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ENERGY

USPAS’23 | Hadron Beam Cooling

Stochastic Cooling: Details and Experimental Realization

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Lecture#2:

- SC details: description by the Fokker-Plank equation, bunched beam cooling
- SC hardware: pick-ups and kickers
- Experimental realization of SC:
    - First tests of SC at CERN
    - SC at FNAL
    - SC for Relativistic Heavy Ion Collider (RHIC) at BNL

References:

- For more information, please refer to the following literature:

[1] F. Caspers, D. Möhl, "*History of stochastic beam cooling and its application in many different projects",* Eur. Phys. J. H 36, 601–632 (2011) DOI: 10.1140/epjh/e2012-20037-8
[2] T. Katayama, N. Tokuda, "*Fokker-Planck approach to stochastic momentum cooling with a notch filter*", 1985
[3] J.M. Brennan, M. Blaskiewicz, K. Mernick, "*New stochastic cooling kickers for RHIC",* 2014.
[4] R. J. Pasquinelli, "Twenty-five years of stochastic cooling experience at FERMILAB", COOL'09, 2009.
[5] J. Marriner, "*Stochastic cooling overview*", Nuclear Instruments and Methods in Physics Research A 532 (2004) 11–18

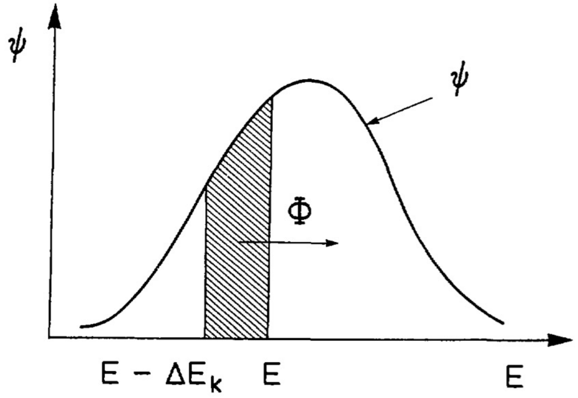
Fokker-Plank equation

- In the case of a momentum cooling, we want to bring the particles to the center of the distribution.
- The particle density will vary throughout the distribution.
- The cooling process will change the density profile ↔ cooling rate will decrease as the density grows.
- Let’s characterize the SC process in terms of particle distribution.

We will attempt to describe the **time evolution of the particle density function**: $\psi(E,t) = \frac{\mathrm{d}N}{\mathrm{d}E}$

We will use **energy as an independent variable** instead of the frequency: $\frac{\mathrm{d}\omega_{\mathrm{rev}}}{\mathrm{d}E} = \eta\omega_{\mathrm{rev}}/\beta p$

Particle flux—number of particles per unit time passing a give energy: $\Phi = \mathrm{d}N/\mathrm{d}t$

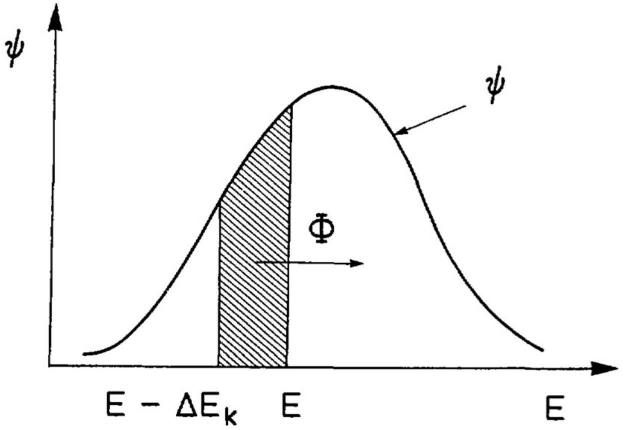


Derivation of a Fokker-Plank Equation

The continuity equation:

particle flux: number of particles crossing E per unit time

$$\frac{\partial \psi}{\partial t} + \frac{\partial \Phi}{\partial E} = 0$$



When the particle energy is corrected in a kicker by ΔE_k:

$$\Phi = f_0 \left[\psi(E)\Delta E_k - \frac{1}{2}\frac{\partial \psi}{\partial E}(\Delta E_k)^2 \right]$$

$$\Delta E_k = \underbrace{\Delta E_c}_{\text{coherent correction}} + \underbrace{\Delta E_{ic}}_{\text{incoherent noise}}$$

We are interested in the motion over a time range much longer than the revolution time:

$$\overline{\Delta E_k} \simeq \Delta E_c$$
$$\overline{(\Delta E_k)^2} \simeq \overline{(\Delta E_{ic})^2}$$

assuming $\overline{\Delta E_{ic}} = 0, \ \overline{(\Delta E_{ic})^2} \gg \overline{(\Delta E_c)^2}$

Then the flux: $\Phi = f_0 \left[\psi \Delta E_c - \frac{1}{2}\frac{\partial \psi}{\partial E}\overline{(\Delta E_{ic})^2} \right]$

$$\boxed{\frac{\partial \psi}{\partial t} + \frac{\partial}{\partial E}(F\psi) - \frac{\partial}{\partial E}\left(D\frac{\partial \psi}{\partial E}\right) = 0 \qquad \text{with} \qquad F \equiv f_0 \Delta E_c, \ D \equiv \frac{1}{2} f_0 \overline{(\Delta E_{ic})^2}}$$

Simplified Fokker-Plank Equation: Coherent Term

$$\frac{\partial \psi}{\partial t} + \frac{\partial}{\partial E}(F\psi) - \frac{\partial}{\partial E}\left(D\frac{\partial \psi}{\partial E}\right) = 0$$
$$F \equiv f_0 \Delta E_c, \; D \equiv \frac{1}{2} f_0 \overline{(\Delta E_{ic})^2}$$

Assume $F = -\frac{E}{\tau_0}$ because $\Delta E_c \propto E$.

Plugging it into the FP equation and dropping incoherent term:

$$\frac{\partial \psi}{\partial t} + \frac{\partial}{\partial E}\left(-\frac{E}{\tau_0}\psi\right) = 0$$

Then one can find a solution: $\psi(E,t) = e^{t/\tau_0}\psi_0(Ee^{t/\tau_0})$

Let's choose a Gaussian initial distribution as an example: $\psi_0(E) = \frac{N}{\sqrt{2\pi}\sigma_0}e^{-E^2/2\sigma_0^2}$

Then we obtain:

$$\psi(E,t) = \frac{N}{\sqrt{2\pi}\sigma(t)}e^{-E^2/2\sigma^2(t)}$$
$$\sigma(t) = \sigma_0 e^{-t/\tau_0}$$

The energy spread decreases exponentially with a time constant τ_0, which we call the single particle cooling time.
The coherent term narrows the distribution – cooling!

