

10648 **Chapter 14**  
10649 **Optical Elements and Keywords, Complements**

10650 **Abstract** This chapter is not a review of the 60+ optical elements of zgoubi's  
10651 library. They are described in the Users' Guide. One aim here is, regarding some of  
10652 them, to briefly recall some aspects which may not be found in the Users' Guide and  
10653 yet addressed, or referred to, in the theoretical reminder sections and in the exercises.  
10654 This chapter is not a review of the 40+ monitoring and command keywords available  
10655 in zgoubi, either. However it reviews some of the methods used, by keywords such  
10656 as MATRIX (computation of transport coefficients from sets of rays), FAISCEAU  
10657 (which produces beam emittance parameters), and others. This chapter in addition  
10658 recalls the basics of transport and beam matrix methods, in particular it provides the  
10659 first order transport matrix of several of the optical elements used in the exercises, in  
10660 view essentially of comparisons with transport coefficients drawn from raytracing,  
10661 in simulation exercises.

10662 **14.1 Introduction**

10663 Optical elements are the basic bricks of charged particle beam lines and accelerators.  
10664 An optical element sequence is aimed at guiding the beam from one location to  
10665 another while maintaining it confined in the vicinity of a reference optical axis.  
10666 Zgoubi library offers of collection of about 100 keywords, amongst which about  
10667 60 are optical elements, the others being commands (to trigger spin tracking, trigger  
10668 synchrotron radiation, print out particle coordinates, compute beam parameters,  
10669 etc.). This library has built over half a century, so it allows simulating most of  
10670 the optical elements met in real life accelerator facilities. Quite often, elements  
10671 available provide different ways to model a particular optical component. A bending  
10672 magnet for instance can be simulated using AIMANT, or BEND, CYCLOTRON,  
10673 DIPOLE[S][-M], FFAG, FFAG-SPI, MULTIPOL, QUADISEX, or a field map and  
10674 TOSCA, CARTEMES or POLARMES to handle it. These various keywords have  
10675 their respective subtleties, though, more on this can be found in the "Optical Elements  
10676 Versus Keywords" Section of the guide [1, pp. 12, 227], which tells "Which optical

10677 component can be simulated. Which keyword(s) can be used for that purpose”. For  
 10678 a complete inventory of optical elements, refer to the “Glossary of Keywords” found  
 10679 at the beginning of PART A [1, page 9] or PART B of the Users’ Guide [1, page 229].

10680 Optical elements in *zgoubi* are actually field models, or field modeling methods  
 10681 such as reading and handling field maps. Their role is to provide the numerical  
 10682 integrator with the necessary field vector(s) to push a particle, and possibly its spin,  
 10683 along a trajectory. The following sections introduce the analytical field models which  
 10684 the simulation exercises resort to.

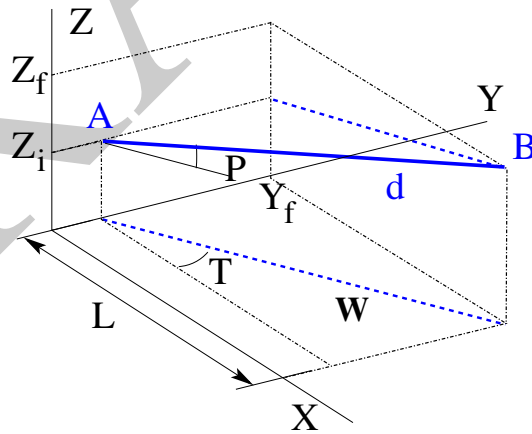
10685 *Zgoubi*’s coordinate nomenclature, as well as the Cartesian or cylindrical refer-  
 10686 ence frames used in the optical elements and field maps, have been introduced in  
 10687 Sect. 1.2 and Fig. 1.5.

## 10688 14.2 Drift Space

10689 This is the DRIFT, or ESL (for the French “ESpace Libre”) optical element, through  
 10690 which a particle moves on a straight line. From the geometry and notations in  
 10691 Fig. 14.1, with  $L$  the length of the drift, coordinate transport satisfies

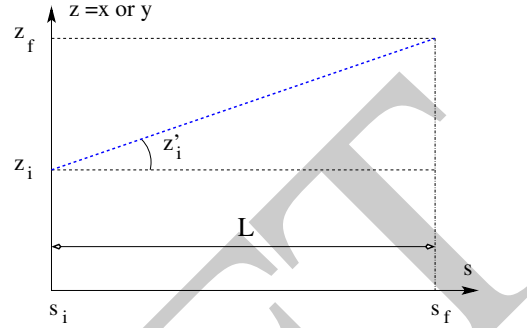
$$\begin{cases} X_f - X_i = L \\ Y_f - Y_i = L \tan T \\ Z_f - Z_i = L \tan P / \cos T \\ \text{path length } d = L / (\cos T \cos P) \end{cases} \quad (14.1)$$

**Fig. 14.1** An  $L$ -long drift in *zgoubi* ( $O;X,Y,Z$ ) frame, with origin at the upstream end of the drift. A particle flies from  $A(Y_i, Z_i)$  to  $B(Y_f, Z_f)$ , at an angle  $P$  to the  $(X, Y)$  plane. Projection  $W$  of its straight path in  $(X, Y)$  plane is at an angle  $T$  to the  $X$  axis



10692 *Linear approach*

10693 Coordinate transport from initial to final position in the linear approximation is  
 10694 written (with  $z$  standing indifferently for  $x$  or  $y$ , subscripts  $i$  for initial and  $f$  for final  
 coordinates) (Fig. 14.2)



**Fig. 14.2** A drift section with length  $L = s_f - s_i$ , and projection of a straight trajectory in the  $(s, z)$  plane, at an angle  $z'$  (standing for  $x'$  or  $y'$ ) to the  $s$  axis

10695

$$\begin{cases} z_f = z_i + L z'_i \\ z'_f = z'_i \\ \delta l_f - \delta l_i = \beta c \delta t = \frac{L}{\gamma^2} \frac{\delta p}{p} \\ \delta p_f / p = \delta p_i / p \end{cases} \quad \text{or, } T_{\text{drift}} = \begin{pmatrix} 1 & L & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & L & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \frac{L}{\gamma^2} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad (14.2)$$

10696 where  $\beta c$  is the particle velocity,  $p = \gamma m \beta c$  its momentum,  $\gamma$  is the Lorentz relativistic factor.  
 10697

10698 **14.3 Guiding**

10699 Beam guiding is in general assured using dipole magnets to provide a field vector  
 10700 normal to a bend plane. Gradient dipoles combine guiding and focusing in a single  
 10701 magnet, this is the case in cyclotrons where the field index is tailored to ensure  
 10702 isochronism, in scaling FFAGs where  $B \propto r^k$  ensures the zero-chromaticity prop-  
 10703 erty. This may also be the case in strong focusing synchrotrons, for instance in the  
 10704 BNL AGS [2], in the CERN PS [3]. Dipole magnets sometimes include a sextupole  
 10705 component for the compensation of chromatic aberrations [4]. Non-linear optical ef-  
 10706 fects may be introduced in addition by shaping entrance and/or exit EFBs, a parabola  
 10707 for instance for  $x^2$  field integral dependence, a cubic curve for  $x^3$  dependence (see  
 10708 Chap. 13).

10709 Low energy beam guiding also uses electrostatic deflectors, shaped to provide a  
 10710 field normal to the trajectory arc, and possibly focusing properties. Plane condensers  
 10711 may be used as well for beam steering, including beam filtering in combination with  
 10712 a magnetic field, and at high energy in addition for such functions as pretzel orbit  
 10713 separation, extraction septa, etc.

10714 Guiding optical elements are dispersive systems: trajectory deflection has a first  
 10715 order dependence on particle momentum.

### 10716 14.3.1 Dipole Magnet, Curved

10717 This is the DIPOLE element (an evolution of the 1972's AIMANT [1]) or variants:  
 10718 DIPOLES, DIPOLE-M. Lines of constant field in the magnet body are isocentric  
 10719 circle arcs. The magnet reference curve is a particular arc, at a reference radius  $R_0$   
 10720 for which the field value is  $B_0$ . The field in the median plane can be written

$$B_Z(r, \theta) = \mathcal{G}(r, \theta) B_0 \left( 1 + N \frac{r - R_0}{R_0} + N' \left( \frac{r - R_0}{R_0} \right)^2 + N'' \left( \frac{r - R_0}{R_0} \right)^3 + \dots \right) \quad (14.3)$$

10721  $N^{(n)} = d^n N / dY^n$  are the field index and derivatives.  $\mathcal{G}(r, \theta)$  describes the azimuthal  
 10722 shape of the field, from a plateau value in the body to zero away from the magnet. It  
 10723 can be written under the form [5]

$$\mathcal{G}(r, \theta) = G_0 F(d(r, \theta)) \quad (14.4)$$

10724 where  $G_0$  a constant factor, and  $F(d)$  a convenient model for the field fall-off, such  
 10725 as the Enge model discussed in Sect. 14.3.3. In that model take  $d(r, \theta)$  the distance  
 10726 from particle location  $(X, Y, Z)$  to the magnet EFB,  $\lambda(r)$  an  $r$ -dependent characteristic  
 10727 extent of the field fall-off (e.g., representing a radial dependence of dipole gap height  
 10728  $\text{gap}(r)$ , such that  $\lambda(r) \approx \text{gap}(r)$ ). The latter allows modeling the  $r$ -dependence of the  
 10729 flutter and its effect on vertical focusing.

