PHY 564
Advanced Accelerator Physics
Lecture 27

Colliders

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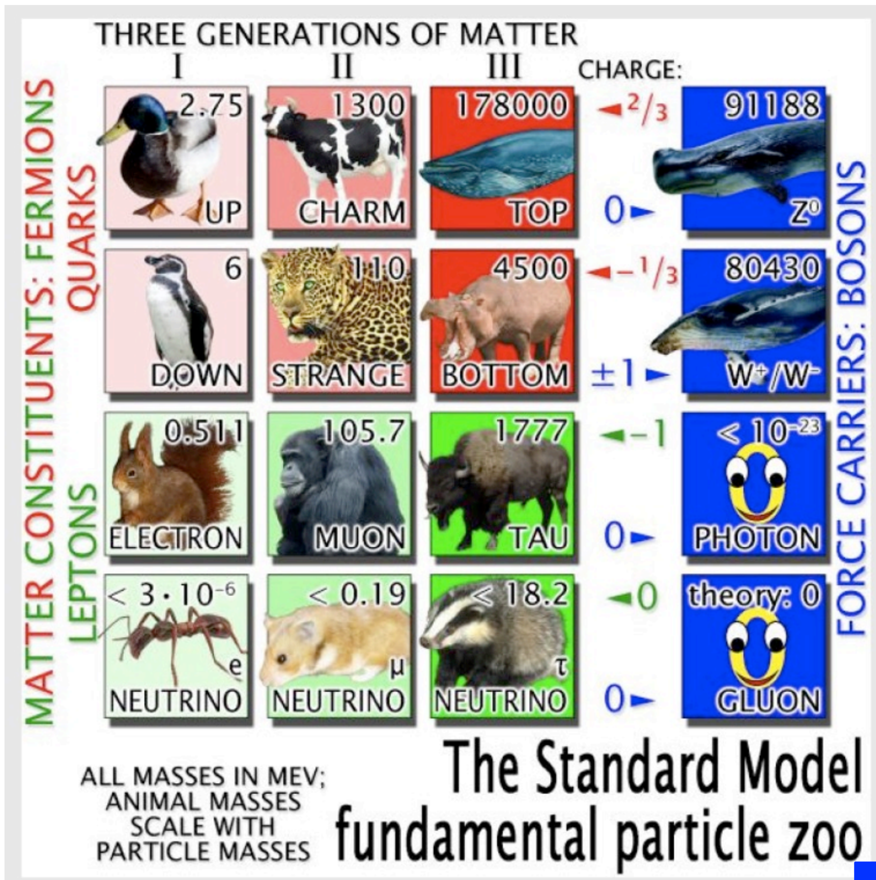
Colliders are probably the most fundamental application of the accelerators. From first days of accelerators they were used for collider experiments to discover new particles (from positrons or antiproton to Higgs) and new forces (weak, strong). There are two main parameter of any collider: the center-of-mass energy of collisions (e.g. energy available for generating new particles) and luminosity, which is defining the collider "productivity". Generally speaking, there are two types of collisions used for generating new particles: (a) beam colliding with a fixed targets (e.g. at rest in the lab, e.g. the detector, frame) and (b) beam-beam collisions (when both colliding species are moving with relativistic velocities in the lab frame. Second type of collisions became the leader in high-energy physics, starting from pioneering electron and electron-position colliders built in Novosibirsk and SLAC in late 1960s.



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Each and every type of accelerators was used for collision experiments with fixed target – they are still popular and are used for, what is now caller, low and medium energy nuclear/particle physics. Fig. 27-1 shows detector for 12 GeV electrons coming from CEBAF recirculating linac and colliding with a fixed target in front of the detector.

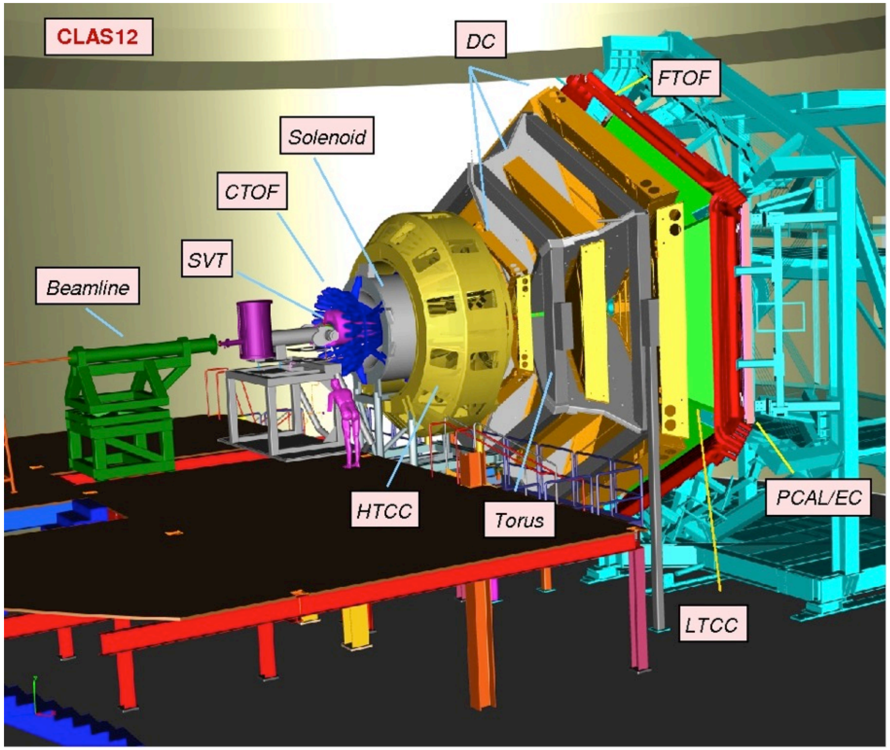


Fig. 27-1. CLAS12 detector at Hall B at CEBAF, Thomas Jefferson National Accelerator Facility, Newport News, VA

Let's consider kinematics of such collisions, shown in Fig.27-2.

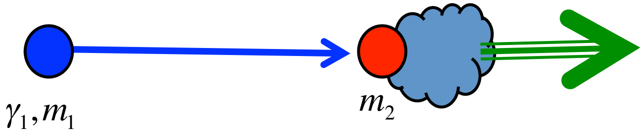


Fig. 27-2. A relativistic particle collides with a particle at rest. Resulting "products", shown as a cloud, carries the total momentum of the incoming particle – it means that there is always non-zero kinetic energy in the rest frame.

As we learned from relativistic mechanics, the maximum energy available for generating particle is given by the relativistic invariant contraction of 4-momentum:

$$P^i = p_1^i + p_2^i = \left(\frac{E_1 + E_2}{c}, \vec{p}_1 + \vec{p}_2 \right);$$
$$E_{c.m.} \equiv Mc^2 = \sqrt{P^i P_i} = \sqrt{\left(E_1 + E_2\right)^2 - c^2\left(\vec{p}_1 + \vec{p}_2\right)^2} \qquad (27\text{-}1)$$

Note that $\vec{p}_1 + \vec{p}_2 \neq 0$ always reduces the available energy. The meaning of (27-1) is to indicate the threshold of the available energy. In other words, if resulting products of collision are at rest, the maximum total mass of the products can not exceed M. Hence, if we are looking for a new particle with mass of 1 TeV, we will need at least 1 TeV c.m. energy. If particles are generated in pairs (for example as particle-antiparticle pair) – we will need 2 TeV c.m. In reality we will always need more that the threshold energy.

Now, let look at the kinematic in Fig.27-2:

$$E_1 = \gamma_1 m_1 c^2; c\vec{p}_1 = \hat{z}\beta_1 E_1; \beta_1 = \sqrt{1-\gamma_1^{-2}}; E_2 = m_2 c^2; \vec{p}_2 = 0; \gamma_2 = 1;$$
$$M^2 = \frac{E_{c.m.}^{\;2}}{c^4} = \left(\gamma_1 m_1 + m_2\right)^2 - \beta_1^2 \gamma_1^2 m_1^2 = m_1^2 + m_2^2 + 2\gamma_1 m_1 m_2; \qquad (27\text{-}2)$$
$$E_{c.m.} = c^2 \sqrt{m_1^2 + m_2^2 + 2\gamma_1 m_1 m_2}.$$

First two terms are just rest energies of the colliding particles and for the most interesting case of highly relativistic collisions $\gamma_1 >> 1$

$$E_{c.m.} \cong c^2 \sqrt{2\gamma_1 m_1 m_2} + O\left(\gamma_1^{-1}\right) \approx \sqrt{2E_1 m_2 c^2} \qquad (27\text{-}3)$$

scaling of the available energy in such collisions is very unfavorable: to increase energy available for generating new particles 10-fold one need to increase accelerator energy 100-fold.

Colliding beams with fixed target has tremendous advantage – you can collide a relatively weak beam with solid or gaseous target and take advantage of eccentrically infinite (just look at Avogadro number of $6^{x}10^{23}$ …) number of particles at rest. This is why for awhile fixed target were the only collisions used for nuclear high energy physics. The energy of accelerators was increasing exponentially by invention of new concepts: from electrostatic MeV-scale in 1930s to GeV scale synchrotrons in 1950s. But eventually machines became so large and scaling so unfavorable that invention of circular colliders became a necessity and later, a reality.

The main disadvantage of colliding beams is that we have a very limited number of particles in a single bunch – typically in 10^{9}-10^{11} range – and necessity to maintain particles from diffusing and eventually being lost. At the same time, the main advantage of colliders is favorable energy scaling when particles moving towards each other:

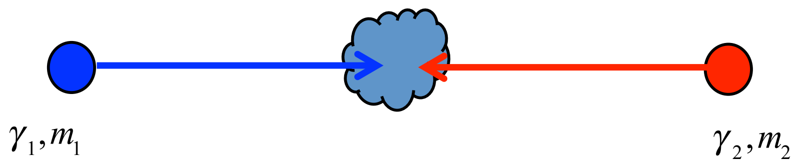


Fig. 27-3. Head-on collision of two particles.

$$E_1 = \gamma_1 m_1 c^2; c\vec{p}_1 = \hat{z}\beta_1 E_1; \beta_1 = \sqrt{1-\gamma_1^{-2}}; E_2 = \gamma_1 m_2 c^2; \vec{p}_2 = -\hat{z}\beta_2 E_2; \beta_{1,2} = \sqrt{1-\gamma_{1,2}^{-2}};$$

$$\gamma_i^2\left(1-\beta_i^2\right) \equiv 1 \rightarrow E_i^2 - c^2\vec{p}_i^{\,2} \equiv \left(m_i c^2\right)^2;$$

$$M^2 = \frac{E_{c.m.}^{\;2}}{c^4} = \left(\gamma_1 m_1 + \gamma_2 m_2\right)^2 - \left(\beta_1\gamma_1 m_1 - \beta_2\gamma_2 m_2\right)^2 = \tag{27-4}$$

$$m_1^2 + m_2^2 + 2\gamma_1\gamma_2 m_1 m_2\left(1+\beta_1\beta_2\right);$$

$$E_{c.m.} = c^2\sqrt{m_1^2 + m_2^2 + 2\left(1+\beta_1\beta_2\right)\gamma_1\gamma_2 m_1 m_2}.$$

and for the most interesting case of highly relativistic collisions $\gamma_i >> 1,\ 1-\beta_i << 1$

$$E_{c.m.} \cong 2c^2\sqrt{\gamma_1\gamma_2 m_1 m_2} + O\left(\gamma_i^{-1}\right) \approx 2\sqrt{E_1 E_2} \tag{27-5}$$

the energy scales as a geometric average of the energies of colliding particles. In this case increasing energies of particles 10-fold give 10-fold increase in c.m. energy.

The most obvious case is of colliding particles with the same mass (electron-positions, proton-proton or proton-antiproton) and the same energy:

$$E_1 = E_2 = E = \gamma mc^2; c\vec{p}_1 = -c\vec{p}_2 = \hat{z}\beta E; P^i = \left(2E, \vec{0}\right);$$
$$E_{c.m.} = 2E; \tag{27-6}$$

e.g. the total energy of colliding particles is available.

