1. The energy loss per turn is given by

$$U_0 = \frac{e^2\beta^3\gamma^4}{3\varepsilon_0\rho} \ . \tag{1}$$

With $\rho = 5m$ and $\gamma = 2GeV / 0.511MeV = 3914$, eq. (1) yields

$$U_0 = \frac{e^2\beta^3\gamma^4}{3\varepsilon_0\rho} = 283.1KeV = 4.536\times10^{-14}J \ . \tag{2}$$

The critical photon energy is given by

$$E_c = \hbar\omega_c \, , \tag{3}$$

where $\hbar$ is the denoted Planck constant and

$$\omega_c = \frac{3}{2}\gamma^3\frac{c}{\rho} \approx 5.392\times10^{18} rad / s \tag{4}$$

is the critical angular frequency of the synchrotron radiation. Inserting eq. (4) into eq. (3) yields

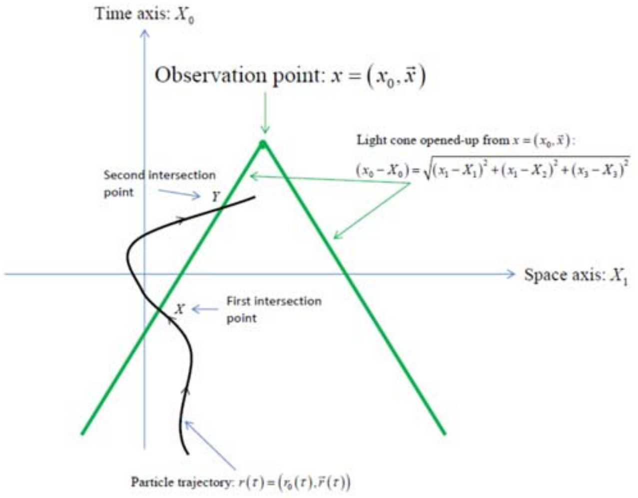
$$E_c \approx 3.549KeV = 5.687\times10^{-16}J \ . \tag{5}$$

The total synchrotron radiation power for a beam is given by the 1-turn energy loss of all particles in the ring divided by the time it takes for one circulation (i.e. the revolution period)

$$P_{beam} = \left(U_0 \cdot N_{ring}\right)\frac{1}{T_{rev}} = \left(U_0 \cdot \frac{I_b}{e}T_{rev}\right)\frac{1}{T_{rev}} = U_0\frac{I_b}{e} \ . \tag{6}$$

where $N_{ring} = I_b T_{rev} / e$ is the total number of electrons in the ring. Inserting eq. (2) and $I_b = 300mA$ into eq. (6) give

$$P_{beam} \approx 84.93KW \ . \tag{7}$$



Since the two intersection points are on the light-cone opened-up by $x=(x_0,\vec{x})$, they satisfy the following equation:

$$(x_0-X_0)-\sqrt{(X_1-x_1)^2+(X_2-x_3)^2+(X_3-x_3)^2}=0\ , \tag{8}$$

and

$$(x_0-Y_0)-\sqrt{(Y_1-x_1)^2+(Y_2-x_3)^2+(Y_3-x_3)^2}=0\ . \tag{9}$$

Subtracting eq. (9) with eq. (8) yields

$$Y_0-X_0=\sqrt{(X_1-x_1)^2+(X_2-x_2)^2+(X_3-x_3)^2}-\sqrt{(Y_1-x_1)^2+(Y_2-x_2)^2+(Y_3-x_3)^2}\ . \tag{10}$$

The three points $\vec{x}$, $\vec{X}$ and $\vec{Y}$ form a triangle and since the difference in the length of any two sides of a triangle is always smaller than the length of the third side, it follows from eq. (10)

$$Y_0-X_0\le\sqrt{(X_1-Y_1)^2+(X_2-Y_2)^2+(X_3-Y_3)^2}\ . \tag{11}$$

The time it takes for the particle to get from $\vec{X}$ to $\vec{Y}$ is given by

$$\Delta t=\frac{Y_0-X_0}{c}\ , \tag{12}$$

and hence the average velocity of the particle during its travelling from $\vec{X}$ to $\vec{Y}$ is

$$\left\langle v_{particle} \right\rangle = \frac{\sqrt{\left(X_1 - Y_1\right)^2 + \left(X_2 - Y_2\right)^2 + \left(X_3 - Y_3\right)^2}}{\Delta t} = \frac{c\sqrt{\left(X_1 - Y_1\right)^2 + \left(X_2 - Y_2\right)^2 + \left(X_3 - Y_3\right)^2}}{Y_0 - X_0} \,. \tag{13}$$

According to eq. (11) , the following relation holds

$$\frac{\sqrt{\left(X_1 - Y_1\right)^2 + \left(X_2 - Y_2\right)^2 + \left(X_3 - Y_3\right)^2}}{Y_0 - X_0} \geq 1 \,, \tag{14}$$

and inserting eq. (14) into eq. (13) yields

$$\left\langle v_{particle} \right\rangle = c\,\frac{\sqrt{\left(X_1 - Y_1\right)^2 + \left(X_2 - Y_2\right)^2 + \left(X_3 - Y_3\right)^2}}{Y_0 - X_0} \geq c \,. \tag{15}$$

Eq. (15) violates special relativity and hence the trajectory of a particle cannot intersect a light-cone twice.

3. The angular distribution of radiation power is given by

$$\frac{dP(t_r)}{d\Omega}=\frac{1}{4\pi\varepsilon_0}\frac{e^2}{4\pi c}\frac{\dot{\beta}^2}{(1-\beta\cos\theta)^3}\left[1-\frac{\sin^2\theta\cos^2\phi}{\gamma^2(1-\beta\cos\theta)^2}\right]. \tag{1}$$

For $\frac{1}{\gamma^4} << \theta << 1$ and $\gamma >> 1$, we can use the following approximation

$$\begin{aligned}1-\beta\cos\theta &\approx 1-\beta\left(1-\frac{1}{2}\theta^2\right)\\ &=1-\beta+\frac{1}{2}\beta\theta^2\\ &=\frac{1}{\gamma^2(1+\beta)}+\frac{1}{2}\theta^2\\ &=\frac{1}{\gamma^2}\left[\frac{1}{2-(1-\beta)}\right]+\frac{1}{2}\theta^2 \quad ,\\ &\approx\frac{1}{2\gamma^2}\left[1+\frac{1-\beta}{2}\right]+\frac{1}{2}\theta^2\\ &\approx\frac{1}{2\gamma^2}\left[1+\frac{1}{4\gamma^2}+\dots\right]+\frac{1}{2}\theta^2\\ &\approx\frac{1}{2\gamma^2}+\frac{1}{2}\theta^2\end{aligned} \tag{2}$$

and eq. (1) becomes

$$\frac{dP(t_r)}{d\Omega}\approx\frac{1}{4\pi\varepsilon_0}\frac{2e^2}{\pi c}\frac{\gamma^6\dot{\beta}^2}{\left(1+\gamma^2\theta^2\right)^3}\left[1-\frac{4\gamma^2\theta^2\cos^2\phi}{\left(1+\gamma^2\theta^2\right)^2}\right]. \tag{3}$$

Since the factor inside the square bracket is between 0 and 1, the angular width of eq. (3) is determined by the factor $\left(1+\gamma^2\theta^2\right)^{-3}$, i.e. the radiation power drops substantially when $\theta\geq\frac{1}{\gamma}$.