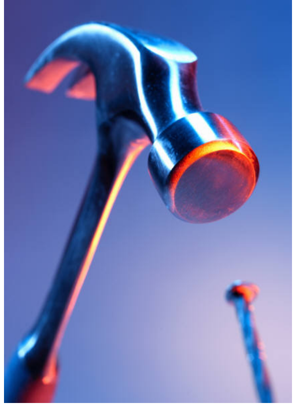
$$\frac{1}{Q_L} = \frac{1+\beta}{Q_0}$$

- β defines how strongly the couplers interact with the cavity
- Large β implies that the power taken out of the coupler is large compared to the power dissipated in the cavity walls:

$$P_{ext} = \frac{V_c^2}{R/Q \cdot Q_{ext}} = \frac{V_c^2}{R/Q \cdot Q_0} \cdot \beta = \beta P_0$$

- The total power needed from an RF power source is expressed as

$$P_{forward} = (\beta + 1) P_0$$



What we learned about RF accelerators ?

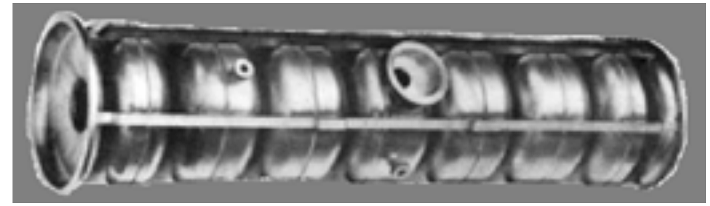
- Several figures of merits are used to characterize accelerating cavities:

$$V_{rf}\,,\ E_{peak}\,,\ H_{peak},\ R_s,\ Q_0,\ Q_{ext},\ R/Q,\ G,\ R_{sh}\ldots$$

- Superconducting RF cavities can have quality factor a million times higher than that of best Cu cavities.
- In a multi-cell cavity every eigen mode splits into a pass-band. The number of modes in each pass-band is equal to the number of cavity cells.
- Coaxial lines and rectangular waveguides are commonly used in RF systems for power delivery to cavities.

SRF cavities for linacs and ERLs

TESLA / ILC / European XFEL 1.3 GHz cavity



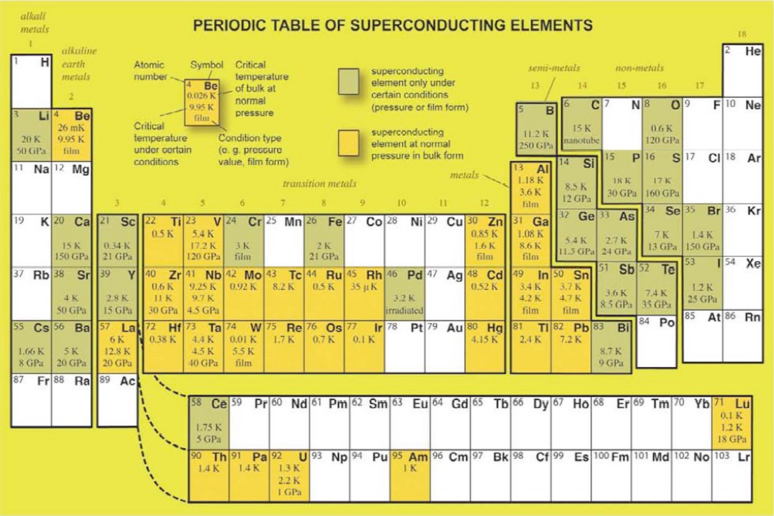
HEPL 1.3 GHz cavity

SNS 805 MHz cavities (*b* = 0.61 and 0.81)

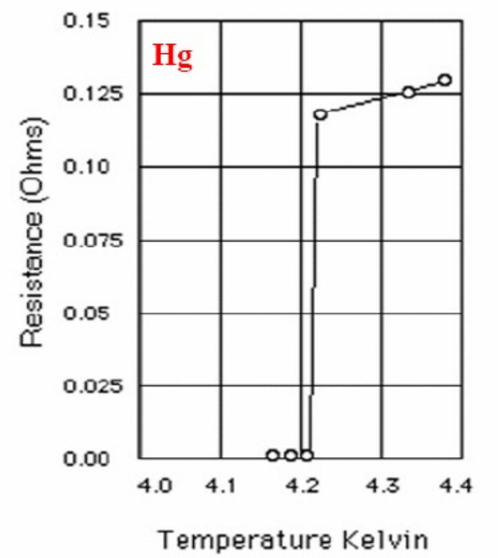
BNL-3 704 MHz cavity

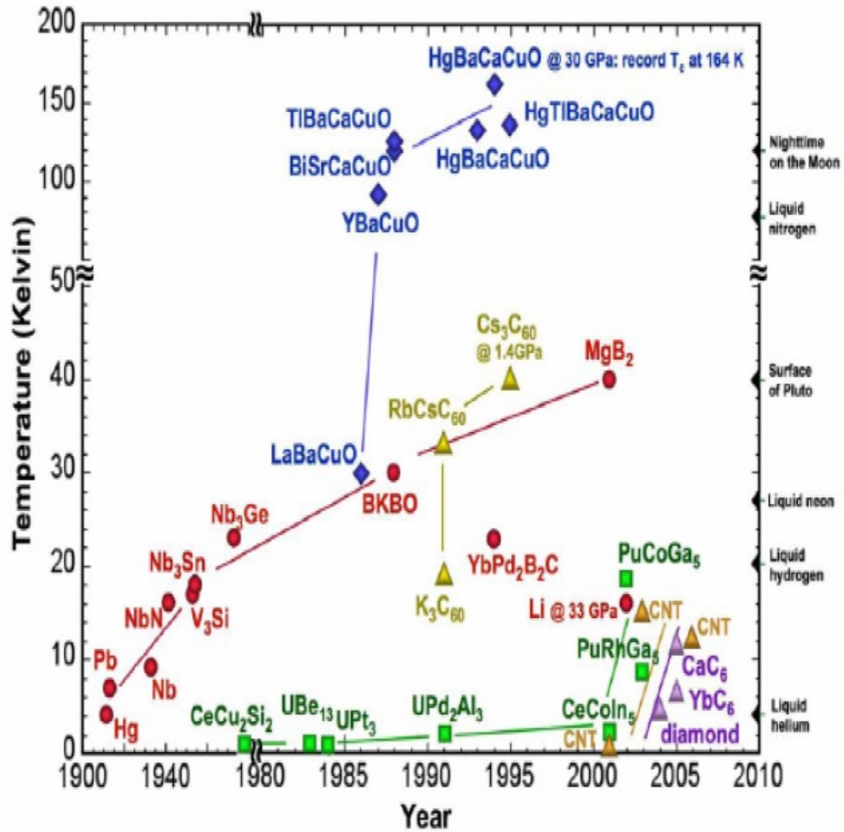
CEBAF Upgrade 1.5 GHz cavity

Discovery of superconductivity: April 8th of 1911



Discovered in 1911 by Heike Kamerlingh Onnes and Giles Holst after Onnes was able to liquify helium in 1908. Nobel prize in 1913





Simplified explanation for zero DC resistivity

- NC
  - Resistance to flow of electric current
  - Free electrons scatter off impurities, lattice vibrations (phonons)
- SC
  - Cooper pairs carry all the current
  - Cooper pairs do not scatter off impurities due to their coherent state
  - Some pairs are broken at T > 0 K due to phonon interaction
  - But super-current component has zero resistance

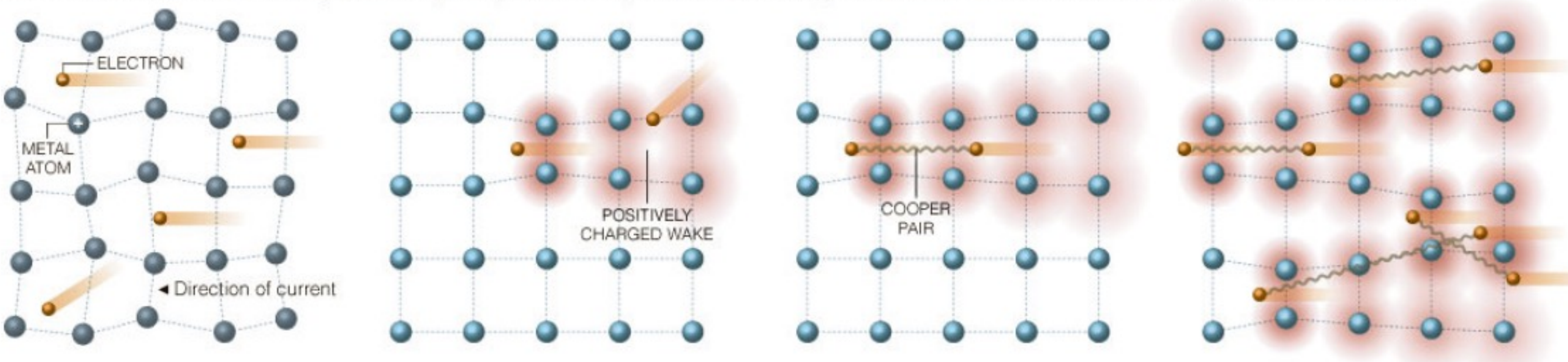
Microscopic theory of superconductivity

Bardeen-Cooper-Schrieffer (BCS) theory (1957).
Nobel prize in 1972

January 7, 2008

Low-Temperature Superconductivity

December was the 50th anniversary of the theory of superconductivity, the flow of electricity without resistance that can occur in some metals and ceramics.



ELECTRICAL RESISTANCE
Electrons carrying an electrical current through a metal wire typically encounter resistance, which is caused by collisions and scattering as the particles move through the vibrating lattice of metal atoms.

CRITICAL TEMPERATURE
As the metal is cooled to low temperatures, the lattice vibration slows. A moving electron attracts nearby metal atoms, which create a positively charged wake behind the electron. This wake can attract another nearby electron.

COOPER PAIRS
The two electrons form a weak bond, called a Cooper pair, which encounters less resistance than two electrons moving separately. When more Cooper pairs form, they behave in the same way.

SUPERCONDUCTIVITY
If a pair is scattered by an impurity, it will quickly get back in step with other pairs. This allows the electrons to flow undisturbed through the lattice of metal atoms. With no resistance, the current may persist for years.

Sources: Oak Ridge National Laboratory; Philip W. Phillips

JONATHAN CORUM/THE NEW YORK TIMES

BCS "theory"

What is the phase coherence?

Incoherent (normal) crowd: each electron for itself

Phase-coherent (superconducting) condensate of electrons

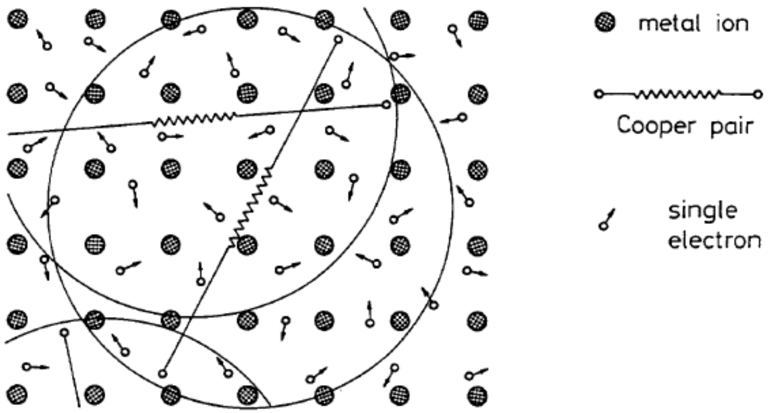


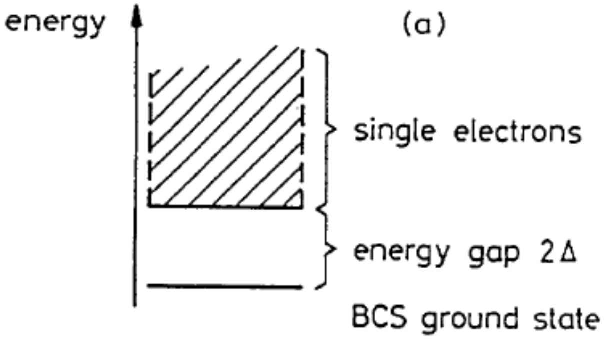
Figure 22: Cooper pairs and single electrons in the crystal lattice of a superconductor. (After Essmann and Träuble [12]).