#### 10730 *Linear approach*

10731 The first order transport matrix of a sector dipole with curvature radius  $\rho$ , deflection  
 10732  $\alpha$  and index  $n$ , in the hard-edge model, writes

$$T_{\text{bend}} = \begin{pmatrix} C_x & S_x & 0 & 0 & 0 & \frac{r_x^2}{\rho}(1 - C_x) \\ C'_x & S'_x & 0 & 0 & 0 & \frac{1}{\rho} S_x \\ 0 & 0 & C_y & S_y & 0 & 0 \\ 0 & 0 & C'_y & S'_y & 0 & 0 \\ \frac{1}{\rho} S_x & \frac{r_x^2}{\rho}(1 - C_x) & 0 & 0 & 1 & \frac{r_x^3}{\rho^2}(\rho\alpha - S_x) \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \text{ with } \begin{cases} C = \cos \frac{\rho\alpha}{r} \\ C' = \frac{dC}{ds} = \frac{1}{\rho} \frac{dC}{d\alpha} = -\frac{S}{r^2} \\ S = r \sin \frac{\rho\alpha}{r} \\ S' = \frac{dS}{ds} = \frac{1}{\rho} \frac{dS}{d\alpha} = C \\ (*)_x : r = \rho / \sqrt{1 - n} \\ (*)_y : r = \rho / \sqrt{n} \end{cases} \quad (14.5)$$

10733 or, explicitly,

$$T_{\text{bend}} = \begin{pmatrix} \cos \sqrt{1-n}\alpha & \frac{\rho}{\sqrt{1-n}} \sin \sqrt{1-n}\alpha & 0 & 0 & 0 & \frac{\rho}{1-n} (1 - \cos \sqrt{1-n}\alpha) \\ -\frac{\sqrt{1-n}}{\rho} \sin \sqrt{1-n}\alpha & \cos \sqrt{1-n}\alpha & 0 & 0 & 0 & \frac{1}{\sqrt{1-n}} \sin \sqrt{1-n}\alpha \\ 0 & 0 & \cos \sqrt{n}\alpha & \frac{\rho}{\sqrt{n}} \sin \sqrt{n}\alpha & 0 & 0 \\ 0 & 0 & -\frac{\sqrt{n}}{\rho} \sin \sqrt{n}\alpha & \cos \sqrt{n}\alpha & 0 & 0 \\ \frac{1}{\sqrt{1-n}} \sin \sqrt{1-n}\alpha & \frac{\rho}{1-n} (1 - \cos \sqrt{1-n}\alpha) & 0 & 0 & 1 & \rho \frac{\sqrt{1-n}\alpha - \sin \sqrt{1-n}\alpha}{(1-n)^{3/2}} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad (14.6)$$

10734 Cancel the index in the previous sector dipole, introduce a wedge angle  $\varepsilon$  at entrance  
 10735 and exit EFBs, introduce the flutter term  $\psi$  to account for dependence of vertical  
 10736 focusing on fringe field extent (see Sect. 14.4.1, Eq. 14.20). The first order transport  
 10737 matrix, accounting for the entrance and exit EFB wedge focusing, then writes

$$T_{\text{bend}} = \begin{pmatrix} \frac{\cos(\alpha-\varepsilon)}{\cos \varepsilon} & \frac{\rho \sin \alpha}{\sin(\alpha-2\varepsilon)} & 0 & 0 & 0 & \frac{\rho(1-\cos \alpha)}{\cos \varepsilon} \\ -\frac{\rho \cos^2 \varepsilon}{\cos(\alpha-\varepsilon)} & \frac{\cos(\alpha-\varepsilon)}{\cos \varepsilon} & 0 & 0 & 0 & \frac{\sin(\alpha-\varepsilon) + \sin \varepsilon}{\cos \varepsilon} \\ 0 & 0 & 1 - \alpha \tan(\varepsilon - \psi) & \rho \alpha & 0 & 0 \\ 0 & 0 & -\frac{\tan(\varepsilon - \psi)}{\rho} (2 - \alpha \tan(\varepsilon - \psi)) & 1 - \alpha \tan(\varepsilon - \psi) & 0 & 0 \\ \sin \alpha & 0 & 0 & 0 & 1 & \rho(\alpha - \sin \alpha) \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad (14.7)$$

10738

### 10739 14.3.2 Dipole Magnet, Straight

10740 This is the MULTIPOL element. Lines of constant field in the magnet body are  
 10741 straight lines. An early instance of a straight dipole magnet is the AGS main dipole  
 10742 (Fig. 9.2), which combines steering and focusing, and features in addition a small sex-  
 10743 tupole defect component [7]. The multipole components  $B_n(X, Y, Z)$  [ $n=1$  (dipole),  
 10744 2 (quadrupole), 3 (sextupole), ...] in the Cartesian frame of the straight dipole derive,  
 10745 by differentiation, from the scalar potential

$$V_n(X, Y, Z) = (n!)^2 \left( \sum_{q=0}^{\infty} (-1)^q \frac{\mathcal{G}^{(2q)}(X)(Y^2 + Z^2)^q}{4^q q!(n+q)!} \right) \left( \sum_{m=0}^n \frac{\sin\left(\frac{m\pi}{2}\right) Y^{n-m} Z^m}{m!(n-m)!} \right) \quad (14.8)$$

10746 where  $\mathcal{G}^{(2q)}(X) = d^{2q} \mathcal{G}(X) / dX^{2q}$ . In the case of pure dipole field for instance

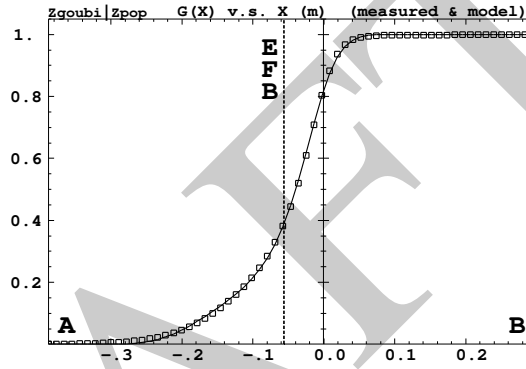
$$V_1(X, Y, Z) = \mathcal{G}(X) Z - \frac{\mathcal{G}''(X)}{8} (Y^2 + Z^2) + \frac{\mathcal{G}^{(4)}(X)}{512} (Y^2 + Z^2) Z \dots \quad (14.9)$$

10747 and

$$\begin{aligned}
 B_X(X, Y, Z) &= -\frac{\partial V_1}{\partial X} = \mathcal{G}'(X)Z - \frac{\mathcal{G}'''(X)}{8}(Y^2 + Z^2) \dots \\
 B_Y(X, Y, Z) &= -\frac{\partial V_1}{\partial Y} = -\frac{\mathcal{G}''(X)}{4}Y + \frac{\mathcal{G}^{(4)}(X)}{256}YZ \dots \\
 B_Z(X, Y, Z) &= -\frac{\partial V_1}{\partial Z} = \mathcal{G}(X) - \frac{\mathcal{G}''(X)}{4}Z + \frac{\mathcal{G}^{(4)}(X)}{512}(Y^2 + 3Z^2) \dots \quad (14.10)
 \end{aligned}$$

10748 The longitudinal form factor  $\mathcal{G}(X)$  accounts for the field fall-offs at the ends of the magnet,  
 10749 it is modeled using the Enge model discussed in Sect. 14.3.3.

**Fig. 14.3** Longitudinal field form factor  $\mathcal{G}(X)$  (normalized to one) in BNL AGS main bend, taken along the magnet reference axis. Solid line: from Eqs. 14.11, 14.12; square markers : measured field data.  $X = 0$  is the origin in the field map frame, the vertical dashed line at  $X_{\text{EFB}} = -5.62$  cm is the location of the EFB



### 10750 14.3.3 Fringe Field, Modeling, Overlapping

10751 A fringe field model is described here, which is resorted to in several optical elements  
 10752 of zgoubi's library.

10753 Field shape at the EFBs of magnetic or electrostatic devices can be simulated  
 10754 using a hard-edge model (the field is assumed to change following a Heaviside step).  
 10755 When using stepwise ray-tracing techniques however, a smooth change of the field  
 10756 can accurately be accounted for. An efficient model is Enge's field form factor [6]

$$F(d) = \frac{1}{1 + \exp P(d)} \quad (14.11)$$

$$P(d) = C_0 + C_1 \left(\frac{d}{\lambda}\right) + C_2 \left(\frac{d}{\lambda}\right)^2 + C_3 \left(\frac{d}{\lambda}\right)^3 + C_4 \left(\frac{d}{\lambda}\right)^4 + C_5 \left(\frac{d}{\lambda}\right)^5$$

10757 where  $d$  is the distance to the field boundary, and  $\lambda \approx$  gap aperture is the extent of  
 10758 the fall-off. The latter is normally commensurate with gap aperture in a dipole, or  
 10759  $r_{\text{pole tip}}/(n-1)$  in a multipole ( $n = 2, 3, \dots$  for quadrupole, sextupole...).

10760 As an illustration, Fig. 14.3 shows  $F(d)$  as matched to the measured end fields of  
10761 BNL AGS main magnet [8, 9], using

$$\lambda = \text{gap aperture} \approx 10 \text{ cm} \quad \text{and} \quad (14.12)$$

$$C_0 = 0.45473, C_1 = 2.4406, C_2 = -1.5088, C_3 = 0.7335, C_4 = C_5 = 0$$

10762 These  $C_i$  coefficient values result from an interpolation to measured field data, which  
10763 are also represented in the figure. The location of the EFB results from the following  
10764 constraint, which is part of the matching: the field integral on the down side of the  
10765 fall-off (the region from A to  $X=0$  in Fig. 14.3) is equal to the complement to 1 of  
10766 the field integral on the rising side of the fall-off ( $X=0$  to B region in the figure),  
10767 which writes

$$\int_{X_A}^{X_{\text{EFB}}} F(X) dX = \int_{X_{\text{EFB}}}^{X_B} dX - \int_{X_{\text{EFB}}}^B F(X) dX \Rightarrow X_{\text{EFB}} = X_B - \int_A^B F(X) dX \quad (14.13)$$

10768 A convenient property of this model is that changing the slope of the fall-off (*i.e.*,  
10769 changing  $\lambda$ ) will not affect the location of the EFB.

10770 Inward fringe field extents may overlap when simulating an optical element  
10771 (Fig. 14.4). A way to ensure continuity of the resulting field form factor in such  
10772 case is to use

$$F = F_E + F_S - 1 \quad \text{or} \quad F = F_E * F_S \quad (14.14)$$

10773 where  $F_E$  ( $F_S$ ) is the entrance (exit) form factor and follows Eq. 14.11. Both expres-  
10774 sions can be extended to more than two EFBs (for instance 4, to account for the 4  
10775 faces of a dipole magnet: entrance and exit faces, inner and outer radial boundaries).  
10776 Note that in that case of overlapping field extents, the field integral is affected, de-  
10777 creasing with more pronounced overlapping, it is therefore necessary to change the  
10778 field value ( $B_0$  in Eq. 14.4 for instance) to recover the proper integrated strength.

### 10779 Overlapping Fringe Fields

10780 Zgoubi allows a superposition technique to simulate the field in a series of neighbor-  
10781 ing magnets. The method consists in computing the mid-plane field at any location  
10782 ( $r, \theta$ ) by adding individual contributions, namely [5]

$$B_Z(r, \theta) = \sum_{i=1, N} B_{Z,i}(r, \theta) = \sum_{i=1, N} B_{Z,0,i} \mathcal{F}_i(r, \theta) \mathcal{R}_i(r) \\ \frac{\partial^{k+l} \mathbf{B}_Z(r, \theta)}{\partial \theta^k \partial r^l} = \sum_{i=1, N} \frac{\partial^{k+l} \mathbf{B}_{Z,i}(r, \theta)}{\partial \theta^k \partial r^l} \quad (14.15)$$

10783 with  $\mathcal{F}_i(r, \theta)$  and  $\mathcal{R}_i(r)$  taken independently for each individual dipole in the series  
10784 (for instance as per Eqs. 10.7, 10.15). Note that, in doing so it is not meant that field  
10785 superposition would apply in reality (FFAG magnets are closely spaced, cross-talk

10786 may occurs), however it appears to allow closely reproducing magnet computation  
10787 code outcomes.