Simplified Fokker-Plank Equation: Incoherent Term

$$\frac{\partial\psi}{\partial t}+\frac{\partial}{\partial E}(F\psi)-\frac{\partial}{\partial E}\left(D\frac{\partial\psi}{\partial E}\right)=0$$

$$F\equiv f_0\Delta E_c,\;\; D\equiv\frac{1}{2}f_0\overline{(\Delta E_{ic})^2}$$

The incoherent term is proportional to the noise: amplifier & beam noise.

stays constant

Beam noise is proportional to the square of the standard deviation of energy error: $\sigma^2(t)=\frac{1}{N}\int\limits_{-\infty}^{\infty}E^2\psi(E,t)\mathrm{d}E$

FP without coherent term:

$$\frac{\partial\psi}{\partial t}-\big(\underbrace{D_a}_{\text{amplifier noise}}+\underbrace{\mathrm{d}_b\sigma^2(t)}_{\text{beam noise}}\big)\frac{\partial^2\psi}{\partial E^2}=0$$

$$\sigma\frac{\mathrm{d}\sigma}{\mathrm{d}t}=\mathrm{d}_b\sigma^2+D_a \longleftarrow \text{r.h.s. is positive}$$

$$\sigma(t)=\sqrt{\sigma_0^2e^{2d_bt}+\frac{D_a}{d_b}\left(e^{2d_bt}-1\right)}$$

When power of the beam noise is much less than the noise from the amplifier, we can set $d_b=0$

Diffusion equation:

$$\frac{\partial\psi}{\partial t}-\left(D_a+\mathrm{d}_b\sigma^2(t)\right)\frac{\partial^2\psi}{\partial E^2}=0\;\; \text{with}\;\; \sigma(t)=\sqrt{\sigma_0^2+2D_at}$$

The incoherent term causes diffusion—heating!

Simplified FP Equation: combining coherent and incoherent terms

FP equation with both terms:

$$\frac{\partial \psi}{\partial t}+\frac{\partial}{\partial E}\left(-\frac{E}{\tau_0}\psi\right)-\left(D_a+\mathrm{d}_b\sigma^2(t)\right)\frac{\partial^2\psi}{\partial E^2}=0$$

Analogous to the last slide derivation:

$$\sigma\frac{\mathrm{d}\sigma}{\mathrm{d}t}=-\left(\frac{1}{\tau_0}-d_b\right)\sigma^2+D_a$$

Then the solution is:

$$\sigma(t)=\sqrt{\sigma_0^2 e^{-2t/\tau}+\tau D_a\left(1-e^{-2t/\tau}\right)},\quad \frac{1}{\tau}=\frac{1}{\tau_0}-d_b$$

In the limit $t\rightarrow\infty$: $\sigma(\infty)=\sqrt{\tau D_a}$

Initial cooling time: $\tau_i\equiv-\frac{\sigma_0}{\dot\sigma(0)}=\tau\left(1-\frac{\sigma^2(\infty)}{\sigma_0^2}\right)^{-1}$

Final energy spread and the initial cooling time are closely related!

Unfortunately, FP equation is not easily amenable to analytic solution, because both F and D depend in a complicated way on the Schottky noise and the beam-feedback effect. However, numerical solution allows a detailed analysis of the cooling process.

$$\frac{\partial\psi}{\partial t}+\frac{\partial}{\partial E}(F\psi)-\frac{\partial}{\partial E}\left(D\frac{\partial\psi}{\partial E}\right)=0$$

Bunched Beam Cooling: reduction of the cooling rate

- Decreasing bunch length increases the sample population and the cooling rate is reducing accordingly.
- Mixing will be much worse, at least for the particles in the center of the bucket.

Let's estimate the cooling rate: replace the bunches by pieces of coasting beams.

$$\frac{1}{\tau} = \frac{W}{N}\left[\underbrace{2g(1-\tilde{M}^{-2})}_{\text{coherent effect (cooling)}} - \underbrace{g^2(M+U/Z^2)}_{\text{incoherent effect (heating)}}\right]$$

can still be used, but use the effective number of particles

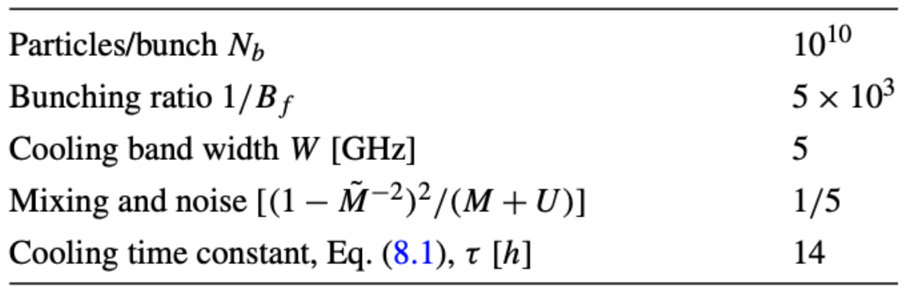
$$N \rightarrow N_b 2\pi R/l_b = N_b/B_b$$

The bunching ratio is usually a big number (~several thousands) → **large bandwidth is needed** to obtain useful cooling rates.

$$\frac{1}{\tau} \leq \frac{W}{N_b/B_b}\left[\frac{(1-\tilde{M}^{-2})^2}{M+U}\right] < \frac{W}{N_b/B_b}$$

- Strong bunch signal at certain harmonics of the revolution frequency up to a cut-off determined by the bunch length $f_b \approx \beta c/l_b$
- The signal is proportional to N_b (kN_b for *k* bunches), and tends to blind the cooling signal that is proportional to $\sqrt{N_b}$

Example for the SPS-collider



Particles/bunch N_b	10^{10}
Bunching ratio $1/B_f$	5×10^3
Cooling band width W [GHz]	5
Mixing and noise $[(1-\tilde{M}^{-2})^2/(M+U)]$	1/5
Cooling time constant, Eq. (8.1), τ [h]	14

Bunched Beam Cooling: mixing issues

The relative speed by which these particles overtake each other will vary sinusoidally and for equal synchrotron frequencies the same particles will meet again and again in the same sample.

- The revolution frequency of each particle is modulated by the much lower synchrotron frequency ω_s
-
- Each Schottky line splits up into a central line $n\omega_{\mathrm{rev}}$ accompanied by satellite lines spaced by ω_s

- Current of a single particle:

$$I = ef_{\mathrm{rev}} \sum_n \sum_k J_k(n\Delta\phi)\exp\left[i\left(n\omega_{\mathrm{rev}}t + k\omega_s t + k\varphi_s\right)\right]$$

peak phase excursion due to the synchrotron motion

- For unbunched beams $k\omega_s t$ is replaced by the initial phase, and the random phase destroys the coherence.

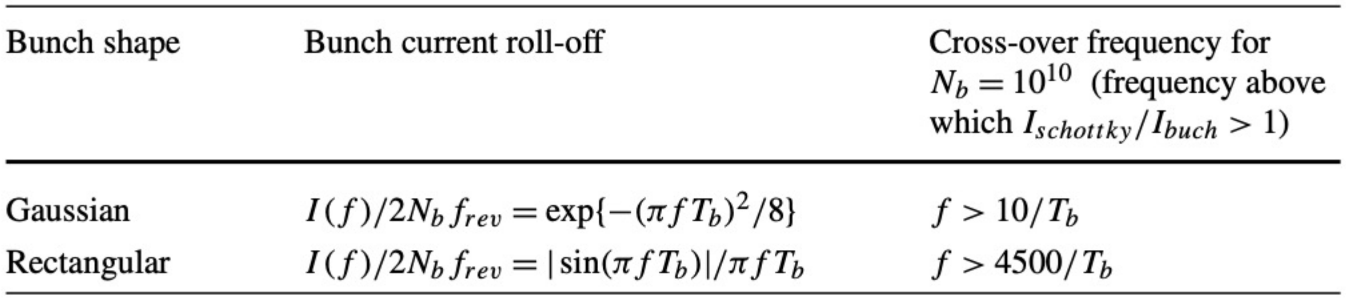
- Only a fraction of the bunched beam Schottky bands is useful for cooling, except when very high frequencies are employed; the distance in frequency of the useful bands is about equal to the inverse of the bunch duration.

