PHY 554. Homework 6

1 (35 pts, each 5 pts). Synchrotron Radiation in NSLS II

Let us calculate the synchrotron radiation related problems in NSLS II. NSLS II adopts DBA lattice (separate function magnets).

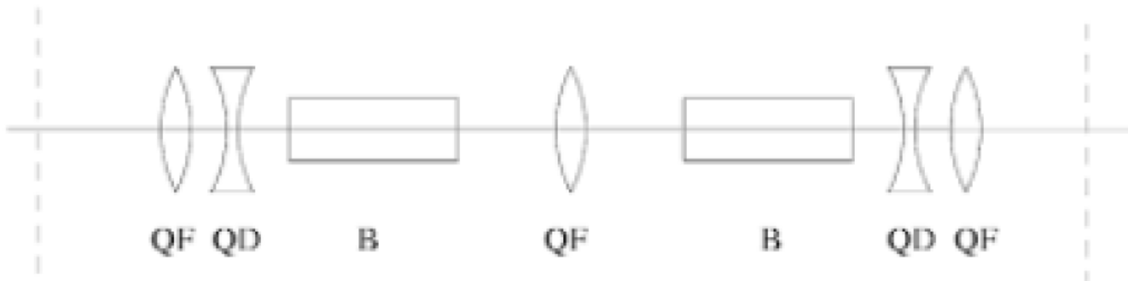

Figure 1: DBA lattice

The parameter of the electron ring is given by the following table.

Table 1: NSLS II parameters

Parameters	Values
Energy [GeV]	3.0
Circumference [m]	780
Number of dipoles	60
Dipole field [T]	0.4
Beam current [A]	0.5
RF frequency [MHz]	499.68
Harmonic number	1320

1. The energy loss due to the synchrotron radiation in all dipoles.
2. If the accelerating phase of the RF cavity is $\pi/6$, what is the minimum RF voltage needed to replenish the loss due to synchrotron radiation?
3. Actually the RF voltage is 3MV. Find the longitudinal tune of NSLS II (Hint: find the ϕ_s)
4. What is the critical radiation frequency of the dipole radiation?
5. Find the partition number in dipoles.
6. Find the longitudinal damping rate E and compare with the period of longitudinal oscillation.
7. Find the equilibrium energy spread of NSLS II.

Solution:

The energy loss due to synchrotron radiation is

$$U_{SR} = C_\gamma E^4 \oint ds/\rho^2/2/\pi = \frac{C_\gamma E^4}{\rho} = 0.287MeV$$

These energy must be compensated by the RF cavity

$$\begin{aligned} eV \sin\phi_s &= U_{SR} \\ V &= 0.57MV \end{aligned}$$

therefore the RF cavity must provide at least 0.57 MV to compensate the synchrotron radiation loss in the dipoles.

The actual voltage is 3MV, the phase should be $\arcsin(0.287/3) = \pi - 0.095$, the synchrotron tune is:

$$\nu_s = \sqrt{\frac{h\eta eV\cos\phi_s}{2\pi E}} = 8.8\times 10^{-3}$$

The time to finish one synchrotron oscillation is $2\pi/(\nu_s\omega_0) = 2.94\times 10^{-4}s$.

The critical frequency of the dipole radiation is given by

$$\omega_c = \frac{3}{2}\frac{c\gamma^3}{\rho} = 3.64\times 10^{18}Hz$$

We then calculate the radiation integrals, taking advantage that $K(s) = 0$ in dipoles:

$$I_2 = \oint 1/\rho^2 ds = 2\pi/\rho = 0.2513m^{-1}$$

$$I_3 = \oint 1/\rho^3 ds = 2\pi/\rho^2 = 1.0\times 10^{-2}m^{-2}$$

$$I_4 = \oint D/\rho^3 ds = 60\int_0^{L_D} s^2/2/\rho^4 ds = 10L_D^3/\rho^4 = 4.59\times 10^{-4}m^{-1}$$

Therefore the partition number $\bar{D} = I_4/I_2 = 1.828\times 10^{-3}$

The longitudinal damping rate

$$\alpha_E = \frac{U_0}{2T_0E}(2+\bar{D}) = 36.79s^{-1}$$

It is much slower than the synchrotron oscillation.

The equilibrium energy spread can be calculated as:

$$\frac{\delta E}{E} = \sqrt{\frac{C_q\gamma^2 I_3}{2I_2 + I_4}} = 5.12\times 10^{-4}$$

2 (5 pts). Equilibrium emittance in LEP

The equilibrium electron emittance at 100 GeV in LEP is about 0.06 mm-mrad. If we want to use the same LEP machine as a storage ring for a 5 GeV synchrotron light source by scaling down the magnet strength proportional to the beam energy, what will be the equilibrium electron emittance? (Hint: bending radius stays the same)

Solution:

Equilibiurm emittance is proportional to γ^2 when bending radius is kept the same. Therefore, the equilibrium emittance for LEP is 0.06 um/400 = 0.15 nm.