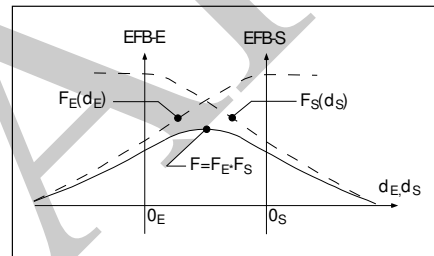
### 10788 Short Optical Elements

10789 In some cases, an optical element in which fringe fields are taken into account (of  
10790 any kind: dipole, multipole, electrostatic, etc.) may be given small enough a length,  
10791  $L$ , that it finds itself in the configuration schemed in Fig. 14.4: the entrance and/or  
10792 the exit EFB field fall-off extends inward enough that it overlaps with the other EFB's  
10793 fall-off. In zgoubi notations, this happens if  $L < X_E + X_S$ . As a reminder [1]: in  
10794 the presence of fringe fields,  $X_E$  (resp.  $X_S$ ) is the stepwise integration extent added  
10795 upstream (resp. added downstream) of the actual extent  $L$  of the optical element.

10796 In such case, zgoubi computes field and derivatives along the element using a  
10797 field form factor  $F = F_E \times F_S$ .  $F_E$  (respectively  $F_S$ ) is the value of the Enge model  
10798 coefficient (Eq. 14.11) at distance  $d_E$  (resp.  $d_S$ ) from the entrance (resp. exit) EFB.

10799 This may have the immediate effect, apparent in Fig. 14.4, that the integrated  
10800 field is not the expected value  $B \times L$  from the input data  $L$  and  $B$ , and may require  
10801 adjusting (increasing)  $B$  so to recover the required  $\int B dl$ .

**Fig. 14.4** A sketch of overlapping entrance field form factor  $F_E(d_E)$  (at the entrance "EFB-E") and exit  $F_S(d_S)$  (at the exit "EFB-S"). The resulting form factor  $F = F_E \times F_S$  is actually accounted for in modeling the field



### 10802 14.3.4 Toroidal Condenser

10803 This is the ELCYLDEF element in zgoubi. With proper parameters, it can be used  
10804 as a spherical, a toroidal or a cylindrical deflector.

Motion along the optical axis, an arc of a circle of radius  $r$  normal to electric field  $\mathbf{E}$ , satisfies

$$Er = v \frac{p}{q} = v(B\rho)$$

10805 with  $p = mv$  the particle momentum,  $q$  its charge and  $(B\rho) = p/q$  the particle  
10806 rigidity.

10807 The first order transport matrix of an electrostatic bend writes



$$T_{\text{condenser}} = \begin{pmatrix} C_x & S_x & 0 & 0 & 0 & \frac{2-\beta^2}{p_x^2} r_0 (1-C_x) \\ C'_x & S'_x & 0 & 0 & 0 & \frac{2-\beta^2}{r_0} S_x \\ 0 & 0 & C_y & S_y & 0 & 0 \\ 0 & 0 & C'_y & S'_y & 0 & 0 \\ -\frac{2-\beta^2}{r_0} S_x & -\frac{2-\beta^2}{p_x^2} r_0 (1-C_x) & 0 & 0 & 1 & r_0 \alpha \left[ \frac{1}{\gamma^2} - \left( \frac{2-\beta^2}{p_x^2} \right)^2 \left( 1 - \frac{S_x}{r_0 \alpha} \right) \right] \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad (14.16)$$

$$\text{with } \begin{cases} \alpha = \text{deflection angle} \\ C = \cos p\alpha \\ C' = \frac{dC}{ds} = -\frac{p^2}{r^2} S \\ S = \frac{r}{p} \sin p\alpha \\ S' = \frac{dS}{ds} = C \\ (*)_x : p = p_x = \sqrt{2 - \beta^2 - r_0/R_0} \\ (*)_y : p = p_y = \sqrt{r_0/R_0} \end{cases}$$

## 10808 14.4 Focusing

10809 Particle beams are maintained confined along a reference propagation axis by means  
10810 of focusing techniques and devices. Methods available in zgoubi to simulate those  
10811 are addressed here.

### 10812 14.4.1 Wedge Focusing

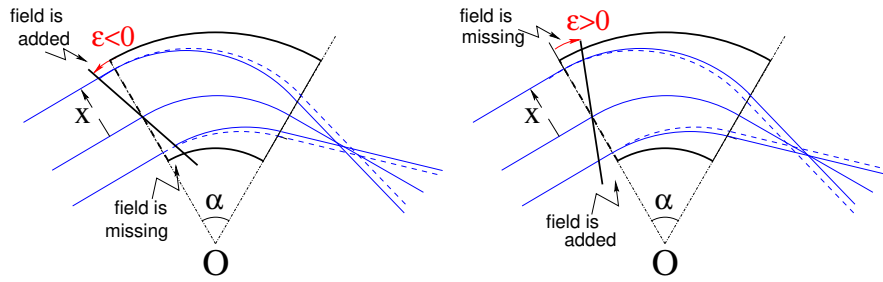
10813 Wedge focusing is sketched in Fig. 14.5. A wedge angle  $\varepsilon$  causes a particle at  
10814 local excursion  $x$  to experience a change  $\int B_y ds = x B_y \tan \varepsilon$  in the field integral,  
10815 compared to the field integral through the sector magnet. In the linear approximation  
10816 this causes a change in trajectory angle

$$\Delta x' = \frac{1}{B\rho} \int B_y ds = x \frac{\tan \varepsilon}{\rho_0} \quad (14.17)$$

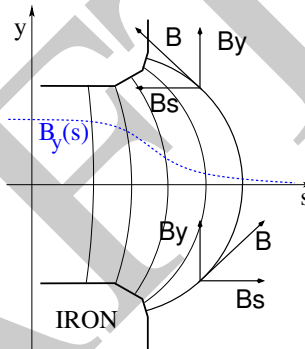
10817 with  $B\rho$  the particle rigidity and  $\rho_0$  its trajectory curvature radius in the field  $B_0$   
10818 of the dipole. Vertical focusing results from the non-zero off-mid plane radial field  
10819 component  $B_x$  in the fringe field region (Fig. 14.7): from (Maxwell's equations)  
10820  $\frac{\partial}{\partial y} \int B_x ds = \frac{\partial}{\partial x} \int B_y ds$  and Eq. 14.17 the change in trajectory angle comes out to  
10821 be

$$\Delta y' = \frac{1}{B\rho} \int B_x ds = -y \frac{\tan \varepsilon}{\rho_0} \quad (14.18)$$

10822 A first order correction  $\psi$  to the vertical kick accounts for the fringe field extent  
10823 (it is a second order effect for the horizontal kick):

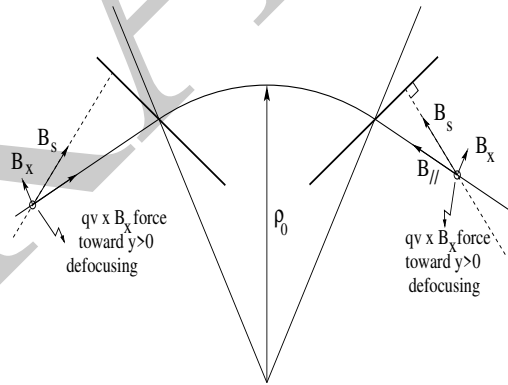


**Fig. 14.5** Left: a focusing wedge ( $\epsilon < 0$  by convention); opening the sector increases the horizontal focusing. Right: a defocusing wedge ( $\epsilon > 0$ ); closing the sector decreases the horizontal focusing. The effect is the opposite in the vertical plane, opening/closing the sector decreases/increases the vertical focusing



**Fig. 14.6** Field components in the  $B_y(s)$  field fall-off at a dipole EFB

**Fig. 14.7** Field components off mid-plane, in the fringe field region at the ends of a dipole ( $y > 0$ , here, referring to Fig. 14.6).  $B_{//}$  parallel to the particle velocity has no effect.  $B_x$  pulls a positively charged particle away from the median plane, under the effect of a  $\mathbf{v} \times \mathbf{B}_x$  force component. Inspection of the  $y < 0$  region gives the same result: the charge is pulled away from the median plane



$$\Delta y' = -y \frac{\tan(\varepsilon - \psi)}{\rho_0} \quad (14.19)$$

10824 with

$$\psi = I_1 \frac{\lambda}{\rho_0} \frac{1 + \sin^2 \varepsilon}{\cos \varepsilon} \quad \text{with} \quad I_1 = \int_{\text{edge}} \frac{B(s)(B_0 - B(s))}{\lambda B_0^2} ds \quad (14.20)$$

10825  $\lambda$  is the fringe field extent,  $I_1$  quantifies the flutter (see Sect. 4.2.1); a longer/shorter  
10826 field fall-off (smaller/greater flutter) decreases/increases the vertical focusing.

10827 *Linear approach*

10828 A wedge focusing first order transport matrix writes

$$T_{\text{wedge}} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{\tan \varepsilon}{\rho} & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -\frac{\tan \varepsilon}{\rho} & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad (14.21)$$

10829 Substitute  $\varepsilon - \psi$  to  $\varepsilon$  in the  $R_{43}$  coefficient, when accounting for fringe field extent  $\lambda$ .

## 10830 14.4.2 Quadrupole

10831 Quadrupoles are the optical lenses of charged particle beams, they ensure confine-  
10832 ment of the beam in the vicinity of the optical axis. Most of the time in beam lines and  
10833 cyclic accelerators, guiding and focusing are separate functions, focusing is assured  
10834 by quadrupoles, magnetic most frequently, possibly electrostatic at low energy.