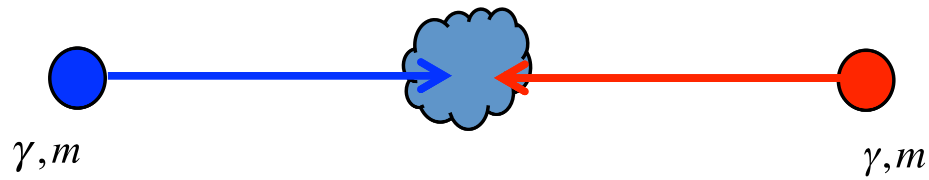


Fig. 27-4. Head-on collision of two particles with the same masses and energies.

It is especially true for electron-position (or proposes muon-anti-muon, $\mu\bar{\mu}$) colliders, where electron and positron can annihilate and make all their energy available for generate other species of particles. It is also true that lepton colliders (e^-e^+ or $\mu\bar{\mu}$) are the cleanest – we collide indeed elementary particles and start with well-defined initial states (including spin) and well defined energies. The main limitation of electron-position circular colliders is relatively low energy (max ~ 100 GeV per beam) when compared with pp colliders (max ~ TeV). Muon colliders, while being studies as a potentially important high energy tool, have two main problems:

(a) we can not generate low emittance muons and have to find a very complex (and untested) cooling techniques to bring the beam quality to acceptable value;

(b) muons are leaving (in their rest-frame) only for 2.2 microseconds – hence, they have to be accelerated and cooled very quickly …

These are indeed very challenging problems.

Collider luminosity. In addition to energy collider has another figure of merit – its luminosity. Goal of any collider is to collide particles and to investigate products of these collisions. The productivity of the collider is defined by how effective it is in producing such collision – it is call luminosity.

As you should know, each event of interest – such as creation of new particles - is described by a cross-section, σ. By definition the cross-section is defined in the rest frame of one type of the particles and is a constant defined by the process of interest (for example creation of Higgs boson). Let's two species of particles with densities $n_{1,2}$ (number of particles per unit volume) and velocities $\vec{\mathrm{v}}_{1,2}$ colliding with each other. Let's consider this process in the rest frame of particle of second type (target) and the other impending this target. The number of events generated in volume dV and interval time dt is defined by a simple formula (coming from definition of σ):

$$d\nu = \sigma \cdot \mathrm{v}_{rel} n_1 n_2 dV dt \tag{27-7}$$

where v_{rel} is relative particle's velocity in this frame. In arbitrary (for example lab-) frame or reference we would have

$$d\nu = A n_1 n_2 dV dt \tag{27-8}$$

where wee need to define A. Since number of created particles (events) $d\nu$ as well as 4-volumen dVd are relativistic invariants, so should be $A n_1 n_2$.

We know that number of particles in the a volume is also invariant, e.g.

$$ndV = n_o dV_o \to \frac{n}{n_o} = \gamma = \frac{\mathrm{E}}{mc^2}; \tag{27-9}$$

where index $_o$ indicated the rest frame of the particles. Hence,

$$\frac{A\mathrm{E}_1\mathrm{E}_2}{c^2 p_1^{\ i} p_{2i}} = \frac{A\mathrm{E}_1\mathrm{E}_2}{\mathrm{E}_1\mathrm{E}_2 - c^2 \vec{p}_1\vec{p}_2} = \mathrm{inv} \tag{27-10}$$

In the rest frame of the "target" (second type of particles)

$$\mathrm{E}_2 = m_2 c^2; \vec{p}_2 = 0; \ \frac{A\mathrm{E}_1\mathrm{E}_2}{c^2 p_1^{\ i} p_{2i}} = \frac{A\mathrm{E}_1\mathrm{E}_2}{\mathrm{E}_1\mathrm{E}_2} = A = \sigma \cdot \mathrm{v}_{rel} \tag{27-11}$$

The same it true in the rest frame of "beam 1". Thus, in arbitrary system

$$A = \sigma \cdot \mathrm{v}_{rel} \frac{c^2 p_1^{\ i} p_{2i}}{\mathrm{E}_1\mathrm{E}_2} \tag{27-12}$$

Now let's look at it in the "target 2" rest frame:

$$\mathrm{E}_1 = \frac{m_1 c^2}{\sqrt{1-\beta_{rel}^{\ 2}}}; \beta_{rel} = \frac{\mathrm{v}_{rel}}{c};$$

$$\frac{p_1^{\ i} p_{2i}}{c^2} = \frac{\mathrm{E}_1\mathrm{E}_2}{c^4} = \frac{m_1 m_2}{\sqrt{1-\beta_{rel}^{\ 2}}} \to \beta_{rel} = \sqrt{1-\left(\frac{m_1 m_2 c^2}{p_1^{\ i} p_{2i}}\right)^2}; \tag{27-13}$$

$$\mathrm{v}_{rel} = c\sqrt{1-\left(\frac{m_1 m_2 c^2}{p_1^{\ i} p_{2i}}\right)^2}$$

We can express in arbitrary frame $p_1^{\ i} p_{2i}$ using 3D beam velocities

$$\frac{p_1^{\ i} p_{2i}}{m_1 m_2 c^2} = \gamma_1 \gamma_2 \left(1 - \vec{\beta}_1 \vec{\beta}_2\right) \equiv \frac{1 - \vec{\beta}_1 \vec{\beta}_2}{\sqrt{\left(1 - \vec{\beta}_1^{\ 2}\right)\left(1 - \vec{\beta}_2^{\ 2}\right)}}; \qquad (27\text{-}14)$$

and after vector manipulations get final expression for v_{rel} :

$$\frac{\mathrm{v}_{rel}}{c} = \sqrt{1 - \frac{\left(1 - \vec{\beta}_1^{\ 2}\right)\left(1 - \vec{\beta}_2^{\ 2}\right)}{\left(1 - \vec{\beta}_1 \vec{\beta}_2\right)^2}} = \frac{\sqrt{\left(1 - \vec{\beta}_1 \vec{\beta}_2\right)^2 - \left(1 - \vec{\beta}_1^{\ 2}\right)\left(1 - \vec{\beta}_2^{\ 2}\right)}}{1 - \vec{\beta}_1 \vec{\beta}_2}$$

$$\left(1 - \left(\vec{\beta}_1 \vec{\beta}_2\right)\right)^2 - \left(1 - \vec{\beta}_1^{\ 2}\right)\left(1 - \vec{\beta}_2^{\ 2}\right) = \left(\vec{\beta}_1 - \vec{\beta}_2\right)^2 + \left(\vec{\beta}_1 \vec{\beta}_2\right)^2 - \left(\vec{\beta}_1^{\ 2}\right)\left(\vec{\beta}_2^{\ 2}\right); \qquad (27\text{-}15)$$

$$\left(\vec{\beta}_1^{\ 2}\right)\left(\vec{\beta}_2^{\ 2}\right) - \left(\vec{\beta}_1 \vec{\beta}_2\right)^2 = \left[\vec{\beta}_1 \times \vec{\beta}_2\right]^2;$$

$$\mathrm{v}_{rel} = c \frac{\sqrt{\left(\vec{\beta}_1 - \vec{\beta}_2\right)^2 - \left[\vec{\beta}_1 \times \vec{\beta}_2\right]^2}}{1 - \vec{\beta}_1 \vec{\beta}_2}.$$

and final expression for event rate:

$$d\nu = \sigma \frac{c^2 \sqrt{\left(p_1^{\ i} p_{2i}\right)^2 - \left(m_1 m_2 c^2\right)^2}}{\mathrm{E}_1 \mathrm{E}_2} n_1 n_2 dV dt; \qquad (27\text{-}16)$$

$$d\nu = \sigma c \sqrt{\left(\vec{\beta}_1 - \vec{\beta}_2\right)^2 - \left[\vec{\beta}_1 \times \vec{\beta}_2\right]^2} n_1 n_2 dV dt.$$

For head-on collision (e.g. beam velocities lay are in the same or opposite direction) we have $\left[\vec{\beta}_1 \times \vec{\beta}_2\right]$=0 and expression is simplified significantly

$$d\nu = \sigma\left|\vec{\mathrm{v}}_1 - \vec{\mathrm{v}}_2\right| n_1 n_2 dVdt. \qquad (27\text{-}17)$$

to a simple subtraction of the beam velocities. It means that for head on collisions $\vec{\mathrm{v}}_1 = \hat{z}\left|\vec{\mathrm{v}}_1\right|; \vec{\mathrm{v}}_2 = -\hat{z}\left|\vec{\mathrm{v}}_2\right|$ we have a sum of velocities:

$$d\nu = \sigma\left(\left|\vec{\mathrm{v}}_1\right| + \left|\vec{\mathrm{v}}_2\right|\right) n_1 n_2 dVdt \qquad (27\text{-}17)$$

Since this condition is also maximizing the c.m. energy, it is the most favorite way to collide the beams, which we will use. Furthermore, in collider we are always collide ultra-relativistic particles and the sum of velocities is simply *2c*. The integral number of the events accumulated during the time period T is given by simple integral and is given by the product of the event cross-section and so-called integral luminosity:

$$\begin{gathered} \mathrm{N}(T) = \sigma \int_0^T dt \int_V \left(\left|\vec{\mathrm{v}}_1\right| + \left|\vec{\mathrm{v}}_2\right|\right) n_1 n_2 \, dV = \sigma \int_0^T L(t)\, dt; \\ L(t) \underset{def}{=} \int_V \left(\left|\vec{\mathrm{v}}_1\right| + \left|\vec{\mathrm{v}}_2\right|\right) n_1(\vec{r},t)\, n_2(\vec{r},t)\, dV; \end{gathered} \qquad (27\text{-}18)$$

where the integral is taken by the volume containing colliding beams.

Let's consider bunch trains of particles coming into the detector with frequency of f_c:

$$n_1(\vec{r},t)=\sum_m N_1 f_1\left(\vec{r}_\perp,s,t-\frac{m}{f_c}-\frac{s}{\mathrm{v}_1}\right);n_1(\vec{r},t)=\sum_m N_2 f_2\left(\vec{r}_\perp,s,s+\mathrm{v}_2 t,t-\frac{m}{f_c}\right) \qquad (27\text{-}19)$$

where N_1 and N_2 are particle's number per bunch and $f_{1,2}$ are bunches distribution functions normalized to unity

$$\int f_{1,2}\,dV=1$$

Note that for simplicity we neglected angular spread of particles and spread in their energies – it can be taken into account for specifics of the generated products, but these details are important for detectors, not for the collider luminosity per se.

Hence, the instant luminosity is given by convolution of the entire trains

$$L(t)=N_1N_2\left(|\vec{\mathrm{v}}_1|+|\vec{\mathrm{v}}_2|\right)\int_V\sum_{m,k} f_1(\vec{r},t-mT_c)\,f_2(\vec{r},t-kT_c)\,dV;\; T_c=1/f_c. \qquad (27\text{-}20)$$

while in practice only bunches with the same time stamp $k=m$ do collide in the detector, in other words the limited volume of the detector is selecting colliding bunches

$$\int_V\sum_{m,k} f_1(\vec{r},t-mT_c)\,f_2(\vec{r},t-kT_c)\,dV=\sum_m\int_V f_1(\vec{r},t-mT_c)\,f_2(\vec{r},t-mT_c)\,dV;$$

$$L(t)=\sum_m L_s(t-mT_c);\; L_s(t)=N_1N_2\left(|\vec{\mathrm{v}}_1|+|\vec{\mathrm{v}}_2|\right)\int_V f_1(\vec{r},t)\,f_2(\vec{r},t)\,dV; \qquad (27\text{-}21)$$

where we defined luminosity of single bunch collision, L_s and assume that number of colliding particles is the same in all bunches (other wise we need to add $N_{1m}N_{2m}$).

We are usually also not interested in details of the time structure of the single bunch collision and we simply can define average luminosity as

$$\int_V \sum_{m,k} f_1(\vec{r},t-mT_c) f_2(\vec{r},t-kT_c)\,dV = \sum_m \int_V f_1(\vec{r},t-mT_c) f_2(\vec{r},t-mT_c)\,dV;$$

$$\bar{L}(t) = \frac{1}{T_c}\int_0^{T_c} L(t+\tau)\,d\tau \equiv \frac{1}{T_c}\int_0^{T_c} L_s(t)\,dt = \frac{f_c \cdot N_1 N_2}{\mathbf{A}} \tag{27-22}$$

$$\mathbf{A}^{-1} = \int_0^{T_c}\int_V \left(|\vec{\mathrm{v}}_1| + |\vec{\mathrm{v}}_2|\right) f_1(\vec{r},t) f_2(\vec{r},t)\,dV\,dt$$

where we defined the effective transverse are of the beam, $\mathbf{A}$. By definition $f_{1,2}$ have dimensionality of inverse volume, L^{-3}, and integral of the product (L^{-6}) over the volume and *vdt* gives L^{+4}, which makes the dimensionality of last integral in (27-22) be L^{-2}. Hence, units for measuring the collider luminosity are in $cm^{-2}\ sec^{-1}$. Product with cross-section (L^2) naturally gives rate of event per second.