- Attraction between electrons with antiparallel momenta **k** and spins due to exchange of lattice vibration quanta (phonons)
- Instability of the normal Fermi surface due to bound states of electron (Cooper) pairs
- Bose condensation of overlapping Cooper pairs in a coherent superconducting state.
- Scattering on electrons does not cause the electric resistance because it would break the Cooper pair

The strong overlap of many Cooper pairs results in the macroscopic phase coherence

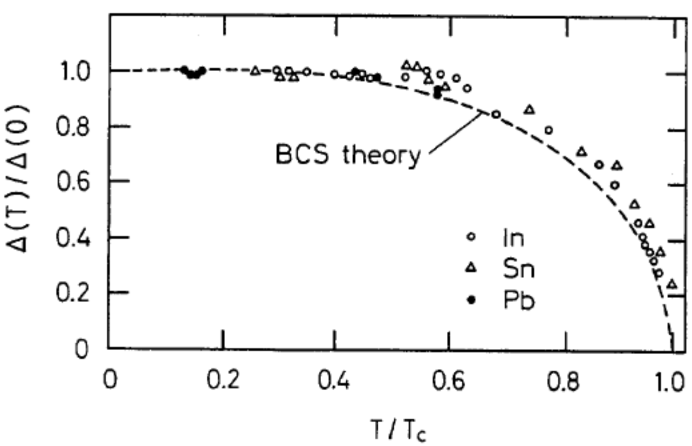
BCS theory

- The BCS ground state is characterized by the macroscopic wave function and a ground state energy that is separated from the energy levels of unpaired electrons by an energy gap. In order to break a pair an energy of 2Δ is needed:



$$n_n \propto \exp\left(-\frac{\Delta}{k_B T}\right)$$

- Temperature dependence of the energy gap according to BCS theory in comparison with experimental data:



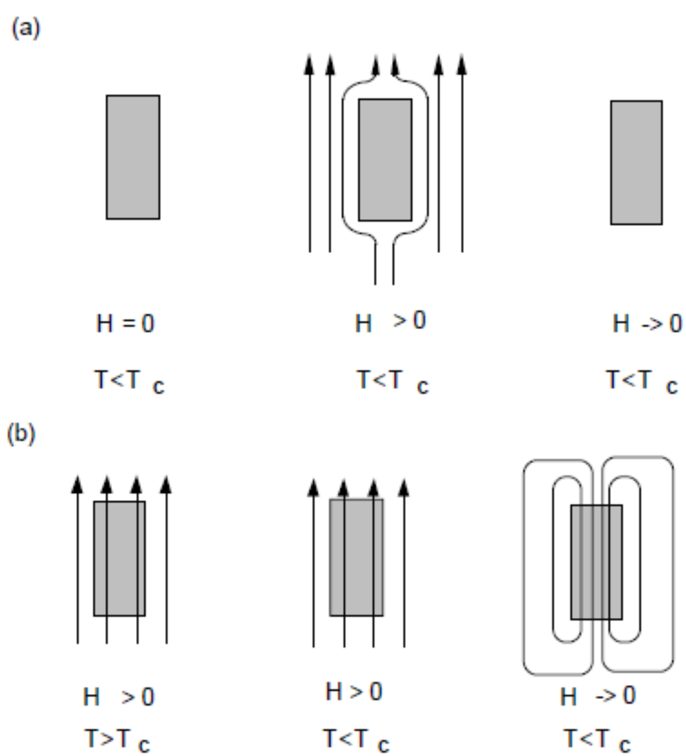
element	Sn	In	Tl	Ta	Nb	Hg	Pb
$\Delta(0)/k_B T_c$	1.75	1.8	1.8	1.75	1.75	2.3	2.15

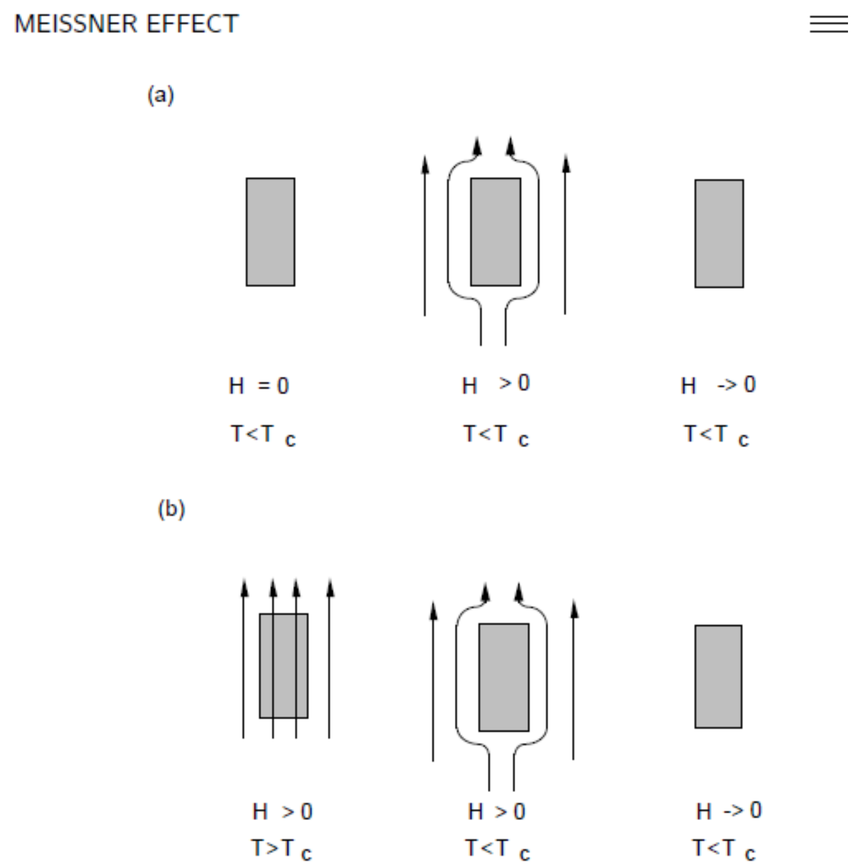
Remarkable prediction!

Meissner effect
perfect conductor ≠ SC

Inside a perfect conductor $\partial \mathbf{B}/\partial t = 0$
But $\mathbf{B}$ = constant is allowed.

In a superconductor (see next)



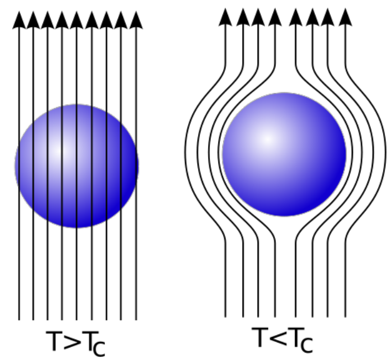


Superconducting state

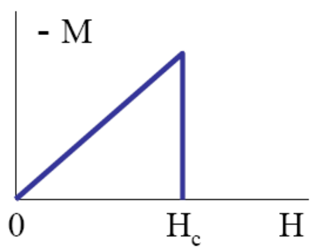
- The superconducting state is characterized by the critical temperature T_c and field H_c

$$H_c(T) = H_c(0) \cdot \left[1 - \left(\frac{T}{T_c}\right)^2\right]$$

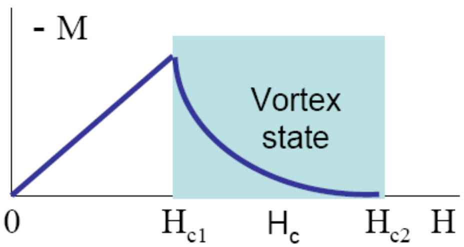
- The external field is expelled from a superconductor if $H_{ext} < H_c$ for Type I superconductors.
- For Type II superconductors the external field will partially penetrate for $H_{ext} > H_{c1}$ and will completely penetrate at H_{c2}



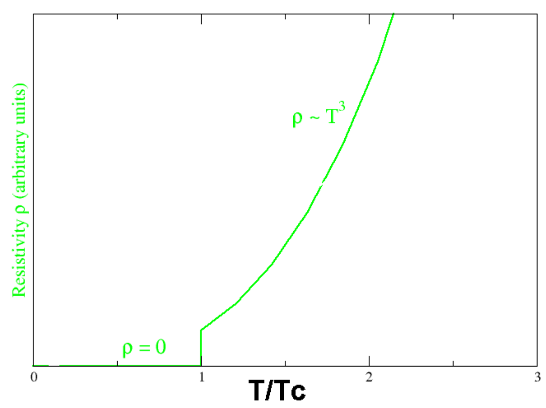
Superconductor in Meissner state = ideal diamagnetic



Complete Meissner effect in type-I superconductors



High-field partial Meissner effect in type-II superconductors

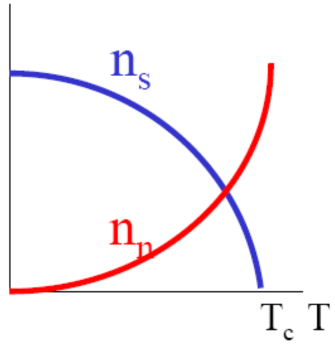


Type-I: Meissner state B = H + M = 0 for H < H_c; normal state at H > H_c

Type-II: Meissner state B = H + M = 0 for H < H_{c1}; partial flux penetration for H_{c1} < H < H_{c2}; normal state for H > H_{c2}

Two fluid model & AC fields

- Two-fluid model: coexisting SC and N "liquids" with the densities $n_s(T) + n_n(T) = n$.
- Electric field E accelerates only the SC component, the N component is short circuited.
- Second Newton law for the SC component: $m d\mathbf{v}_s/dt = e\mathbf{E}$ yields the first London equation:



$$n_\mathrm{n} \propto \exp\left(-\frac{\Delta}{k_B T}\right)$$

Two fluid model considers both superconducting and normal conducting components:

- At $0 < T < T_c$ not all electrons are bonded into Cooper pairs. The density of *unpaired*, "normal" electrons is given by the Boltzmann factor

$$n_n \propto \exp\left(-\frac{\Delta}{k_B T}\right)$$

  where 2D is the energy gap around Fermi level between the ground state and the excited state.

- Cooper pairs move without resistance, and thus dissipate no power. In DC case the lossless Cooper pairs short out the field, hence the normal electrons are not accelerated and the SC is lossless even for $T > 0$ K.

Superconducting part of AC current $\boxed{\vec{F} = m\vec{a}}$

- The Cooper pairs are electrons and do have an inertial mass
- They cannot follow an AC electromagnetic fields instantly and do not shield it perfectly.
- A residual EM field will acts on the unpaired electrons causing power dissipation.

First London equation

$$\boxed{\begin{aligned} &\vec{F} = -e\vec{\mathbf{E}} = m\vec{a} \\ &\vec{j}_s = -en_s\vec{\mathrm{v}} \Rightarrow \frac{\partial \vec{j}_s}{\partial t} = -en_s\vec{a} \\ &\Rightarrow i\omega\vec{j}_s = n_s\frac{e^2\vec{\mathbf{E}}}{m} \end{aligned}}$$

$$\Rightarrow \quad \sigma_s = \frac{n_s}{i\omega}\frac{e^2}{m}$$

- Using Maxwell equation $\nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$ we obtain

$$\frac{\partial}{\partial t}\left(\frac{m}{n_s e^2}\nabla \times \mathbf{J}_s + \mathbf{B}\right) = 0$$

Second London equation

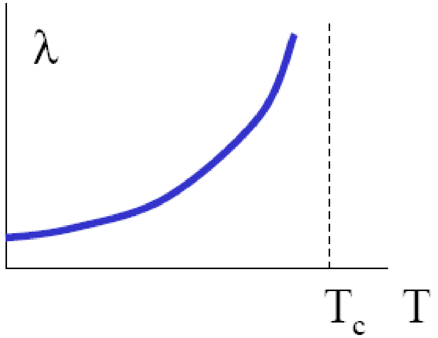
- The Meissner effect requires $\vec{\mathbf{B}} = -\dfrac{m}{n_s e^2}\nabla \times \vec{\mathbf{J}}_s$

London penetration depth

- Using the Maxwell equations, $\nabla\times \mathbf{E} = -\mu_0\partial_t\mathbf{H}$ and $\nabla\times \mathbf{H} = \mathbf{J}_s$ we obtain the second London equation:

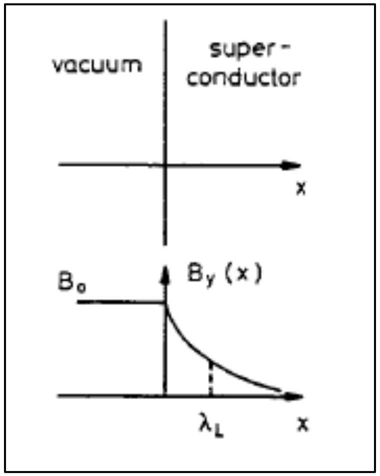
$$\lambda^2\nabla\mathbf{H} - \mathbf{H} = 0$$

- London penetration depth: $$\lambda = \left(\frac{m}{e^2 n_s(T)\mu_0}\right)^{1/2}$$



- It is important to understand that this equation is not valid in a normal conductor. The depth is frequency independent!
- If we consider a simple geometry, a boundary between a superconductor and vacuum, then the solution is

$$B_y(x) = B_0 \exp\left(-x/\lambda_L\right)$$



- Magnetic field does not stop abruptly, but penetrates into the material with exponential attenuation. The penetration depth is quite small, 20 – 50 nm.
- According to BCS theory not single electrons, but pairs are carriers of the super-current. However, the penetration depth remains unchanged: 2e/2m=e/m.

