

# Influence of energy jitter to PCA-based CeC (updated)

G. Wang

# Estimate Cooling Force for PCA-based CeC: Longitudinal Electric Field in the Kicker Section

The electric potential induced by the line density perturbation is determined by the following equations

$$\frac{1}{r} \left[ \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \varphi(r, z) \right) \right] + \frac{\partial^2}{\partial z^2} \varphi(r, z) = \frac{1}{\epsilon_0} \rho_2(z) f_{\perp}(r)$$

If we take the transverse distribution of the electrons as

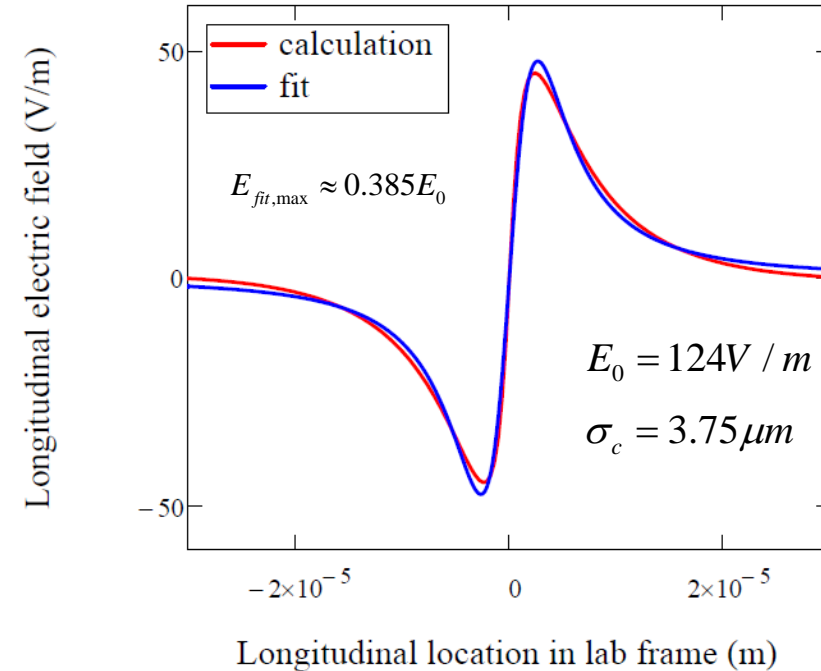
$$f_{\perp}(r) = \frac{1}{\pi a^2} H(a - r)$$

The electric field can be solved as

$$E_z(r, z) = -\frac{\partial \varphi}{\partial z} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{E}_z(r, k_z) e^{ik_z z} dk_z$$

$$\tilde{E}_z(r) = -ik_z \frac{\tilde{\rho}_2(k_z)}{\pi \epsilon_0}$$

$$\times \left[ I_0(k_z r) \int_{r/a}^1 \eta K_0(k_z a \cdot \eta) d\eta + K_0(k_z r) \int_0^{r/a} \eta I_0(k_z a \cdot \eta) d\eta \right]$$



For easy implementation into ion tracking code, we use the fitting formula:

$$E_{fit}(z) = E_0 \cdot \frac{z}{\sigma_c} \left[ 1 + \frac{z^2}{\sigma_c^2} \right]^{-3/2}$$