Let's calculate luminosity for Gaussian distribution of the beams assuming that there is no focusing elements in the collision area (typical for detectors) and β-functions have waist is the center of the detector, s=0:

$$\beta_{kx,y}(s) = \beta^*_{kx,y} + \frac{s^2}{\beta^*_{kx,y}}; k=1,2;\ \sigma_{kx,y}(s) = \sqrt{\varepsilon_{kx,y}\beta_{kx,y}(s)};$$

$$f_k = \frac{1}{(2\pi)^{3/2}\sigma_{kx}\sigma_{ks}\sigma_{ks}} \exp\left(-\frac{1}{2}\left(\frac{x^2}{\sigma_{kx}{}^2} + \frac{y^2}{\sigma_{ky}{}^2} + \frac{(s \pm \mathrm{v}_k t)^2}{\sigma_{ks}{}^2}\right)\right). \tag{27-23}$$

We will first execute integration in x and y:

$$I_x = \int_{-\infty}^{\infty} dx \exp\left(-\frac{1}{2}\left(\frac{x^2}{\sigma_{1x}^{\ 2}}+\frac{x^2}{\sigma_{2x}^{\ 2}}\right)\right) = (2\pi)^{1/2}\sigma_x;\ \sigma_x^{\ -2} = \sigma_{1x}^{\ -2}+\sigma_{2x}^{\ -2};$$

$$\sigma_x = \frac{\sigma_{1x}\sigma_{2x}}{\sqrt{\sigma_{1x}^{\ 2}+\sigma_{2x}^{\ 2}}} \to \frac{1}{2\pi\sigma_{1x}\sigma_{2x}} I_x = \frac{1}{\sqrt{2\pi}\sqrt{\sigma_{1x}^{\ 2}+\sigma_{2x}^{\ 2}}} \qquad (27\text{-}24)$$

$$-//- \quad \frac{1}{2\pi\sigma_{1y}\sigma_{2y}} I_y = \frac{1}{\sqrt{2\pi}\sqrt{\sigma_{1y}^{\ 2}+\sigma_{2y}^{\ 2}}}$$

$$\int dx\,dy \to \frac{1}{2\pi\sigma_{1x}\sigma_{2x}}\frac{1}{2\pi\sigma_{1y}\sigma_{2y}} I_x I_y = \frac{1}{2\pi\sqrt{\varepsilon_{1x}\beta_{1x}+\varepsilon_{2x}\beta_{2x}}\sqrt{\varepsilon_{1y}\beta_{1y}+\varepsilon_{2y}\beta_{2y}}}.$$

Note that β-functions are functions of the s but not t and we can make one more integral, assuming that bunch tails do not extend beyond the time between collisions:

$$\int \exp\left(-\frac{1}{2}\left(\frac{(s-\mathrm{v}_1 t)^2}{\sigma_{1s}^{\ 2}}+\frac{(s+\mathrm{v}_2 t)^2}{\sigma_{2s}^{\ 2}}\right)\right)dt; \sigma_s^{\ 2} = \frac{\sigma_{1s}^{\ 2}\sigma_{2s}^{\ 2}}{\sigma_{1s}^{\ 2}+\sigma_{2s}^{\ 2}}; \sigma_t^{\ 2} = \frac{\sigma_{1s}^{\ 2}\sigma_{2s}^{\ 2}}{\mathrm{v}_2^{\ 2}\sigma_{1s}^{\ 2}+\mathrm{v}_1^{\ 2}\sigma_{2s}^{\ 2}};$$

$$\frac{(s-\mathrm{v}_1 t)^2}{\sigma_{1s}^{\ 2}}+\frac{(s+\mathrm{v}_2 t)^2}{\sigma_{2s}^{\ 2}} = \frac{s^2}{\sigma_s^{\ 2}}+\frac{t^2}{\sigma_t^{\ 2}} - 2st\left(\frac{\mathrm{v}_1}{\sigma_{1s}^{\ 2}}-\frac{\mathrm{v}_2}{\sigma_{2s}^{\ 2}}\right);$$

$$\frac{t^2}{\sigma_{2t}^{\ 2}} - 2st\left(\frac{\mathrm{v}_1}{\sigma_{1s}^{\ 2}}-\frac{\mathrm{v}_2}{\sigma_{2s}^{\ 2}}\right) = \frac{(t-as)^2}{\sigma_t^{\ 2}} - s^2\frac{a^2}{\sigma_t^{\ 2}};\ a = \sigma_t^{\ 2}\left(\frac{\mathrm{v}_1}{\sigma_{1s}^{\ 2}}-\frac{\mathrm{v}_2}{\sigma_{2s}^{\ 2}}\right) = \frac{\mathrm{v}_1\sigma_{2s}^{\ 2}-\sigma_{1s}^{\ 2}\mathrm{v}_2}{\mathrm{v}_1^{\ 2}\sigma_{2s}^{\ 2}+\mathrm{v}_2^{\ 2}\sigma_{1s}^{\ 2}}; \qquad (27\text{-}25)$$

$$g_s = \frac{1}{2\pi\sigma_{1s}\sigma_{2s}}\int \exp\left(-\frac{1}{2}\left(\frac{(s-\mathrm{v}_1 t)^2}{\sigma_{1s}^{\ 2}}+\frac{(s+\mathrm{v}_2 t)^2}{\sigma_{2s}^{\ 2}}\right)\right)dt = \frac{1}{\sqrt{2\pi}\tilde{\sigma}_s}\exp\left(-\frac{s^2}{2\tilde{\sigma}_s^{\ 2}}\right);$$

$$\tilde{\sigma}_s^{\ 2} = \frac{\mathrm{v}_2^{\ 2}\sigma_{1s}^{\ 2}+\mathrm{v}_1^{\ 2}\sigma_{2s}^{\ 2}}{(\mathrm{v}_1+\mathrm{v}_2)^2};\ \int g_s\,ds = 1$$

Hence, the remaining integral is

$$\mathbf{A}^{-1} = \frac{1}{2\pi\sqrt{\varepsilon_{1x}\beta^*_{1x} + \varepsilon_{2x}\beta^*_{2x}}\sqrt{\varepsilon_{1y}\beta^*_{1y} + \varepsilon_{2y}\beta*_{2y}}}\int g(s)\,ds = \frac{1}{4\pi\sigma^*_x\sigma^*_y}h\left(\beta^*_{1,2x,y},\sigma_s\right)$$

$$\sigma^{*\,2}_{x,y} = \frac{\varepsilon_{1x,y}\beta^*_{1x,y} + \varepsilon_{2x,y}\beta^*_{2x,y}}{2};\; h\left(\beta^*_{1,2x,y},\sigma_s\right) = \int g(s)\,ds;\; g(s) = g_x g_y g_s;$$

$$g_{x,y} = \sqrt{\frac{\varepsilon_{1x,y}\beta^*_{1x,y} + \varepsilon_{2x,y}\beta^*_{2x,y}}{\varepsilon_{1x,y}\beta_{1x,y}(s) + \varepsilon_{2x,y}\beta_{2x,y}(s)}} = \left(1 + \frac{s^2}{\tilde{\beta}^{*2}_{x,y}}\right)^{-1/2} = g_{x,y}\left(\frac{s}{\tilde{\beta}^*_{x,y}}\right) \qquad (27\text{-}26)$$

$$\tilde{\beta}^{*2}_{x,y} = \beta^*_{1x,y}\beta^*_{2x,y}\cdot\frac{\varepsilon_{1x,y}\beta^*_{1x,y} + \varepsilon_{2x,y}\beta^*_{2x,y}}{\varepsilon_{2x,y}\beta^*_{1x,y} + \varepsilon_{1x,y}\beta^*_{2x,y}};$$

$$\bar{L}(t) = \frac{f_c\cdot N_1 N_2}{4\pi\sigma^*_x\sigma^*_y}h\left(\frac{\beta^*_{x,y}}{\tilde{\sigma}_s}\right).$$

where we introduce so called hour-glass effect function which takes into account length of the bunches. Carrying out some parameters, which is overestimation of accuracy is indeed unnecessary and we can use

$$\left|\vec{\mathrm{v}}_1\right| = \left|\vec{\mathrm{v}}_2\right| = c;\; \sigma_s = c\sigma_t = \sqrt{2}\tilde{\sigma}_s;$$

$$g_s = \frac{1}{\sqrt{2\pi\left(\sigma_{1s}^{\,2} + \sigma_{2s}^{\,2}\right)}}\exp\left(-\frac{s^2}{\sigma_{2s}^{\,2} + \sigma_{1s}^{\,2}}\right); \qquad (27\text{-}27)$$

The first term in expression for luminosity is its maximum value, which could be achieved for a very short bunches:

$$\tilde{\beta}^*_{x,y} >> \tilde{\sigma}_s; g_{x,y} \cong 1; \int g(s)\,ds = 1$$

$$\sigma^{*\ 2}_{x,y} = \frac{\varepsilon_{1x,y}\beta^*_{1x,y} + \varepsilon_{2x,y}\beta^*_{2x,y}}{2}; \qquad (27\text{-}28)$$

$$\bar{L}(t) \cong \frac{f_c \cdot N_1 N_2}{4\pi\sigma^*_x\sigma^*_y} = \frac{f_c \cdot N_1 N_2}{2\pi\sqrt{\varepsilon_{1x}\beta^*_{1x} + \varepsilon_{2x}\beta^*_{2x}}\sqrt{\varepsilon_{1y}\beta^*_{1y} + \varepsilon_{2y}\beta*_{2y}}}$$

with even more simple formulae for beam with equal emittances and β-functions:

$$\beta^*_{1,2x,y} = \beta^*_{x,y}; \varepsilon_{1,2x,y} = \varepsilon_{x,y} \Rightarrow \bar{L}(t) = \frac{f_c \cdot N_1 N_2}{4\pi\sqrt{\varepsilon_x\beta^*_x}\sqrt{\varepsilon_y\beta^*_y}} h \qquad (27\text{-}29)$$

or round beam (as in typical hadron colliders):

$$\beta^*_{x,y} = \beta^*; \varepsilon_{x,y} = \varepsilon \Rightarrow \bar{L}(t) = \frac{f_c \cdot N_1 N_2}{4\pi\varepsilon\beta^*} h(\beta^*, \sigma_s). \qquad (27\text{-}30)$$

Thus, one can increase luminosity buy increasing frequency of collision (e.g. increasing average beam currents $I_1, I_2, L \sim \sqrt{I_1 I_2}$), increasing number of particles per bunch (more effective way since $L \sim I_1 I_2$), deducing β^* (can be limited by beam optics or by bunch lengths, σ_s) or reducing emittance(s).

Beam-beam effects and limits. Two colliding beams sample strongly non-linear transverse EM fields induced by the opposite beam. It is very important to observe that particles also sample EM fields generated by their our bunch, but its effect is relativistically suppressed. This can be clearly demonstrated in a following way: moving particle generates transverse electric and magnetic field (check your E&M)

$$\vec{B}_\perp = \frac{\mathrm{v}}{c}[\hat{z}\times\vec{E}_\perp]; \vec{\mathrm{v}}=\hat{z}\mathrm{v} \tag{27-31}$$

and a particle moving in the same direction and with the same velocity experiencing Lorentz force of:

$$\frac{d\vec{p}_\perp}{dt} = e\left(\vec{E}_\perp + \left[\frac{\vec{\mathrm{v}}}{c}\times\vec{B}_\perp\right]\right) = \vec{E}_\perp + \left(\frac{\mathrm{v}}{c}\right)^2\left[\hat{z}\times[\hat{z}\times\vec{E}_\perp]\right]$$

$$\left[\hat{z}\times[\hat{z}\times\vec{E}_\perp]\right] = \hat{z}\underset{0}{\left(\hat{z}\vec{E}_\perp\right)\downarrow} - \vec{E}_\perp\left(\hat{z}\hat{z}\right) = -\vec{E}_\perp; \tag{27-32}$$

$$\frac{d\vec{p}_\perp}{dt} = e\vec{E}_\perp\left(1-\left(\frac{\mathrm{v}}{c}\right)^2\right) = \frac{e\vec{E}_\perp}{\gamma^2}$$

which is relativistically suppressed by huge factor γ^2. Note that for colliders $\gamma \propto 10^2 - 10^4$ are typical. In contrast, particle on the colliding course $\vec{\mathrm{v}}=-\hat{z}\mathrm{v}$ experiences Lorentz force of

$$\frac{d\vec{p}_\perp}{dt} = e\left(\vec{E}_\perp - \left[\frac{\vec{\mathrm{v}}}{c}\times\vec{B}_\perp\right]\right) = \vec{E}_\perp\left(1+\left(\frac{\mathrm{v}}{c}\right)^2\right) \tag{27-33}$$

which for ultra relativistic particles is simply doubles that of electric field. Since these EM fields are generated by the beams themselves, they are strongly non-linear with typical scale of the filed variations defined by the transverse beam size.