Bunched Beam Cooling: amplifier saturation

The central bands, for k = 0, will add up linearly for all particles, as long as $n\Delta\phi$ is small compared with 1. At lower *n* values the signal will be much stronger than the Schottky signals from the satellite bands, which are proportional to $N^{1/2}$. The sensitive amplifier needed for the cooling systems may therefore be saturated.

Solution:

- utilize filters with steep notches at the revolution harmonics
- choose high harmonic numbers so that the systematic bunch-signal at is sufficiently decreased—requires dependence of *n* on the bunch shape.



Bunch shape	Bunch current roll-off	Cross-over frequency for $N_b = 10^{10}$ (frequency above which $I_{schottky}/I_{buch} > 1$)
Gaussian	$I(f)/2N_b f_{rev} = \exp\{-(\pi f T_b)^2/8\}$	$f > 10/T_b$
Rectangular	$I(f)/2N_b f_{rev} = \lvert\sin(\pi f T_b)\rvert/\pi f T_b$	$f > 4500/T_b$

To avoid the strong unwanted bunch signals, one prefers to work at frequencies (much) above the cross-over; therefore, the cooling band to be chosen depends strongly on the bunch-shape assumed.

Pick-Up Technologies

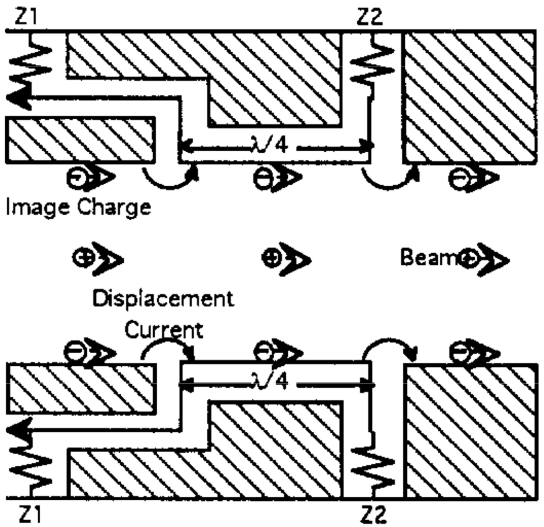
Phased arrays:

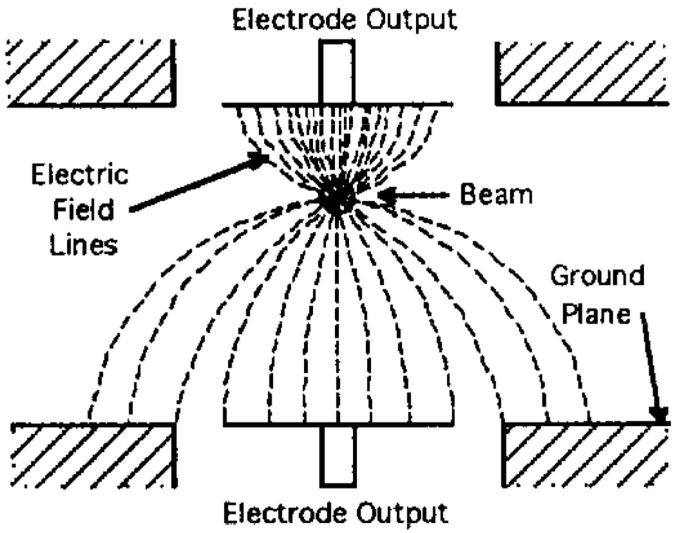
- Consist of many individual pickups of relatively low impedance. The outputs of a series (up to 100’s) of pickups are added to achieve the desired sensitivity.
- The mostly widely used structure is a stripline pickup. These pickups are generally constructed from a plate placed parallel to the beam and the vacuum chamber with connections at the upstream and downstream end of the plate.
- One advantage of using a large array of pickups is that it is relatively easy to adapt the array to a varying beam velocity.

Traveling wave structures:

- The beam induces a wave that travels at the beam velocity and grows as the wave and the beam travel the length of the structure.
- Maintaining synchronism between the beam and the traveling wave results in a tradeoff between bandwidth and sensitivity in these devices. The signals from traveling wave pickups can be added, but often a single unit has enough sensitivity.
- Relatively tight constraints on synchronism make it difficult to adapt this type of pickup to varying beam velocities.

Pick-Up Technologies: Stripline





The beam current passing through:

$$i(z,t) = i_b e^{i(kz-\omega t)}$$

Pickup output voltage:

$$V_p = f Z_p i_b \sin(kl)$$

$$\uparrow$$

fraction of the image
current in the stripline

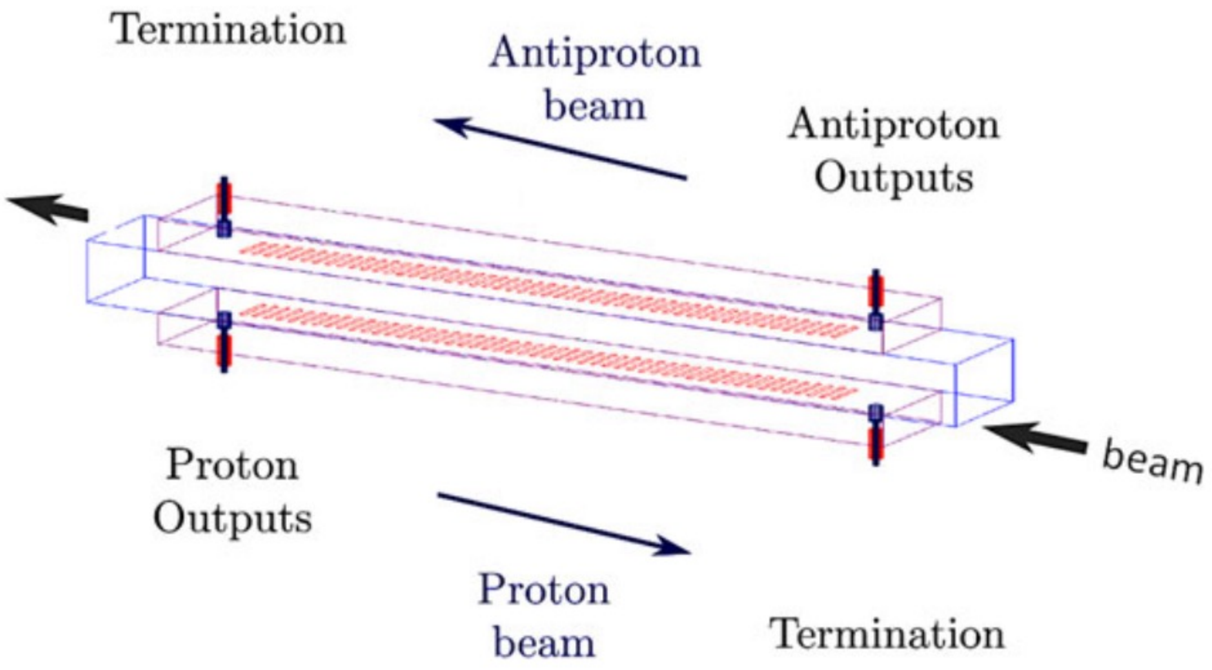
- The peak in the response occurs at $kl = \pi/2$ .Thus, the length is chosen to be a ¼ wavelength long at the band center frequency.
- The fraction *f* of image current that is sensed by the pickup may be estimated by assuming the pickup is infinitely long in *z* an solving the wave equation for the scalar potential:

$$\left(\frac{1}{c}\right)^2 \frac{\partial^2 \Phi}{\partial t^2} - \nabla^2 \Phi = \rho(x,y) e^{j(kz-\omega t)}$$

$$\Phi = \Theta(x,y) e^{j(kz-\omega t)} \longrightarrow \nabla^2 \Theta - \left[k^2 - \left(\frac{\omega}{c}\right)^2\right] \Theta = \rho(x,y)$$