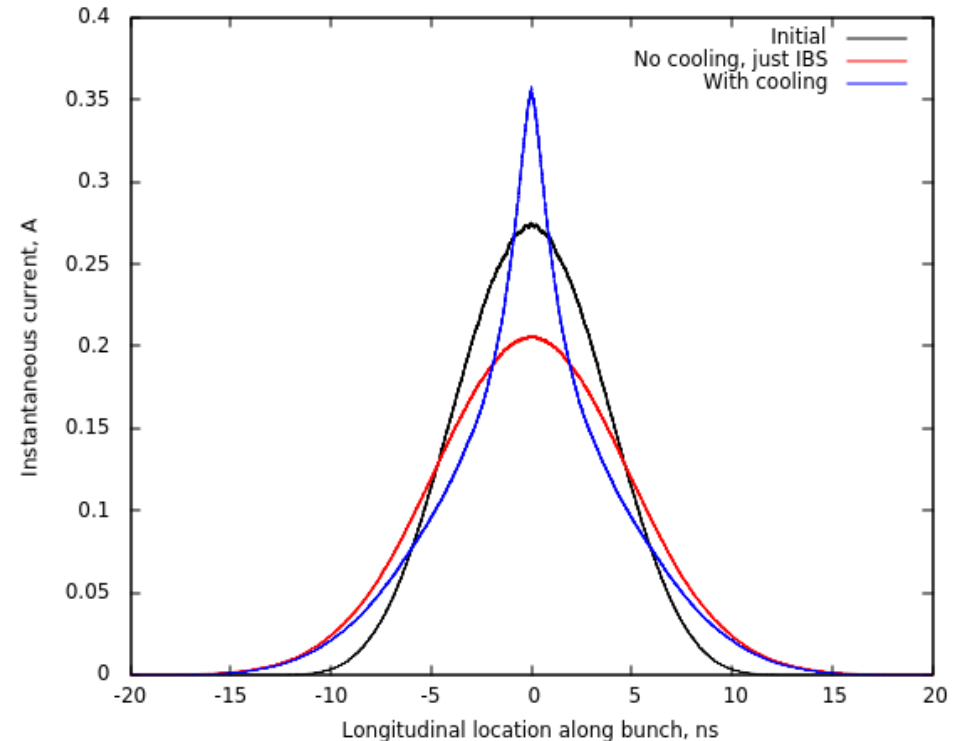
# Single-pass Kick and Tracking Results for PCA-based CeC

$$\Delta\gamma_{j,N} = -g_\gamma \frac{(D \cdot \delta_{j,N})}{\sigma_c} \left[ 1 + \frac{(D \cdot \delta_{j,N})^2}{\sigma_c^2} \right]^{-3/2} + g_\gamma \sqrt{\frac{3\pi}{8} \rho_{ion}(z_{j,N}) \sigma_c} \cdot X_{j,N} + \frac{g_\gamma}{Z_i} \sqrt{\frac{3\pi}{8} \rho_e(z_{j,N}) \sigma_c} \cdot Y_{j,N}$$

$$g_\gamma = Z_i e E_0 L_k / (A_i m_u c^2)$$

Parameters used in ion tracking	
$E_0^*$	62 V/m
$\sigma_c$	3.75 $\mu\text{m}$
$Z_{ion}$	79
Ion bunch intensity	2E8
$D (R_{56})$	1.2 cm

\* $E_0$  is reduced by a factor of 2 to account for reduced cooling for ions with large betatron amplitude