Electric field of a Gaussian bunch can be found in its co-moving frame and than transferred into the lab frame using Lorentz transformation for the EM fields. In the co-moving the bunch length is increasing by the factors of γ . While even in the lab-frame bunch length is much larger than its transverse size, it is definitely true in the beam-frame for all operating colliders. Thus, we have to solve Poisson equation for

$$\Delta\varphi = -4\pi\rho; \rho = \frac{\rho_o}{2\pi\sigma_x\sigma_y}\exp\left(-\frac{1}{2}\left(\frac{x^2}{\sigma_x}+\frac{y^2}{\sigma_y}\right)\right) \tag{27-34}$$

which can be solve using Fourier transform:

$$\iint ..\exp\left(\vec{k}\vec{r}\right)dxdy/(2\pi)^2 \to \varphi\left(\vec{k}\right) = 4\pi\rho\left(\vec{k}\right)/\vec{k}^2$$

$$\frac{1}{\vec{k}^2} = \int_0^\infty e^{-\vec{k}^2 t}dt \equiv \frac{1}{4}\int_0^\infty e^{-\frac{\vec{k}^2 t}{4}}dt$$

and scaling it by 1/4th we get:

$$\varphi\left(\vec{k}\right) = 4\pi\int_0^\infty \rho\left(\vec{k}\right)e^{-\vec{k}^2 t}dt; \;\; \varphi\left(\vec{r}\right) = \pi\int_0^\infty dt\iint e^{-i\vec{k}\vec{r}}\rho\left(\vec{k}\right)e^{-\frac{\vec{k}^2 t}{4}}dk_x dk_y.$$

Then, for a long Gaussian bunch with linear density of $\rho_o\left(z\right) = eZN\cdot e^{-\frac{\overline{z}^2}{2\overline{\sigma}_z^2}}/\left(\sqrt{2\pi}\overline{\sigma}_z\right)$.

$$\rho = \rho_o(z)\frac{1}{2\pi\sigma_x\sigma_y}e^{-\frac{x^2}{2\sigma_x^2}-\frac{y^2}{2\sigma_y^2}}; \quad \rho(k) = \frac{1}{(2\pi)^2}\cdot e^{-\frac{k_x^2\sigma_x^2}{2}-\frac{k_y^2\sigma_y^2}{2}};$$

after trivial integration,

$$\varphi(\vec{r}) = \pi\rho_o(z)\frac{1}{(2\pi)^2}\cdot\int_0^\infty dt\iint e^{-i\vec{k}\vec{r}}e^{-\frac{k_x^2\sigma_x^2}{2}-\frac{k_y^2\sigma_y^2}{2}}e^{-\frac{\vec{k}^2 t}{4}}dk_x dk_y; \;\; \int_{-\infty}^{\infty} e^{-ik_x x}e^{-\frac{k_x^2\left(2\sigma_x^2+t\right)}{4}}dx = \sqrt{\frac{4\pi}{2\sigma_x^2+t}}e^{-\frac{x^2}{2\sigma_x^2+t}};$$

we get the desirable result identical to (A7):

$$\varphi(\vec{r}) = \frac{eZN}{\sqrt{2\pi}\overline{\sigma}_z}e^{-\frac{z^2}{2\sigma_z^2}}\cdot\int_0^\infty \frac{e^{-\frac{x^2}{2\sigma_x^2+t}-\frac{y^2}{2\sigma_y^2+t}}}{\sqrt{\left(2\sigma_x^2+t\right)\left(2\sigma_y^2+t\right)}}dt. \tag{27-35}$$

where t is integration parameter, not the time!

What is true about (27-35) is that the field is indeed very nonlinear and effect on colliding particles is reducing when they go outside of the beam core. Expansion of the potential near the beam axis gives a simple expression:

$$\bar{\varphi}(x,y,\bar{z}) = -\frac{eN}{\sqrt{2\pi}\bar{\sigma}_z} e^{-\frac{\bar{z}^2}{2\bar{\sigma}_z^2}} \cdot \frac{2}{\sigma_x+\sigma_y}\left(\frac{x^2}{\sigma_x}+\frac{y^2}{\sigma_y}\right) + O^4 . \qquad (27\text{-}36)$$

with electric field simply calculated by

$$\vec{E}_{bf} = -\vec{\nabla}\bar{\varphi}(x,y,\bar{z}) = \frac{eN}{\sqrt{2\pi}\bar{\sigma}_z} e^{-\frac{\bar{z}^2}{2\bar{\sigma}_z^2}} \cdot \left(\hat{x}\frac{x}{\sigma_x(\sigma_x+\sigma_y)} + \hat{y}\frac{y}{\sigma_y(\sigma_x+\sigma_y)}\right) \qquad (27\text{-}37)$$

e.g. the beams of the same charge sign will experience defocusing (repulsion, for example *pp*) and beams of the opposite charge signs will experience focusing (for example in electron-position collides). Lorentz transferring the field into the lab frame and applying the formulae for the tune shifts we already derived we have (q is the charge of the particle in the opposite beam):

$$\vec{E} = \gamma \vec{E}_{bf}; \vec{B} = \gamma\beta\left[\hat{z}\times\vec{E}_{bf}\right];$$

$$\Delta Q_{2x,y} = -\frac{qeN_1}{2\pi\gamma m_2 c^2}\int \frac{\beta_{2x,y}}{\sigma_{1x,y}(\sigma_{1x}+\sigma_{1y})} e^{-\frac{s^2}{2\sigma_{1s}^2}} \frac{ds}{\sqrt{2\pi}\sigma_{1s}}; \qquad (27\text{-}38)$$

$$\Delta Q_{1x,y} = -\frac{qeN_2}{2\pi\gamma m_1 c^2}\int \frac{\beta_{1x,y}}{\sigma_{2x,y}(\sigma_{2x}+\sigma_{2y})} e^{-\frac{s^2}{2\sigma_{2s}^2}} \frac{ds}{\sqrt{2\pi}\sigma_{2s}};$$

For a short bunches, $\sigma_s << \beta_{x,y}$ we can move the expression from the integral and have an approximation for the beam-beam tune shifts:

$$\Delta Q_{2x,y} \cong -\frac{qeN}{2\pi\gamma m_2 c^2}\frac{\beta^*_{2\ x,y}}{\sigma^*_{1x,y}\left(\sigma^*_{1x}+\sigma^*_{1y}\right)} \ \ \& \ \ 1 \Leftrightarrow 2 \tag{27-39}$$

Considering again round beams with the same β-functions and emittance, we would have bema-beam tune shifts of:

$$\Delta Q_{1,2} = -\frac{qeN_{2,1}}{4\pi\gamma m_{1,2}c^2}\int \frac{\beta(s)}{\varepsilon\beta(s)} e^{-\frac{s^2}{2\sigma_s^2}} \frac{ds}{\sqrt{2\pi}\sigma_s} = -\frac{qeN_{2,1}}{4\pi\gamma m_{1,2}c^2\varepsilon}$$

$$\Delta Q_{1,2} == \pm\frac{N_{2,1}r_{1,2}}{4\pi\gamma\varepsilon}; \ \ r_{1,2} = \frac{e^2}{m_{1,2}c^2}. \tag{27-40}$$

Note that the beam-beam tune shift is inverse proportional to emittance.

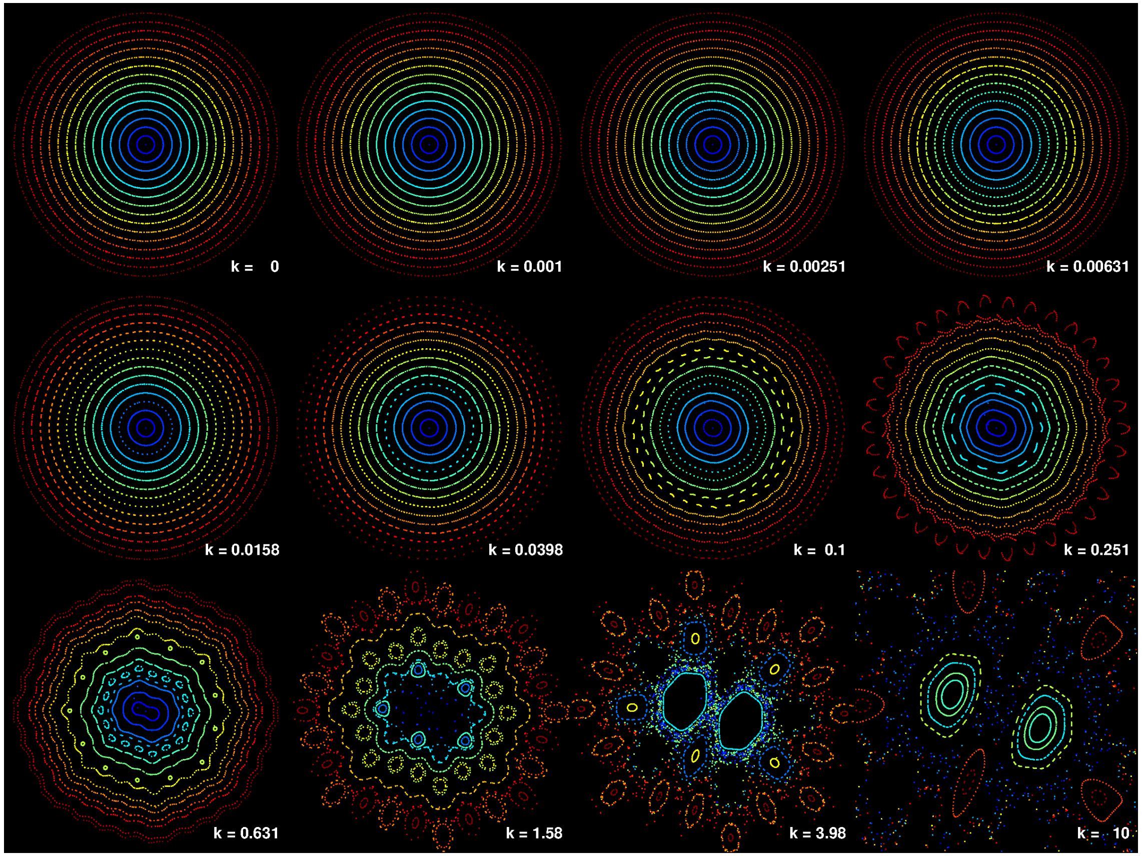
Since the tune shift is reducing with the amplitude (particles far away from the beam see field ~ 1/r), the bema-beam tune tuned shift is equal to the tune spread in the beam. Hence, some of the particles can get to strong resonance and get lost (or move to large amplitudes). Otherwise, which frequently happens in 6D phase space, a stochastic layer occurs and particle diffuse to large amplitudes, the beam emittances and sized increase and luminosity is reduced.

In any case, accelerator physicist tried multiple tricks (including colliding 4 bunches with two electron-position beams) and found that there is practical limitation for ring-ring colliders:

(27-41)

1. For hadron ring where damping is absent, maximum achievable tune shifts are $\left|\Delta Q_h\right| \le 0.02$;
2. For lepton colliders with strong damping (~ 1,000 turns) $\left|\Delta Q_l\right| \le 0.1$.

It is easy to make it worth, but so far nobody manage to exceed these limits.



Modification of the phase space structure with the increase of the beam-bema tune shift.