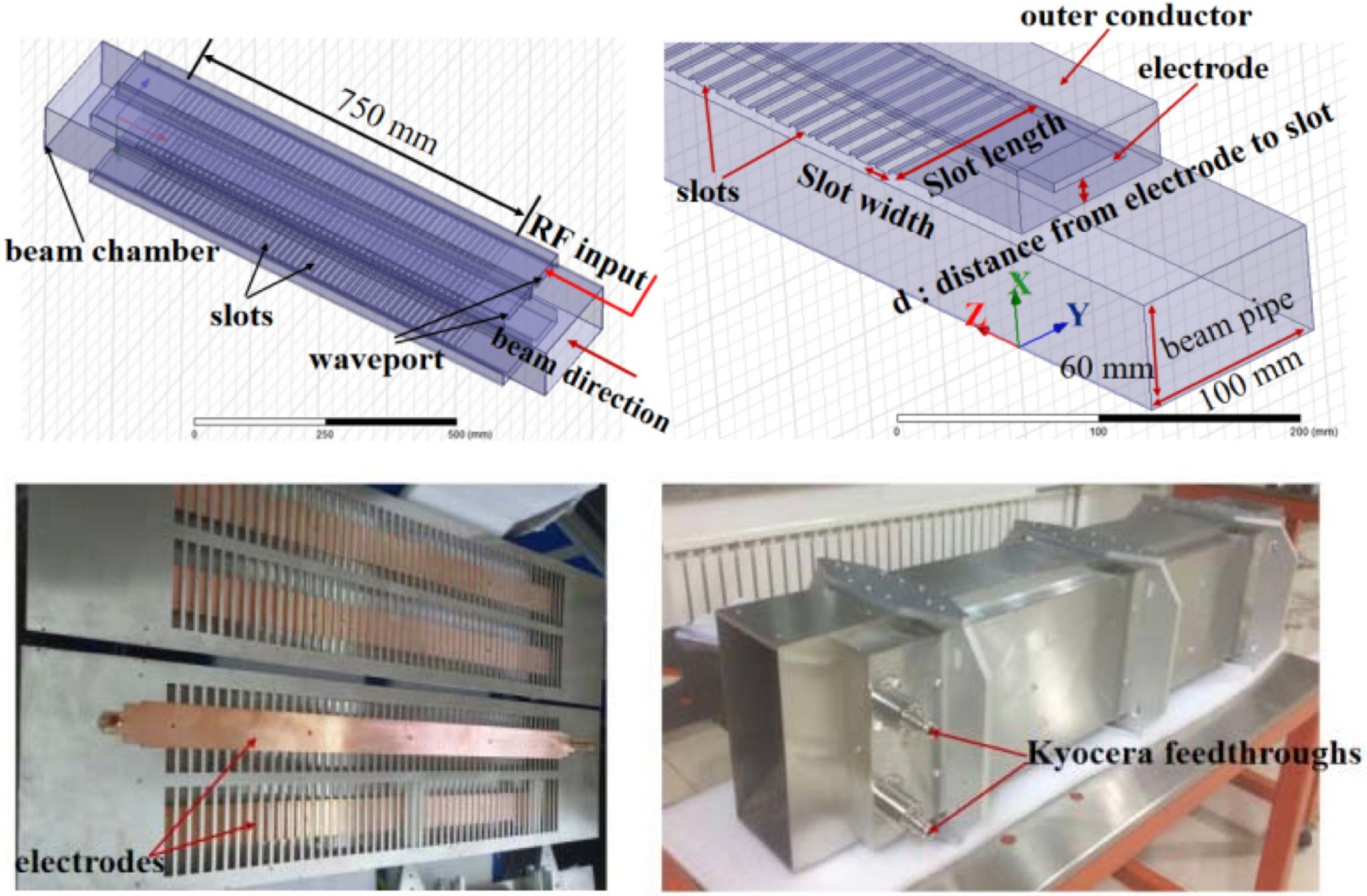
For simple geometries can be solved analytically.

Pick-Up Technologies: Slotted Waveguide Arrays

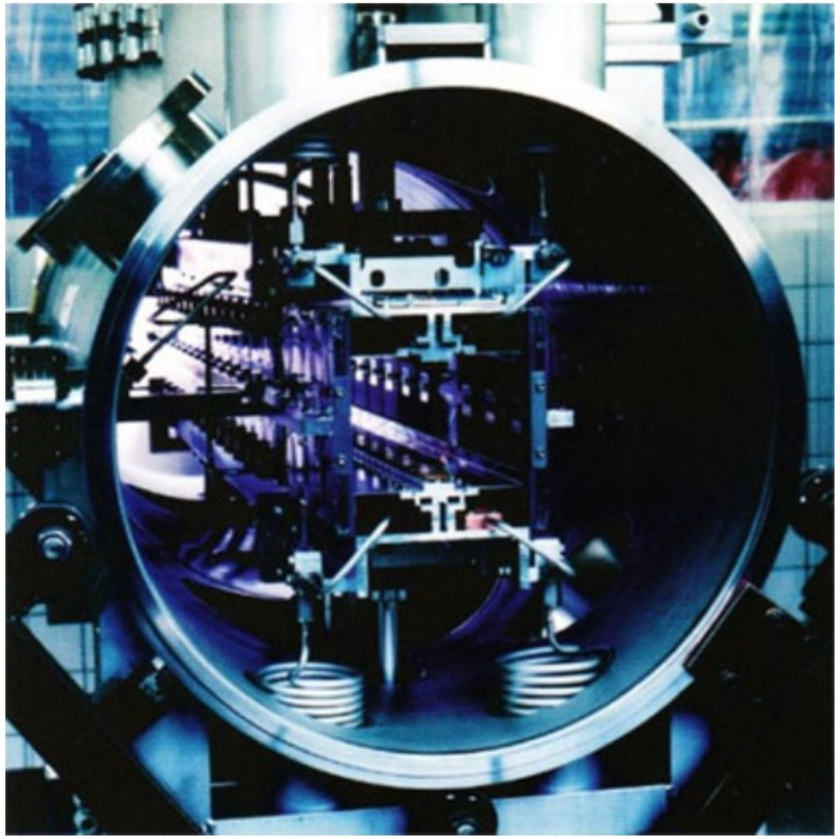


- Two rectangular waveguides are coupled to a rectangular beam pipe by a series of slots.
- The transverse signal is derived from the difference between the two waveguides and the momentum signal is derived from the sum of the two waveguides.
- The image current flows along the walls of the beam pipe excites EM waves in the slots which excite travelling modes in the side waveguides and beampipe.

Kicker Technologies



Faltin traveling wave-type kicker prototype



The interior of a COSY vertical kicker tank, with λ/4 electrodes (loop couplers) above and below the beam center and ferrite material to the right and left to damp unwanted microwaves.