AC current in two-fluid model

- To calculate the surface impedance of a superconductor we take into account both the "superconducting" electrons n_s and "normal" electrons n_n in the two-fluid model
- There is no scattering, thus $\vec{j}_{s,n} = -n_{s,n} e \vec{\mathbf{v}}_{s,n}$ and we already got this of n_s

$$\boxed{m\frac{\partial \vec{\mathbf{v}}_s}{\partial t} = -e\vec{\mathbf{E}} \;\Rightarrow\; \frac{\partial \mathbf{j}_s}{\partial t} = \frac{n_s e^2}{m}\vec{\mathbf{E}}}$$

- Or in an AC field

$$\vec{j}_s = -i\frac{n_s e^2}{m\omega}\vec{\mathbf{E}} = -i\sigma_s \vec{\mathbf{E}} \quad \text{or} \quad \vec{j}_s = \frac{-i}{\omega\mu_0\lambda_L^2}\mathbf{E}$$

- The total current is simply a sum of currents due to two "fluids":

$$\vec{j} = \vec{j}_n + \vec{j}_s = \left(\sigma_n - i\sigma_s\right)\vec{\mathbf{E}}$$

- Thus one can apply the same treatment to a superconductor as was used for a normal conductor before with the substitution of the newly obtained conductivity.

$$\sigma_s = \frac{n_s}{i\omega}\frac{e^2}{m}$$

Surface impedance of superconductors

- We expect the real part of the surface resistance to drop exponentially below T_c.
- The surface impedance

$$Z_s = \sqrt{\frac{\omega\mu_0}{2\sigma}}(1+i) \Rightarrow \sqrt{\frac{\omega\mu_0}{2(\sigma_n - i\sigma_s)}}(1+i)$$

- The penetration depth

$$\delta = \frac{1}{\sqrt{\pi f \mu_0 \sigma}} \Rightarrow \frac{1}{\sqrt{\pi f \mu_0 (\sigma_n - i\sigma_s)}}$$

- Note that $1/\omega$ is of the order of 100 ps whereas the relaxation time for normal conducting electrons if of the order of 10 fs. Also, $n_s \gg n_n$ for $T \ll T_c$, hence $\sigma_n \ll \sigma_s$.
- Then

$$\delta \approx (1+i)\lambda_L\left(1 + i\frac{\sigma_n}{2\sigma_s}\right) \quad \text{and} \quad H_y = H_0 e^{-x/\lambda_L} e^{-ix\sigma_n/2\sigma_s\lambda_L}$$

- The fields decay rapidly, but now over the London penetration depth, which is much shorter than the skin depth of a normal conductor.
- For the impedance we get

$$Z_s \approx \sqrt{\frac{\omega\mu_0}{\sigma_s}}\left(\frac{\sigma_n}{2\sigma_s} + i\right) \quad X_s = \omega\mu_0\lambda_L \quad R_s = \frac{1}{2}\sigma_n\omega^2\mu_0^2\lambda_L^3$$

BCS surface resistivity

- Let us take a closer look at the surface impedance

$$Z_s \approx \sqrt{\frac{\omega\mu_0}{\sigma_s}}\left(\frac{\sigma_n}{2\sigma_s}+i\right) \quad X_s = \omega\mu_0\lambda_L \quad R_s = \frac{1}{2}\sigma_n\omega^2\mu_0^2\lambda_L^3$$

- One can easily show that $X_s >> R_s \rightarrow$ the superconductor is mostly reactive.
- The surface <u>resistivity</u> is proportional to the <u>conductivity</u> of the normal fluid! That is if the normal-state resistivity is low, the superconductor is more lossy.
  - *Analogy: a parallel circuit of a resistor and a reactive element driven by a current source. Observation: lower Q for cavities made of higher purity Nb.*
- While this explanation works for all practical purposes, it is a simplification.
- For real materials instead of the London penetration depth we should use an effective penetration depth, which is

$$\lambda = \lambda_L\sqrt{\frac{\xi_0}{\xi}}\ ,$$

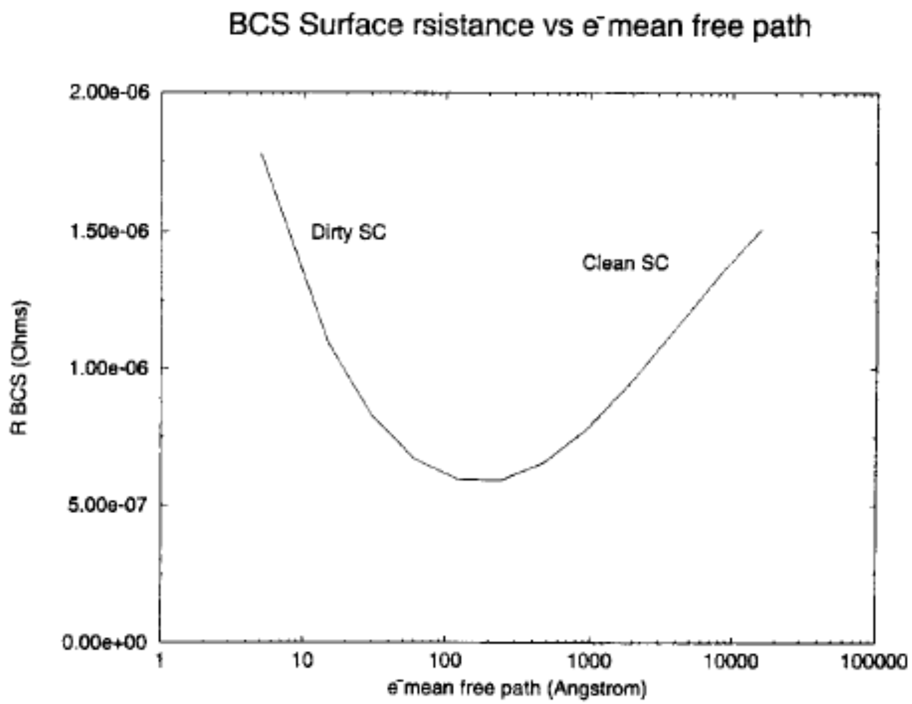
  where ξ_0 and ξ are the coherence lengths of the pure and real materials respectively.
- In the real material the coherence length is given by

$$\xi^{-1} = \xi_0^{-1} + l^{-1},$$

  where l is the electron mean free path.

BCS surface resistivity (2)

- Let us now consider two extremes
    1. For clean superconductors, $l >> \xi_0$, thus $R_{BCS} \sim l$. For very clean materials the equation is not valid anymore and BCS theory predicts roughly constant surface resistivity.
    2. For dirty superconductors, $l << \xi_0$, thus $\xi \cong l$, and we get $R_{BCS} \sim l^{-1/2}$.
- Between the clean and dirty limits R_{BCS} reaches a minimum, when the coherence length and mean free path are approximately equal



BCS surface resistivity vs. T

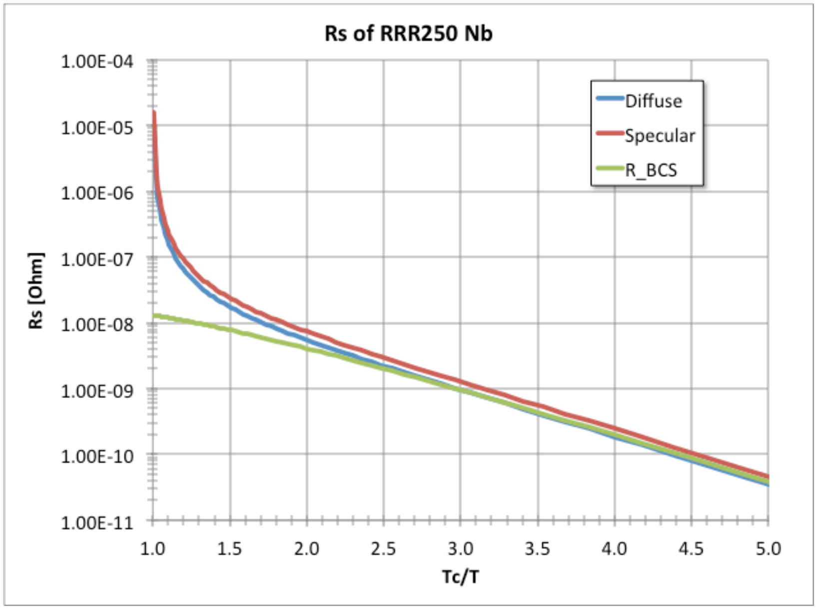
- Calculation of surface resistivity must take into account numerous parameters. Mattis and Bardeen developed theory based on BCS, which predicts

$$R_{BCS} = A\frac{\omega^2}{T}e^{-\left(\frac{\Delta}{k_B T_c}\right)\frac{T_c}{T}},$$

where *A* is the material constant.

- While for low frequencies (≤ 500 MHz) it may be efficient to operate at 4.2 K (liquid helium at atmospheric pressure), higher frequency structures favor lower operating temperatures (typically superfluid LHe at 2 K, below the lambda point, 2.172 K).
- Approximate expression for Nb:

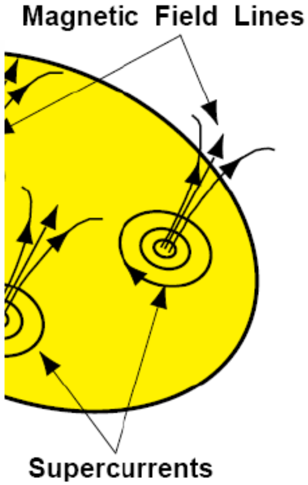
$$R_{BCS} \approx 2\times10^{-4}\left(\frac{f[\mathrm{MHz}]}{1500}\right)^2\frac{1}{T}e^{\left(\frac{-17.67}{T}\right)}[\mathrm{Ohm}]$$



- Above *~T_c/2*, this formula is not valid and one have to perform more complicated calculations. The plots show comparison of the surface resistivity calculated using the formula with more precise calculation using Halbritter's program **`SRIMP`**.
- In this program the Nb mean free path (in Angstroms) is assumed to be approximately **60×RRR**.

Trapped magnetic flux

- Ideally, if the external magnetic fields is less than H_{c1}, the DC flux will be expelled due to Meissner effect. In reality, there are lattice defects and other inhomogeneities, where the flux lines may be "pinned" and trapped within material.



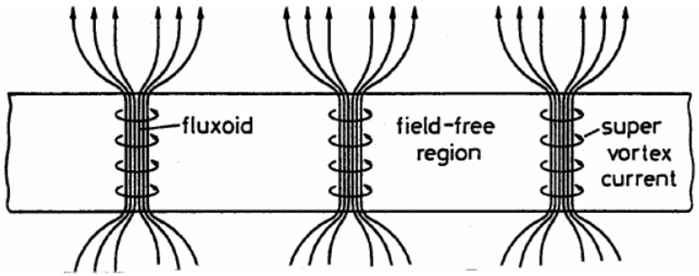
- The resulting contribution to the residual resistance

$$R_{mag} = \frac{H_{ext}}{2H_{c2}} R_n$$

- For high purity (RRR=300) Nb one gets

$$R_{mag} = 0.3(n\Omega) H_{ext}(mOe)\sqrt{f(GHz)}$$

- Earth's field is 0.5 G, which produces residual resistivity of 150 nOhm at 1 GHz and $Q_0 < 2\text{x}10^9$
- Hence one needs magnetic shielding around the cavity to reach quality factor in the 10^{10} range.
- Usually the goal is to have residual magnetic field of less than 10 mG.



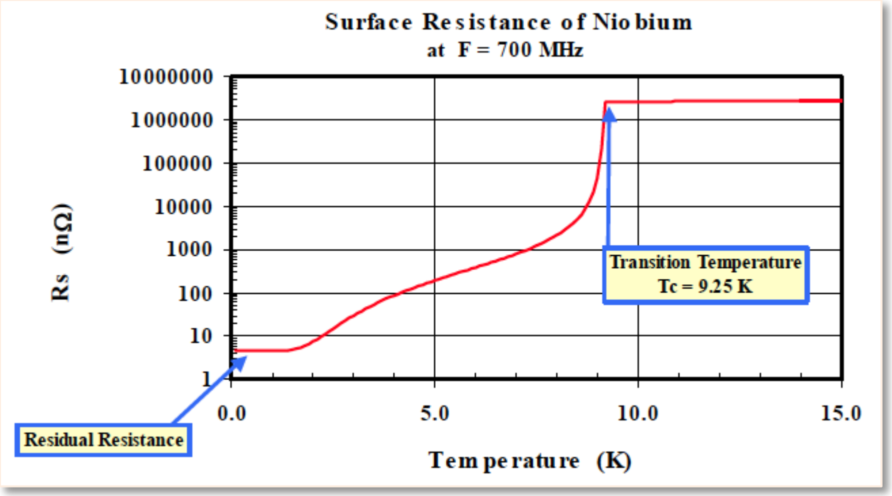
Residual surface resistivity

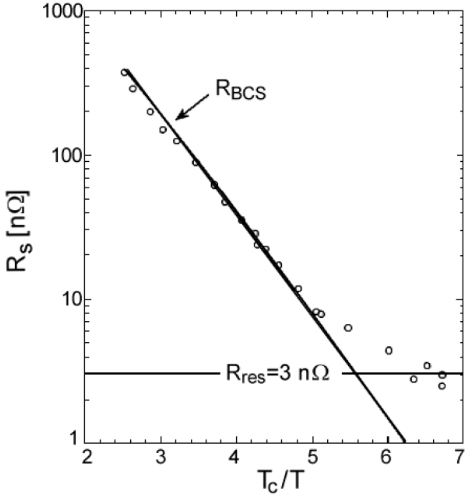
- At low temperatures the measured surface resistivity is larger than predicted by theory:

$$R_s = R_{BCS}(T) + R_{res}$$

where R_{res} is the temperature independent residual resistivity.
It can be as low as 1 nOhm, but typically is ~10 nOhm.

- Characteristics:
  - no strong temperature dependence
  - no clear frequency dependence
  - can be localized
  - not always reproducible
- Causes for this are:
  - magnetic flux trapped in at cool-down
  - dielectric surface contaminations (chemical residues, dust, adsorbents)
  - NC defects & inclusions
  - surface imperfections
  - hydrogen precipitates





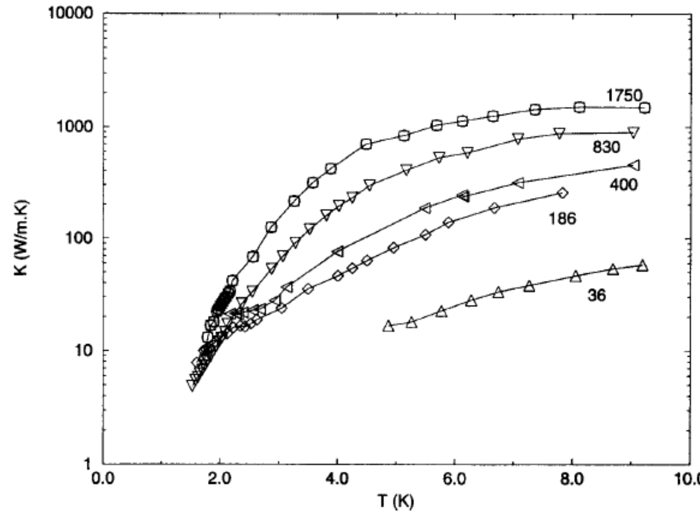
RRR

- Residual Resistivity Ratio (RRR) is a measure of material purity and is defined as the ratio of the resistivity at 273 K (or at 300 K) to that at 4.2 K in normal state.
- High purity materials have better thermal conductivity, hence better handling of RF losses.
- The ideal RRR of niobium due to phonon scattering is 35,000. Typical “reactor grade” Nb has RRR ≈ 30. Nb sheets used in cavity fabrication have RRR ≥ 200.

$$\lambda(4.2K) \approx 0.25 \cdot RRR \quad [W/(m \cdot K)]$$

$$RRR = \left(\sum_i f_i / r_i \right)^{-1}$$

where f_i denote the fractional contents of impurity i (measured in weight ppm) and the r_i the corresponding resistivity coefficients, which are listed in the table below.