Thus, both reducing beam emittances and increasing the number particles per bunch can be used for increasing luminosity when tune shifts do not exceed the limits imposed by the beam dynamics.

Types of colliders

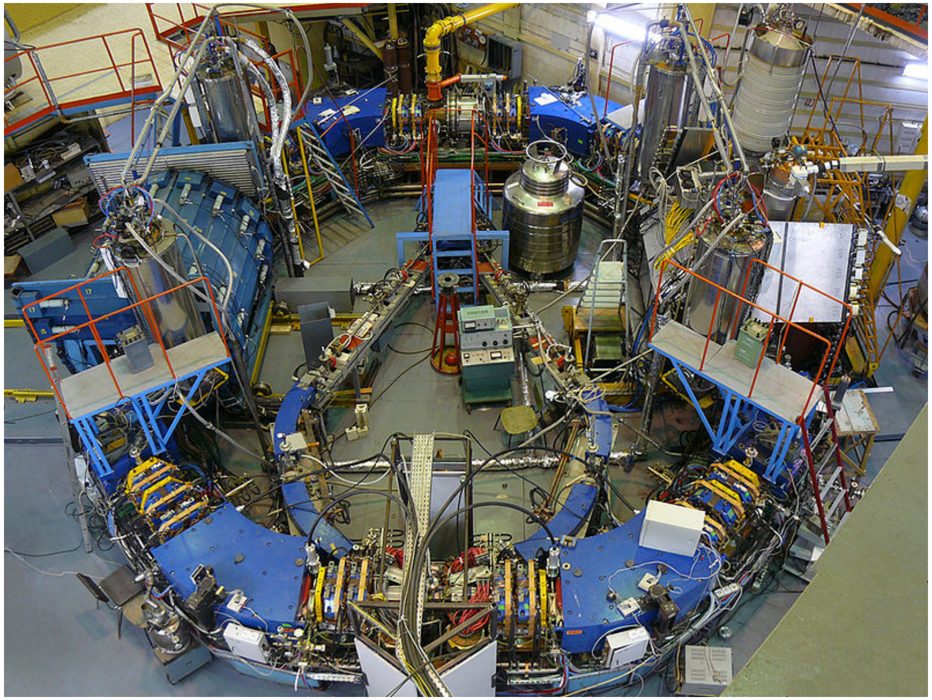


Fig. 27-5. A relatively compact electron-position collider VEPP-2000 (2 GEV c.m.). Electrons and positions have the same energy and circulating in the same storage ring in opposite directions.

A traditional (e.g. low cost!) electron collider has a single storage ring where electron and positions with the same energies rotate in opposite direction. It is rather easy to see that changing the sign of the charge and direction of motion provides for a perfect two-species accelerator:

$$\frac{d\vec{p}}{dt} = \frac{e}{c}\left[\vec{\mathrm{v}} \times \vec{B}\right]; \left\{ \begin{array}{c} e \to -e \\ \vec{\mathrm{v}} \to \vec{\mathrm{v}} \end{array} \right\} \Rightarrow \frac{d\vec{p}_{e^+}}{dt} = \frac{d\vec{p}_{e^-}}{dt} \Leftrightarrow \frac{d\vec{p}_{e^+}}{ds} = \frac{d\vec{p}_{e^-}}{ds} \tag{27-42}$$

which a simple indication that in magnetic fields trajectory of electrons are bilateral symmetric to those of positions. It also means that the orbit and the focusing of two beams is the same: a simple inversion proves this

$$s_{e^+} \to -s_{e^-};\ \vec{p}_{e^+}(-s) \to -\vec{p}_{e^-}(s); \vec{r}(-s)_{e^+} \to r_{e^-}(s). \tag{27-43}$$

The same is true for protons and antiprotons – as it was done for Tevatron in FermiLab.

Colliding beams – as we will see later in this lecture- is rather violent process and therefore should be limited only to detectors. Hence, beams of particles and antiparticles should be separated outside the detectors. As we observed above, it cannot be done by magnetic fields and electrostatic fields are used for such separation:

$$\frac{d\vec{p}}{dt} = e\vec{E} + \frac{e}{c}\left[\vec{\mathrm{v}} \times \vec{B}\right]; \left\{ \begin{array}{c} e \to -e \\ \vec{\mathrm{v}} \to \vec{\mathrm{v}} \end{array} \right\} \Rightarrow \frac{d\vec{p}_{e^+}}{dt} = \frac{d\vec{p}_{e^-}}{dt} + 2e\vec{E} \qquad (27\text{-}44)$$

The equal energy configuration was used from the dawn of circular colliders (operating at 100s of MeV) till era of the Large Electron Positron collider (LEP, 104.5 GeV per beam) at CERN.

One the most recent advances in circular electron-position colliders are so-called B-factories, which focus of detailed studies of short-leaving (femto-seconds) B-mesons (5.28 GeV) and their products. In this case using asymmetric collisions with different energies (3.5 GeV – low-energy ring and 8 GeV – high energy ring) of electrons and positions provides for significant velocity of the products and allows for physical separation of the decays. There was two such colliders: PEP at SLAC (CA) and KEKB at Tsukuba, Japan (Fig. 27- – the later is still operating and process physicist results. Naturally having two independent rings provide additional flexibility, but in return it requires organizing beam crossing using a dedicated techniques.

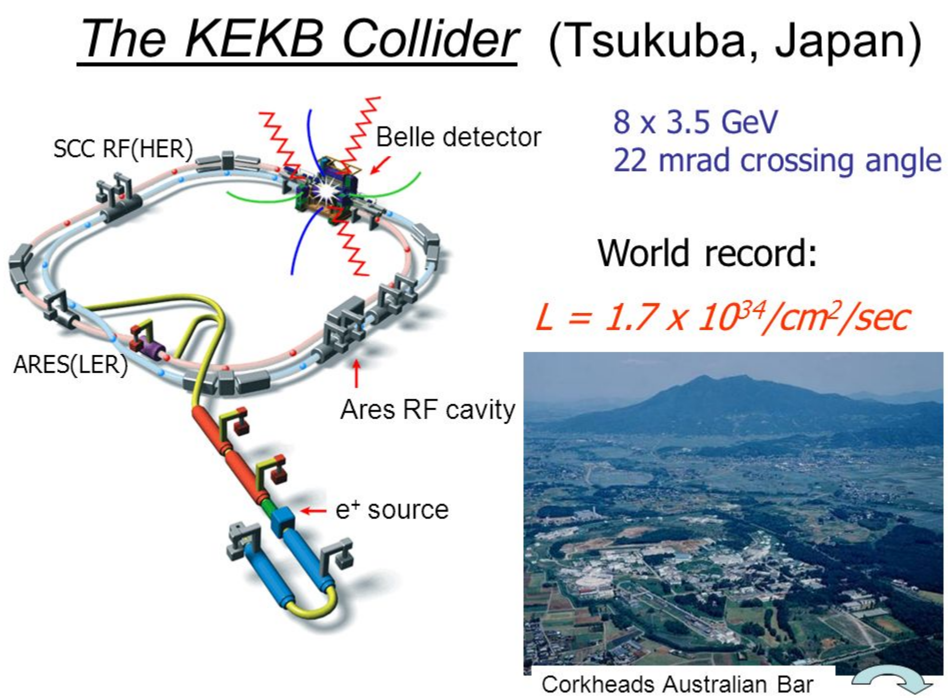


Fig. 27-6. Head-on collision of two particles with the same masses and energies.

In contrast with lepton colliders, in proton pp (as well as proton-anti-proton, $p\bar{p}$) colliders we do not collide elementary particles: protons are comprised of three quarks, myriad if gluons as well so-called sea-quarks (these "elementary" constituencies are frequently called partons). Thus, we are colliding two cups of soup with poorly defined initial state and broad range of parton's energies. Richard Feynman had an excellent joke about colliding protons: "Scattering of protons on protons is like colliding Swiss watches to find out how they are built?" This is one of the reasons why proton-antiproton collision offer very limited complimentary information when compared with proton-proton colliders. Hence, all current and planned hadron colliders are based on pp collisions.

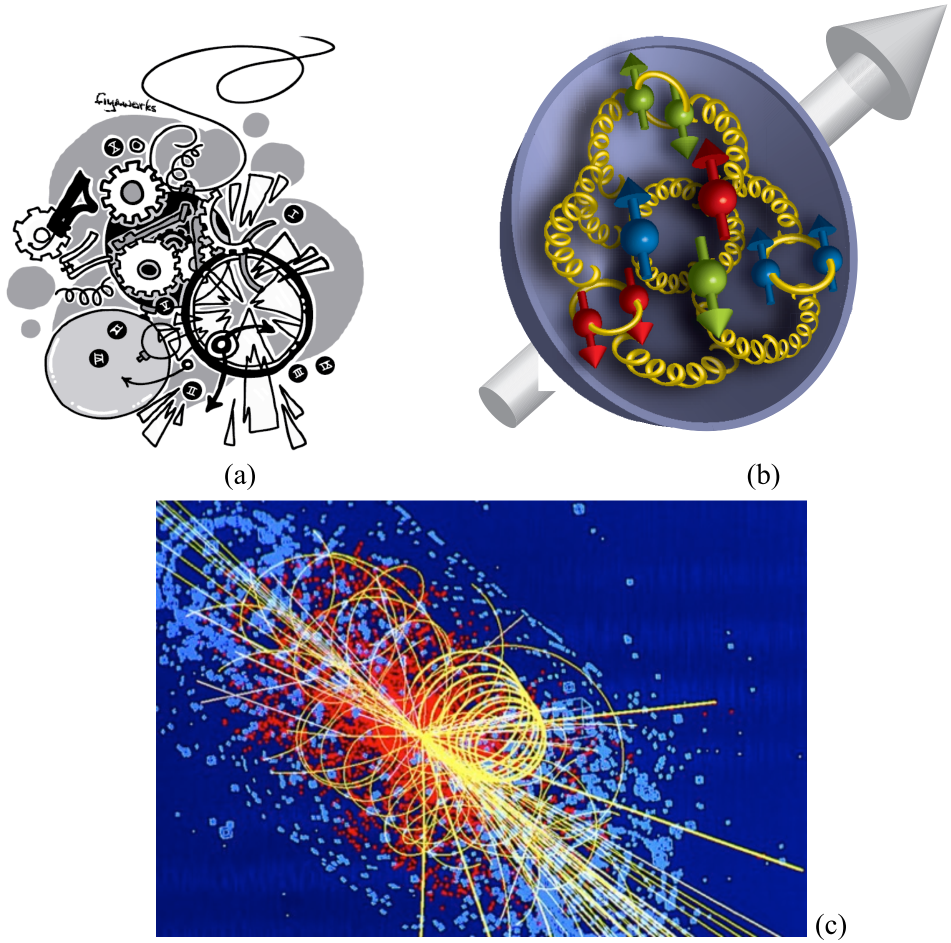


Fig. 27-7. (a) Colliding Swiss watches ale Feynman, (b) Artistic representation of a polarized proton; (c) A Higgs event in a single pp collision even in ATLAS detector at LHC, CERN

Colliding proton or positively charge ion beams require two separate storage rings with opposite sign of the guiding (bending) magnetic fields. In this case particle's energy not necessarily should be identical, but putting beams into collision is relatively straightforward: a dipole magnet on common orbit provides opposite bending for contra-propagating beams with the same sign of charge (pp or p and Au^{+79}) – see Fig. 27-8. In Relativistic Heavy Ion Collider (RHIC), BNL DX magnet provides separation of hadron beam orbits for mean colliding in one of it six IPs. RHIC is colliding large variety of ions, including polarize protons, deuterons, light and heavy ions (Al, Cu, Au, U…) in essential any desirable combination. Colliding ions adds even more "carrots and potatoes" into the already soupy picture, but it opens a gate-way to condensed matter properties of strong-interacting matter. RHIC, while being only one heavy energy polarized proton collider (250 GeV per beam), was first to discover quark-gluon plasma in heavy ion collisions (with 100 GeV/u).