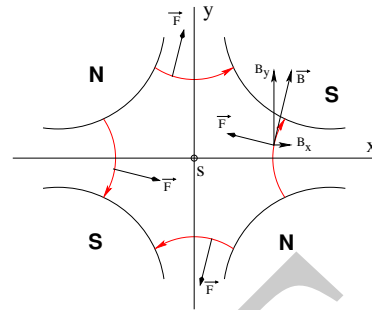
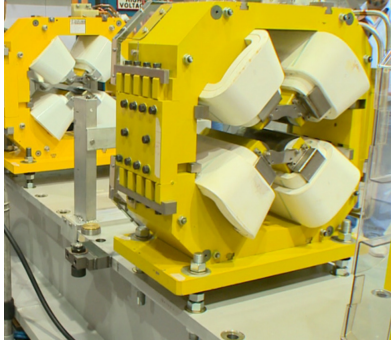
10835 The field in quadrupole lenses results from hyperbolic equipotentials,  $V = axy$ .  
10836 Pole profiles follow these equipotentials, in a  $2\pi/4$ -symmetrical arrangement for  
10837 technological simplicity.

### 10838 14.4.2.1 Magnetic Quadrupole

10839 Magnetic quadrupoles are the optical lenses of high energy beams.

10840 The theoretical field in a quadrupole can be derived from Eq. 14.8 for the scalar  
10841 potential, with  $n = 2$  which yields

$$V_2(X, Y, Z) = \mathcal{G}(X)YZ - \frac{\mathcal{G}''(X)}{12} (Y^2 + Z^2)YZ + \frac{\mathcal{G}^{(4)}(X)}{384} (Y^2 + Z^2)^2YZ - \dots \quad (14.22)$$



**Fig. 14.8** Left: a quadrupole magnet [11]. Right: field lines and forces (assuming positive charges moving out of the page) over the cross section of an horizontally focusing / vertically defocusing quadrupole

10842 and

$$B_X(X, Y, Z) = -\frac{\partial V_2}{\partial X} = \mathcal{G}'(X)YZ - \frac{\mathcal{G}'''(X)}{12}(Y^2 + Z^2)YZ + \dots \quad (14.23)$$

$$B_Y(X, Y, Z) = -\frac{\partial V_2}{\partial Y} = \mathcal{G}(X)Z - \frac{\mathcal{G}''(X)}{12}(3Y^2 + Z^2)Z + \dots \quad (14.24)$$

$$B_Z(X, Y, Z) = -\frac{\partial V_2}{\partial Z} = \mathcal{G}(X)Y - \frac{\mathcal{G}''(X)}{12}(Y^2 + 3Z^2)Y + \dots \quad (14.25)$$

10843  $\mathcal{G}(X)$  is given by Eq. 14.4 whereas

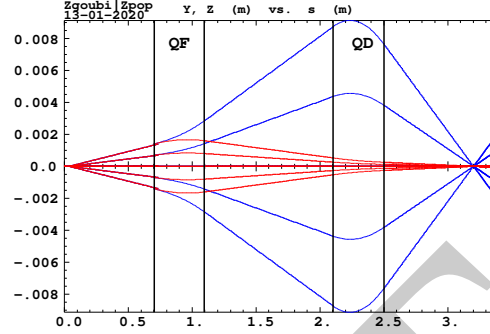
$$G_0 = \frac{B_0}{a} \quad \text{and} \quad K = \frac{G_0}{B\rho} \quad (14.26)$$

10844 define respectively the quadrupole gradient and strength, the latter relative to the  
 10845 rigidity  $B\rho$ . The quadrupole is horizontally focusing and vertically defocusing if  
 10846  $K > 0$ , and the reverse if  $K < 0$ , this is illustrated in Fig. 14.9 which shows the effect  
 10847 of a doublet of quadrupoles with focusing strengths of opposite signs.

10848 *Linear approach*

10849 The first order transport matrix of a quadrupole with length  $L$ , gradient  $G$  and  
 10850 strength  $K = G/B\rho$  writes

**Fig. 14.9** Horizontal and vertical projections of particle trajectories across a stigmatic quadrupole doublet. The first quadrupole (QF) is horizontally focusing ( $K > 0$ ; thus vertically defocusing), the second one (QD) has reverse sign ( $K < 0$ ) and reverse effect



$$T_{\text{quad}} = \begin{pmatrix} C_x & S_x & 0 & 0 & 0 & 0 \\ C'_x & S'_x & 0 & 0 & 0 & 0 \\ 0 & 0 & C_y & S_y & 0 & 0 \\ 0 & 0 & C'_y & S'_y & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \frac{L}{\gamma^2} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{with} \quad \begin{cases} C_x = \cos L\sqrt{K}; C'_x = \frac{dC_x}{dL} = -KS_x \\ S_x = \frac{1}{\sqrt{K}} \sin L\sqrt{K}; S'_x = \frac{dS_x}{dL} = C_x \\ C_y = \cosh L\sqrt{K}; C'_y = \frac{dC_y}{dL} = KS_y \\ S_y = \frac{1}{\sqrt{K}} \sinh L\sqrt{K}; S'_y = \frac{dS_y}{dL} = C_y \end{cases} \quad (14.27)$$

10851  $K > 0$  for a focusing quadrupole (by convention, in the  $(x, x')$  plane, thus defocusing  
 10852 in the  $(y, y')$  plane). Permute the horizontal and vertical  $2 \times 2$  sub-matrices in the  
 10853 case of a *defocusing* quadrupole.

10854 **14.4.2.2 Electrostatic Quadrupole**

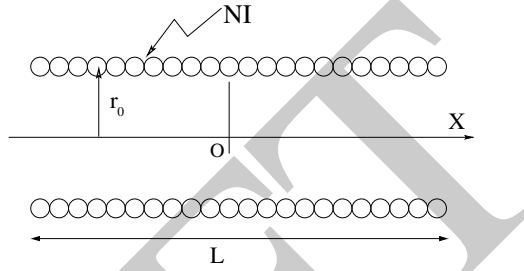
10855 The hypotheses are those of Sect. 2.2.2: paraxial motion, field normal to velocity, etc.  
 10856 Take the notations of Eqs. 2.25, 2.26 for the field and potential, case of electrodes  
 10857 in the horizontal and vertical planes (Fig. 2.14). Electrode potential is  $\pm V/2$ , pole  
 10858 tip radius  $a$ , so that  $K = -V/2a^2$  in Eq. 2.26. The equations of motion then write

$$\begin{cases} \frac{d^2x}{ds^2} + K_x x = 0 \\ \frac{d^2y}{ds^2} + K_y y = 0 \end{cases} \quad \text{with} \quad K_x = -K_y = \frac{-qV}{a^2 m v^2} = \pm \frac{V}{a^2} \underbrace{\frac{1}{|E\rho|}}_{\text{electrical rigidity}} \quad (14.28)$$

10859 The transport matrix is the same as for the magnetic quadrupole, Eq. 14.27, taken  
 10860 for that  $K$  value.

10861 **14.4.3 Solenoid**

10862 Assume a solenoid magnet with longitudinal axis (OX). In a cylindrical frame  
 10863 ( $O; X, r, \phi$ ), Fig. 14.10 ( $r$  is the radial coordinate, the angle  $\phi$  is taken in the X-  
 10864 normal plane),  $B_\phi(X, r, \phi) \equiv 0$ . Take solenoid length  $L$ , mean coil radius  $r_0$  and an  
 10865 asymptotic field  $B_0 = \mu_0 NI/L$ , with  $NI$  = number of ampere-Turns,  $\mu_0 = 4\pi \times 10^{-7}$ .  
 The asymptotic field value is defined by



**Fig. 14.10** A sketch of a solenoid, and quantities used to define it

10866

$$\int_{-\infty}^{\infty} B_X(X, r < r_0) dX = \mu_0 NI = B_0 L \quad \text{independent of } r \quad (14.29)$$

10867 There is a variety of methods to compute the field vector  $\mathbf{B}(X, r)$ . Opting for one  
 10868 in particular may be a matter of compromise between computing speed and field  
 10869 modeling accuracy. A simple model is the on-axis field

$$B_X(X, r = 0) = \frac{B_0}{2} \left[ \frac{L/2 - X}{\sqrt{(L/2 - X)^2 + r_0^2}} + \frac{L/2 + X}{\sqrt{(L/2 + X)^2 + r_0^2}} \right] \quad (14.30)$$

10870 with  $X = r = 0$  taken at the center of the solenoid. This model assumes that the coil  
 10871 thickness is small compared to its mean radius  $r_0$ . The magnetic length comes out  
 10872 to be

$$L_{\text{mag}} \equiv \frac{\int_{-\infty}^{\infty} B_X(X, r < r_0) dX}{B_X(X = r = 0)} = L \sqrt{1 + \frac{4r_0^2}{L^2}} > L \quad (14.31)$$

so satisfying

$$\text{on-axis } B_X(X = r = 0) = \frac{\mu_0 NI}{L \sqrt{1 + \frac{4r_0^2}{L^2}}} \xrightarrow{r_0 \ll XL} \frac{\mu_0 NI}{L}$$

10873 Maxwell's equations and Taylor expansions provide the off-axis field  $\mathbf{B}(X, r) =$   
 10874 ( $B_X(X, r), B_r(X, r)$ ). One has in particular in the  $r_0 \ll XL$  limit,

$$B_X(X, r) = \frac{\mu_0 NI}{L} \quad \text{and} \quad B_r(X, r) = \frac{-r}{2} \frac{dB_X}{dX} \quad (14.32)$$

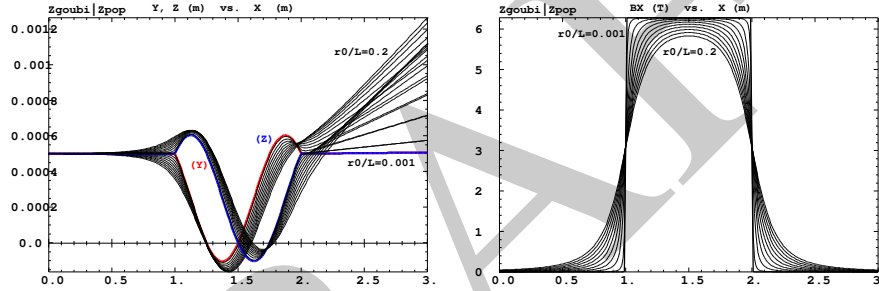
10875 An other way to compute the field vector  $\mathbf{B}(X, r)$  is the elliptic integrals technique  
 10876 developed in [12], which constructs  $B_X(X, r)$  and  $B_r(X, r)$  from respectively

$$B_X(X, r) = \frac{\mu_0 NI}{4\pi} \frac{ck}{r} \mathcal{X} \left[ K + \frac{r_0 - r}{2r_0} (\Pi - K) \right] \quad (14.33)$$

$$B_r(X, r) = \mu_0 NI \frac{1}{k} \sqrt{\frac{r_0}{r}} \left[ 2(K - E) - k^2 K \right]$$

wherein  $K$ ,  $E$  and  $\Pi$  are the three complete elliptic integrals,  $\mathcal{X}$  is an  $X$ - and  $L$ -dependent form factor, and

$$k = 2\sqrt{r_0 r} / \sqrt{(r_0 + r)^2 + \mathcal{X}^2}; \quad c = 2\sqrt{r_0 r} / (r_0 + r)$$



