

### Homework 3. Due September 25

#### Problem 1. 10 points. Sylvester formula – dipole/quadrupole

For an uncoupled transverse motion with constant energy and Hamiltonian of a bending magnet with quadrupole term (e.g. field gradient):

$$\tilde{h}_n = \frac{p_x^2 + p_y^2}{2} + f \frac{x^2}{2} + g \frac{y^2}{2};$$

$$f = [K_o^2 - K_1]; g = -K_1; K_o = -\frac{e}{p_o c} B_y; K_1 = -\frac{e}{p_o c} \frac{\partial B_y}{\partial x}$$

- Define all cases for eigen values of D.
- Use Sylvester formula for one-dimensional motions (x and y) when  $f \neq 0; g \neq 0$ ; (non-degenerated cases) and write explicit form of the 2x2 transport matrices.
- Consider a case of pure quadrupole:  $K_o = 0$ , no bending
- Do the same as above using 4x4 matrix formulation (2D case) and show that results are identical

#### Problem 2. 10 points. Sylvester formula, SQ-quadrupole

For a coupled transverse motion with constant energy and Hamiltonian of a SQ-quadrupole:

$$\tilde{h}_n = \frac{p_x^2 + p_y^2}{2} + Nxy; \quad N = \frac{e}{p_o c} \frac{\partial B_x}{\partial x}$$

- Use Sylvester formula and find matrix of SQ-quadrupole.
- Consider a “standard approach” – turn coordinates 45-degrees (use rotation matrix), to turn SQ-quad into a “normal”. Then make the product of 45-degree turn, quad matrix, -45 degrees turn. Show that the matrix is the same as in case (a).

Note: for point (b), consider a rotation is around z-axis: it will rotate x,y and  $p_x, p_y$ :

$$\begin{pmatrix} x^1 \\ p_x^1 \\ y^1 \\ p_y^1 \end{pmatrix} = R \begin{pmatrix} x \\ p_x \\ y \\ p_y \end{pmatrix} = \begin{bmatrix} I \cdot \cos \varphi & I \cdot \sin \varphi \\ -I \cdot \sin \varphi & I \cdot \cos \varphi \end{bmatrix} \begin{pmatrix} x \\ p_x \\ y \\ p_y \end{pmatrix}; \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Thus

$$x^1 = x \cos \varphi + y \sin \varphi; \quad y^1 = -x \sin \varphi + y \cos \varphi$$

$$p_x^1 = p_x \cos \varphi + p_y \sin \varphi; \quad p_y^1 = -p_x \sin \varphi + p_y \cos \varphi$$

Do not forget that you need inverse matrix of R as well.

In rotated coordinates with  $\varphi = \pi/2$  the Hamiltonian will have a decoupled form of one in quadrupole and you easily can calculate the matrix  $M_Q$ . Finally, you need to rotate back to initial coordinates.  $M_{SQ} = R^{-1} \cdot M_Q \cdot R$