Home Work PHY 554 #6. **Due October 19, 2016**

HW 1 (4 points): Superconducting RF pillbox cavity operating at 2K temperature would quench when the surface magnetic field reaches above 0.1 T (e.g. 1,000 Gs or 1,000 Oe).

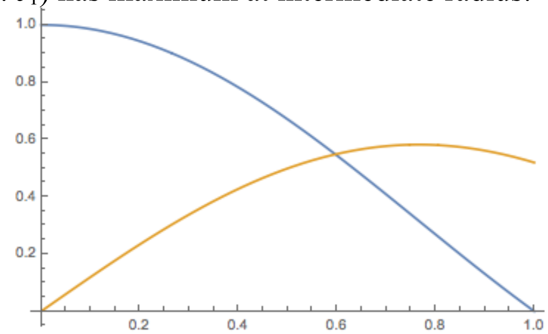
(a) For such pillbox cavity operating in fundamental TM_{010} mode find maximum attainable accelerating electric field on axis of the cavity;
(b) For Rs = 5 nanoOhm, calculate thermal losses in such cavity operating at 20 MV/m (Hint do not forget side walls!)

Solution:
(a) In pillbox cavity (slide 24, Lecture 10)

$$\mathbf{E}_z = E_o \cdot J_o\left(2.405\frac{r}{a}\right)\sin(\omega t);$$

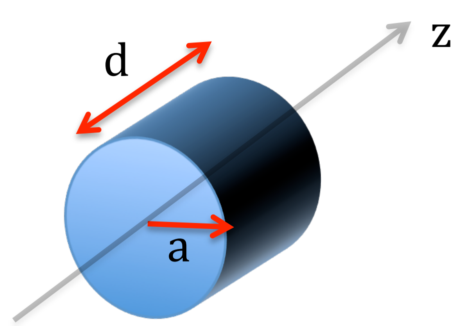
$$\mathbf{B}_\theta = E_o \cdot J_1\left(2.405\frac{r}{a}\right)\cos(\omega t)$$

and magnetic field (e.g. J_1) has maximum at intermediate radius:



$$J_1(x)_{\max} = 0.581865 \quad at\, x = 1.84118$$

e.g. we have limit $\mathbf{B}_\theta = 0.581865 E_o \le 10^3 Gs$. Now we just need to remember that 1 Gs (CGS system) is equal to 299.79 V/c ~ 30kV/m. Hence we get limitation for accelerating field on axis ***Eo < 51.5 MV/m.***



(b) We need to integrate on 3 surfaces: front and back faces (circles with radius *a*) and a cylinder of radius *a* and length *d*. Reversing the relations, 20 MV/m is equivalent to B=667 Gs. Corresponding H_o=5.31 10^4 A/m.

$$\mathbf{H}_\theta = \frac{\mathbf{B}_\theta}{\mu_o} = H_o \cdot J_1\left(2.405\frac{r}{a}\right)\cos(\omega t);$$

$$P_{loss} = \frac{1}{2}R_s \oint\!\!\oint da\left|\vec{\mathbf{H}}_{//}\right|^2 = R_s\frac{H_o}{2}\left(2\oint\!\!\oint_{circle} J_1^2\left(2.405\frac{r}{a}\right)da + J_1^2(2.405)\oint\!\!\oint_{culinder} da\right)$$

$$P_{loss} = 2\pi R_s\frac{H_o^{\ 2}}{2}\left(2a^2\int_0^1 J_1^2(2.405x)x\,dx + adJ_1^2(2.405)\right)$$

$$\int_0^1 J_1^2(2.405x)x\,dx = 0.134757;\quad J_1^2(2.405) = 0.269475;$$

$$P_{loss} \simeq 2\pi R_s\frac{H_o^{\ 2}}{2}\cdot 0.27\left(a^2+ad\right) \approx 0.85\cdot H_o^{\ 2}\left(a^2+ad\right)$$

or, in other terms:

$$P_{loss}[W] \simeq 12*\left(a^2+ad\right)[m^2]$$

With ratio between the RF wavelength and the pillbox-cavity of

$$a = \frac{2.405}{2\pi}\lambda_{RF};\ \lambda_{RF} = \frac{c}{f_{rf}};$$

and assuming a maximum gain form the cavity with $d = \lambda_{RF}/2$, the losses become

$$P_{loss}[W] \simeq 12\cdot\frac{2.405}{2\pi}\left(\frac{2.405}{2\pi}+\frac{1}{2}\right)\lambda_{RF}^{\ 2}[m^2] \approx 4\lambda_{RF}^{\ 2}[m^2]$$

e.g. if Rs is a constant, than the losses are increasing with reducing the frequency! This is the case of rather low frequencies when the so-called residual resistivity is larger that regular SRF surface resistance.

As an example, for 500 MHz pillbox cavity we would have RF wavelength of 0.6 m, and losses will be 1.44 W.

HW 2 (6 points): For SRF Nb cavity the London penetration depth is equal to 40 nanometers.

(a) What is the density of superconducting electrons, n_s? ;
(b) For surface magnetic field of 500 Gs or 500 Oe, find the density of surface current
(c) For frequency of 1 GHz, find value of electric field on the surface of the superconductor
(d) Assuming conductivity on normal component (non-superconducting electron) of Nb is 3 x 10^8 S/m (e.g. conductivity of 6x 10^6 S/m at room temperature multiplied by RRR of 50), find what is the value of the normal component of the surface current.

Hint: assume that the superconducting conductivity is significantly higher than normal part.

Solution: Use lecture 11, slides 27-31

(a) Use your favorite unit system, mine is clearly CGS:

$$\lambda = \sqrt{\frac{m}{\mu_o e^2 n_s}}(SI) = \sqrt{\frac{1}{4\pi r_e n_s}}(CGS); \quad r_e = \frac{e^2}{mc^2}$$

with classical radius of electron being 2.8 10^{-13} cm, λ= 4 10^{-6} cm

$$n_s = \frac{1}{4\pi r_e \lambda^2} = 1.76 \cdot 10^{22}\, cm^{-3}$$

(b) Again, in CGS units

$$\oint H\, dl = \frac{4\pi}{c} I \;\rightarrow\; \frac{I}{l} = \frac{c}{4\pi} H;$$

and in SI units

$$J_s = \frac{I}{l}\left[\frac{A}{m}\right] = H\left[\frac{A}{m}\right] = \frac{10^3 H[Gs]}{4\pi} \approx 40\left[\frac{kA}{m}\right]$$

(c) The current density at the surface

$$j_s = \frac{J_s}{\lambda} = \approx 10^{12}\left[\frac{A}{m^2}\right]; \; E = \mu_o j_s \cdot \omega\lambda^2 = 12.6\; V/m$$

(d) we need to remember expression for Ohms law (lecture 10, slide 8)

$$j_n = \sigma_n E = 3.8 \cdot 10^9\, A/m^2; \;\; J_n = j_n \lambda = 150\; A/m$$