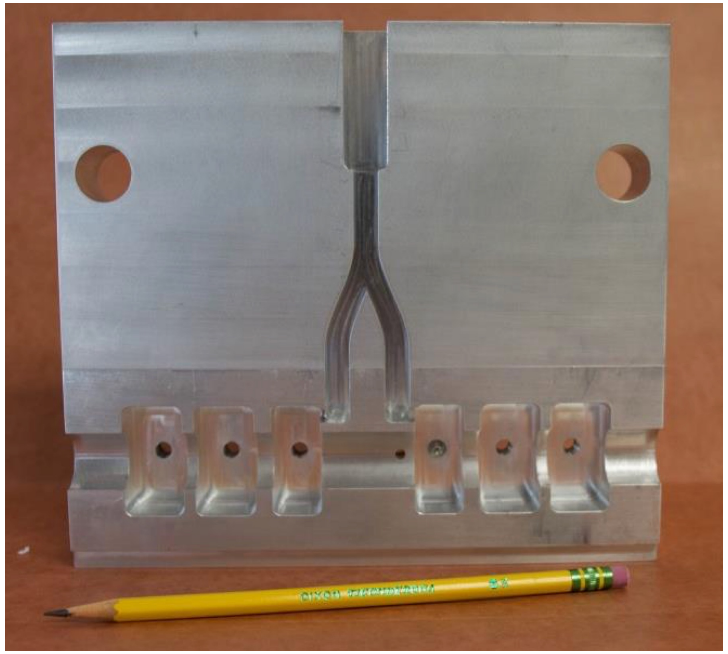
Cooling Kickers for RHIC

Cooling in a collider:

- Beam is bunched:
  1. The local particle density is high → a large system bandwidth is required for practical cooling rate.
  2. The beam effective frequency spectrum is sparse, only harmonics of 1/(bunch length) being non-redundant.
- The RHIC system employs narrowband kickers whose resonant frequencies match these harmonics. This approach energy averages the drive power needed to excite the kickers, thereby reducing the power amplifier size requirement by an order of magnitude.

Kicker cavity:

- 6 TM_{010} rectangular cells, tuned to one of 16 frequencies, 6.0, 6.2...8.8, 9.0 GHz.
- It is split into two sections on the vertical median plane. The symmetry of fields in the TM_{010} ensures that no currents cross the median plane → no ohmic contact between the two halves is necessary.
- The beam bore diameter is 20 mm when in operation, but the two halves separate by ~70 mm during filling and ramping of the collider. Each cavity is driven with a 40 W solid state amplifier and develops ~ 3 kV of kick.

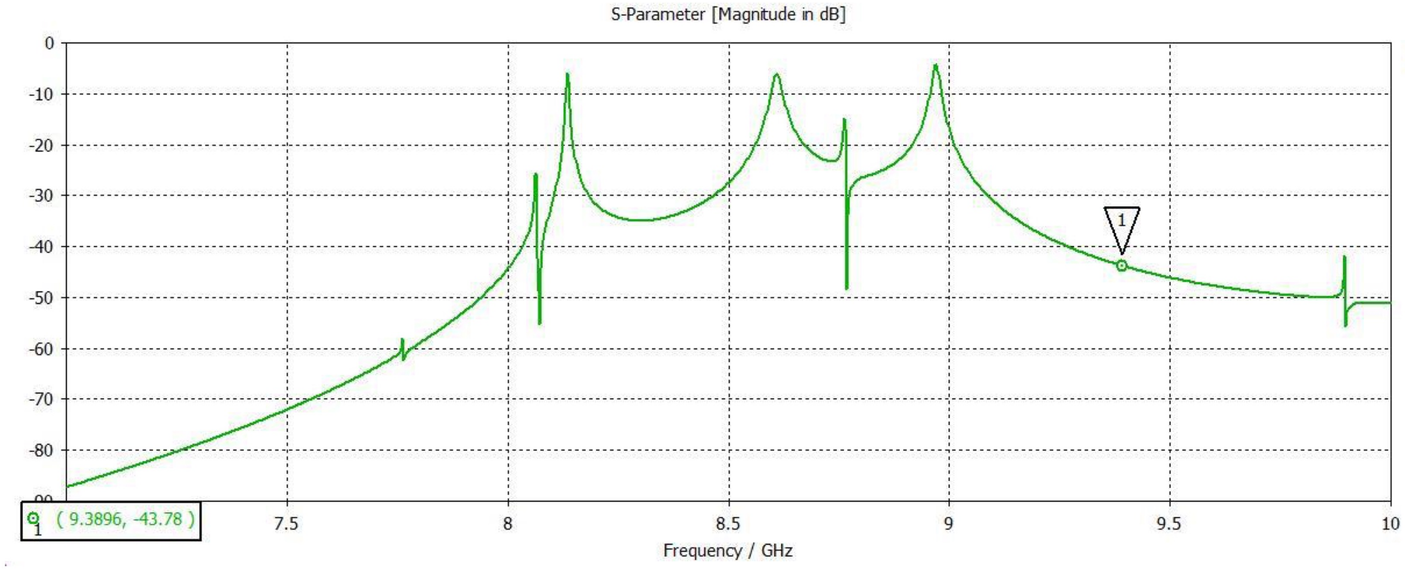


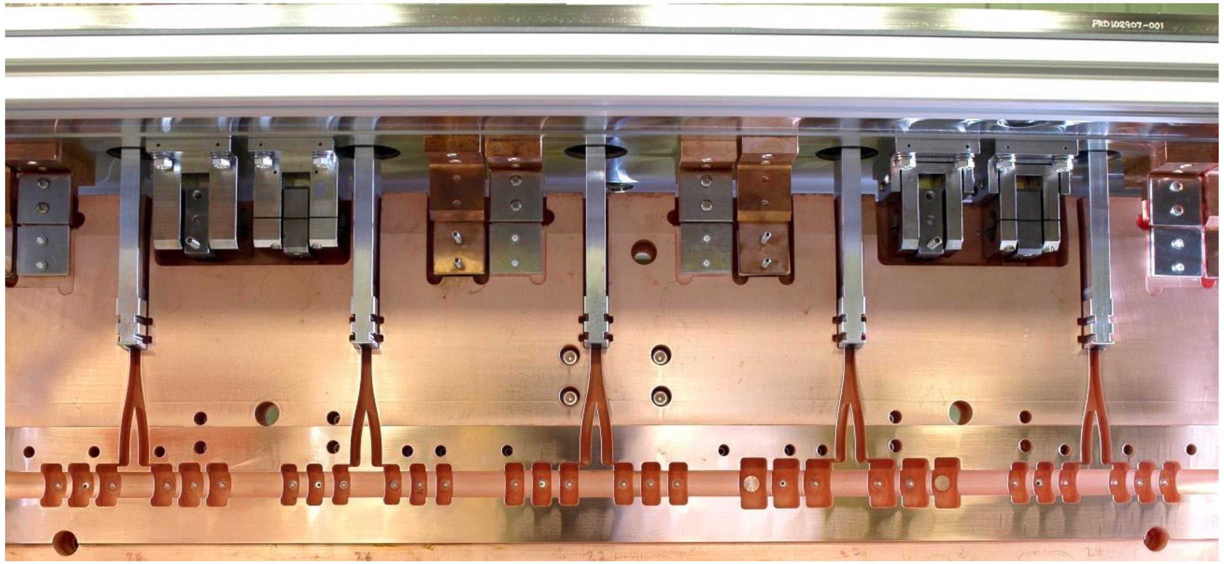
Prototype of 6 GHz 6-cell kicker cavity. One half of the cavity is shown. Waveguide splits to feed the two 3-cells in phase.