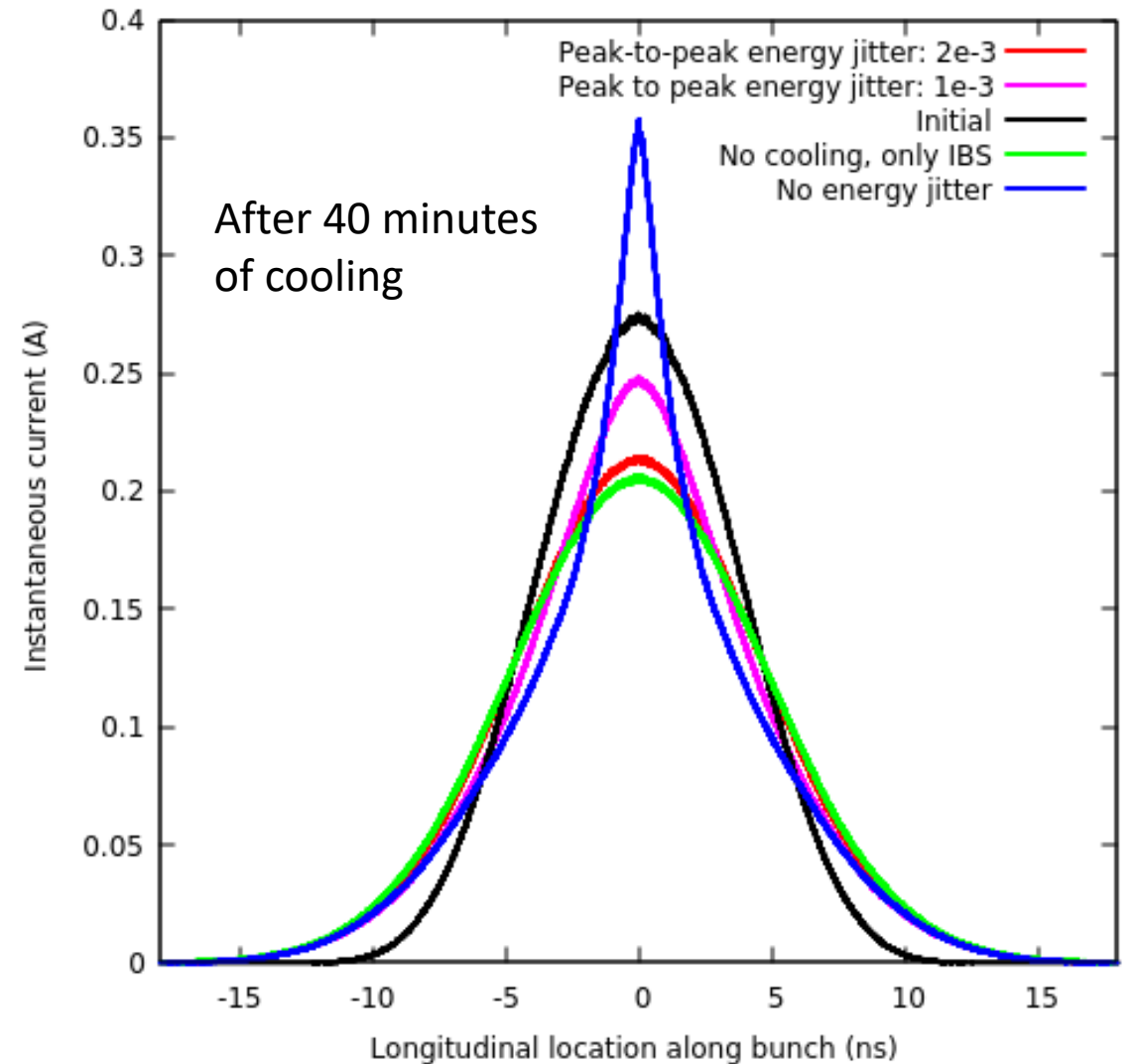


# Influence of energy jitter

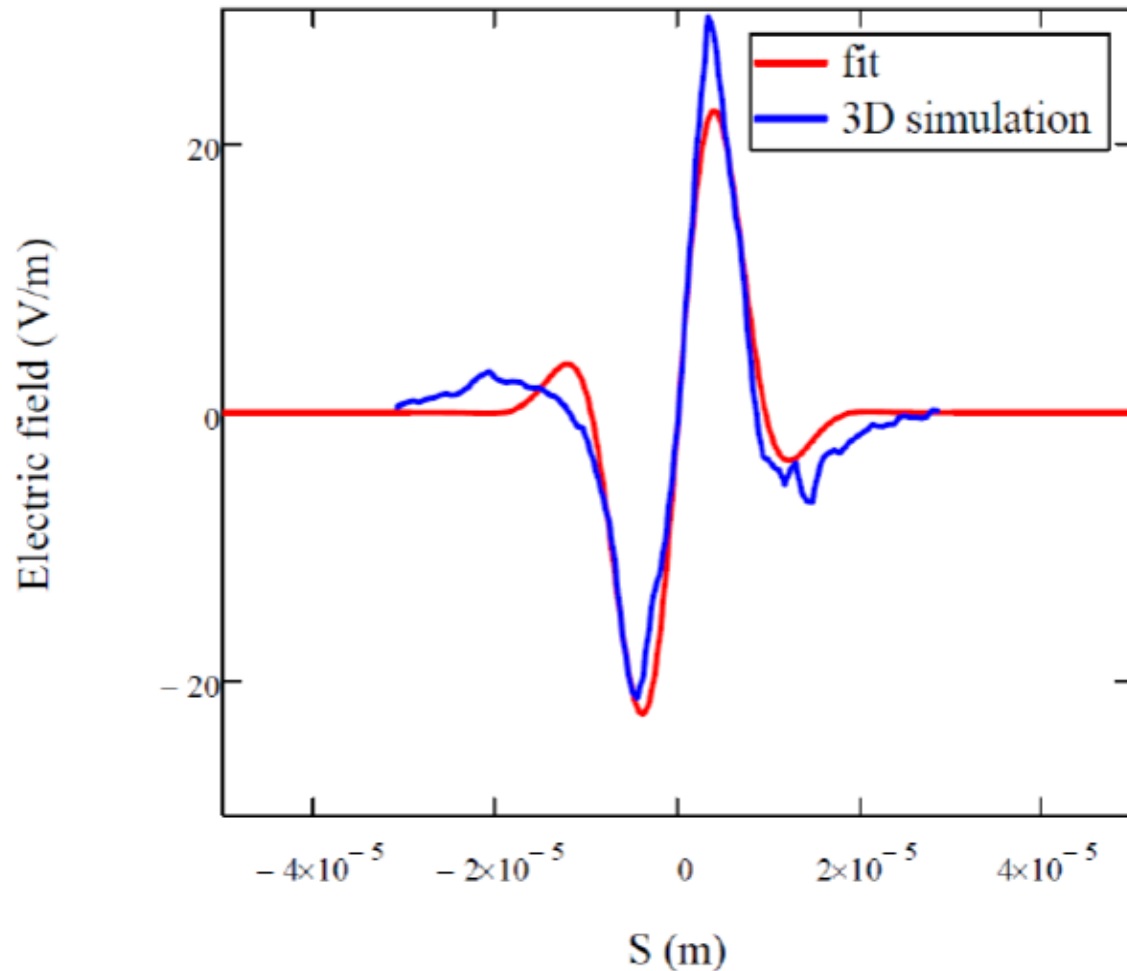
$$\Delta\gamma_{j,N} = -g_\gamma \frac{D \cdot (\delta_{j,N} + \Delta\delta_N)}{\sigma_c} \left[ 1 + \frac{\left( \left[ D \cdot (\delta_{j,N} + \Delta\delta_N) \right] \right)^2}{\sigma_c^2} \right]^{-3/2}$$

$$+ g_\gamma \sqrt{\frac{3\pi}{8} \rho_{ion}(z_{j,N}) \sigma_c} \cdot X_{j,N} + \frac{g_\gamma}{Z_i} \sqrt{\frac{3\pi}{8} \rho_e(z_{j,N}) \sigma_c} \cdot Y_{j,N}$$

$\Delta\delta_N$  is the energy jitter for turn N, which is a random number uniformly distributed in the interval  $\left[ -\frac{\Delta\delta_{\max}}{2} : \frac{\Delta\delta_{\max}}{2} \right]$