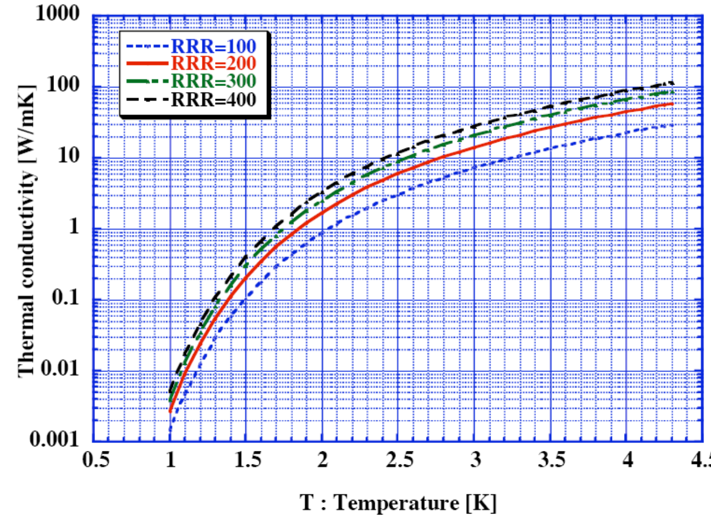
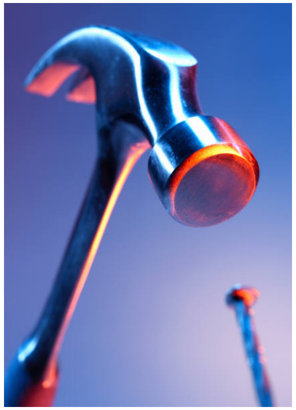


Table II Weight factor r_i of some impurities (see equation (4))

Impurity atom i	N	O	C	H	Ta
r_i in 10^4 wt. ppm	0.44	0.58	0.47	0.36	111

Worth remembering…



- The superconducting state is characterized by the critical temperature and magnetic field.
- There are Type I and Type II superconductors.
- Two-fluid model and BCS theory explain surface resistivity of superconductors.
- Nb is a material of choice in either bulk form or as a film on a copper substrate.
- Other materials are being investigated.
- At low temperatures residual resistivity limits performance of superconducting cavities.
- There are several phenomena responsible for the deviation of “real world” losses from theoretical predictions.
- Material quality (impurities, mechanical damage) plays important role.
- Performance of SC cavities is dependent on the quality of a thin surface layer.

Main non-trivial/nonlinear effects and the limits of SRF linacs?

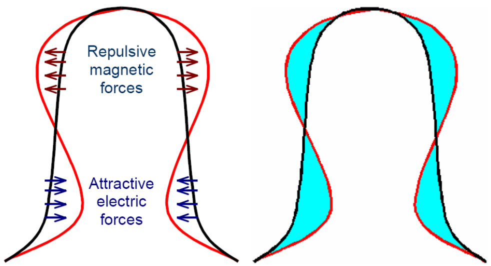
With SR cavities capable of $Q_o \sim 10^{10}$, 850 MHz SRF cavity can have bandwidth of the resonance bandwidth of 0.1 Hz (e.g. it would ring for about 10 seconds without external RF source!).
While being the result of excellent conductivity, it makes cavity susceptible to small mechanical size change – 1 nanometer change in a cavity ~ 1 meter in size could cause ~ 10 Hz change – e.g. 100-fold the bandwidth, and take it completely out of the resonance…

Low level RF system (and cavity tuning system) is used to keep cavity both at resonance, stable and under control. In addition, depending on the application, the cavity Q is reduced to by using strong external coupling. For ERLs it is typical to have : $Q_{ext} \sim 10^8$. It turns bandwidth into a measurable few Hz range.

Side note: if mechanical hand watch would have $Q=10^{10}$,
it would not require rewinding for about 300 years… and it would be a really good astronomical instrument.

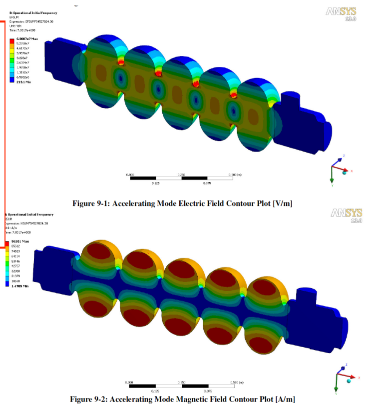
Ponderomotive effects: radiation pressure

- Ponderomotive effects are nothing else but changes of the cavity shape and its frequency caused by the electromagnetic field (radiation) pressure:



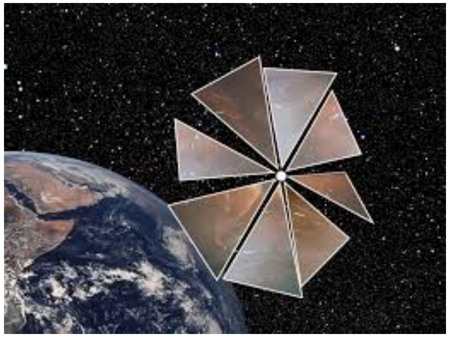
$$P_{Rad} = \frac{1}{4}\left(\mu_0 H^2 - \varepsilon_0 E^2\right)$$

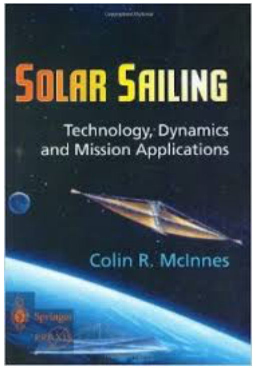
Typical SRF linac pressure
~ 100- 1000 N/m^2



  - Static Lorentz detuning (CW operation)
  - Dynamic Lorentz detuning (pulsed operation)

Solar sailing… again

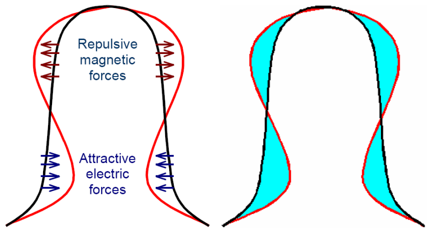




- This effect is called **Lorentz de-tuning** and should be taken into account in the RF control system to make it stable

Lorentz detuning & its compensation

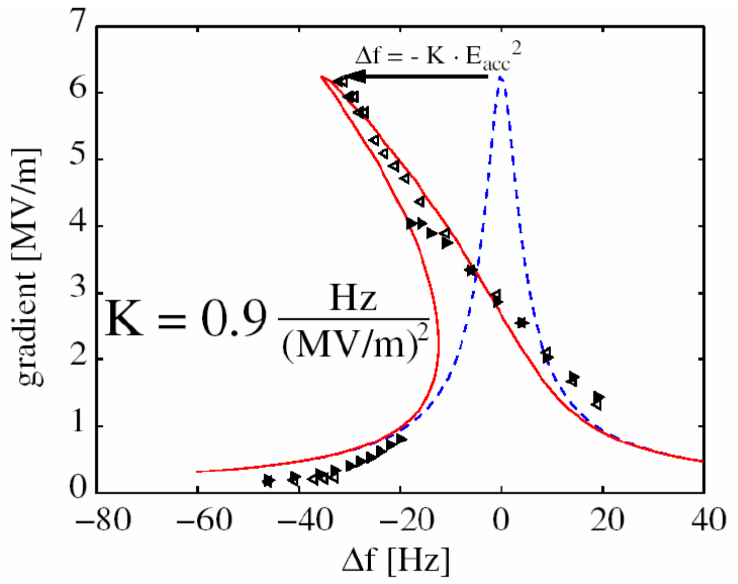
$$P_{Rad} = \frac{1}{4}\left(\mu_0 H^2 - \varepsilon_0 E^2\right)$$



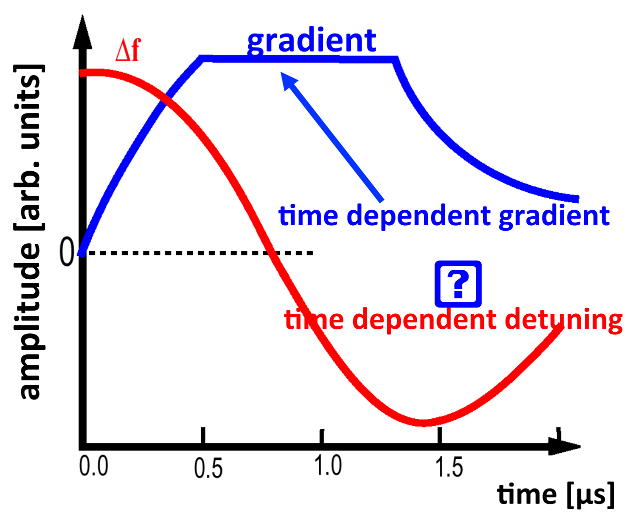
TESLA 9-cell cavity

$$\frac{\Delta f_L}{f} \approx \frac{1}{4W}\int_{\Delta V}\left(\mu_0 H^2 - \varepsilon_0 E^2\right) dv$$

CW-mode operation



Pulsed at 23.5 MV/m



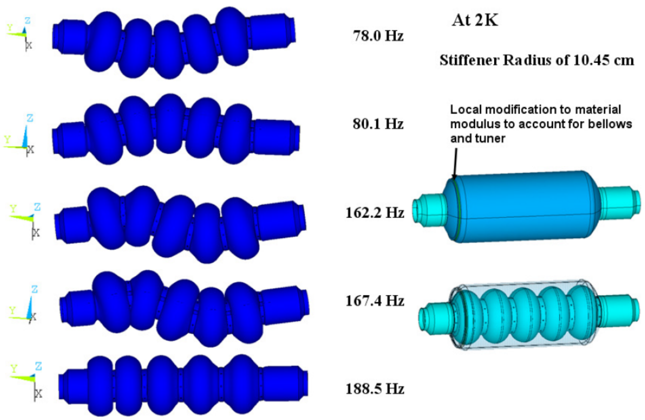
Microphonics

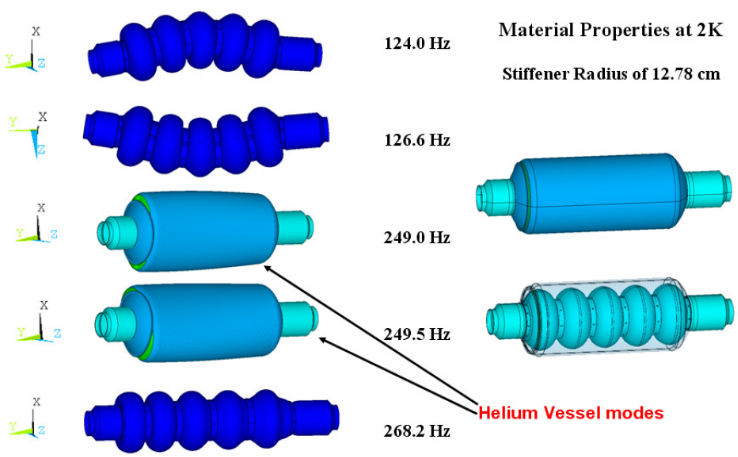
$$\frac{\Delta f_L}{f} \approx \frac{1}{4W} \int_{\Delta V} \left(\mu_0 H^2 - \varepsilon_0 E^2 \right) dv$$

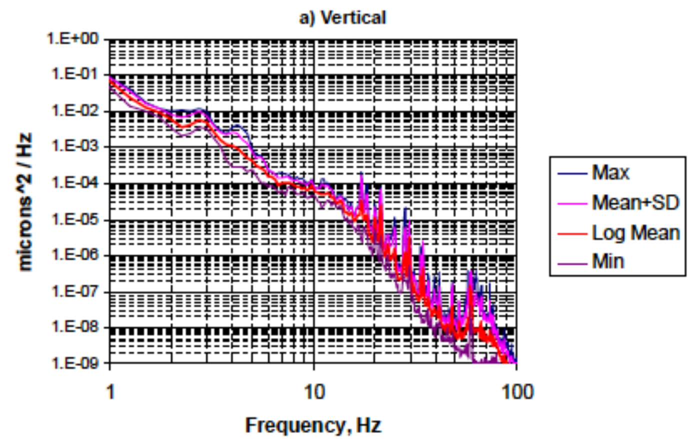
- General word "Microphonics" is used in SRF circles to describe any changes in cavity shaped and size caused by external sources:

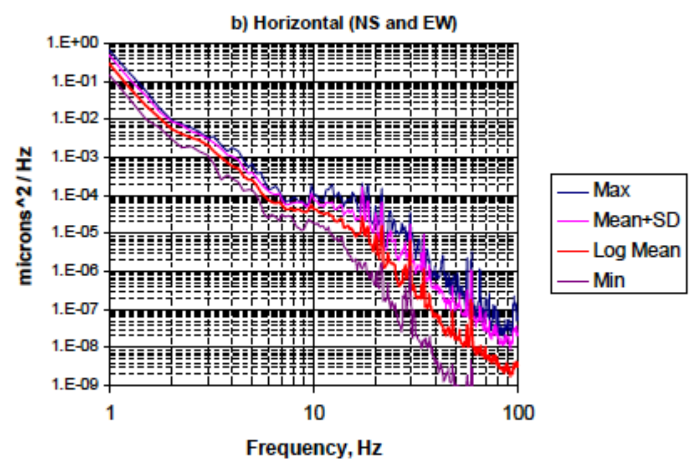
– Vibrations of the structures and cavity walls, including acoustic noise (hence the term!)