Cooling Kickers for RHIC: RF structure

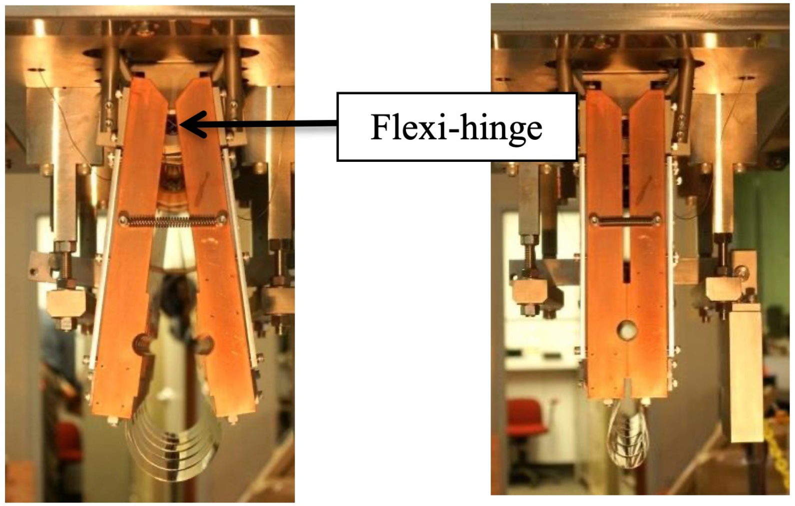
- The 6-cell cavities: two 3-cells operating in π-mode and the two 3-cells are separated by one wavelength.
- They are fed by a TE_{10} waveguide with a 3 dB H-field split. This drives the two innermost cells in phase.
- The rectangular shape of the cells facilitates the machining. Adjustments are made to individual cell dimensions in order to equalize the field strength and thereby maximize the shunt impedance. Each cell has two fine tuners to achieve frequency accuracy of 1 MHz.
- The material is copper-plated aluminum. The purpose of copper plating is to reduce the secondary electron yield. The benefit for reduced power dissipation is insignificant.

Mode spectrum of 9 GHz 3-cell substructure.

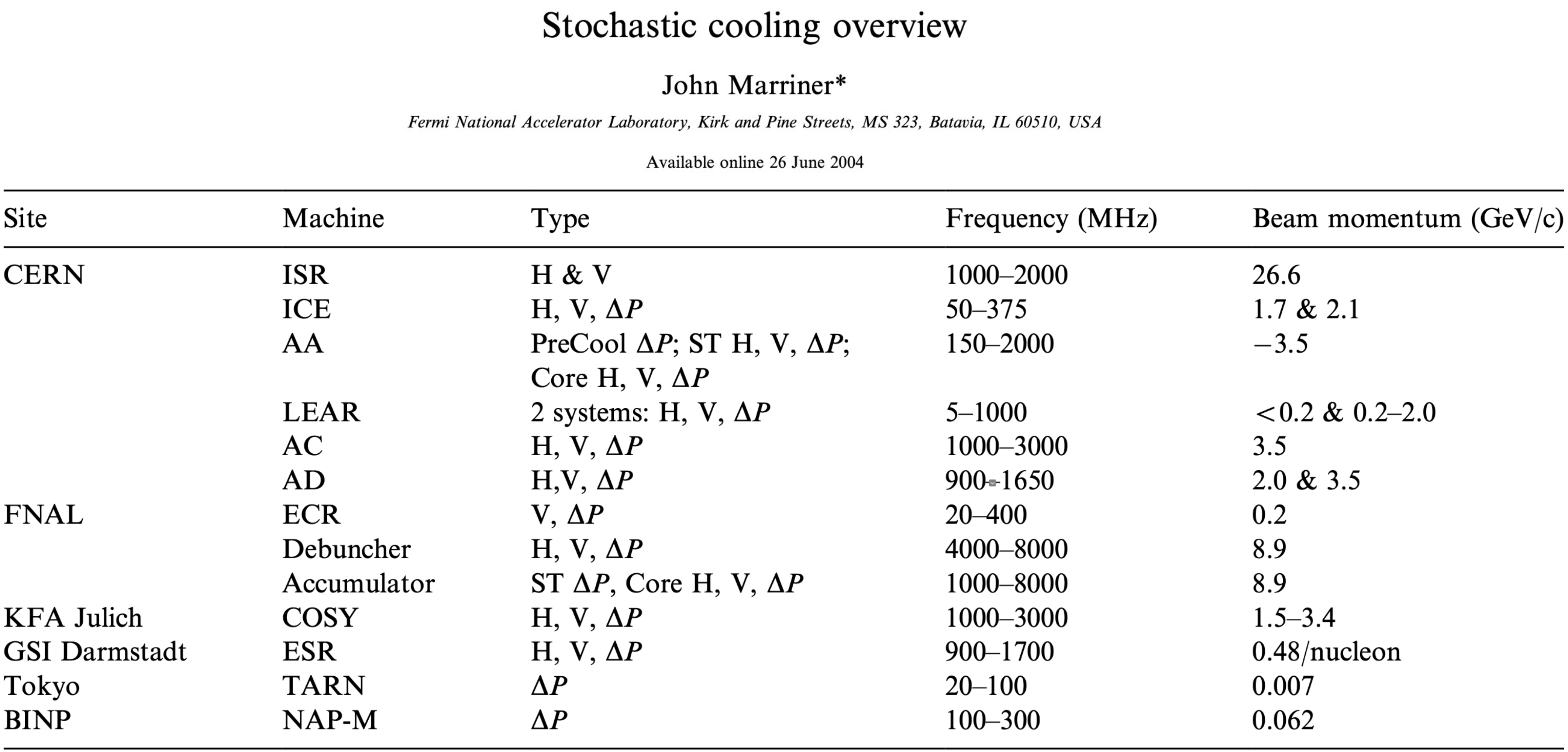




The cavities split on the vertical median plane and open, left, for filling and ramping then close, right, for operation during store.



Experimental realization of SC



Stochastic cooling overview

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Available online 26 June 2004

Site	Machine	Type	Frequency (MHz)	Beam momentum (GeV/c)
CERN	ISR	H & V	1000–2000	26.6
	ICE	H, V, ΔP	50–375	1.7 & 2.1
	AA	PreCool ΔP; ST H, V, ΔP; Core H, V, ΔP	150–2000	-3.5
	LEAR	2 systems: H, V, ΔP	5–1000	<0.2 & 0.2–2.0
	AC	H, V, ΔP	1000–3000	3.5
	AD	H,V, ΔP	900–1650	2.0 & 3.5
FNAL	ECR	V, ΔP	20–400	0.2
	Debuncher	H, V, ΔP	4000–8000	8.9
	Accumulator	ST ΔP, Core H, V, ΔP	1000–8000	8.9
KFA Julich	COSY	H, V, ΔP	1000–3000	1.5–3.4
GSI Darmstadt	ESR	H, V, ΔP	900–1700	0.48/nucleon
Tokyo	TARN	ΔP	20–100	0.007
BINP	NAP-M	ΔP	100–300	0.062