**Fig. 14.11** Left: Horizontal (Y) and vertical (Z) projections of a particle trajectory across a  $L = 1$  m solenoid, with additional 1 m extents upstream and downstream of the coil to account for the extended field fall-offs. The particle is launched with zero incidence, from transverse position  $Y = Z = 0.5$  mm. Sample solenoid radius/length values in the range  $0.001 \leq r_0/L \leq 0.2$  show that only for smallest  $r_0/L = 0.001$  does the trajectory end with  $Y = Z = 0.5$  mm and quasi-zero incidence (the thicker Y(X) and Z(X) curves), whereas greater  $r_0/L$  causes final Y(X) and Z(X) to be offset. Right: field  $B_X(X, r)$  experienced along the trajectory for the various  $r_0/L$  values, the steep fall-off case is for  $r_0/L = 0.001$

10877 As an illustration, Fig. 14.11 displays a trajectory across a  $L = 1$  m solenoid  
 10878 and its field fall-offs, and the field experienced along that trajectory, in the  
 10879 axial model of Eq. 14.30. In the paraxial approximation, a pitch requires a distance  
 10880  $l = 2\pi/K$ , with  $K = B_0/B\rho$  the solenoid strength, which is a condition satisfied here  
 10881 if the fringe field extent is short enough (solenoid radius  $r_0$  is small enough).

10882 *Linear approach*

10883 The equations of motion write, to the first order in the coordinates, in respectively  
10884 the central region (field  $B_s$ ) and at the ends (at  $s = s_{\text{EFB}}$ ),

$$\begin{cases} x'' - K z' = 0 \\ z'' + K x' = 0 \end{cases} \quad \text{and} \quad \begin{cases} x'' - \frac{K}{2} z \delta(s - s_{\text{EFB}}) = 0 \\ z'' + \frac{K}{2} x \delta(s - s_{\text{EFB}}) = 0 \end{cases} \quad (14.34)$$

10885 The first order transport matrix of a solenoid with length  $L$  writes

$$T_{\text{sol}} = \begin{pmatrix} C^2 & \frac{2}{K} SC & SC & \frac{2}{K} S^2 & 0 & 0 \\ -\frac{K}{2} SC & C^2 & -\frac{K}{2} S^2 & SC & 0 & 0 \\ -SC & -\frac{2}{K} S^2 & C^2 & \frac{2}{K} SC & 0 & 0 \\ \frac{K}{2} S^2 & -SC & -\frac{K}{2} SC & C^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \frac{L}{\gamma^2} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{with} \quad \begin{cases} K = \frac{B_s}{B\rho} \\ C = \cos \frac{KL}{2} \\ S = \sin \frac{KL}{2} \end{cases} \quad (14.35)$$

10886 A solenoid rotates the decoupled axis longitudinally by an angle  $\alpha = KL/2 =$   
10887  $B_s L / 2B\rho$ .

## 10888 14.5 Data Treatment Keywords

### 10889 14.5.1 Concentration Ellipse: FAISCEAU, FIT[2], MCOBJET, ...

10890 It is often useful to associate the projection of a particle bunch in the horizontal,  
10891 vertical or longitudinal phase space with an *rms* phase space concentration ellipse  
10892 (CE). Various keywords in *zgoubi* resort to concentration ellipses:

- 10893 - FAISCEAU for instance prints out, in *zgoubi.res*, CE parameters drawn from
- 10894 particle coordinates,
- 10895 - random particle distributions by MCOBJET are defined using CE parameters,
- 10896 - ellipse parameters computed from CEs are possible constraints in FIT[2] pro-
- 10897 cedures.

10898 Transverse phase space graphs by *zpop* also compute CEs.

10899 The CE method is resorted to in various exercises, for instance for comparison  
10900 of the ellipse parameters it gets from the *rms* matching of a bunch, with theoretical  
10901 beam parameters derived from first order transport formalism (such as computed  
10902 from rays by MATRIX, or TWISS).

10903 The CE method used in these various keywords and data treatment procedures is  
10904 the following. Let  $z_i(s)$ ,  $z'_i(s)$  be the phase space coordinates of  $i = 1, n$  particles in  
10905 a set observed at some azimuth  $s$  along an optical sequence. The second moments  
10906 of the particle distribution are



$$\begin{aligned}\overline{z^2}(s) &= \frac{1}{n} \sum_{i=1}^n (z_i(s) - \overline{z}(s))^2 \\ \overline{zz'}(s) &= \frac{1}{n} \sum_{i=1}^n (z_i(s) - \overline{z}(s))(z'_i(s) - \overline{z'}(s)) \\ \overline{z'^2}(s) &= \frac{1}{n} \sum_{i=1}^n (z'_i(s) - \overline{z'}(s))^2\end{aligned}\quad (14.36)$$

10907 From these, a concentration ellipse is defined, encompassing a surface  $S_z(s)$ , with  
10908 equation

$$\gamma_c(s)z^2 + 2\alpha_c(s)zz' + \beta_c(s)z'^2 = S_z(s)/\pi \quad (14.37)$$

10909 Noting  $\Delta = \overline{z^2}(s)\overline{z'^2}(s) - \overline{zz'}^2(s)$ , the ellipse parameters write

$$\gamma_c(s) = \frac{\overline{z'^2}(s)}{\sqrt{\Delta}}, \quad \alpha_c(s) = -\frac{\overline{zz'}(s)}{\sqrt{\Delta}}, \quad \beta_c(s) = \frac{\overline{z^2}(s)}{\sqrt{\Delta}}, \quad S_z(s) = 4\pi\sqrt{\Delta} \quad (14.38)$$

10910 With these conventions, the *rms* values of the  $z$  and  $z'$  projected densities satisfy

$$\sigma_z = \sqrt{\beta_z \frac{S_z}{\pi}} \quad \text{and} \quad \sigma_{z'} = \sqrt{\gamma_z \frac{S_z}{\pi}} \quad (14.39)$$

## 10911 14.5.2 Transport Coefficients: MATRIX, OPTICS, TWISS, etc.

10912 Zgoubi does not know about matrix transport, it does not define optical elements  
10913 by a transport matrix, it defines them by electrostatic and/or magnetic fields in  
10914 space (and time possibly). Well, except for a couple of optical elements, for instance  
10915 TRANSMAT, which pushes particle coordinates using a matrix, or SEPARA, an  
10916 analytical mapping through a Wien filter. Zgoubi does not transport particles using  
10917 matrix products either, it does that by numerical integration of Lorentz force equation  
10918 through these  $\mathbf{E}$  and/or  $\mathbf{B}$  fields.

10919 However it is often useful to dispose of a matrix representation of an optical  
10920 element or a beam line, or of paraxial parameters drawn from the first or second order  
10921 one-turn mapping of a ring accelerator. Several commands in zgoubi perform the  
10922 required treatment to derive these informations from particle coordinates. Examples  
10923 are MATRIX: computation of matrix transport coefficients up to 3rd order, from  
10924 initial and current coordinates of a particle sample. OPTICS transports a beam  
10925 matrix, given its initial value using OBJET[KOBJ=5.1] (see Sect. 14.5.2.2). TWISS  
10926 derives a periodic beam matrix from a 1-turn mapping of a periodic sequence, and  
10927 transports it from end to end so generating the optical functions along the sequence  
10928 (Sects. 14.5.2.2, 14.5.2.3).

10929 These capabilities are resorted to in the exercises. It may be required for instance  
10930 to compare transport coefficients derived from raytracing, with the matrix model

10931 of the optical element(s) concerned. Or to compute a periodic beam matrix in a  
 10932 periodic optical sequence, this is how betatron functions are produced, often for the  
 10933 mere purpose of comparisons with matrix code outcomes, or with expectations from  
 10934 analytical models.

#### 10935 14.5.2.1 Coordinate Transport

10936 In the Gauss approximation (*i.e.*, trajectory angle  $\theta \sim \sin \theta$ ), particles follow paths  
 10937 which can be described with simple functions: parabolic, sinusoidal or hyperbolic.  
 10938 A consequence is that a string of optical elements, and coordinate transport through  
 10939 the latter, can be handled with a simple mathematics toolbox. Taylor expansion (also  
 10940 known as transport) techniques are part of it, whereby a coordinate excursion  $v_{2i}$   
 10941 (with index  $i = 1 \rightarrow 6$  standing for  $x, x', y, y', \delta s$  or  $\delta p/p$ ) from some reference  
 10942 trajectory at a location  $s_2$  along the line is obtained from the excursions  $v_{1i}$  at an  
 10943 upstream location  $s_1$ , via

$$v_{2i} = \sum_{j=1}^6 R_{ij} v_{1j} + \sum_{j,k=1}^6 T_{ijk} v_{1j} v_{1k} + \sum_{j,k,l=1}^6 v_{1ijkl} v_{1j} v_{1k} v_{1l} + \dots \quad (14.40)$$

10944 This Taylor development can be written under matrix form, for instance to the first  
 10945 order in the coordinates, for non-coupled motion,

$$\begin{pmatrix} x \\ x' \\ y \\ y' \\ \delta s \\ \delta p/p \end{pmatrix}_2 = \begin{pmatrix} T_{11} & T_{12} & 0 & 0 & 0 & T_{16} \\ T_{21} & T_{22} & 0 & 0 & 0 & T_{26} \\ 0 & 0 & T_{33} & T_{34} & 0 & T_{36} \\ 0 & 0 & T_{43} & T_{44} & 0 & T_{46} \\ 0 & 0 & 0 & 0 & T_{55} & T_{56} \\ 0 & 0 & 0 & 0 & T_{65} & T_{66} \end{pmatrix} \begin{pmatrix} x \\ x' \\ y \\ y' \\ \delta s \\ \delta p/p \end{pmatrix}_1 = T(s_2 \leftarrow s_1) \begin{pmatrix} x \\ x' \\ y \\ y' \\ \delta s \\ \delta p/p \end{pmatrix}_1 \quad (14.41)$$

10946 These are the quantities which such keywords as MATRIX [1, *cf.* Sect. 6.5] and  
 10947 OPTICS [1, *cf.* Sect. 6.4] compute, from particle coordinates. Most of the time they  
 10948 are resorted to for mere comparison with theoretical matrices such as recalled in  
 10949 Sects. 14.2-14.4.