– Liquid Helium Pressure fluctuations





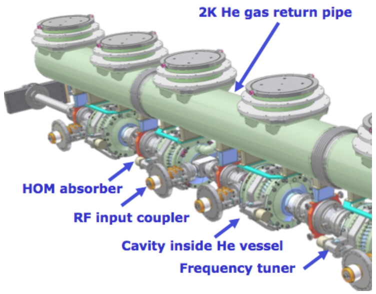


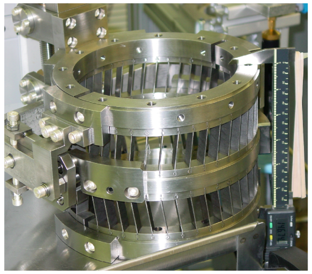


Microphonics

$$\frac{\Delta f_L}{f} \approx \frac{1}{4W} \int_{\Delta V} \left(\mu_0 H^2 - \varepsilon_0 E^2 \right) dv$$

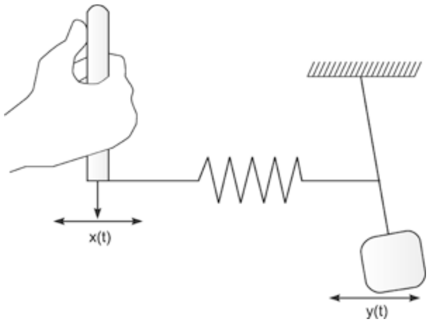
- Change in cavity's resonance frequency is compensated two ways:
  - Low frequency "noise" is taken out by cavity mechanical tuner (stepper motor + piezo)





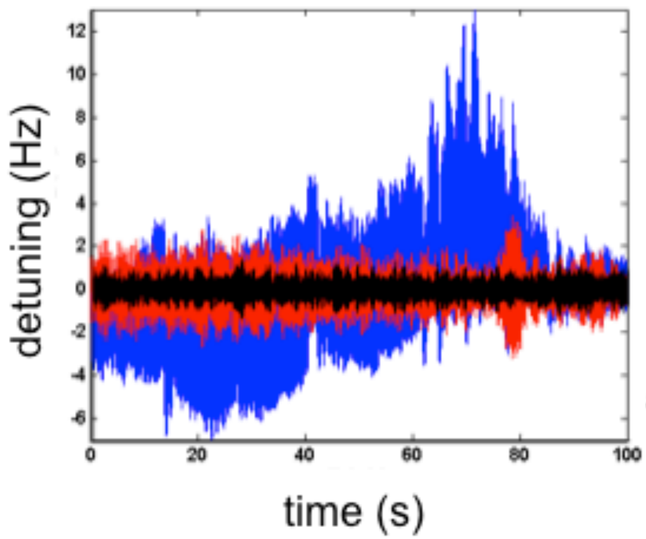
  - High frequency "noise" is taken care by forcing the cavity to oscillate at right frequency and right phase
  - It is done by measuring the "oscillator" phase with respect to the reference and by pushing and pulling it using power form RF transmitter
  - RF transmitter is periodically pushing power in and pulling it out – it is finishing in the dummy load. Typical power need for this is in tens of kW
  - Compare it with few watts dissipated in the walls

$$P_{RF} > 2\pi W \cdot \Delta f_{HF}$$



- A cavity with W ~ 400 J would require ~ 50 kW per cavity to fight 10 Hz frequency shift caused by micro-phonics

Microphonics compensation with Saclay I tuner

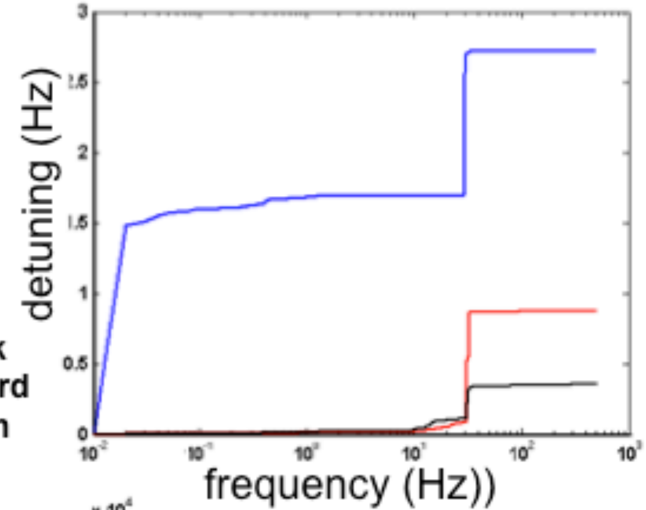


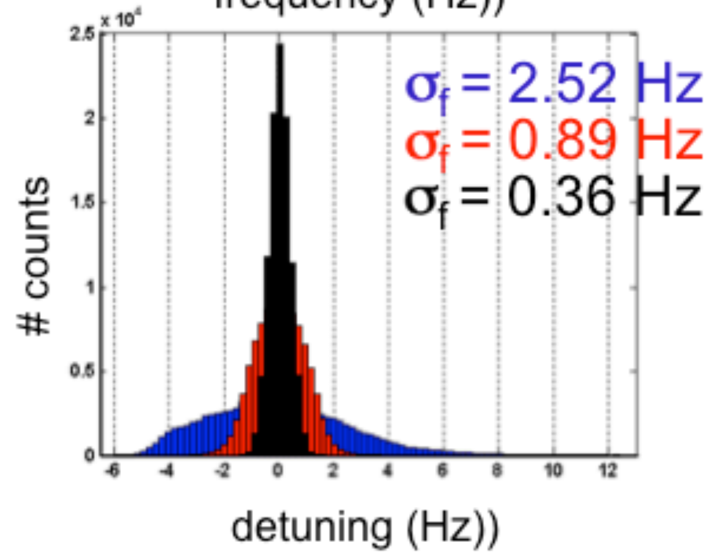
ΣFFT

w/o feedback

with feedback

with feedback
and feed-forward
compensation

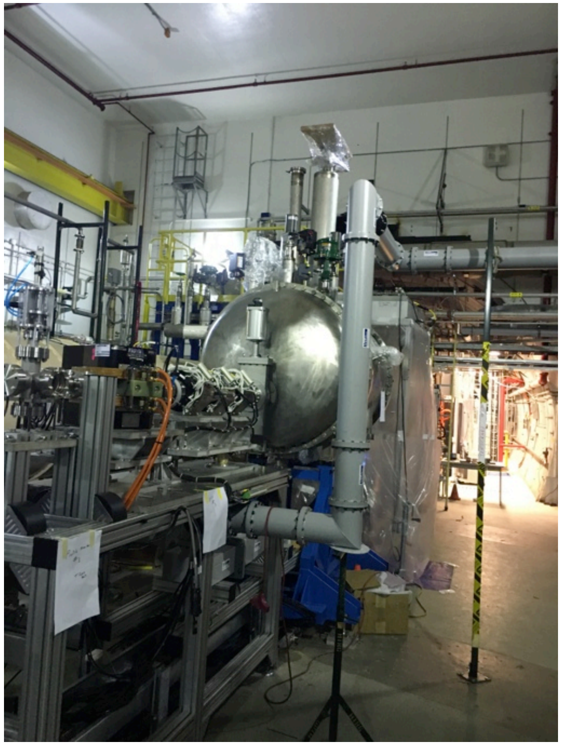


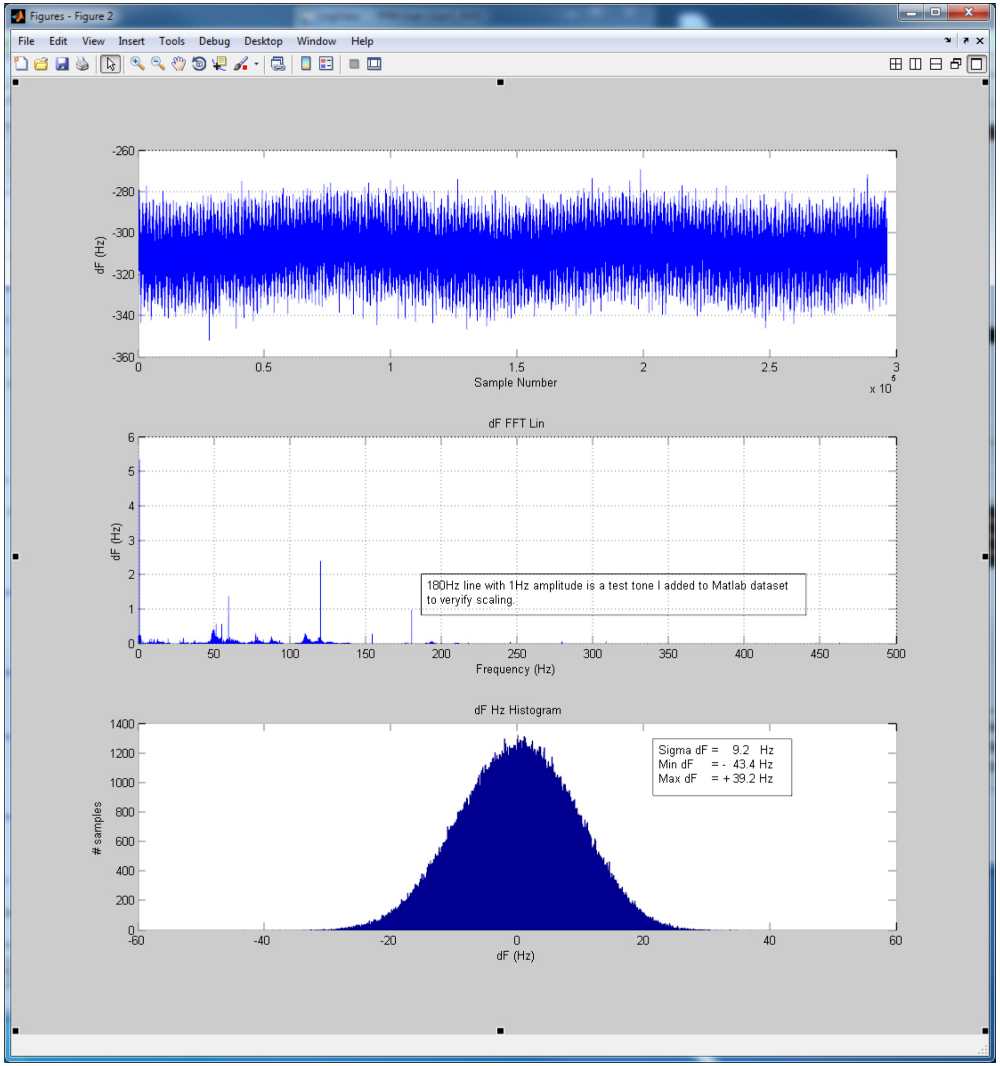


Microphonics measurements
done at HoBiCaT

Is this the limit?
What is the piezo resolution?

CeC 704 MHz 5 cell linac





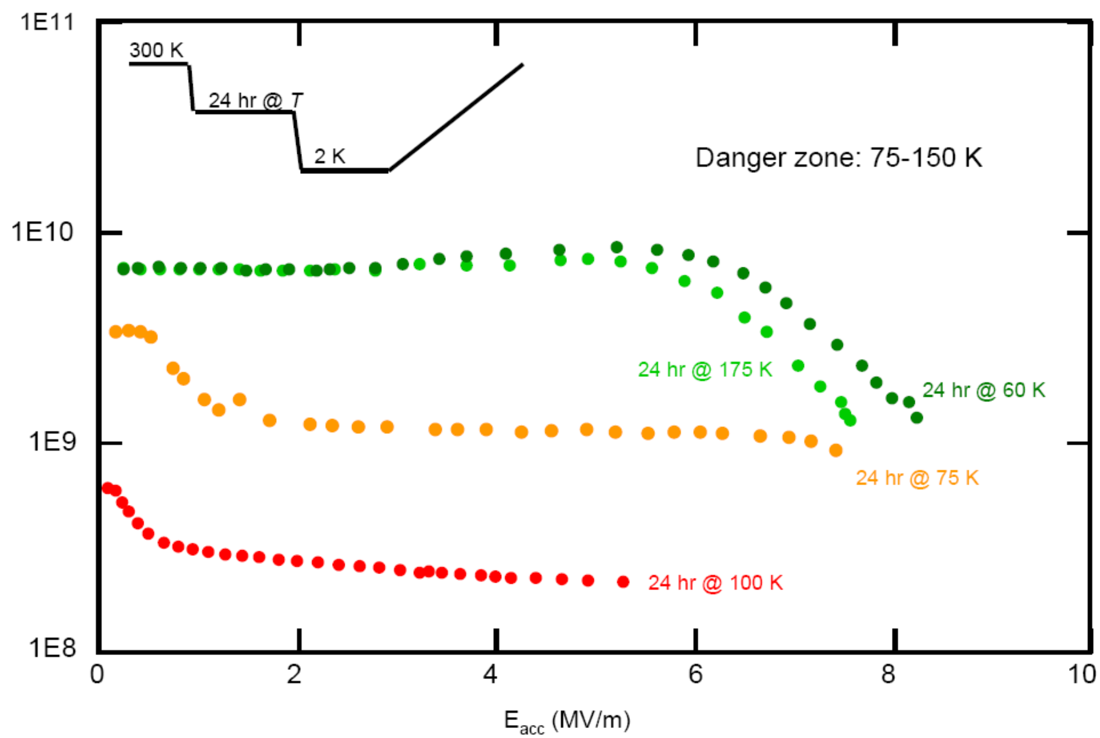
Q disease…. avoidable

The hydrogen dissolved in bulk niobium can under certain conditions during cool-down precipitate as a lossy hydride at the niobium surface. It has poor superconducting properties: *Tc* = 2.8 K and *Hc* = 60 Oe. This is known as the "*Q*-disease."