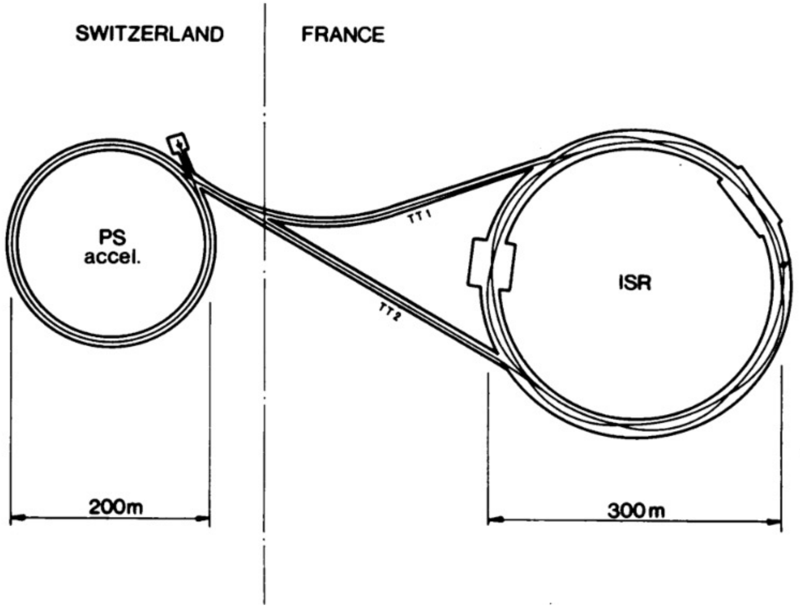
- In general, SC applications are to improve source brightness and accumulate particles; to improve the rate of beam interaction with a target.

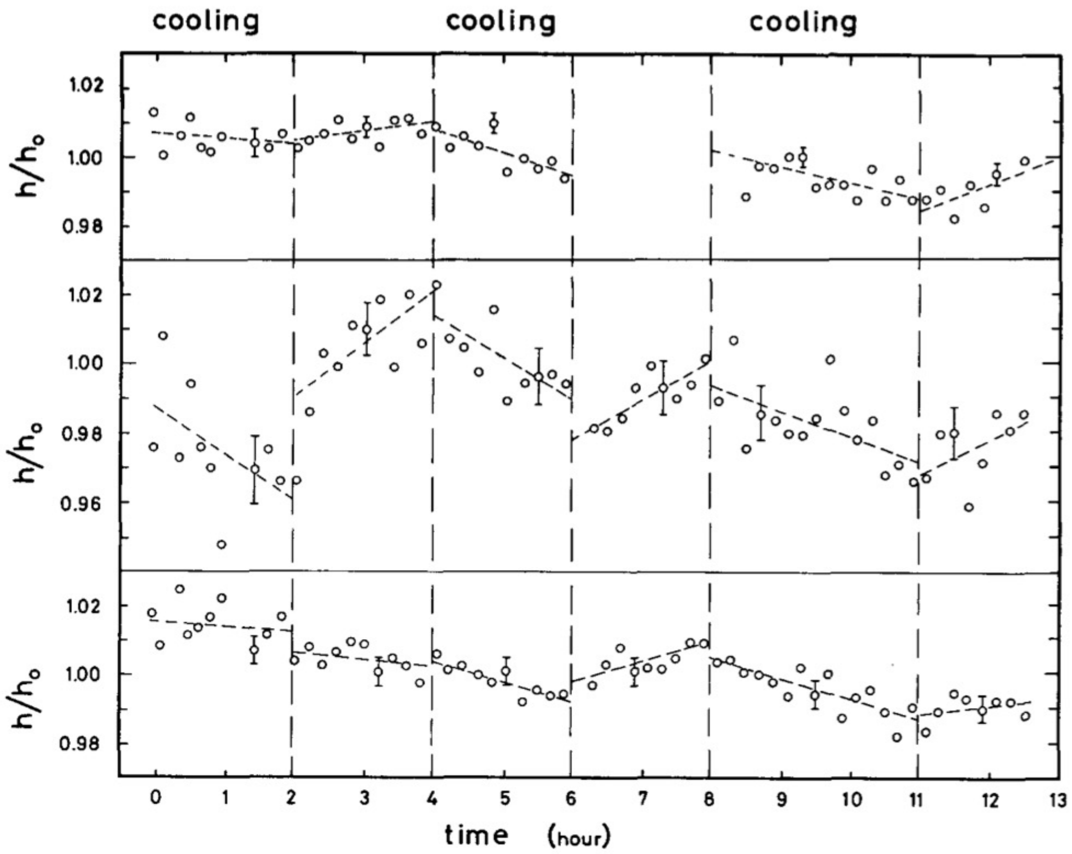
- Many of the systems have been modified several times throughout the years.

First practical realization of SC: CERN Intersecting Storage Rings (ISR)

1974: betatron cooling:

- a pair of 3 cm long loop pickups give an output proportional to the vertical displacement.
- Transmission path is 6 m shorter to allow proper timing (coax followed the inner wall of the tunnel)
- Long-term effects in 0.91 A (1.8x10^{13} protons) was monitored by the luminosity increase.
- The average cooling rate of the effective beam height was ~2% per hour.