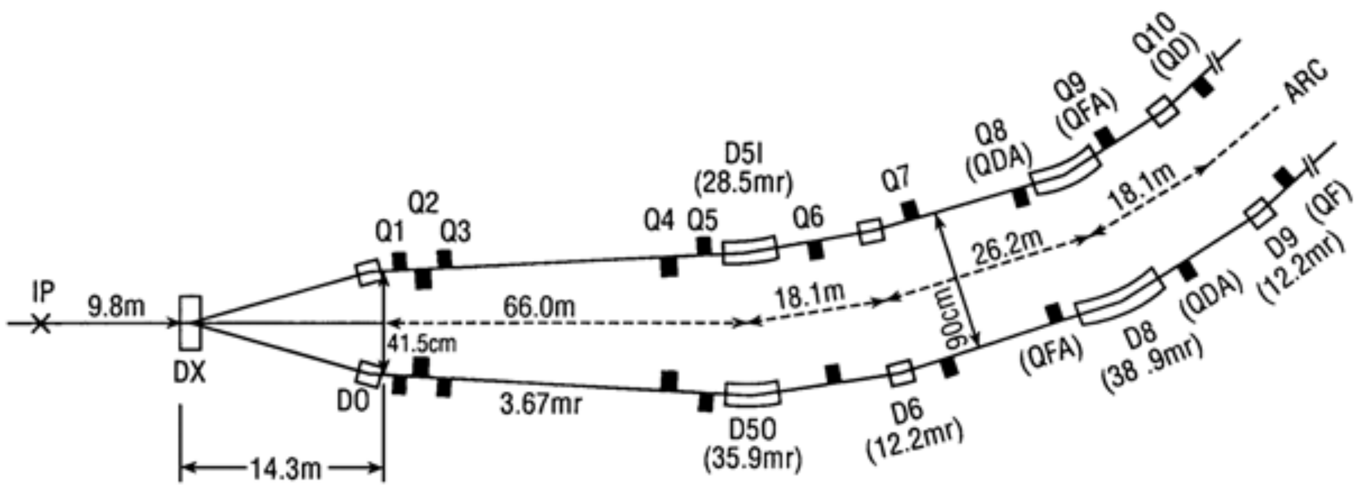
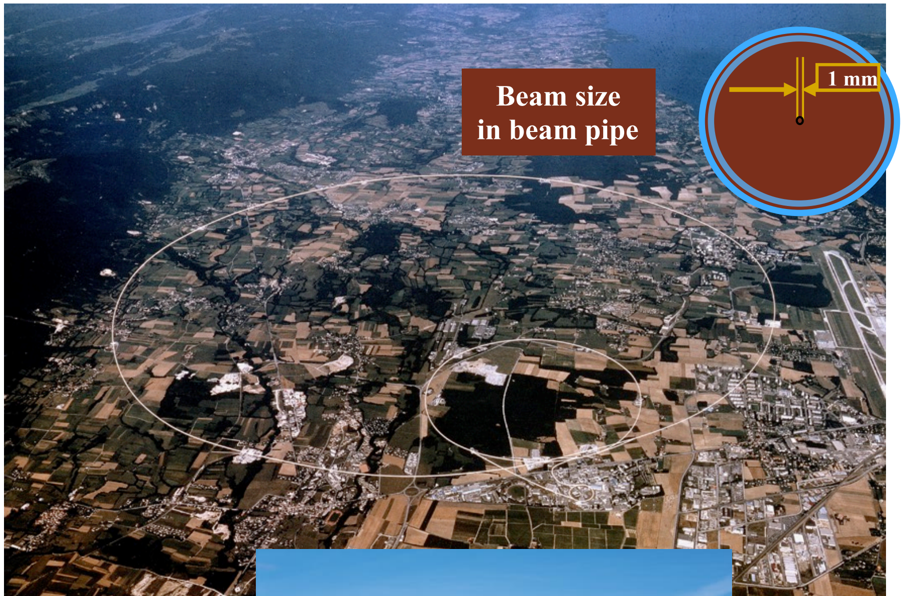


Fig. 27-8. RHIC IR section (to be exact ½ of the IR).

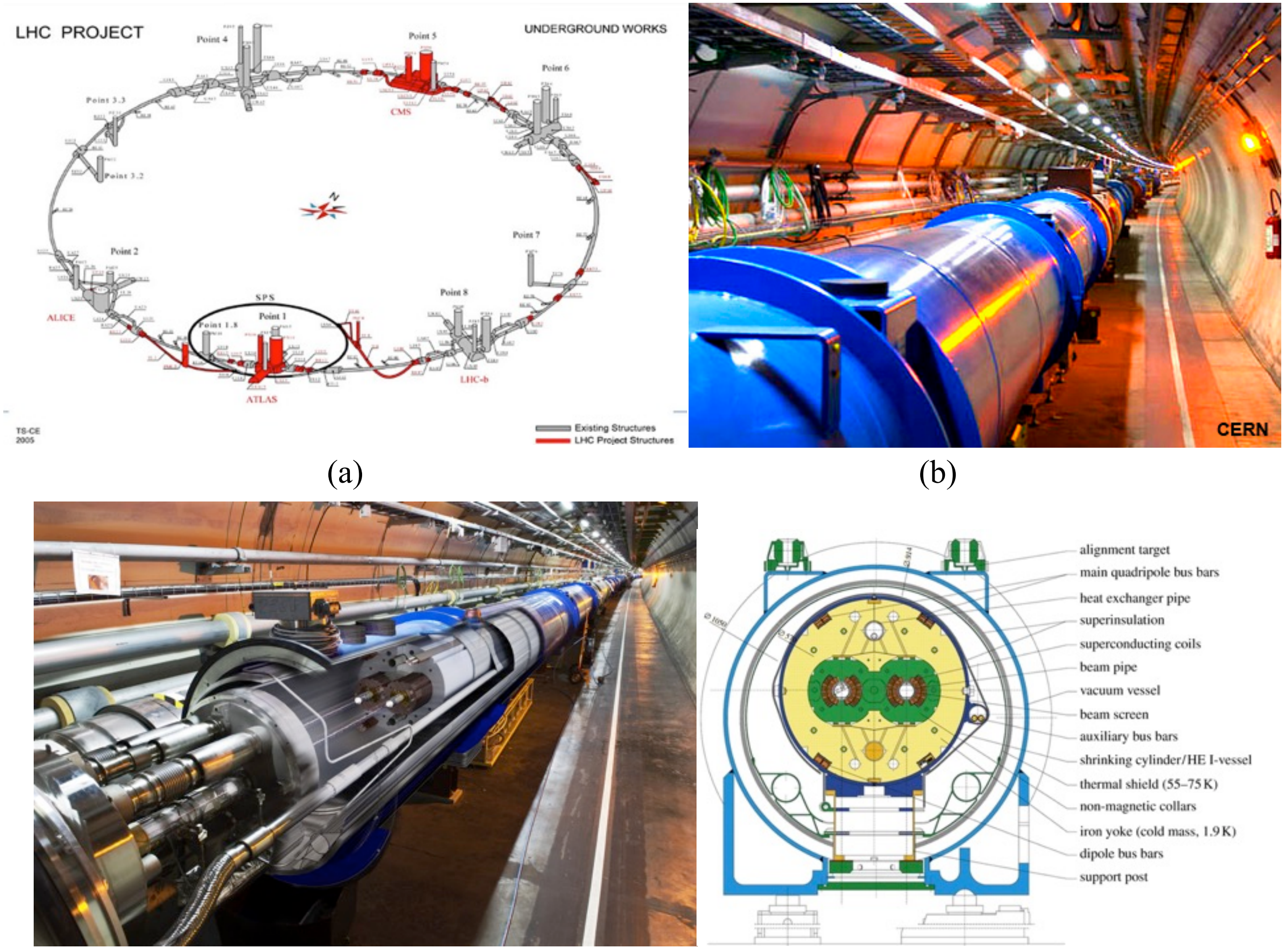
While RHIC has two separate super-conducting rings with individual cryostats, LHC has slightly different design by using so-called “two in one magnet” where two rings have common support and common cryostats – see Fig. 28-10.



Time	2007-
Circumference [km]	26.7
Energy [GeV]	7000 p 580000 Pb
Particles	p-p Pb-Pb
Peak luminosity [$10^{30}cm^{-2}s^{-1}$]	**10000**

350MJ stored energy per beam inside super-conducting magnets = kinetic energy of 20 fully loaded class 8 trucks (120,000lbs) at 55mi/hr

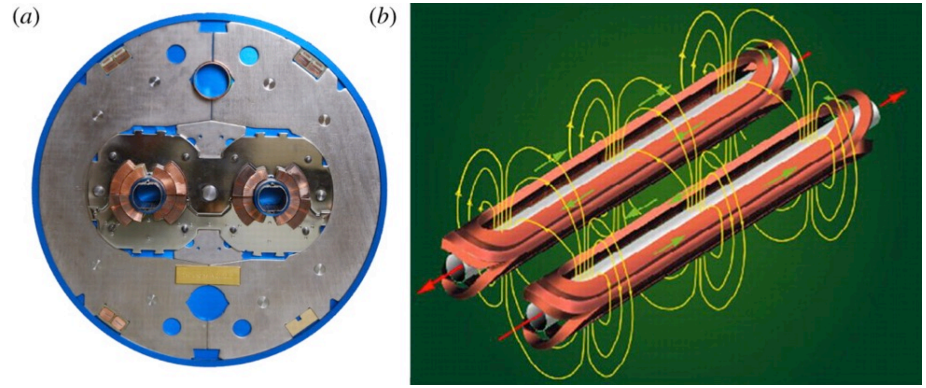
Fig. 27-9. Some trivial facts about Large Hadron Collider, CERN



(a) (b)

(c) (d)

Fig. 27-10. Detailed views of the Large Hadron Collider, CERN. (a) LHC structure and collision points; (b) LHC tunnel with its super-conducting magnets inside a single cryostat; (c) 3D rendering of LHC rings with two separate vacuum chambers; (d) the 'two-in-one" LHC superconducting magnets with opposite field polarity in two rings.



In addition to pp collisions, LHC also collides fully stripped lead ions (PbPb), which also generate quark-gluon plasma with slightly higher temperature than in RHIC.

Before changing the topic of hadron colliders, it is worth mentioning that probability of a single parton to carry majority of proton's energy and momentum is very small and vast majority of of partons have energy ~ 1/6 of that of the proton. Hence, "majority of elementary collisions" are happening at approximately 1/6 of pp c.m. energy. Thus. 7 TeV LHC pp colliders has about the same energy reach as would be reached by 1 TeV lepton collider.

Still the collider community is discussing next step in circular colliders – CERN version is simply called Future Circular Collider (FCC). A 100-km circumference collider would dwarf 28-km LHC and could collide protons with 50x50 TeV (100 TeV c.m.) energy, while the tunnel can be also suitable for electron-position circular collider with c.m. energy up to 350 GeV. A slightly less ambitious project is under discussion in China.

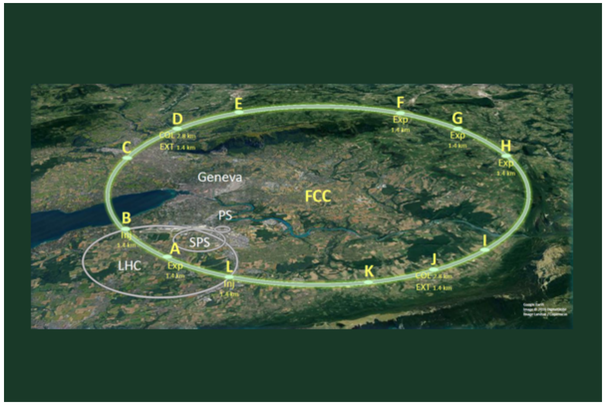


Fig. 27-11. Possible location of FCC.

One more question is rising from Feynman's observation of pp collision – is it possible to collide protons with electrons and extract additional information. The answer is surprisingly – yes – and this is one of the driving forces behind electron-hadron colliders. A circular ring-ring lepton-hadron (30 GeV electron and positions on 800 GeV protons) collider HERA had been built and operated at DESY, but after a number of very successful experiments it has been decommissioned. HERA had a relatively low luminosity when compared with other colliders, it did not operated ion and its protons were not polarized.

The later is one of the main goals for future US electron-ion collider (EIC) and its BNL option simply called eRHIC: https://www.bnl.gov/rhic/eic.asp. This collider would add polarised electron to the variety of species collided in RHIC. One the possibility to reach very high luminosity (e.g. productivity) in eRHIC is to use energy recovery linac of accelerating and colliding polarized electron beam with hadron circulating in RHIC ring (see Fig. 27-12). In contrasts with ring-ring collider, electrons collide with hadrons only once and "can be heavily abused". In other words, limitation for collision effect seen by electron beam can be removed and the collisions can be much more effective.

