Homework 11. Due October 19

Problem 1. 4 x 4 points. FODO cell.
Consider a general FODO cell comprised of two quadrupoles F and D separated by two drift sections, e.g. the structure below:

\[ F : K_F = \frac{e}{pc} \frac{\partial B_y}{\partial x}, l_F; \]
\[ O1 : l_1 \]
\[ D : K_D = \frac{e}{pc} \frac{\partial B_y}{\partial x}, l_D; \]
\[ O2 : l_2 \]

(a) write matrix (both x and y or 4x4) of general FODO cell (not assuming any limitations on K F,D).
(b) write stability criteria (for x and y) for periodic lattice built of this FOD cell. Hint – do not try to solve it!
(c,d) make transition to short lens approximation and assume equal strength of

\[ l_F K_F = -K_D l_D = \frac{1}{f} = \text{const}, l_{F,D} \to 0 \]
\[ l = l_1 = l_2 \]

and

(c) show that both x and y motion can be stable (e.g. prove so called strong focusing: combination of focusing and defocusing length can provide focusing in both directions);
(d) define (e.g solve) the stability criteria for such cell.

Problem 2. 2x5 points. Find not-trivial solution for building an unit 2x2 transport matrix out of repeating cells:

\[ M^4 = I; M \neq I \]

(a) show that one of the solutions \( \text{trace}(M) = 0 \); Hint: used \( M^2 = -I \);
(b) for a “symmetric” FODO cell and finite length equally strong quadrupoles \( K_F = -K_D = K; l_F = l_D = L; l_1 = l_2 = l \) write the condition that \( M_x^4 = M_y^4 = I \), e.g. the 4x4 transport matrix is unit.