# Updates with 3D simulated field for cooling



$$E_{fit}(z) = E_0 \exp\left(-\frac{z^2}{2\sigma_c^2}\right) \sin(k_0 z),$$

$$E_0 = 28.1 \text{ V} / \text{m}$$

$$\sigma_c = 6.475 \mu\text{m}$$

$$k_0 = 3.276 \times 10^5 \text{ m}^{-1} (\lambda_0 = 19.18 \mu\text{m})$$

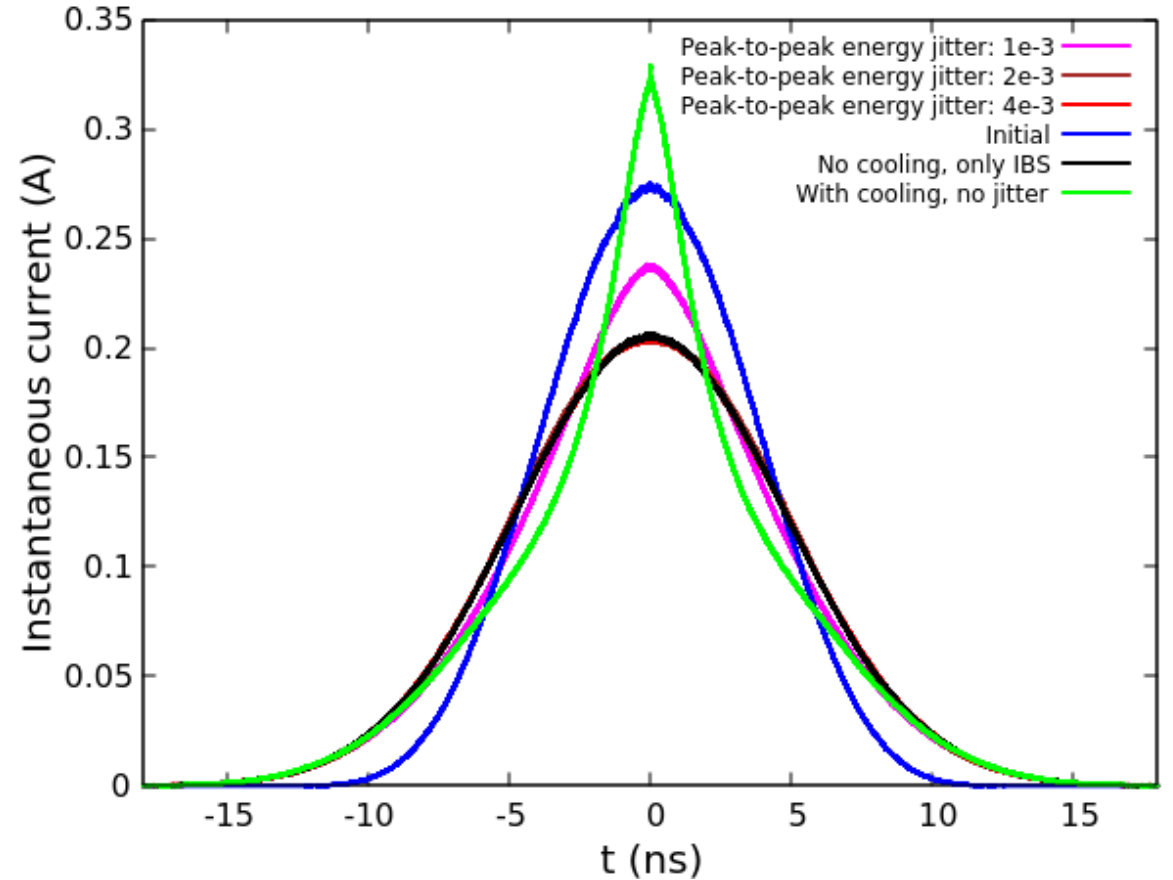
# Simulation results

$$\Delta\gamma_{j,N} = -g_\gamma \sin\left[k_0 D \cdot (\delta_{j,N} + \Delta\delta_N)\right] \exp\left[-\frac{D^2 \cdot (\delta_{j,N} + \Delta\delta_N)^2}{2\sigma_c^2}\right]$$

$$+ g_\gamma \sqrt{\frac{3\sqrt{\pi}}{2} \rho_{ion}(z_{j,N}) \sigma_c (1 - e^{-k_0^2 \sigma_c^2})} \cdot X_{j,N}$$