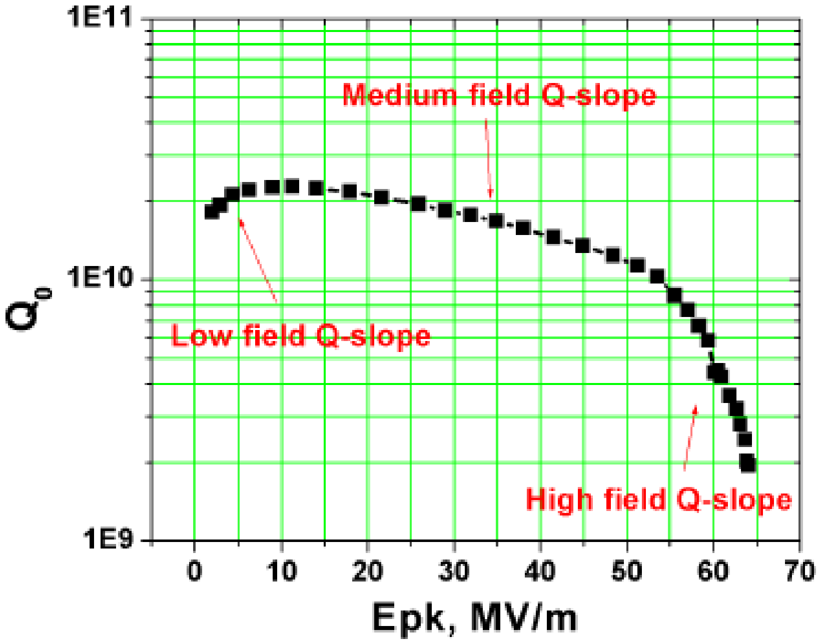
At temperatures above 150 K too high concentration of hydrogen is required to form the hydride phase (10^3 – 10^4 ppm). However, in the temperature range from 75 to 150 K the required hydrogen concentrations drops to as low as 2 ppm while its diffusion rate remains significant. This is the danger zone.

Mitigation:

- rapid cool-down through the danger temperature zone;
- degassing hydrogen by heating the Nb cavity in vacuum of better than 10^{-6} Torr at 600°C for 10 hrs or at 800°C for 1 to 2 hrs.;
- keep the acid temperature below 15°C during chemical etching.

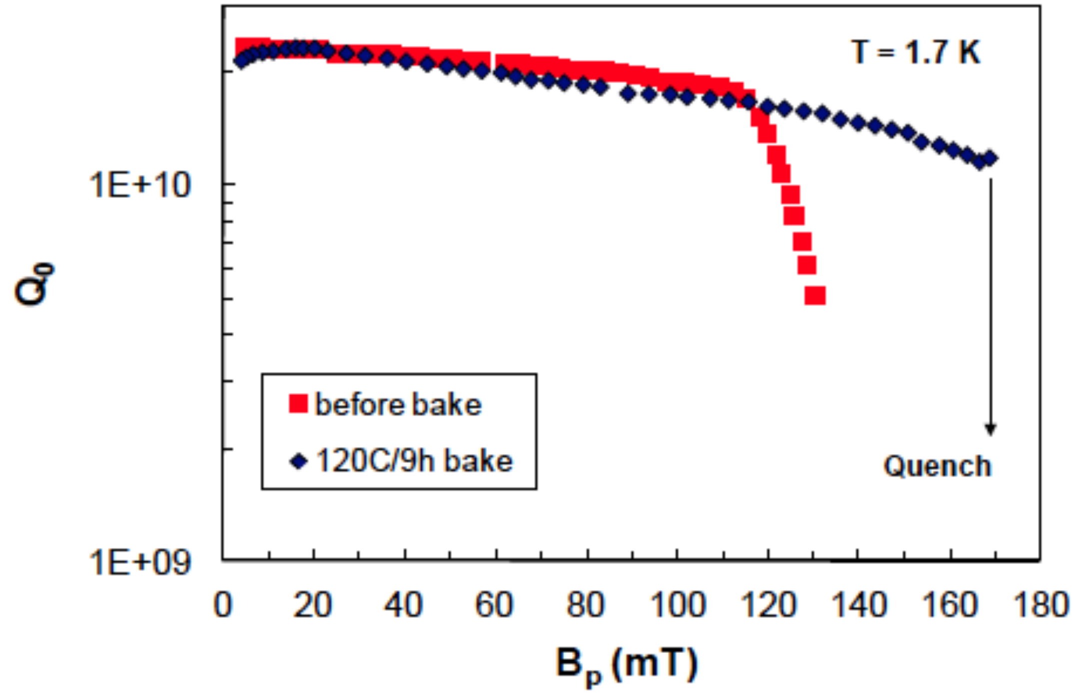


Q slopes



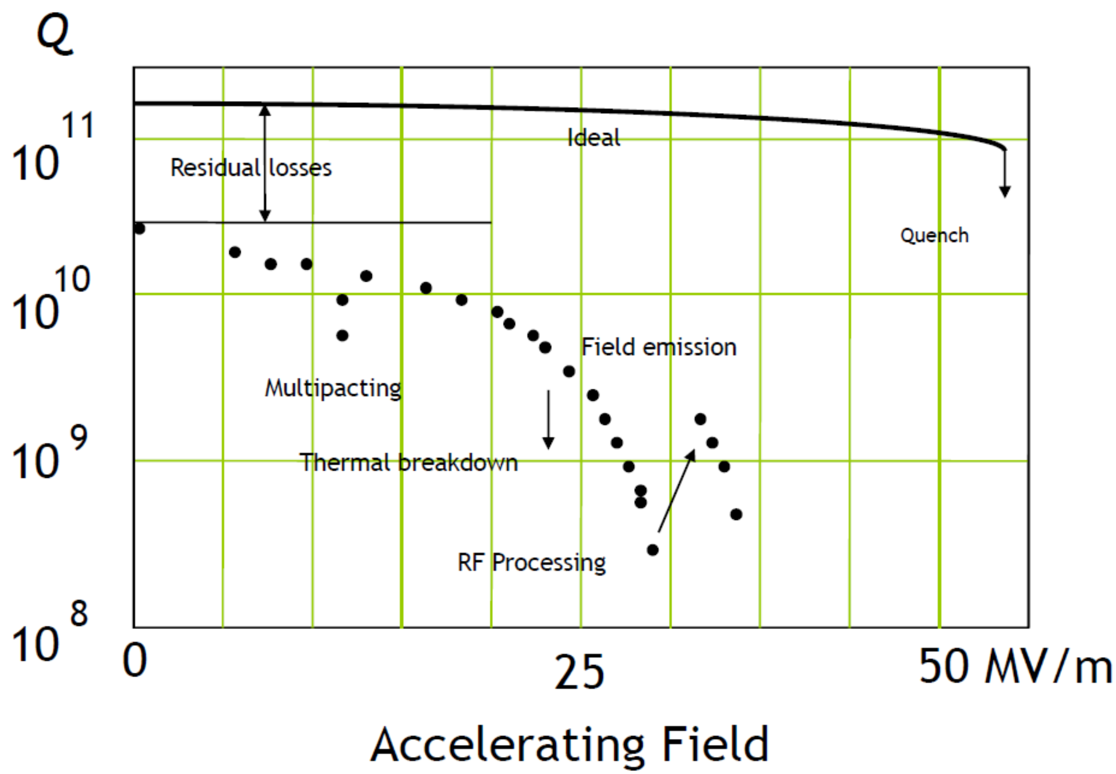
- The observed *Q* of a niobium cavity shows several interesting features with increasing field. As there is still no commonly accepted explanation of physics behind each of the *Q*-slopes
- In the low-field region (B_{pk} < ~20 mT) *Q* surprisingly increases by up to 50% between ~2 mT and ~20 mT. This does not present any limitation of cavity performance.
- Mild baking generally enhances the low-field *Q*-slope.
- At medium fields (~20 mT < B_{pk} < ~90 mT) Q gradually decreases by about 20% to 50%, a common feature of all Nb cavities.
- This is generally attributed to a combination of surface heating and "nonlinear" BCS resistance. Mild baking (100 – 120^{o}C for 48 hrs) usually decreases this *Q*-slope.

Q slopes (2)



- There is a sharp, exponential Q-drop at the highest fields (B_{pk} > ~90 mT)
- While some of the models provide a good fit of the experimental data, none of them has clearly provided a physical explanation of the phenomenon.
- This is still highly active area of basic SRF research. It was found empirically that mild baking helps under certain conditions.
- That the beneficial effect of mild baking on the high-field Q-slope is found to be dependent on the material (fine-grain or ingot Nb) and treatment combination (EP, BCP or post-purification) suggests that multiple mechanisms are involved….
- This is what I call "chemistry", e.g. I do not fully understand what it is?
- *Eventually superconductivity quenches due to a thermal instability at defects.*

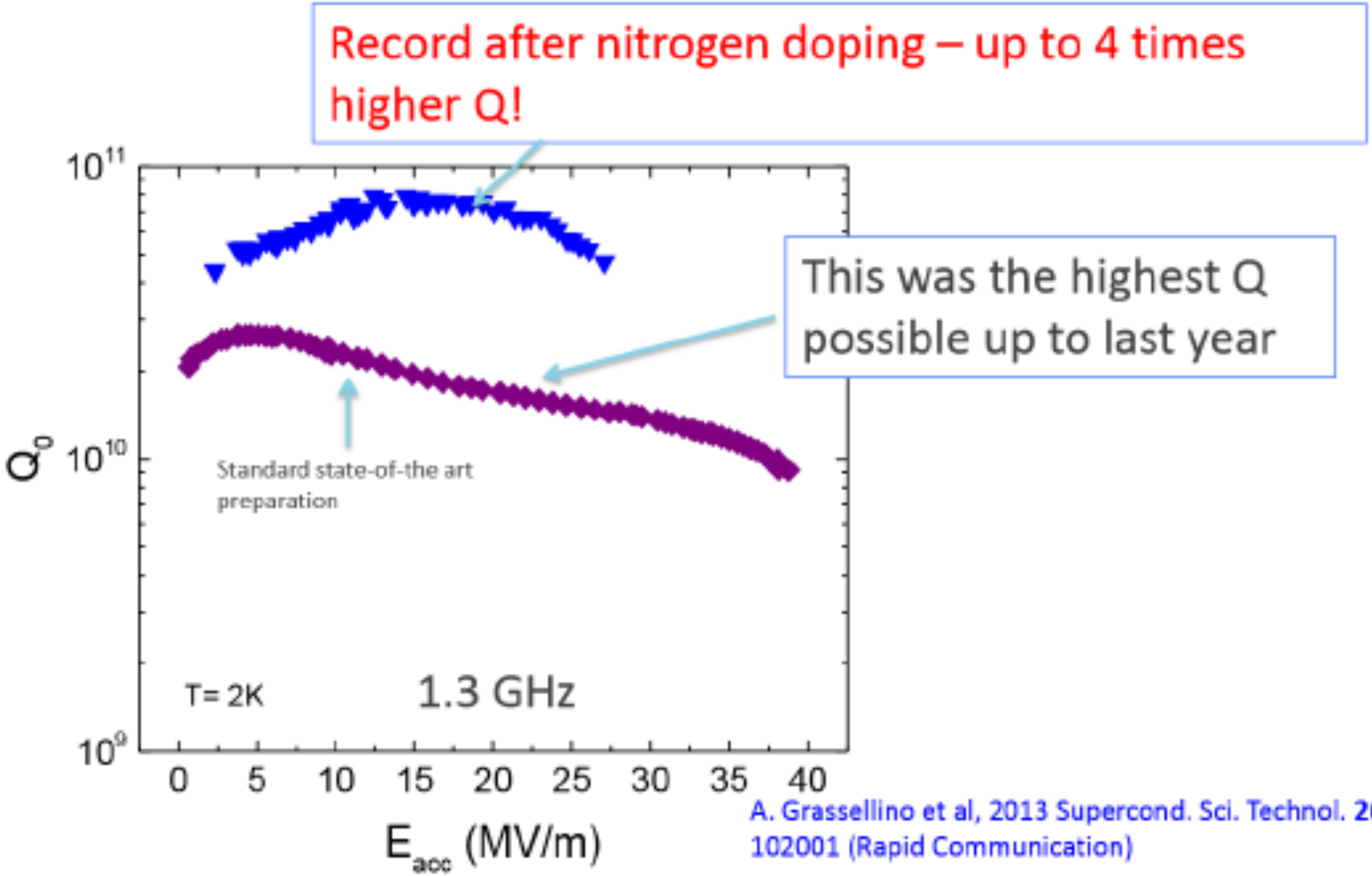
Q vs. *E*: real world



- It is customary to characterize performance of superconducting cavities by plotting dependence of their quality factor on either electric field (accelerating or peak surface) or peak magnetic field.
- *Q* vs. *E* plots is a "signature" of cavity performance.
- At low temperatures measured *Q* is lower than predicted by BCS theory.
- There are several mechanisms responsible for additional losses. Some of them are well understood and preventable, some are still under investigation.

