Problem 1.

Show that for $\hat{C} \ll 1$, the eigenvalue of the growing mode for the 1-D FEL (cold beam) can be approximated as

$$\lambda = a_0 + a_1 \hat{C} + a_2 \hat{C}^2$$

with

$$a_0 = \frac{\sqrt{3}}{2} + i \frac{1}{2}, \quad a_1 = -i \frac{2}{3}, \quad a_2 = -\frac{1}{9} \left( \frac{\sqrt{3}}{2} - i \frac{1}{2} \right)$$

Problem 2.

Assuming the saturation of a FEL takes place at the condition

$$\Omega_{p, sat} = \sqrt{\frac{eE_{sat} \theta_s \omega}{\gamma^2 cE_0}} \approx \frac{1}{L_G} = \sqrt{3} \Gamma$$

show that the radiation power at saturation is given by

$$P_{sat} = \varepsilon_0 c E_{sat}^2 A = \chi \cdot \rho \cdot \frac{E_0}{e} I_e$$

and find the numerical coefficient $\chi$. 