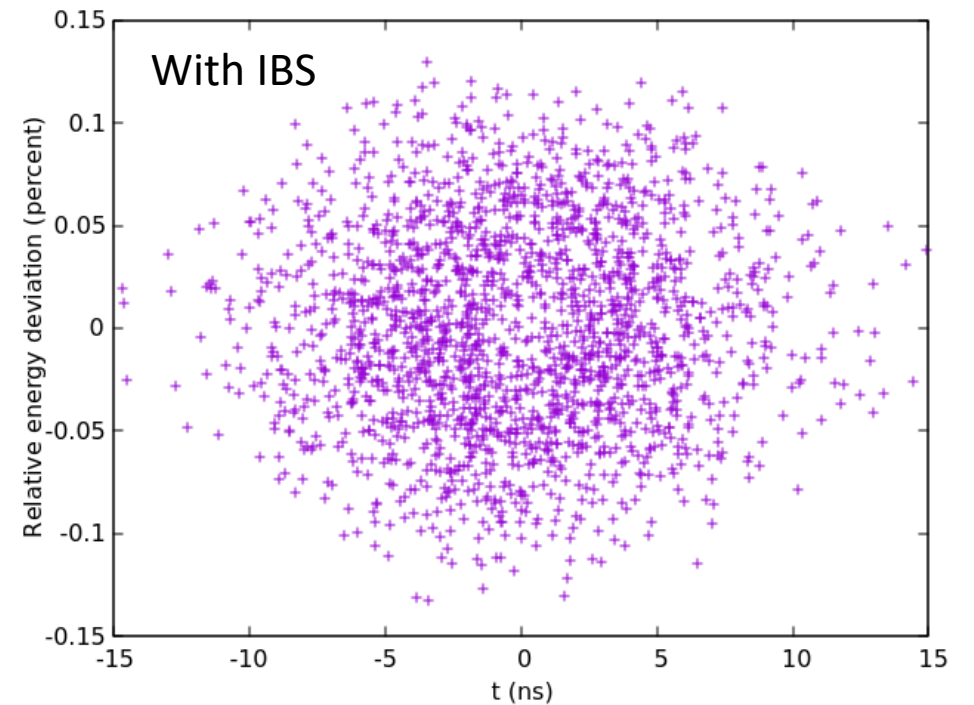
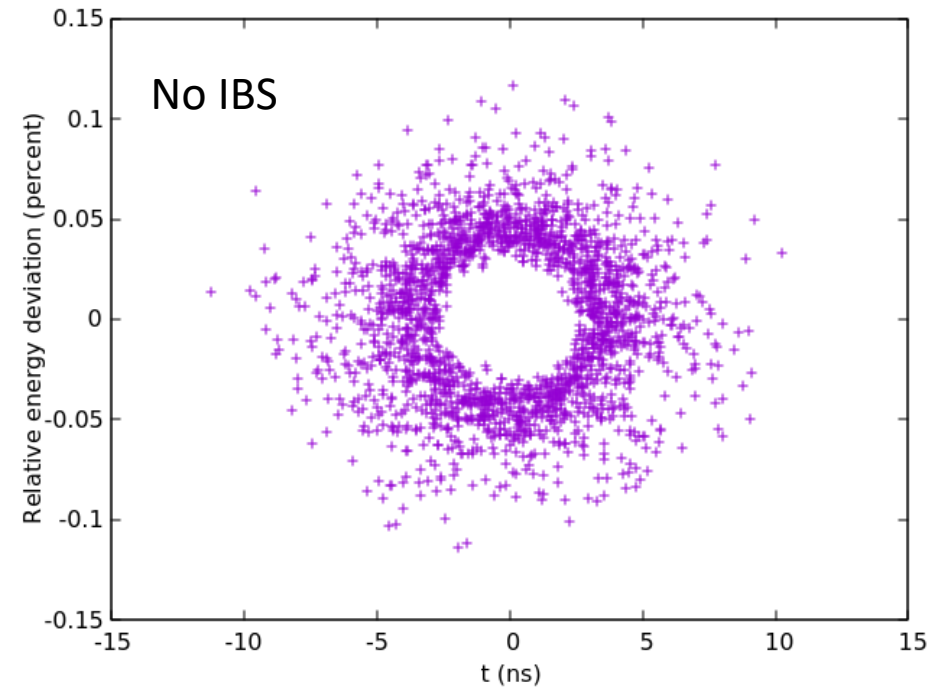
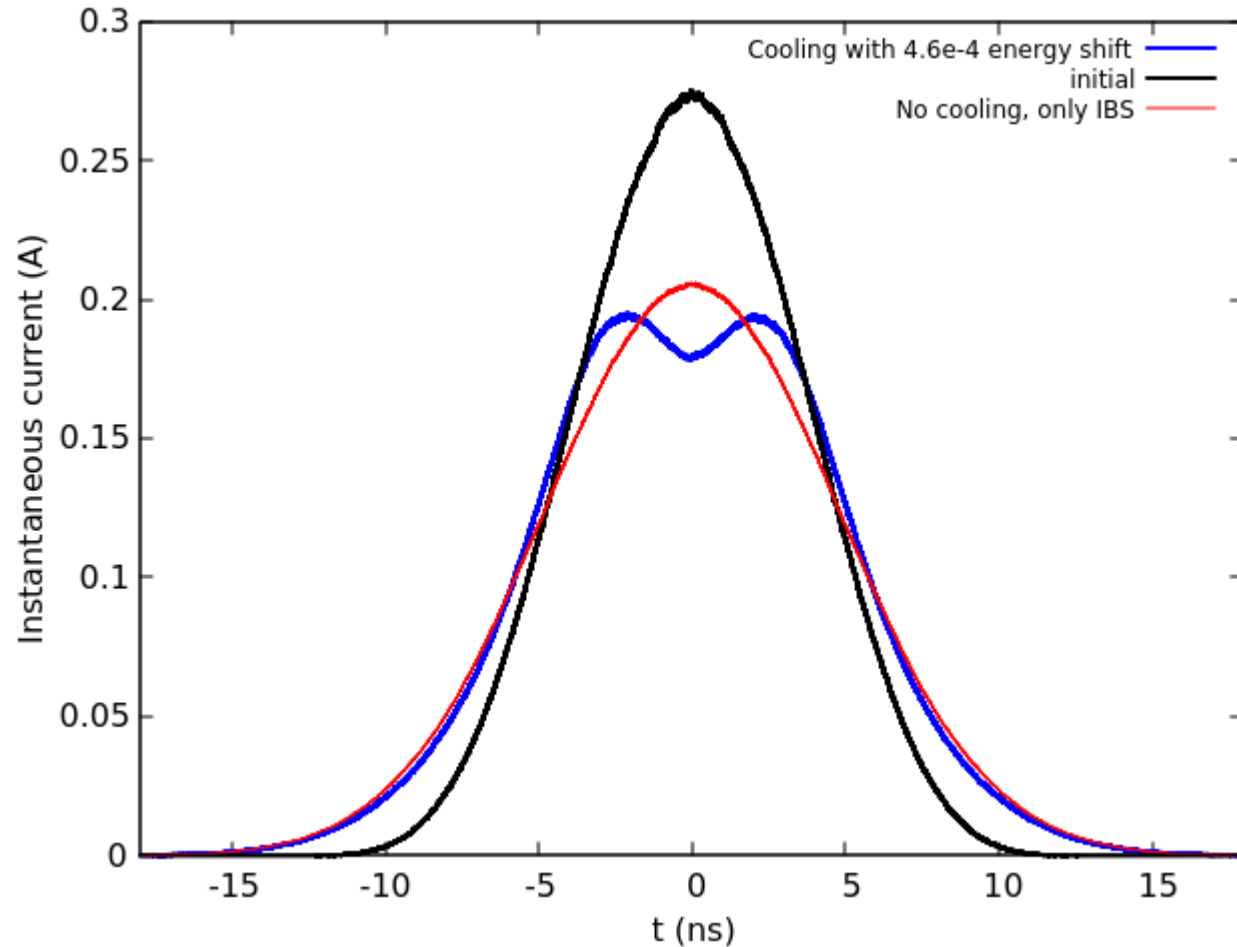
$$+ \frac{g_\gamma}{Z_i} \sqrt{\frac{3\sqrt{\pi}}{2} \rho_e(z_{j,N}) \sigma_c (1 - e^{-k_0^2 \sigma_c^2})} \cdot Y_{j,N}$$