And also real potential for breakthroughs

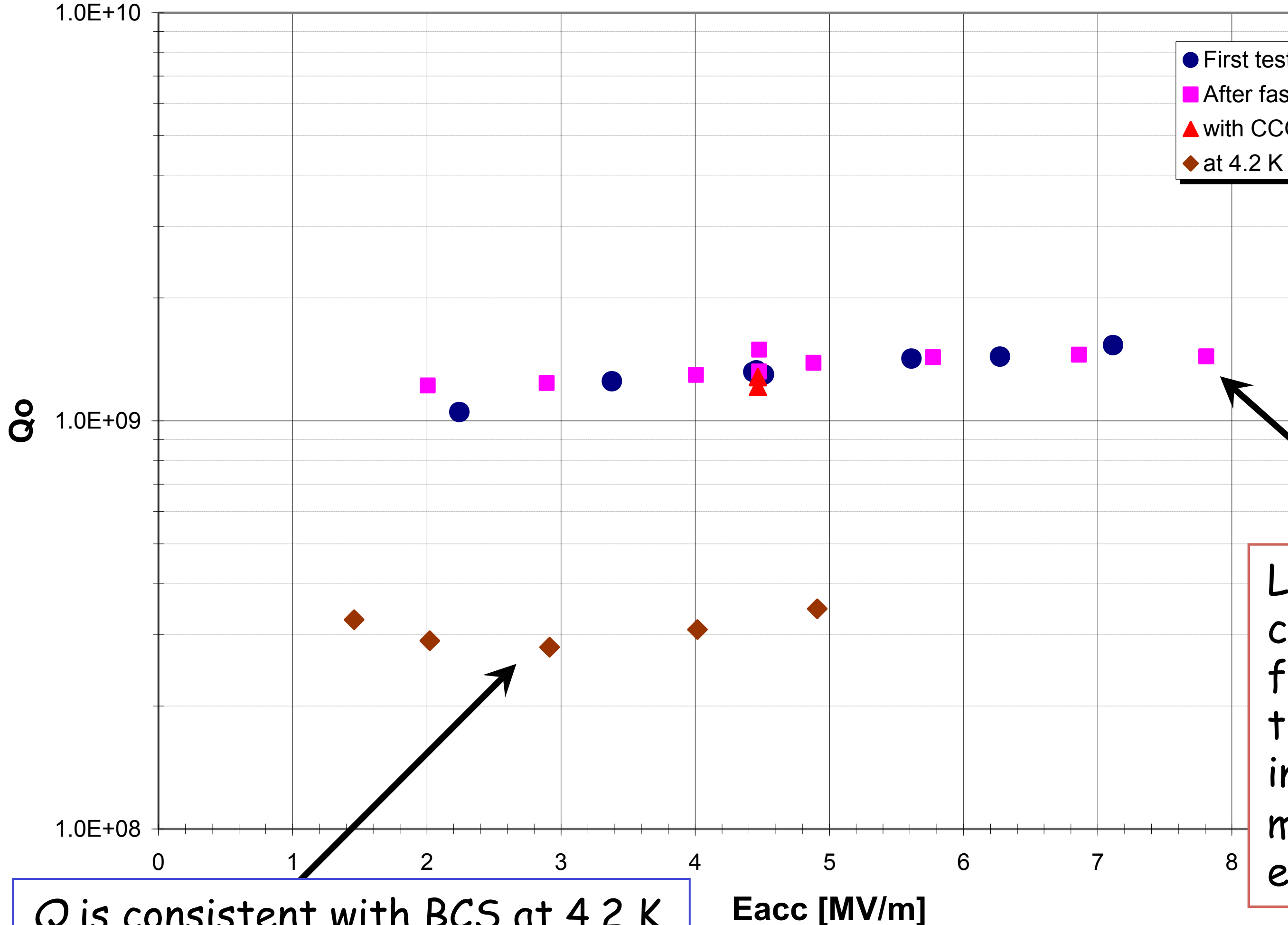
Nitrogen doping: a breakthrough in Q

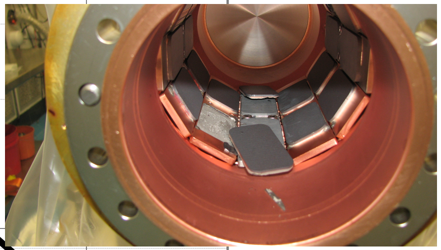


A. Grassellino et al, 2013 Supercond. Sci. Technol. **26** 102001 (Rapid Communication)

Surface contamination: Cornell U

1.3 GHz two-cell cavity with attached ferrite beam loads.
Expected to measure $Q > 10^{10}$



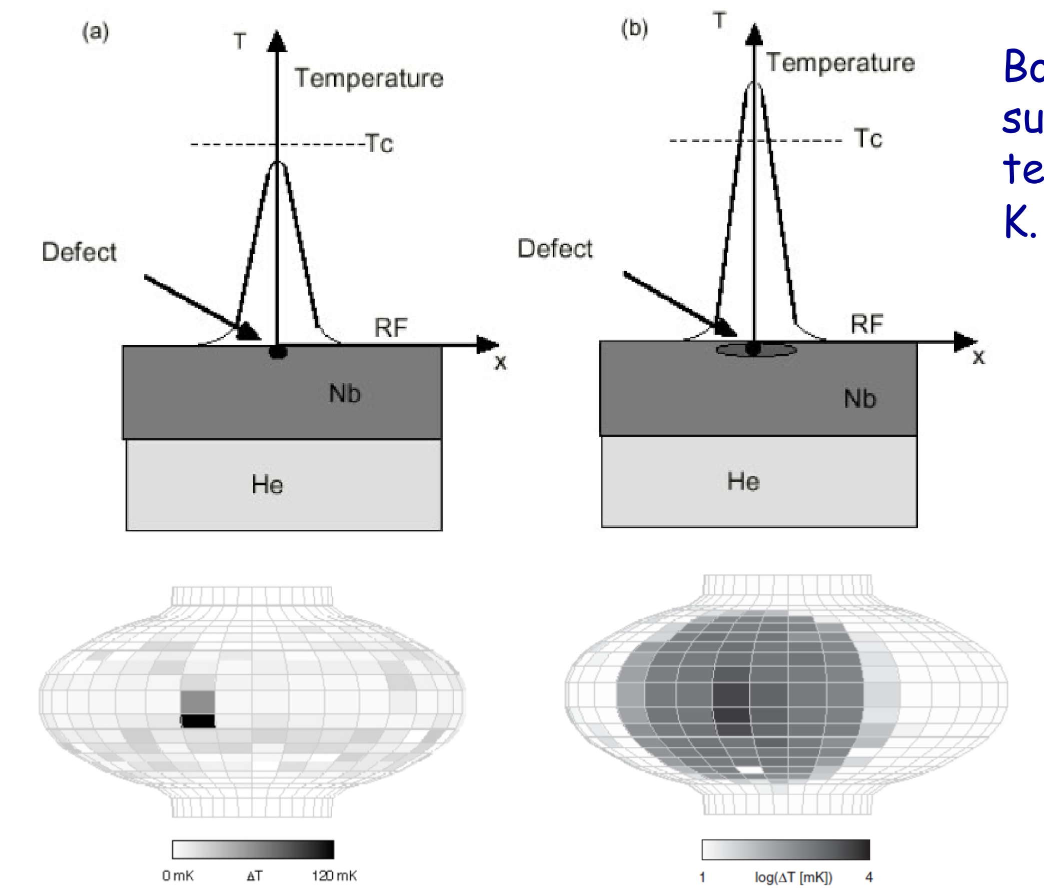


Low Q due to cavity contamination by dust from broken ferrite tiles. Simulations indicate that one ~1 mm size particle is enough to cause this!

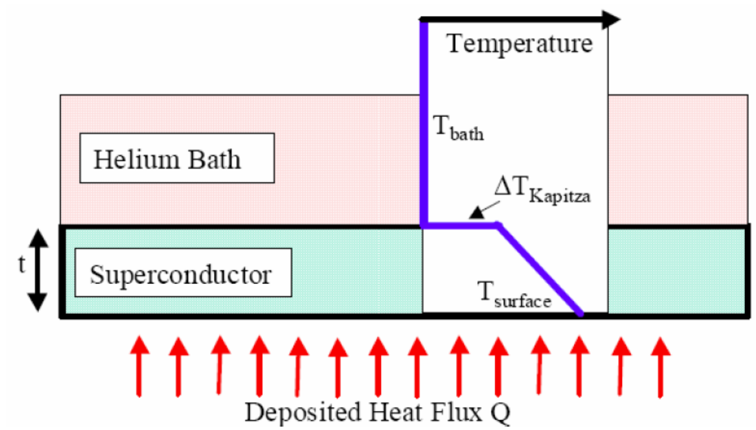
Q is consistent with BCS at 4.2 K

Thermal breakdown

- If there is a localized heating, the hot area will increase with field. At a certain field there is a thermal runaway and the filed collapses (loss of superconductivity or quench).
- Thermal breakdown occurs when the heat generated at the hot spot is larger than that can be evacuated to the helium bath.

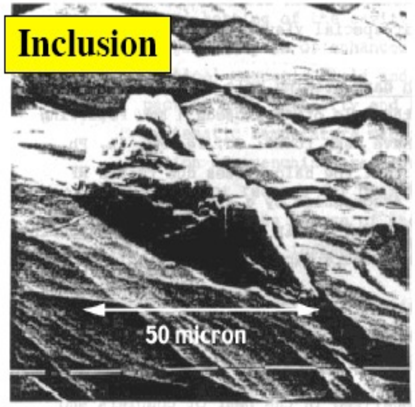


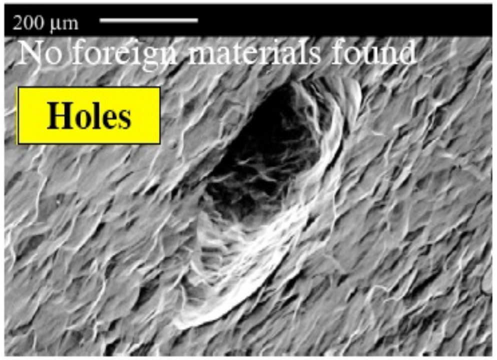
Both the thermal conductivity and the surface resistivity of Nb are highly temperature dependent between 2 and 9 K.



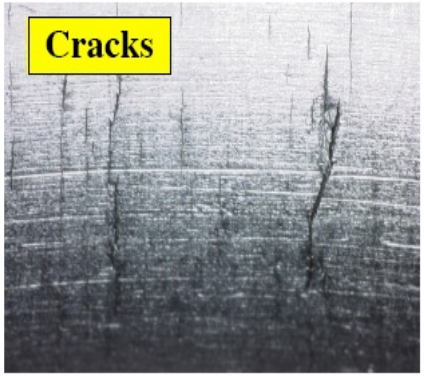
$$(T - T_{bath}) = Q\left[\frac{t}{\Lambda} + \frac{1}{h_k}\right]$$

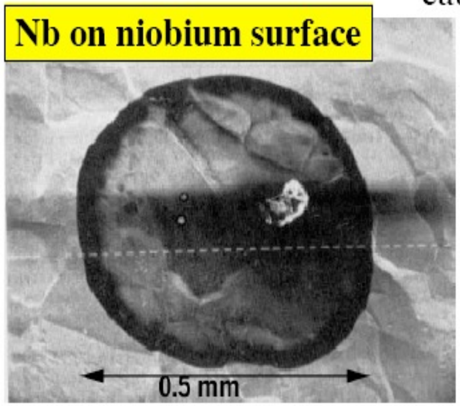
Examples of surface defects

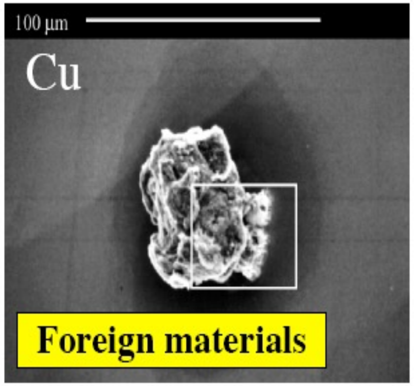


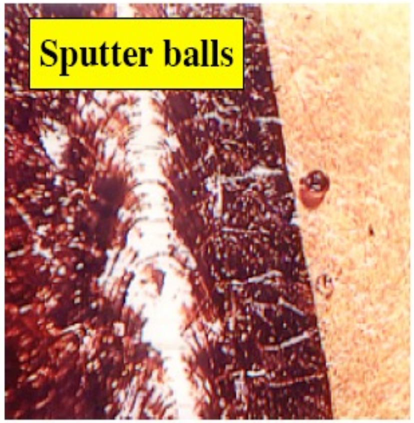


Surface defects, holes can also cause TB







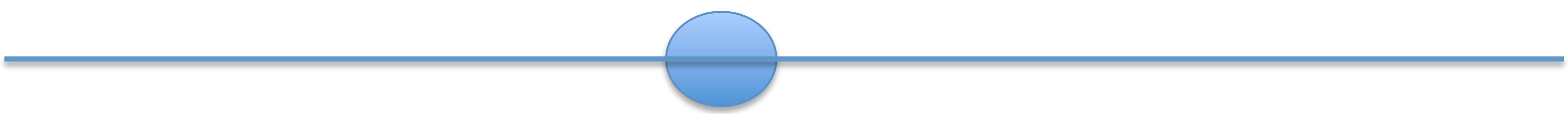


Surface defects can cause:

