Home Work PHY 554 #10. Due – changed to March 30, 2020

HW 1 (10 points): Let us calculate the synchrotron radiation related problem in NSLS II. NSLS II adopts DBA lattice (separate function magnets). Here are the parameters:

Table 1: NSLS II parameters

Parameter	Value
Energy [GeV]	3.0
Circumference [m]	780
Number of dipoles	60
Dipole field [T]	0.4
Beam current [A]	0.5
RF frequency [MHz]	499.68
Harmonic number	1320

From the design parameters, we can calculate the following parameters:

Note: In DBA lattice, dispersion D and dispersion slope D0 are zero at one end of dipoles and non-zero at the other end of the dipole. Find dispersion function inside the dipole magnet.

• What is the compaction factor α_c of the ring?
• What the energy loss per turn due to synchrotron radiation in the dipoles
• For accelerating phase $\pi/6$ in the RF cavity, what is minimum RF voltage
is required?
How much is the power needed to compensate losses of the beam energy for synchrotron radiation? (neglect RF losses in the cavity – they are super-conducting)
• For actual RF voltage of 3MV, find the longitudinal tune of NSLS II ring
• Find the partition number $\bar{D}$ due to synchrotron radiation in dipole.
• Find the longitudinal damping rate α_E and compare with the period of synchrotron oscillations.
• Find the equilibrium energy spread of NSLS II.

Hint: all dipoles in NSLS II are identical with uniform field ($\partial B_y / \partial x = 0$). First, find what is the length of the dipole. Calculate the dispersion in the dipole starting from D=0 and D'=0, and calculate all other integrals for this dipole. The rest – just multiply them by number of dipoles and use formulae from the lecture 13.

Answers:
Answer:
NSLS II has 60 dipoles to form a closed loop, therefore each dipole bends 6 degree, which is $2\pi/60 = 0.105\ rad$. The radius of the dipole can be found as $pc = eB\rho$, therefore $\rho = 3GeV/c/0.4 = 25m$. The length of each dipole is $L_D = 2\pi\rho/60 = 2.618$m.

Since each dipole at one end has D = 0 and D'= 0 (dero dispersion in straight sections), we can calculate the dispersion function in the dipole from this end using small angle approximation:

$$\begin{bmatrix} D(s) \\ D'(s) \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & l & \rho\theta^2/2 \\ 0 & 1 & \theta \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} D(0)=0 \\ D'(0)=0 \\ 1 \end{bmatrix} \rightarrow D(s) = \frac{s^2}{2\rho}$$

The compaction factor is

$$\alpha_c = \frac{1}{C}\oint \frac{D(s)}{\rho} ds = \frac{60}{C}\int_0^{L_D} \frac{s^2}{2\rho^2} ds = \frac{10}{C}\frac{L_D{}^3}{\rho^2} = 3.68\cdot 10^{-4}.$$

The energy loss due to synchrotron radiation is

$$U_{SR} = \frac{C_\gamma E^4}{2\pi}\oint \frac{ds}{\rho^2} = \frac{C_\gamma E^4}{\rho} = 0.287\, MeV.$$

These energy must be compensated by the RF cavity, for 30-degree synchronous phase

$$eV_{RF}\sin\varphi_s = U_{SR};\ \varphi_s = \frac{\pi}{6}; \sin\varphi_s = \frac{1}{2} \rightarrow V_{RF} = 0.574\, MV$$

the RF cavity voltage is 0.574 MV. The actual voltage is 3MV, the synchronous phase should be $\varphi_s = arcsin(0.287/3) = \pi - 0.095$, the synchrotron tune is:

$$\nu_s = \sqrt{\frac{h\eta eV\cos\varphi_s}{2\pi E}} = 8.8\cdot 10^{-3}.$$

The synchrotron oscillation period is $T_s = \frac{1}{f_{rec}\nu_s} = \frac{C}{c\nu_s} = 2.94\cdot 10^{-4}\sec$.

Critical energy of synchrotron radiation from the dipole magnet will be

$$\omega_c = \frac{3}{2}\frac{c\gamma^3}{\rho} = 3.64\cdot 10^{18}\, Hz.$$

Since dipoles do not have gradient, focusing is very weak ($K=1/\rho^2$) and we can used K=0 approximation in dipoles. Then integral are easy to evaluate:

$$I_2 = \oint \frac{ds}{\rho^2} = \frac{2\pi}{\rho} = 0.2513\ m^{-1};\ I_2 = \oint \frac{ds}{\rho^3} = \frac{2\pi}{\rho^2} = 10^{-2}\ m^{-2};$$

$$I_3 = \oint \frac{Dds}{\rho^3} = 60 \int_0^{L_D} \frac{s^2 ds}{\rho^4} = 10 \frac{L_D{}^3}{\rho^4} = 4.59 \cdot 10^{-4} m^{-1}$$

The damping partition number is, as expected, is very small:

$$\bar{D} = \frac{I_4}{I_2} = 1.83 \cdot 10^{-3},$$

And the longitudinal damping rate

$$\alpha_E = \frac{U_{SR}}{2T_o E}\left(2 + \bar{D}\right) = 36.79\ \sec^{-1};\ \tau_E = \frac{1}{\alpha_E} = 0.027 \sec,$$

e.g. much slower than synchrotron oscillations. Finally, using formula from our lecture, we can calculate RMS energy spread in the beam

$$\frac{\sigma_E}{E} = \sqrt{\frac{\gamma^2 C_q I_3}{2I_2 + I_3}} = 5.12 \cdot 10^{-4}.$$