#### 10950 14.5.2.2 Beam Matrix

10951 OPTICS and TWISS keywords cause the transport of a beam matrix. The former  
 10952 requires initial beam ellipse parameters: these are provided as part of the initial  
 10953 object definition, by OBJET. The latter first derives a periodic beam matrix from  
 10954 initial and final particle coordinates resulting from raytracing throughout an optical  
 10955 sequence. Basic principles are recalled here, regarding the way these keywords work  
 10956 in zgoubi. They are resorted to quite often in the exercises.

10957 In the linear approximation, the transverse phase space ellipse associated with a  
 10958 particle distribution (for instance, the concentration ellipse, Sect. 14.5.1) is written  
 10959 (with  $z$  standing for indifferently  $x$  or  $y$ )

$$\gamma_z(s)z^2 + 2\alpha_z(s)zz' + \beta_z(s)z'^2 = \frac{\varepsilon_z}{\pi} \quad (14.42)$$

10960 in which the ellipse parameters

$$\beta_z(s), \alpha_z(s) = -\frac{1}{2} \frac{d\beta_z}{ds}, \gamma_z(s) = \frac{1 + \alpha_z^2}{\beta_z} \quad (14.43)$$

10961 are functions of the observation location  $s$  along the optical sequence. The surface  
 10962  $\varepsilon_z$  of the ellipse is an invariant if the beam travels in magnetic fields, however field  
 10963 non-linearities, phase space dilution, etc. may distort the distribution and change the  
 10964 surface of its *rms* matching concentration ellipse. In the presence of acceleration or  
 10965 deceleration the invariant quantity is  $\beta\gamma\varepsilon_z$  instead, with  $\beta = v/c$  and  $\gamma$  the Lorentz  
 10966 relativistic factor.

10967 The ellipse Eq. 14.42 can be written under the matrix form

$$[z, z'] \sigma_z(s) \begin{bmatrix} z \\ z' \end{bmatrix} = \mathbf{1} \quad (14.44)$$

10968 with  $\sigma_z$  the beam matrix:

$$\sigma_z = \frac{\varepsilon_z}{\pi} \begin{pmatrix} \beta_z(s) & -\alpha_z(s) \\ -\alpha_z(s) & \gamma_z(s) \end{pmatrix} \quad (14.45)$$

10969 The ellipse parameters can be transported from  $s_1$  to  $s_2$  using

$$\sigma_{z,2} = T \sigma_{z,1} \tilde{T} \quad (14.46)$$

10970 with  $T = T(s_2 \leftarrow s_1)$  the transport matrix (Eq. 14.41) and  $\tilde{T}$  its transposed. This can  
 10971 also be written under the form

$$\begin{pmatrix} \beta_z \\ \alpha_z \\ \gamma_z \end{pmatrix}_2 = \begin{pmatrix} T_{11}^2 & -2T_{11}T_{12} & T_{12}^2 \\ -T_{11}T_{21} & T_{21}T_{12} + T_{11}T_{22} & -T_{12}T_{22} \\ T_{21}^2 & -2T_{21}T_{22} & T_{22}^2 \end{pmatrix}_{s_2 \leftarrow s_1} \begin{pmatrix} \beta_z \\ \alpha_z \\ \gamma_z \end{pmatrix}_1 \quad (14.47)$$

10972 (subscripts 1, 2 normally hold for horizontal plane motion,  $z = x$ : change to 3, 4  
 10973 for vertical motion,  $z = y$ ). This beam matrix formalism can be extended to the  
 10974 longitudinal phase space and coordinates  $(\delta s, \delta p/p)$ . Thus a  $6 \times 6$  beam matrix can  
 10975 be defined,

$$\sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} & 0 & 0 & 0 & \sigma_{16} \\ \sigma_{21} & \sigma_{22} & 0 & 0 & 0 & \sigma_{26} \\ 0 & 0 & \sigma_{33} & \sigma_{34} & 0 & \sigma_{36} \\ 0 & 0 & \sigma_{43} & \sigma_{44} & 0 & \sigma_{46} \\ 0 & 0 & 0 & 0 & \sigma_{55} & \sigma_{56} \\ 0 & 0 & 0 & 0 & \sigma_{65} & \sigma_{66} \end{pmatrix} \quad (14.48)$$

10976 This can be generalized to non-zero anti-diagonal terms, if motions are coupled.

### 10977 14.5.2.3 Periodic Structures

10978 In the hypothesis of an  $S$ -periodic structure: a long beam line with repeating pattern,  
10979 a cyclic accelerator, transverse motion stability requires the transport matrix over a  
10980 period, from  $s$  to  $s + S$  to satisfy

$$[T_{ij}](s + S \leftarrow s) = I \cos \mu + J \sin \mu \quad (14.49)$$

10981 where  $\mu = \int_{(s)} ds/\beta$  is the betatron phase advance over the period (independent of  
10982 the origin),

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \text{ is the identity matrix, } J = \begin{pmatrix} \alpha_z(s) & \beta_z(s) \\ -\gamma_z(s) & -\alpha_z(s) \end{pmatrix} \text{ (and } J^2 = -I) \quad (14.50)$$

## 10983 14.6 Exercises

### 10984 14.1 Magnetic Sector Dipole

10985 Solution: page 599.

10986 (a) Simulate a  $\rho = 0.5$  m radius,  $\alpha = 60$  degree sector dipole with  $n=-0.6$  field  
10987 index, in both cases of hard edge and of soft fall-off fringe field model. Find the  
10988 reference arc, such that  $\int_{\text{arc}} B ds = BL$  with  $L$  the arc length in the hard-edge model  
10989 and  $B$  the field along that arc.

10990 Make sure that the reference arc has the expected length.

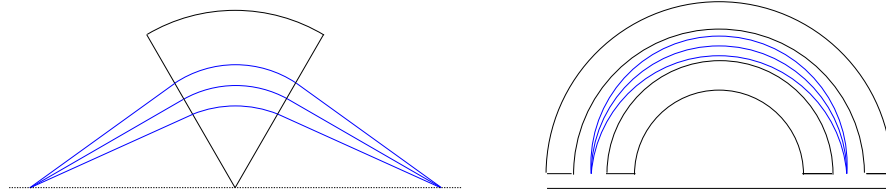
10991 Produce the field along the reference arc, for a few different values of the fringe-  
10992 field extent.

10993 (b) A possible check of the first order: OBJET[KOBJ=5], MATRIX[IORD=1,IFOC=0]  
10994 can be used to compute the transport matrix from the rays. Compare what it gives  
10995 with theory.

10996 (c) Consider a sector dipole with parallel gap, uniform field. Show the well known  
10997 geometrical property of point-to-point focusing represented in Fig. 14.12.

10998 Produce the aberration curve  $x'(x)$  in the horizontal phase-space at the image  
10999 plane.

11000 Test the convergence of the numerical solution versus integration step size.



**Fig. 14.12** Symmetric point to point focusing, case of a 60 deg or a 180 deg sector dipole

11001 (d) Transport a proton along the reference axis, injected with its spin tangent to  
 11002 the axis. Compare spin rotation with theory.

11003 Test the convergence of the numerical solution versus integration step size.

11004 **14.2 Quadrupole Doublet**

11005 Solution: page 604.

11006 Reproduce Fig. 14.9.

11007 **14.3 Solenoid**

11008 Solution: page 605.

11009 An introduction to SOLENOID.

11010 (a) Reproduce Fig. 14.11. Use both field models of Eqs. 14.30, 14.33 and compare  
 11011 their outcomes, including the first order paraxial transport matrices, and some higher  
 11012 order coefficients as well (computed from in and out trajectory coordinates).

11013 (b) Compare final coordinates in (a) with outcomes from the first order transport  
 11014 formalism (Sect. 14.4.3).

11015 (c) Make a 1-dimensional (on-axis) field map of a  $r_0 = 10$  cm,  $L = 1$  m solenoid  
 11016 (namely, a map  $B_{X,i}(X_i)$  of the field at the nodes of a X-mesh with mesh size  
 11017  $X_{i+1} - X_i$ ). Reproduce the trajectory in (a) (case  $r_0 = 10$  cm) using that field map,  
 11018 with the keyword BREVOL. Check the convergence of the final particle coordinates,  
 11019 using the field map, depending on the mesh size.

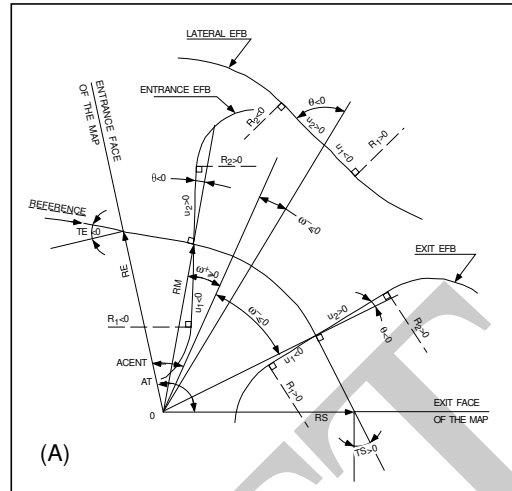
11020 **14.7 Solutions of Exercises of Chapter 3: Optical Elements and**  
 11021 **Keywords, Complements**

11022 **14.1 Magnetic Sector Dipole**

11023 DIPOLE input data.

11024 (a) A simulation of a  $\rho = 0.5$  m radius, 60 degree sector dipole with  $n=-0.6$   
 11025 field index, in the hard-edge field model, is given in Tab. 14.1. A simulation which  
 11026 includes fringe fields is given in Tab. 14.2.

11027 A major difference between the two is in the angular extent of the field domain,  
 11028 AT, in order to allow encompassing the fringe field extents, however there is more,  
 11029 as follows.



**Fig. 14.13** Parameters used to define the geometry of a dipole magnet with index, using DIPOLE [13, see DIPOLE]

11030 *Hard edge model*

The effective field boundaries (EFB) have to be placed on the angular opening limits, which means, in the representation of Fig. 14.13, and according to the users' guide [13, see DIPOLE],

$$\omega^+ = ACENT > 0, \quad \omega^- = -AT + ACENT < 0, \quad \omega^+ - \omega^- = AT > 0$$

11031 Otherwise, in the case AT would be greater than the magnet deflection angle  $\alpha =$   
 11032 60 deg, particles would jump from zero field to plateau field value over the EFB,  
 11033 and so miss part of the field integral. Note that for mere code-specific, geometry  
 11034 computation reasons, it also requires that  $ACENT=AT/2$ , so that, *in fine*,  $\omega^+ =$   
 11035  $-\omega^- = AT/2$ .