$\Delta\delta_N$  is the energy jitter for turn N, which is a random number uniformly distributed in the interval  $\left[-\frac{\Delta\delta_{\max}}{2}, \frac{\Delta\delta_{\max}}{2}\right]$ .



# Influence of unmatched energy between electrons and ions

$$\frac{\gamma_e - \gamma_{ion}}{\gamma_{ion}} = 4.64 \times 10^{-4} \quad \Delta z = 6.5 \mu m$$



# Summary

- For uniform energy jitter, peak-to-peak relative amplitude of 0.1% should still allow us to observe some cooling, assuming no additional noise in the electron beam other than Poisson noise;
- If the peak-to-peak amplitude of the jitter increases above 0.2%, the cooled bunch behaves similarly as the reference bunch, i.e. no cooling can be observed.
- Further increase of energy jitter amplitude to 0.4% will not noticeably heat electron bunch.
- In the absence of excessive noise in electrons and energy jitter, energy mismatch on the level of  $5e-4$  can noticeably change the longitudinal profile of the ion bunch, even with IBS.



Backup slides

# Energy kick in cooling section

$$\begin{aligned}\Delta E_j &= -Z_i e E_0 l_1 \sin[k_0 D \cdot \delta_j] \exp\left[-\frac{(D \cdot \delta_j)^2}{2\sigma_c^2}\right] \\ &\quad - Z_i e E_0 l_1 \sum_{i \neq j} \sin\left[k_0 (D \cdot \delta_j + \varsigma_j - \varsigma_i)\right] \exp\left[-\frac{(D \cdot \delta_j + \varsigma_j - \varsigma_i)^2}{2\sigma_c^2}\right], \\ &= \Delta E_{coh,j} + \Delta E_{inc,j}\end{aligned}$$

# Diffusive kick at cooling section

$$\begin{aligned}
 \langle \Delta E_{inc,j}^2 \rangle &\equiv (Z_i e E_0 l_1)^2 \sum_{i,k \neq j} \left\langle \begin{aligned} &\sin \left[ k_0 (D \cdot \delta_j + \varsigma_j - \varsigma_i) \right] \sin \left[ k_0 (D \cdot \delta_j + \varsigma_j - \varsigma_k) \right] \\ &\times \exp \left[ -\frac{(D \cdot \delta_j + \varsigma_j - \varsigma_i)^2 + (D \cdot \delta_j + \varsigma_j - \varsigma_k)^2}{2\sigma_c^2} \right] \end{aligned} \right\rangle \\
 &= (Z_i e E_0 l_1)^2 \sum_{i \neq j} \left\langle \sin^2 \left[ k_0 (z_j - \varsigma_i) \right] \exp \left[ -\frac{(z_j - \varsigma_i)^2}{\sigma_c^2} \right] \right\rangle, \\
 &+ (Z_i e E_0 l_1)^2 \sum_{i \neq k \neq j} \left\langle \sin \left[ k_0 (z_j - \varsigma_i) \right] \sin \left[ k_0 (z_j - \varsigma_k) \right] \exp \left[ -\frac{(z_j - \varsigma_i)^2 + (z_j - \varsigma_k)^2}{2\sigma_c^2} \right] \right\rangle
 \end{aligned}$$

# Diffusive kick at cooling section

$$\begin{aligned}\langle \Delta E_{inc,j}^2 \rangle &= (Z_i e E_0 l_1)^2 \sum_{i \neq j} \left\langle \sin^2 [k_0 (z_j - \zeta_i)] \exp \left[ -\frac{(z_j - \zeta_i)^2}{\sigma_c^2} \right] \right\rangle \\ &= (Z_i e E_0 l_1)^2 \int_{-\infty}^{\infty} \rho(\zeta_i) \sin^2 [k_0 (z_j - \zeta_i)] \exp \left[ -\frac{(z_j - \zeta_i)^2}{\sigma_c^2} \right] d\zeta_i \\ &\approx (Z_i e E_0 l_1)^2 \rho(\zeta_j) \int_{-\infty}^{\infty} \sin^2 [k_0 (z_j - \zeta_i)] \exp \left[ -\frac{(z_j - \zeta_i)^2}{\sigma_c^2} \right] d\zeta_i \\ &= \frac{\sqrt{\pi} \rho(\zeta_j) \sigma_c}{2} (Z_i e E_0 l_1)^2 [1 - \exp(-k_0^2 \sigma_c^2)]\end{aligned}$$