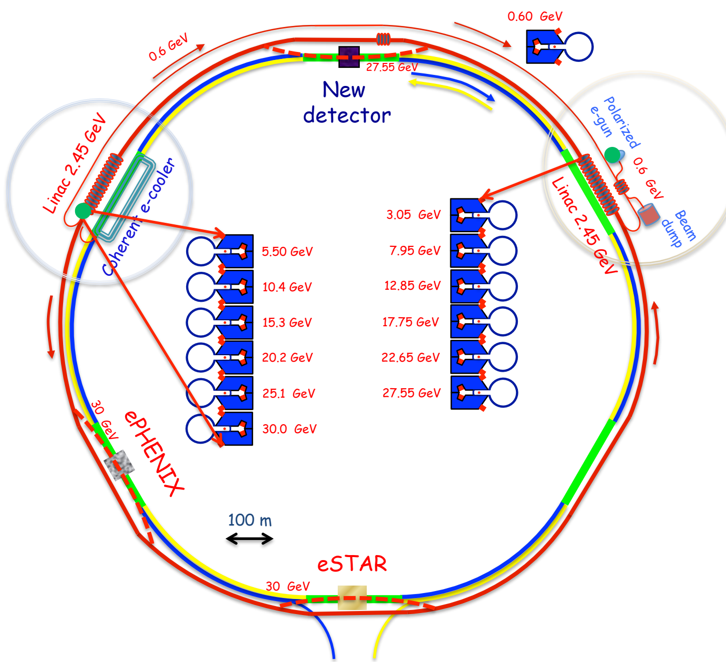


Fig. 27-12. One of ERL-based eRHIC layouts.

This is a good point to discuss three types of modern accelerators using in fundamental research and some applications:

1. Linear (RF) accelerators (called linacs), including recirculating linacs; storage rings – we briefly discussed this topics in our class;
2. Storage rings and circular accelerators – we discussed this is length;
3. Energy recovery linacs.

Frequently accelerator complexes supporting either a circular collider or a light source consists of a chain of accelerators – see for example Fig. 27-13. It always include a source of particles to be accelerated (electrons, positrons, protons, ions, antiprotons – note that antiparticles require a special accelerator of particles to be damped onto a target for generate them, a very elaborate process, especially for antiprotons), followed typically by a linear RF accelerator. The accelerator chain can include a synchrotron (or synchrotrons) – a circular rings with fast ramping magnetic field. Finally, a storage ring can either inject the beam on energy of operation (typical for modern light sources) or slowly ramp energy from injection to operation energy. Accelerators are connected by transport channels, where particles pass from one accelerator to the other.

Beam of particles in a collider is typically consists of multiple bunches defined by separatrices of RF system. Filling process of the storage ring is typically consists of injecting individual (or short trains) of bunches. Bunch extraction (called ejection) from and injection to a circular accelerator is done by pulsed magnets or electric devices called kickers. The goal of injection kicker is to suddenly change the injecting bunch trajectory from outside the accelerator aperture onto the design orbit without disturbing the other bunches circulating in the ring. The purpose of ejection kicker just the opposite – to kick a bunch from the ring orbit into the aperture and magnets of the transport beam line.

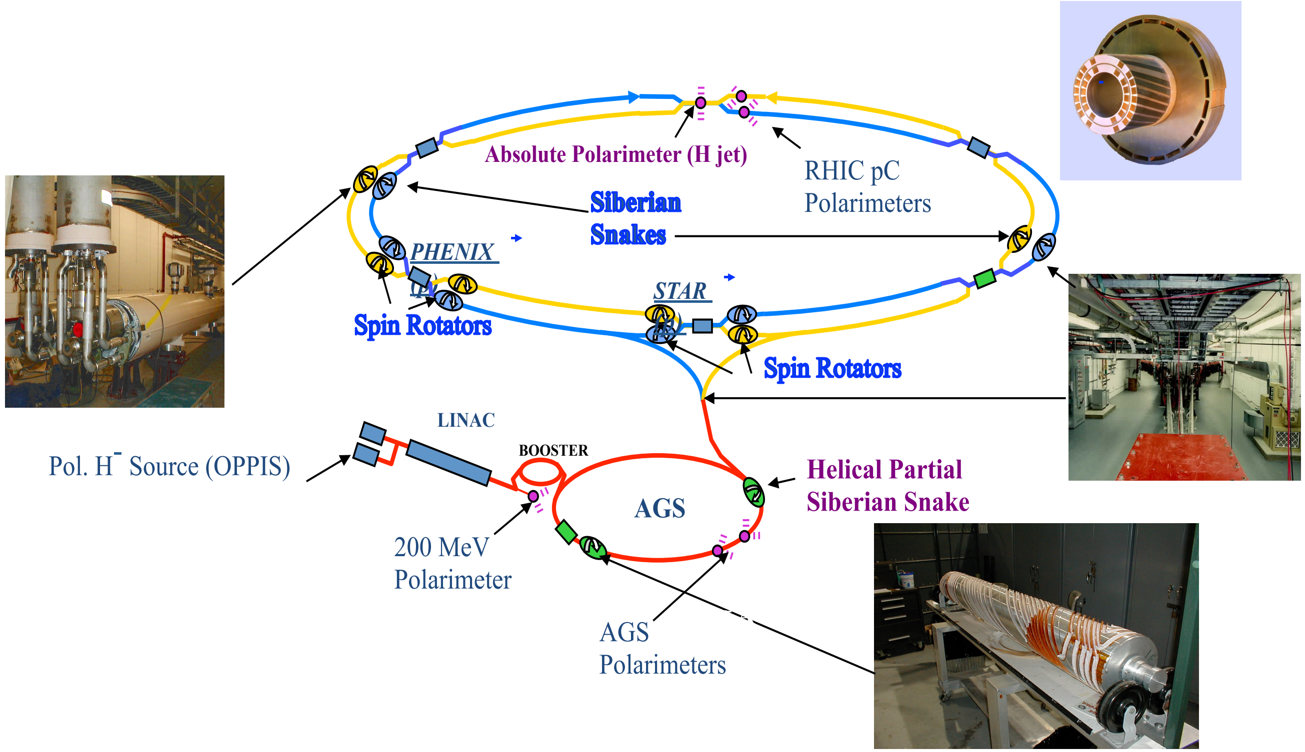
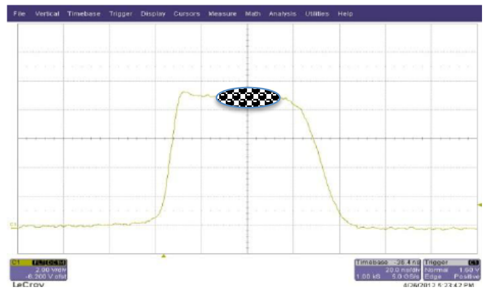
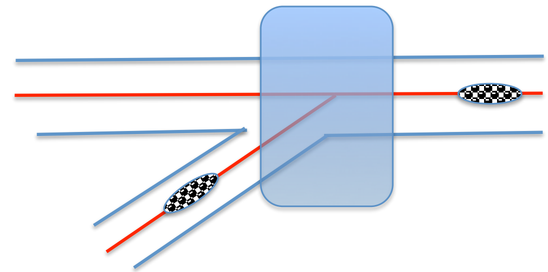


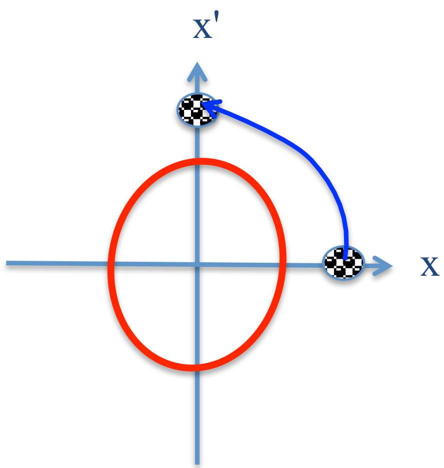
Fig. 27-13. RHIC accelerator complex supporting 250 GeV x 250 GeV polarized proton collisions: it has polarized proton source, a 200 MeV linac, two synchrotrons (booster and AGS – the first strong focusing synchrotron in the world operating since July 1960s…) to accelerate protons upto 30 GeV for injection is two RHIC rings. Final acceleration to beam energy (typically 100 or 250 GeV) for collision occurred in RHIC. While energy ramping in synchrotrons happens very quickly (sub-seconds), acceleration in RHIC required ramping magnetic field in superconducting magnets and the acceleration to the energy of collisions takes quite a few minutes. After that beams are colliding for many hours till collision rate decreases to the level requiring new injection cycle.

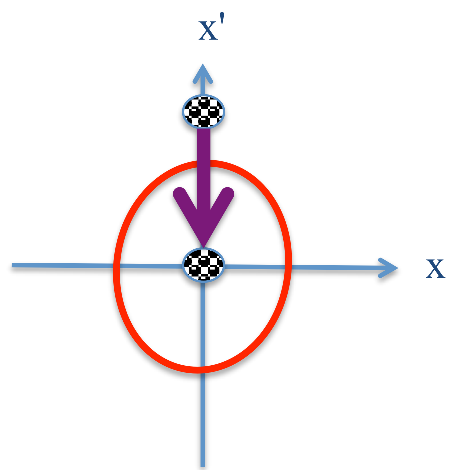
Here we need a short interlude: Poincare proved a theorem that in system with confined motion in time-independent system, the particle will come infinitely close to its position at t=0. Which means that particles coming from the outside the vacuum chamber of a storage ring will eventually (in practice – in few turns) will hit the wall of the chamber and get lost. Hence, one need to violate this by introducing time-dependent kicker – see Fig. 27-14.





1. (b)





(c) (d)

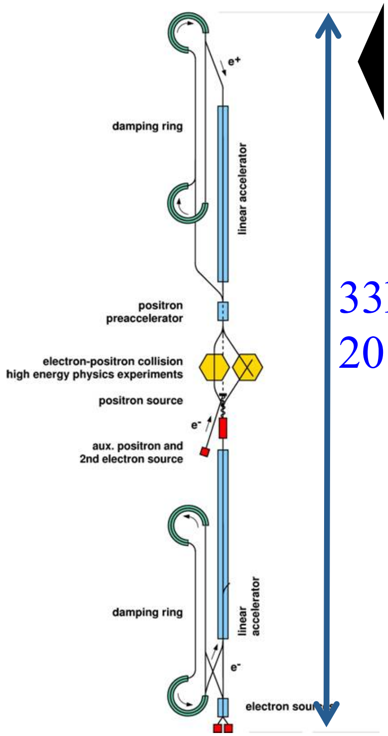
Fig. 27-14. (a) Typical pulse in RHIC injection kicker; (b) schematic (e.g. approximate) action of the injection kicker – it puts injecting bunch coming from outside the ring aperture onto (or close to) the design orbit; (c) and (d) illustrate the action of beam optics and the kicker in 1D injection process.

Red ellipse shows the acceptance of the ring, e.g. the coordinates and angles of the particles, which can calculate inside the ring without being lost. Specifically, this aperture is nothing else that trajectory of the particle in the phase space with ultimately large amplitude of oscillation. The “optics” transferring displacement of the injecting bunch from outside the vacuum chamber to angle x’, which is still outside the acceptance – remember that trajectories in the phase space cannot cross! The kicker corrects the angle of the bunch and it can circulate abound the ring without being lost. The kicker field has to be turned off – otherwise we return to the same problem imposed by Poincare’ theorem.

Let’s return to discussion of circular colliders: their strength is using the same particles for collision for billions of turns and replacing them only when they are depleted either by collisions or by inevitable losses or by diffusion. In electron-position colliders (an for proton in future FCC!) one also need to compensate for energy losses for synchrotron radiation – but this is only small portion of the reactive beam energy. Reactive power of 2 A 10 GeV electron beam circulating in one of KEKB-factory ring carried 10 GW of reactive power! In LHC 7 TeV beam has 0.6 A average current – its reactive energy is 4.2 TW.

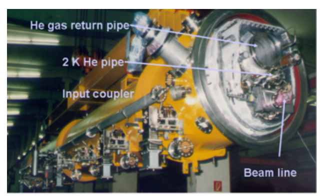
The main drawback, is the limits imposed on beam-beam effects. In contrast, linear colliders collide beams only once and therefore can “abuse” them.