11036 *Soft edge model*

AT has to be greater than the magnet deflection angle  $\alpha = 60$  deg in order to encompass the fringe field extent beyond the entrance and exit EFBs, so that, in the representation of Fig. 14.13, and according to the users' guide,

$$ACENT > \omega^+, \quad |\omega^-| < AT - ACENT$$

Integration-wise, particles will smoothly traverse the field fall-off regions, step by step, no field discontinuity there. Note that motion integration accuracy requires the step size to be small enough, compared to the fringe field extent. In the notations of Fig. 14.13, the resulting additional optical axis lengths  $l_E$  and  $l_S$  within the AT sector, on entrance and exit side respectively, to account for the field fall-offs, write

$$l_E = RM \times \tan(ACENT - \omega^+), \quad l_S = RM \times \tan[AT - (ACENT - \omega^-)]$$

Checking back one fortunately finds

$$\underbrace{\operatorname{atan}\left(\frac{l_E}{RM}\right)}_{\substack{\text{entrance} \\ \text{fringe field}}} + \underbrace{\omega^+ - \omega^-}_{\text{magnet body}} + \underbrace{\operatorname{atan}\left(\frac{l_S}{RM}\right)}_{\substack{\text{exit} \\ \text{fringe field}}} = AT$$

It also results from the fringe field modeling that the reference trajectory (which is ideally the trajectory that coincides with  $R=RM$  in the body of the magnet) enters the  $AT$  sector at radius  $RE$ , with an incidence  $TE$ . These two quantities have to be accounted for in setting the entrance and exit reference frames, however this is user's matter, regarding the choice of reference frames: most often (in synchrotron rings for instance) the reference curve is  $R=RM$ , so that  $Y$  and  $T$  coordinates of the reference particle are zero (the moving frame has its origin at the origin of the polar frame in which the field is defined, and rotates with the particle, clockwise in Fig. 14.13 representation). Thus, one has to set

$$TE = -(ACENT - \omega^+) < 0, \quad RE = RM/\cos TE$$

Note that, because of the small deflection due to fringe fields,  $RS$  and  $TS$  need be adjusted if the **DIPOLE** process has to end up with the reference particle featuring zero  $Y$  and  $T$  coordinates. Expectedly, that would be satisfied with  $RS$  and  $TS$  values near

$$TS = AT - (ACENT - \omega^-) > 0, \quad RS = RM/\cos TS$$

11037 The radius  $R$  of the reference arc, such that  $\int_{\text{arc}} B ds = BL$  with  $L$  the arc length in  
 11038 the hard-edge model, has to be found. Same thing for the arcs at  $\pm 0.1\%$  momentum  
 11039 offset. **FIT** can be used for that.

11040 (b) First order transport.

11041 This is left to the reader. Theoretical matrices are given in Eqs. 14.6, 14.7.

11042 Refer to exercises in earlier chapters, such comparison is often performed.

11043 (c) Point-to-point focusing.

11044 The hard-edge model **DIPOLE** of Tab. 14.1 can be used, with the following  
 11045 modifications and addenda in order to simulate the symmetric  $60^\circ$  sector and  
 11046 drifts configuration of Fig. 14.12:

11047 - add **OBJET**[**KOBJ**=1,**IMAX**=41] so to generate 41 particles launched with  
 11048  $T_0 \in [-20, 20]$  mrad, like so:

```

11049 'OBJET'
11050 64.62444403717985
11051 1
11052 1 41 1 1 1 1
11053 0. 1. 0. 0. 0. 0.
```

**Table 14.1** Input data file: definition of a dipole with index in the hard-edge field model. Definition of the [#S\_60dSectDip\_hardE:#E\_60dSectDip\_hardE] segment, mostly for the purpose of possible further INCLUDE. This file is used under the name sectorDIP\_hardE.inc in subsequent exercises

```

! File sectorDIP.inc (hard-edge, here)
'MARKER' #S_60dSectDip_hardE ! Label should not exceed 20 characters.
'DIPOLE' ! Analytical definition of a dipole field.
2 ! IL=2, only purpose is to log trajectories to zgoubi.plt, for further plotting.
60. 50. ! Sector angle AT; reference radius RM.
30. 5. -0.6 0. 0. ! Reference azimuthal angle ACN; BM field at RM; indices, N=-0.6 at RM=50cm.
0. 0. ! EFB 1 is hard-edge,
4 .1455 2.2670 -.6395 1.1558 0. 0. ! hard-edge only possible with sector magnet.
0. 0. ! EFB 2.
4 .1455 2.2670 -.6395 1.1558 0. 0.
-30. 0. 1.E6 -1.E6 1.E6 1.E6
0. 0. ! EFB 3 (unused).
0. 0. 0. 0. 0. 0. 0. 0.
0. 0. 1.E6 -1.E6 1.E6 1.E6 0.
4 10.
0.5 ! Integration step size. The smaller, the more accurately the orbits close.
2 0. 0. 0. 0. ! Magnet positioning RE, TE, RS, TS.
'MARKER' #E_60dSectDip_hardE ! Label should not exceed 20 characters.
'END'

```

**Table 14.2** Input data file: definition of a dipole with index in the soft-edge field model. The field extent in the Enge model (Eq. 14.11) is taken to be  $g = 5$  cm ( $\lambda_E = \lambda_S = g$  in Users' Guide's notations), so subtended by an angle  $\text{atan}(g/RM) = 5.71059$  deg, thus well comprised in a 10 deg angular aperture. ACENT value is free, 30 deg as adopted here is arbitrary, it is just left to the value it was given in the hard edge settings (Tab. 14.1). This input includes the definition of the [#S\_60dSectDip\_softE:#E\_60dSectDip\_softE] segment. This file is used under the name sectorDIP\_softE.inc in subsequent exercises

```

! File sectorDIP.inc (soft-edge, here)
'MARKER' #S_60dSectDip_softE ! Label should not exceed 20 characters.
'DIPOLE' ! Analytical definition of a dipole field.
2 ! IL=2, only purpose is to log trajectories to zgoubi.plt, for further plotting.
80. 50. ! Sector angle AT=60 deg deflection+2*10deg for fringes; reference radius RM.
30. 5. -0.6 0. 0. ! Reference angle ACENT (arbitrary value); field at RM; indices, N=-0.6 at RM=50cm.
5. 0. ! Entry EFB: lambda-gap=5 cm, well comprised in  $RM \cdot \tan(10\text{deg})$ ; same gap at all R -> nappa=0.
4 .1455 2.2670 -.6395 1.1558 0. 0. ! Enge coefficients at entry.
20. 0. 1.E6 -1.E6 1.E6 1.E6 ! omega*+ = +20 deg from ACENT leaves 10deg room (8.8cm) for entry fringe.
5. 0. ! Exit EFB: lambda-gap=5 cm, well comprised in  $RM \cdot \tan(10\text{deg})$ ; same gap at all R -> nappa=0.
4 .1455 2.2670 -.6395 1.1558 0. 0. ! Enge coefficients at exit.
-40. 0. 1.E6 -1.E6 1.E6 1.E6 ! omega*- = -40 deg from ACENT leaves 10deg room (8.8cm) for exit fringe.
0. 0. ! EFB 3 (unused).
0. 0. 0. 0. 0. 0. 0. 0.
0. 0. 1.E6 -1.E6 1.E6 1.E6 0.
4 10.
0.5 ! Integration step size. The smaller, the more accurately the orbits close.
2 0. 0. 0. 0. ! Magnet positioning RE, TE, RS, TS.
'MARKER' #E_60dSectDip_softE ! Label should not exceed 20 characters.
'REBELOTE'
'END'

```

11054 50. 0. 0. 0. 0. 3.8685052339

- 11055 - following OBJET add a drift with length  $RM/\tan(30^\circ) = 86.6025403784$  cm,
- 11056 - in DIPOLE: set the field index to zero,
- 11057 - following DIPOLE add a drift with length  $RM/\tan(30^\circ)$ ,
- 11058 - add AUTOREF[I=3,I1=1,I2=2,I3=3] after DIPOLE: that will cause computation
- 11059 of the location of the waist formed by particles 1, 2 and 3,
- 11060 - add FAISTORE[FNAME=zgoubi.fai,IP=1] after AUTOREF, before END. This
- 11061 logs particle data at that location.

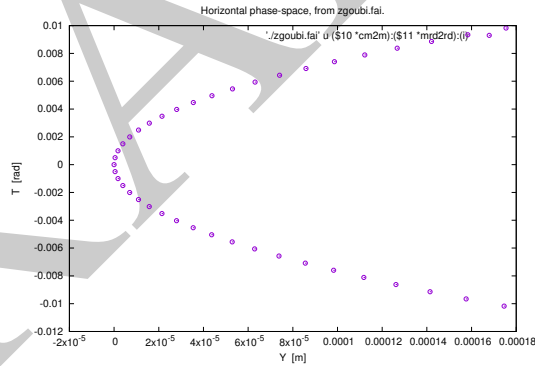
11062 In the execution listing zgoubi.res one finds:



**Table 14.3** Input data file: find closed orbits, using FIT or FIT2, and log stepwise data in zgoubi.plt. Closed orbits are found for the reference particle (a particle with rigidity  $B\rho = 5_{[kG]} \times 50_{[cm]} kG cm$ ) and for particles with  $\pm\delta p/p$  momentum offset. FIT starts with initial  $Y_0$  radius values resulting from a hard edge model, *i.e.*,  $Y_0 = B\rho/B = 250_{[kG cm]}/5_{[kG]}$  and  $\pm 0.1\%$ . This file produces the field along these trajectories, an effect of DIPOLE[IL=2]. The [#S\_60dSectDip\_softE:#E\_60dSectDip\_softE] segment of Tab. 14.2 is INCLUDED; simply substitute [#S\_60dSectDip\_hardE:#E\_60dSectDip\_hardE] (as defined in Tab. 14.1) to work with the hard edge model instead

```
Uniform field sector with index. Field on orbits at different momenta.
'MARKER' DIPOLEfield_S ! Just for edition purposes.
! First stage: find closed orbit at 1 MeV, for some k value.
'OBJET'
64.62444403717985 ! Reference Brho ("BORO" in the users' guide) -> 200keV proton.
2 ! Particles are defined one by one.
3 1 ! 3 particles, classified in a single momenta set.
50. 0. 0. 0. 0. 3.8685052339 'o' ! Y_0=50cm is hard edge case -> 2.9886MeV proton.
50.125472 0. 0. 0. 0. 3.8723737392 'p' ! +0.001 mom. offset. Circular orbit Y_0 is hard edge case.
49.875465 0. 0. 0. 0. 3.8646367287 'm' ! -0.001 mom. offset. Circular orbit Y_0 is hard edge case.
1 1 1 ! As many '1' as there are particles (that dates from programs on punched cards!
'INCLUDE'
1
./sectorDIP.inc[#S_60dSectDip_softE:#E_60dSectDip_softE] ! DIPOLE with fringe, RM=50cm n=-0.6.
./sectorDIP.inc[#S_60dSectDip_hardE:#E_60dSectDip_hardE] ! DIPOLE with hard-edge, RM=50cm n=-0.6.
'FIT' ! This matching procedure finds the closed orbit radius.
3 nofinal
2 30 0 .9 ! Variable : Y_0. Variation allowed up to 90%.
2 40 0 .9 ! Variable : Y_0. Variation allowed up to 90%.
2 50 0 .9 ! Variable : Y_0. Variation allowed up to 90%.
3 1e-15 99 ! Penalty; max numb of calls to the function.
3.1 1 2 #End 0. 1. 0 ! Constraint : Y_final=Y_0, particle 1.
3.1 2 2 #End 0. 1. 0 ! Constraint : Y_final=Y_0, particle 2.
3.1 3 2 #End 0. 1. 0 ! Constraint : Y_final=Y_0, particle 3.
'MARKER' DIPOLEfield_E ! Just for edition purposes.
'END'
```