- Enhanced residual losses
- Premature quench
- Field emission

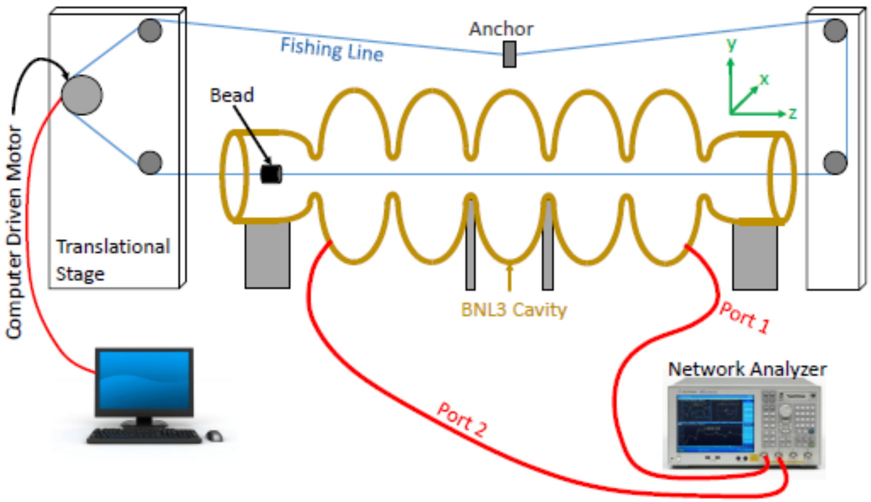
Measuring cavities: bead pilling

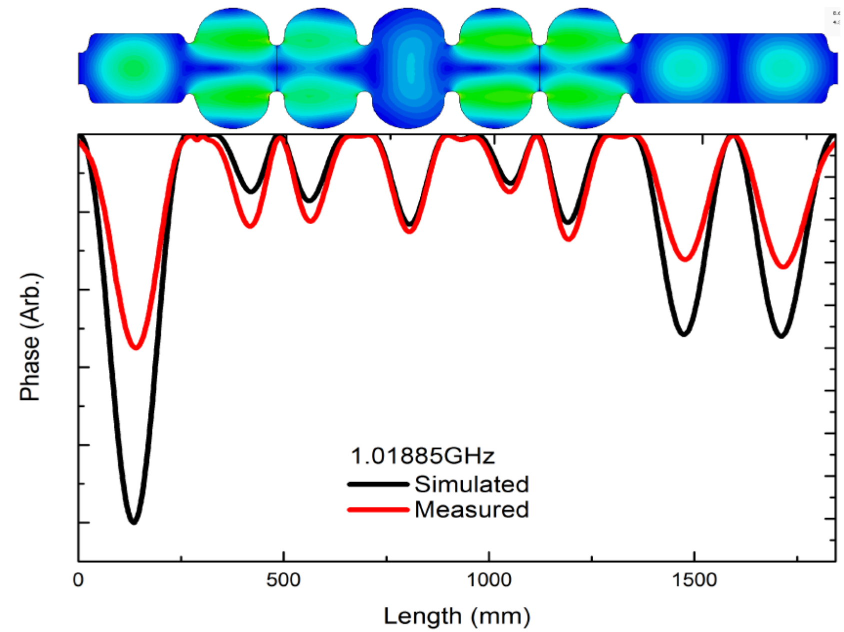
$$\frac{\Delta f}{f_0} = -\frac{1}{4W}\iiint_{\Delta V}\left(\mu_0 H^2 - \varepsilon_0 E^2\right)dV$$

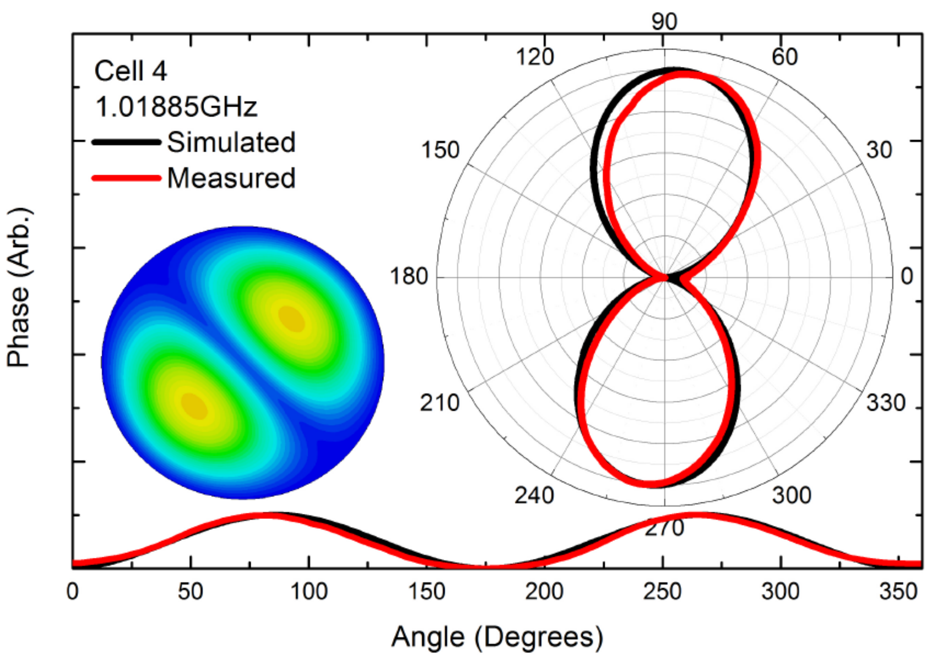


$$\frac{\Delta f}{f_0} = \left(\varepsilon_0 - 1\right)\frac{1}{4W}\iiint_{\Delta V} E^2\, dV \cong \left(\varepsilon_0 - 1\right)\frac{E^2}{4W}\Delta V$$

3-D bead-pulling to identify HOMs

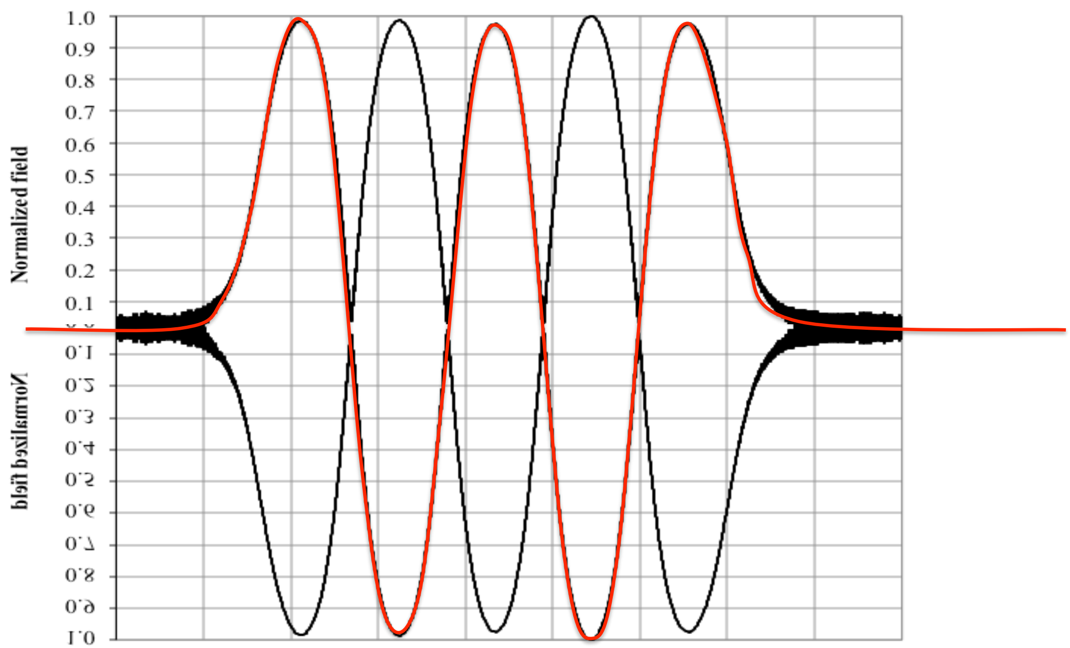






- A 3-D bead-pulling setup with capability of motion in longitudinal, radial and azimuthal directions was developed to map the field in the cavity;
- Using this setup, HOMs were identified and compared with simulation results;
- The detachable design of beam pipes allows us to measure/compare HOMs' information due to different layouts.

Results for CeC 5-cell BNL-3 linac



How many cells?

- What limit number of cells we can use? Why not to use 100 cells or even 1000 instead of 1,5,7,9…
- The answer is two-fold:
  - Narrowly spaced bands
  - Difficulty to tune
  - Trapped HOMs

How many cells?

– Trapped HOMs with Q~10^{10} can make any ERL beam unstable

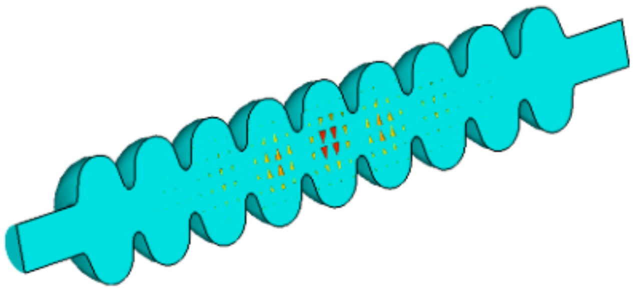
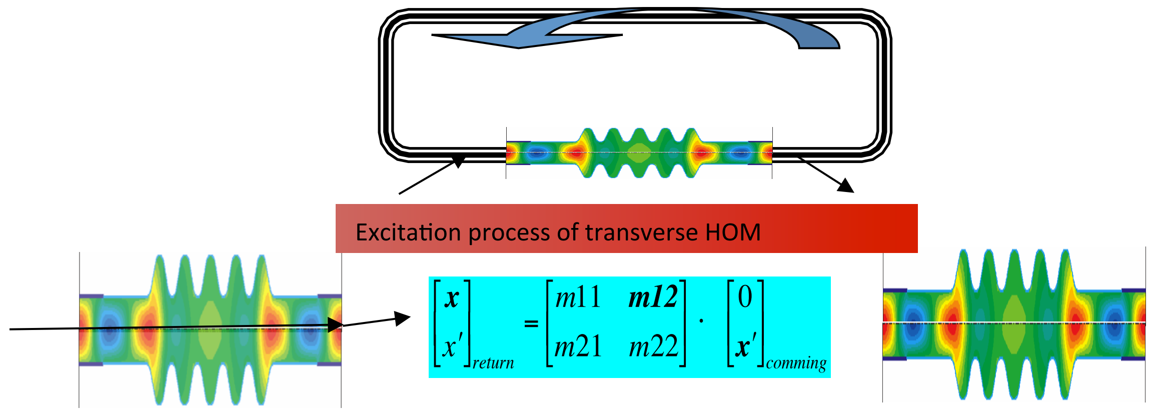


Figure 1: internally trapped mode in TESLA cavity



- First attempts to build ERL using SRF cavities resulted in transverse (TBBU) instability at micro-amp level of current
- 5-cell BNL-3 cavity with HOM damping can support tens of amps of beam current in a single-pass ERL and hundreds of mA in 16-pass ERL
- 7-cell Cornell cavity was designed for 100 mA single pass ERL

Simple picture to remember

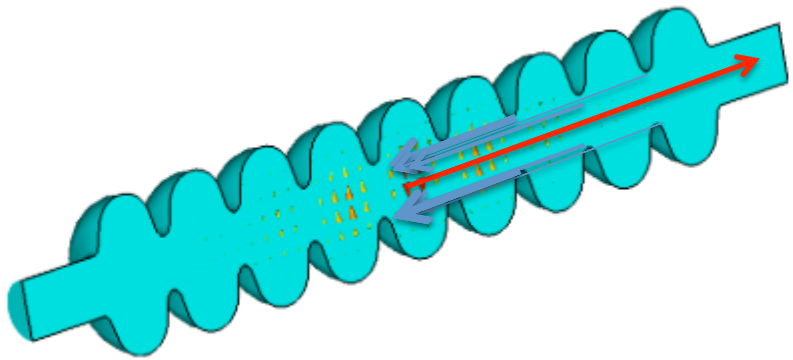
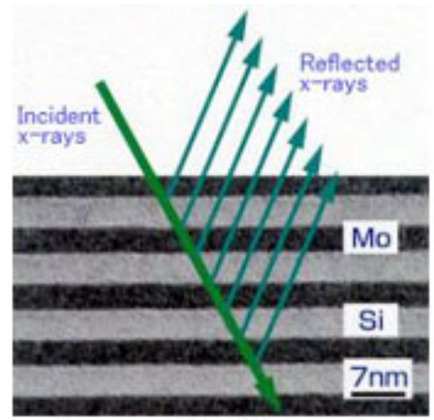
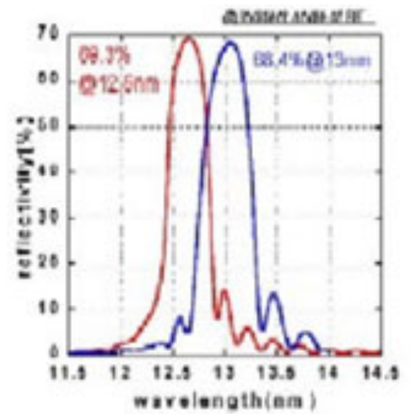


Figure 1: internally trapped mode in TESLA cavity

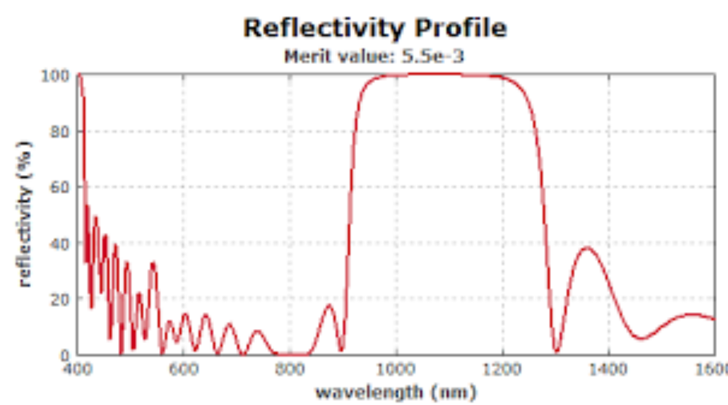


Cross sectional image of Mo/Si multilayer mirror and x-ray reflection mechanism



Reflectivity of fabricated multilayer mirrors for EUV

- Trapped mode when propagating towards the ends of the cavity is reflected from each iris
- Even though the reflection coefficients are not 100%, when the amplitudes of the all reflections add coherently it can result in 100% reflection of the wave
- Physics is similar to a near perfect reflectivity (can be 99.999….%!) of multi-layer dielectric mirror, while reflectivity from each layer is relatively modest



Conclusions

- We discussed in some details why and how RF accelerators (linacs) work
- We figured out that multi-cell cavities is a natural way of building-up the energy gain of the linac. Optimum number of cells depends on application.
- Pulsed room-temperature Cu linacs can operate at ~ 100 MV/m
- SRF CW linacs can have 10-fold higher energy gain when compared with room-temperature RF linacs
- We learned main characteristics of the RF accelerators
- We made a glimpse into the fascinating world of super-conducting linear accelerators