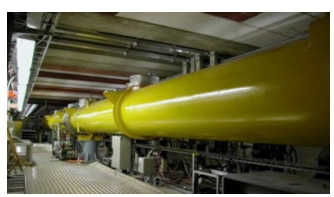
Instead of tune shift the number used in linear colliders is “beam disruption”, representing phase advance of betatron oscillation while bunches collide.



TESLA TDR
500 GeV (800 GeV)

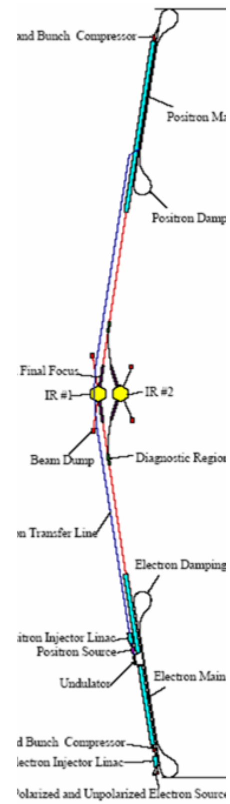
33km
20 miles





47 km
30 miles

US Options Study
500 GeV (1 TeV)



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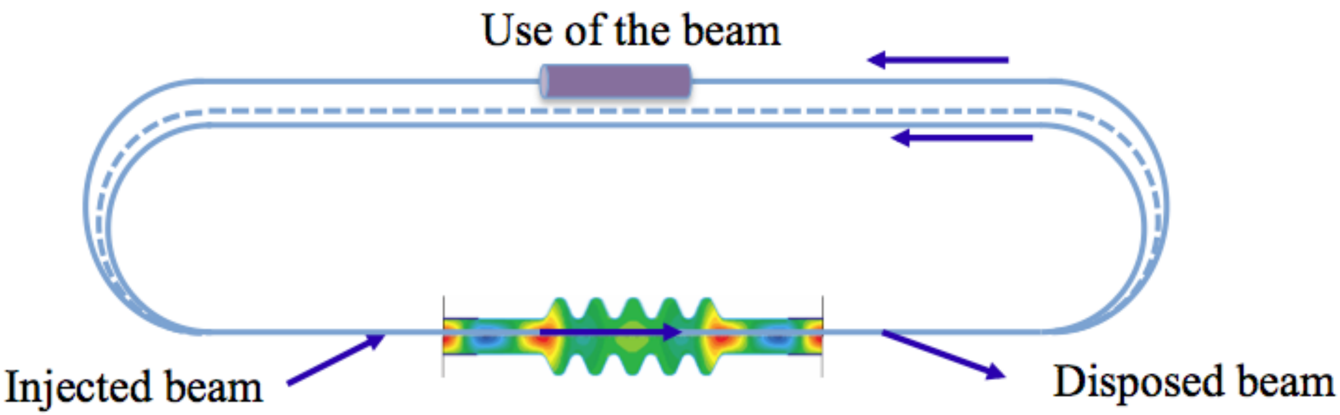
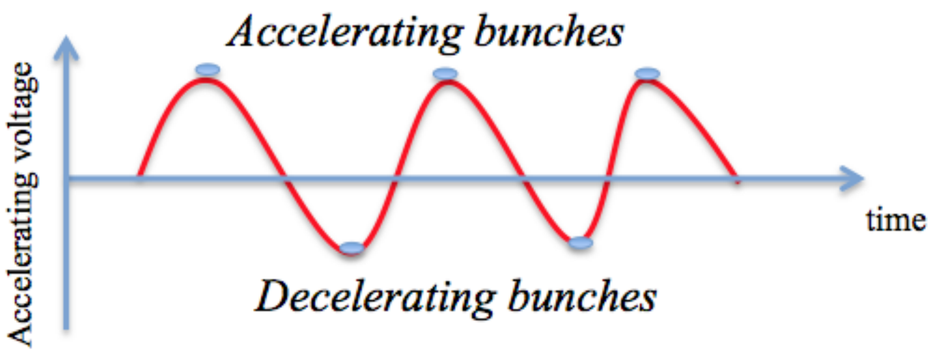
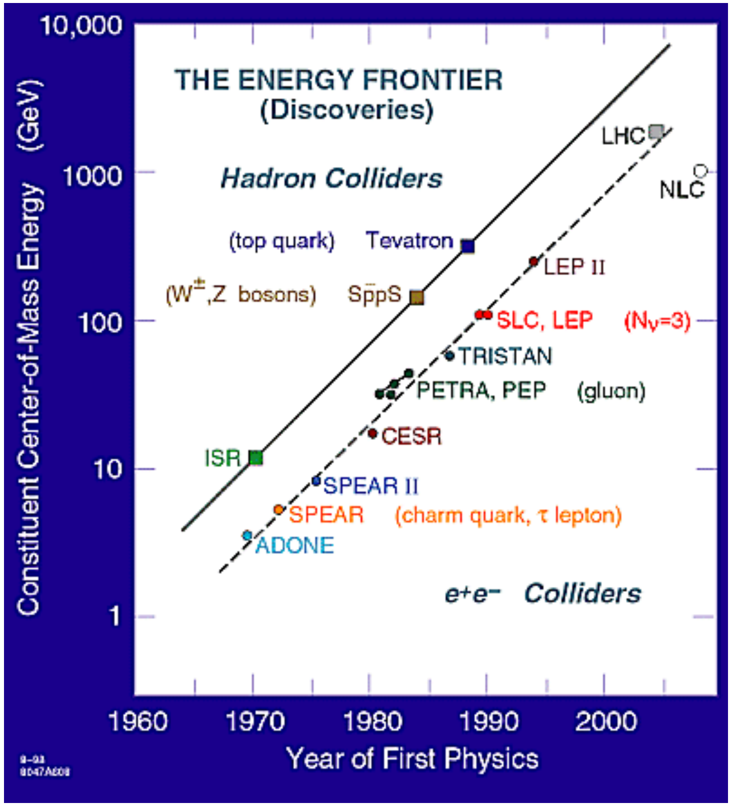
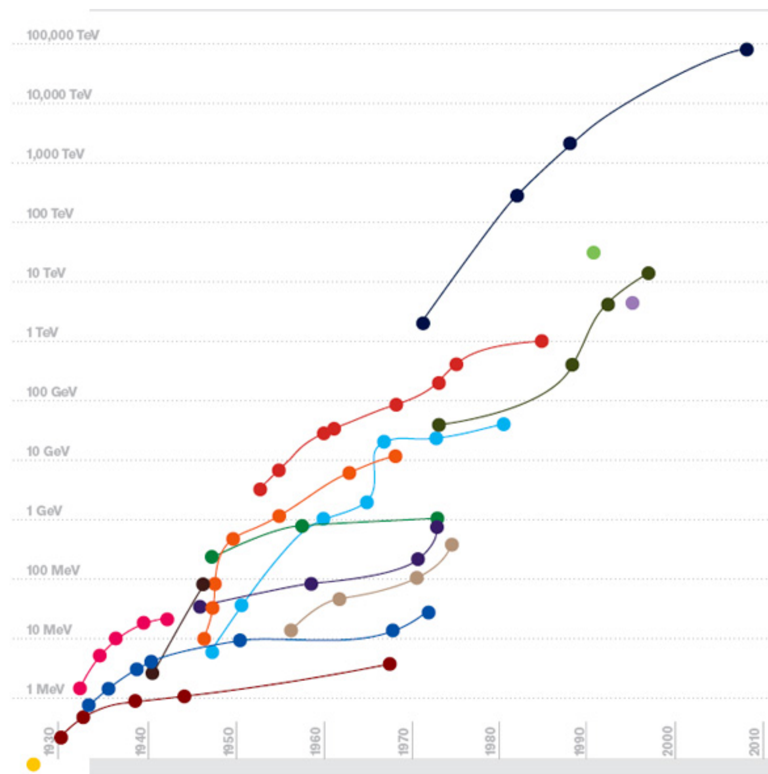
The main draw-back of linear colliders that they have to throw away the colliding beams and this is unfortunately only option for TeV scale electron-positions colliders. At low energies, as in eRHIC, we can take the most of the beam energy back by decelerating it using SRF linac – the mode which is called Energy Recovery Linac, or ERL.





Livingston plots were created in the time when energy of accelerators was growing exponentially from 1930s to 1060s, and new types of accelerators appeared every 5-10 years…. It was just enough to continue fixed target experiments… Than came circular colliders and trend continued for awhile, but last big increase in the c.m. energy (from Tevatron to LHC) fell of the exponent.

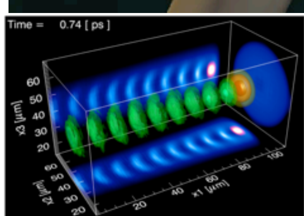




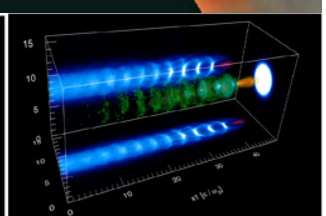
Livingston plot of c.m. energy vs time and corresponding “fix target” energy. Naturally there is no accelerators (probably except cosmic) with 100,000 TeV energy…

At the moment, we are reaching limit (especially $) in building colliders with much higher energies, but hope exists – plasma accelerators are advancing fast and may be just a break we need for c.m. energy run in 21st century

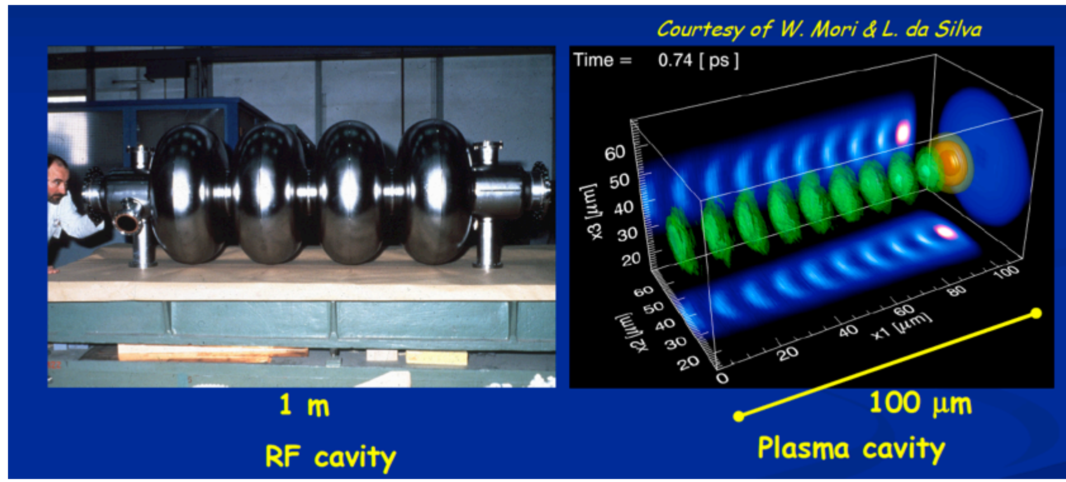
Plasma accelerators

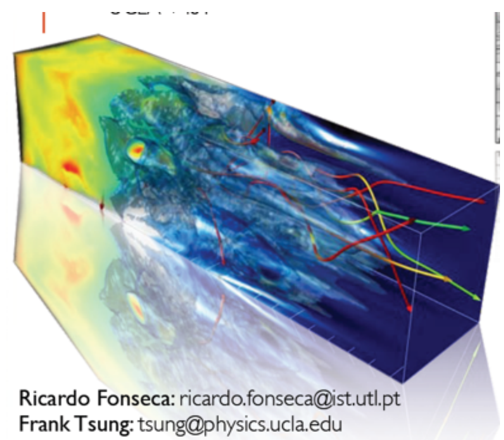


Laser Wake



Electron beam Wake





What we learned today

- We discussed two main figures of merit for colliders – the c.m. energy and luminosity
- We found that head-on collision is the most effective way to maximize the energy of collision
- We defined and derived expressions for collider luminosity and discussed how it can be maximized
- We derived the EM field of the colliding beam and derived the beam-beam tune shift. Without prove (which is impossible!) we defined what is the maximum attainable tune beam-beam tune shift can be tolerated in hadron and lepton circular colliders
- We discussed various types of colliders including circular, linear and their combination
- THIS IS THE END of FALL 2017 PHY 564 Class