**Fig. 14.14** Aberration curve at the focal point of a 180 deg uniform field dipole: a second order (sextupole) aberration,  $Y \propto T^2$ , typical of a bend non-linearities



```
11063 6 Keyword, label(s) : AUTOREF
11064 Change of reference, horizontal, XC= -0.00011588 cm, YC = 49.999999 cm, A= -0. deg
11065 TRAJ 1 IEX,D,Y,T,Z,P,S,time : 1 3.869 -1.1786E-16 0. 0. 0. 225.56 9.44931E-02
```

11066 This indicates that AUTOREF confirms expectations: it found the waist  
 11067 - at  $XC = 0$ , which means right at the end of the downstream drift,  
 11068 - at a radial excursion  $YC = 50 cm$  as expected (the origin of the Y axis is at  
 11069 DIPOLE curvature center),

11070 - with the reference frame  $X$  axis at an angle  $A = 0$  to particle 1 direction of  
 11071 motion.  
 11072 QED.

11073 The following gnuplot script can be used to print the horizontal phase space  $T(Y)$   
 11074 at the image plane (Fig. 14.14)

```
11075 cm2m = 1e-2; mrd2rd = 1e-3
11076 plot './zgoubi.fai' u ($10 *cm2m):($11 *mrd2rd) w p ps .9 pt ; pause 2
```

11077 In the case of an  $\alpha = 180$  deg dipole, the previous input data file can be used,  
 11078 changing DIPOLE angles to  $AT = \omega^+ - \omega^- = 180$  deg with for instance  $\omega^+ =$   
 11079  $-\omega^- = 90$  deg. Remove the drifts in order to obtain the 180 deg sector configuration  
 11080 of Fig. 14.12.

11081 *Step size:*

11082 The method is the same as in exercise 2.2 (b), case of a toroidal condenser, which  
 11083 can be referred to.

11084 (d) Spin precession.

11085 Add SPNTRK[KSO=1] at the beginning of the input data file to track spin, starting  
 11086 aligned on the  $X$  axis. Tracking spin also requires PARTICUL, in order to define  
 11087 particle's mass, charge and anomalous magnetic moment.

11088 The theoretical value of the spin precession angle in the moving frame is  $G\gamma\alpha$   
 11089 (Eq. 3.32), with  $\alpha = \pi/3$  or  $\alpha = \pi$  in the previous two deflection cases considered.

11090 This is the value which the stepwise integration produces.

#### 11091 **14.2 Quadrupole Doublet**

11092 The input data file for this problem is given in Tab. 14.4.

**Table 14.4** Input data file: a double-focus quadrupole doublet

```

100 particles on an ellipse, through drift
'OBJET'
1000.
2
9 1
0. 0. 0. 0. 0. 1. 'o'
0. 1. 0. 0. 0. 1. 'a'
0. -1. 0. 0. 0. 1. 'b'
0. 2. 0. 0. 0. 1. 'c'
0. -2. 0. 0. 0. 1. 'd'
0. 0. 0. 1. 0. 1. 'e'
0. 0. 0. -1. 0. 1. 'f'
0. 0. 0. 2. 0. 1. 'g'
0. 0. 0. -2. 0. 1. 'h'
1 1 1 1 1 1 1 1 1 1
'FAISCEAU'

'MARKER' dum .plt
'DRIFT'
70. split 100 2
'QUADRUPO' QF
2
40. 10. 4.7907188 ! 11.1111
0. 0.
6 .1122 6.2671 -1.4982 3.5882 -2.1209 1.723
0. 0.
6 .1122 6.2671 -1.4982 3.5882 -2.1209 1.723
1.
1 0 0 0
'DRIFT'
100. split 100 2
'QUADRUPO' QD
2
40. 10. -4.7907188 ! -11.1111
0. 0.
6 .1122 6.2671 -1.4982 3.5882 -2.1209 1.723
0. 0.
6 .1122 6.2671 -1.4982 3.5882 -2.1209 1.723
1.
1 0 0 0
'DRIFT'
70. split 100 2
'MARKER' dum .plt
'FAISCEAU'

! 'FIT'
! 2
! 5 12 0 .4 ! This FIT procedure
! 7 12 0 .4 ! varies QF and QD fields so to get
! 4 1E-15 ! common focus point in both planes, 3.2 meters downstream of the object.
! 3 6 2 #End 0. 1. 0
! 3 11 2 #End 0. 1. 0
! 3 2 4 #End 0. 1. 0
! 3 3 4 #End 0. 1. 0

'IMAGE'
'IMAGEZ'

'DRIFT'
20. split 100 2

'END'

```

11093 **14.3 Solenoid**

11094 (a) The paraxial trajectory pitch is  $l = 2\pi B\rho/B_0$  (Sect. 14.4.3). Take  $L = 1$  m  
 11095 (Fig. 14.11) and  $B\rho = 1$  T m for simplicity, thus  $B_0 = 2\pi$  T. Assume a particle  
 11096 launched from  $Y = Z = 1$  mm with zero incidence. Scan the solenoid radius value  
 11097 in the range  $1 \leq r_0 \leq 200$  mm to reproduce the figure. The data to be plotted  
 11098 ( $X, Y, Z, B_x$ ) are read from zgoubi.plt.

11099 The beam optics model is given in Tab. 14.5. Note the use of KOBJ=2 in OBJET,  
 11100 which allows creating particles in an arbitrary number (just one, here), with arbitrary  
 11101 initial coordinates. REBELOTE[IOPT=1] is used to repeat the sequence, varying  
 11102 the parameter  $R_0$  under SOLENOID.

**Table 14.5** Input data file: a 1 m long solenoid, with 1 m upstream and downstream fringe field extents. The initial coil radius is  $r_0 = 0.1$  cm, it is scanned (by REBELOTE) over the range  $1 \leq r_0 \leq 20$  cm. For each  $r_0$  a particle is launched with initial position  $Y = Z = 1$  mm and initial angles  $T = P = 0$

```

A 1 meter long solenoid.
'MARKER' opticallyLmmtsProbSolenoid_S
'OBJET'
1000.
2
! OBJET style KOBJ=2.
1 1
0.1 0. 0.1 0. 0. 1. 'o' ! Initial coordinates Yo, To, Zo, Po, Xo, Do.
1
'SOLENOID'
200 ! Log particle data to zgoubi.plt, every other 100 steps.
100. .1 62.8318530718 ! length (cm); radius (cm); field (kG); [MODL=1] default.
100. 100. ! Extent of integration regions upstream and downstream of coil.
.01
1 0. 0. 0.
'FAISCEAU'
'REBELOTE' ! Used to repeat the sequence.
10 0.1 0 1 ! Repeat 10 times.
1
SOLENOID 11 1.:20. ! Vary parameter 11 (= R0) under SOLENOID.
'MARKER' opticallyLmmtsProbSolenoid_E
'END'

```

**Table 14.6** Input data file: track a particle along the central axis of the solenoid, to generate a 3 m long, 1D field map, with mesh step 5 cm

```

! A 3 meter long solenoid field map.
'MARKER' opticallyLmmtsProbSolenoid_C_S
'OBJET'
1000.
2
! OBJET style KOBJ=2.
1 1
0. 0. 0. 0. 0. 1. 'o' ! Initial coordinates Yo, To, Zo, Po, Xo, Do.
1
'SOLENOID'
200 ! Log particle data to zgoubi.plt, every other 100 steps.
100. .1 62.8318530718 ! length (cm); radius (cm); field (kG); [MODL=1] default.
100. 100. ! Extent of integration regions upstream and downstream of coil.
5.
1 0. 0. 0.
'FAISCEAU'
'END'

```

11103 (b) To allow comparison, theoretical matrices (Eq. 14.35) must be computed for  
 11104 the theoretical length,  $L$ , of the matrix transport solenoid model. Tracking must  
 11105 extend upstream and downstream of the solenoid, over a distance much greater than  
 11106 the solenoid diameter (the latter determines the field fall extent, Eq. 14.30).

11107 (c) A 1-dimensional (on-axis) field map of the solenoid field,  $B_{X,i}(X_i)$ , can simply  
 11108 be generated by tracking a particle along the solenoid axis. It has to extend upstream  
 11109 and downstream of the solenoid, over a distance much greater than the solenoid  
 11110 diameter. The integration step size will be the mesh size, take it in the centimeter  
 11111 range ( $\lesssim r_0$ ), 5 cm here. An intermediate stage is necessary, which consists in  
 11112 reading  $X, B_X(X)$  from `zgoubi.plt` and re-writing it in a dedicated ASCII file in a  
 11113 format proper for use by the keyword BREVOL.

11114 The input file to generate the field and log to `zgoubi.plt` is given in Tab. 14.6.

11115 Similar exercises, generating a 1D field map and using BREVOL, can be found  
 11116 be found in `zgoubi` sourceforge repository [14].

**Table 14.7** Input data file: track a particle in the solenoid, in a similar manner to the input data file of Tab. 14.6, using a field map model instead

```

A 1 meter long solenoid, 3 meter long field map.
'OBJET'
1000.
2
1 1
0. 0. 0. 0. 0. 1. 'o'
1
'BREVOL'
0 0
1. 1.
Test solenoid 1D field map
61                                     ! Number of nodes of the 1D mesh.
solenoid_1meter.map
0 0. 0. 0.
2
1.
1 0 0 0
'FAISCEAU'
'END'

```

## 11117 References

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