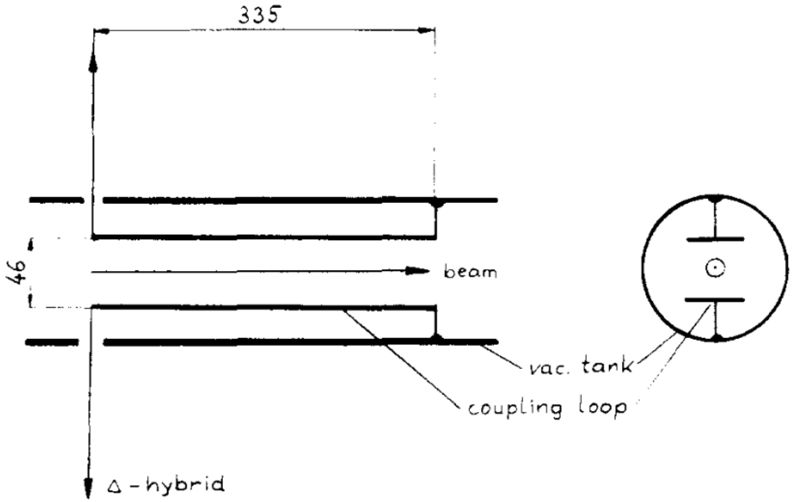
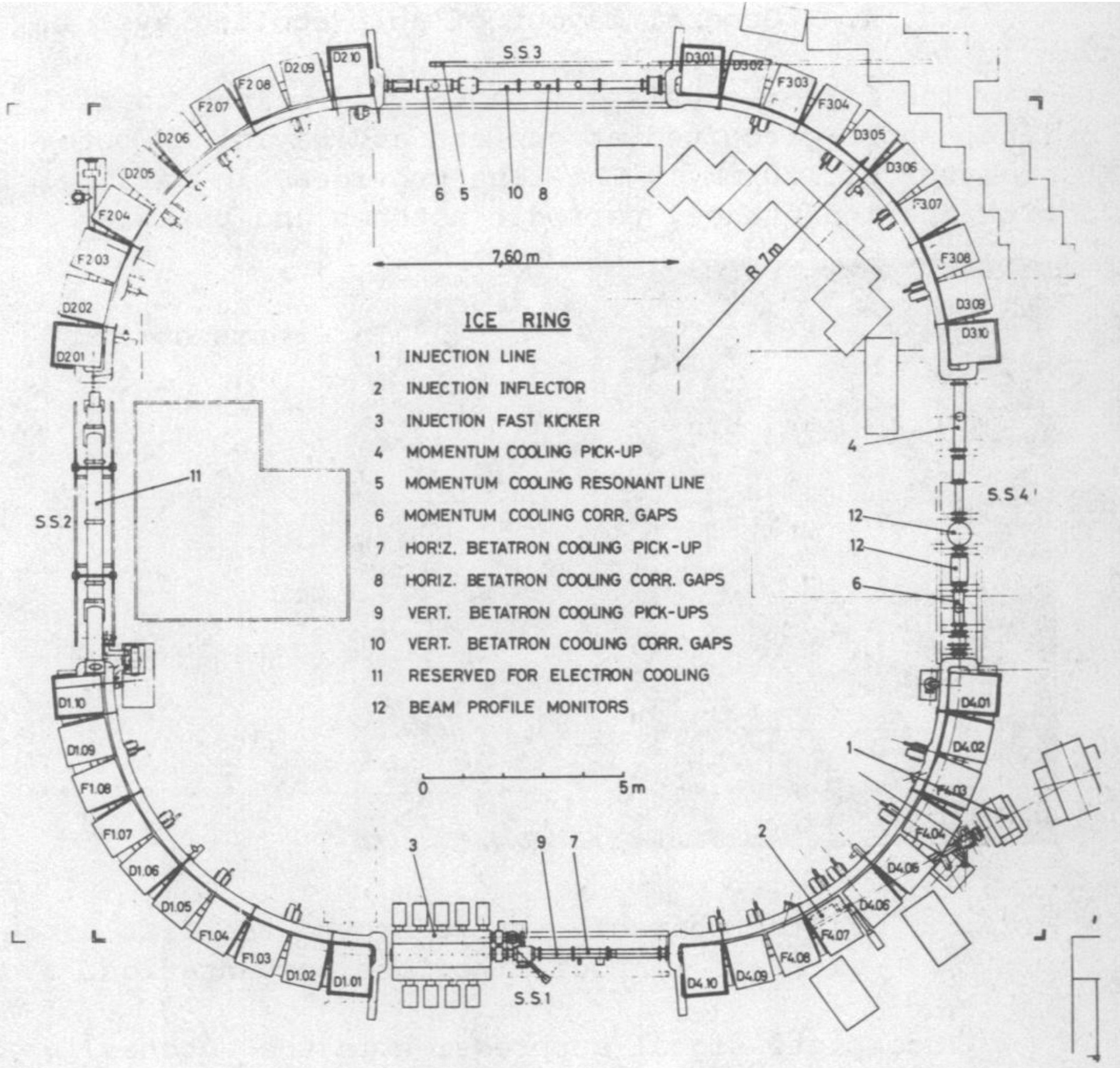
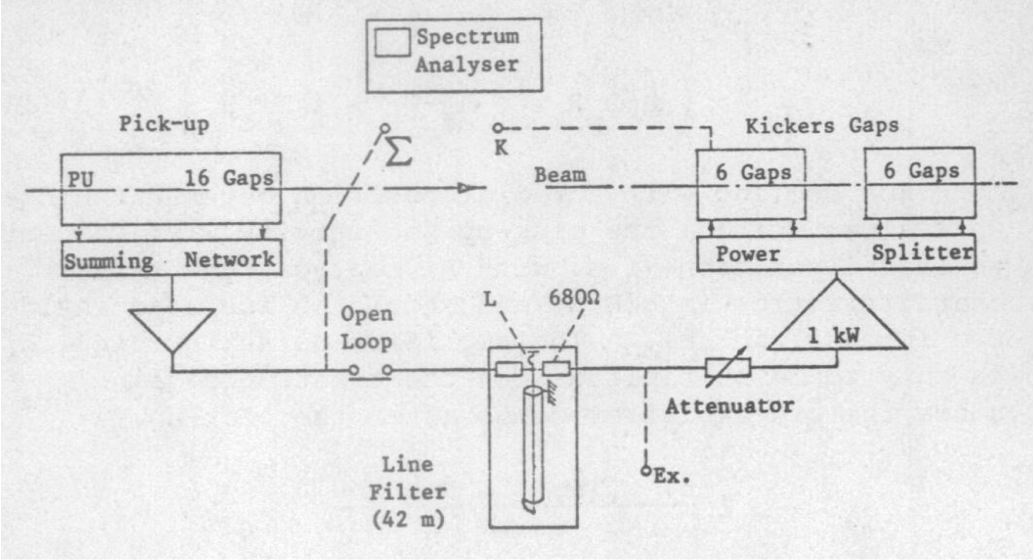


Fig. II.3. Loop coupler pick-up.

Experiments on SC in ICE (Initial Cooling Experiment)



- ICE – strong focusing machine with 4 magnetic sectors and 4 straight sections. For SC experiment it was run at 1.73 and 2.1 GeV/c.
- 16 sum pick-ups are used for momentum cooling.
- The signal is filtered and amplified by a 1 kW amplifier.
- Kickers: 12 accelerating gaps, similar to the pick-up gaps. The correctors are in two groups of six, separated by half a horizontal betatron wavelength to cancel the betatron heating caused by momentum corrections.



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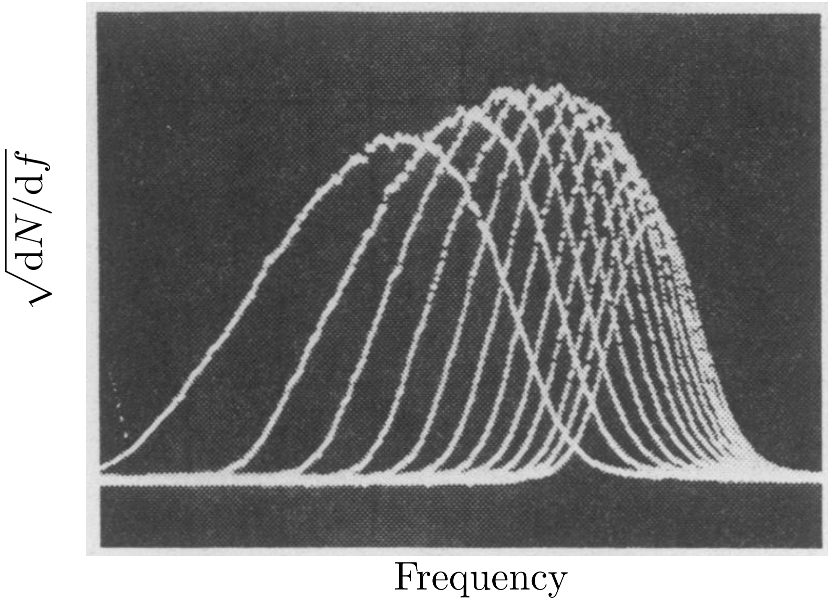
Experiments on SC in ICE (Initial Cooling Experiment)

First evidence of longitudinal SC in ICE (1977)
Longitudinal Shottky scans with 1 min intervals



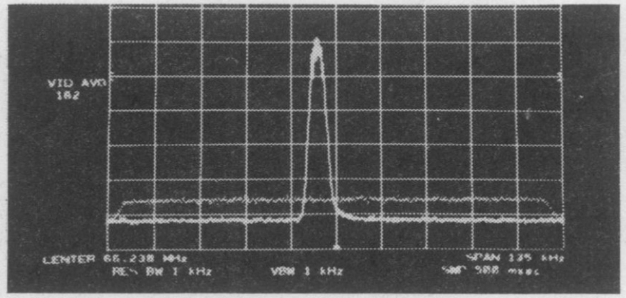
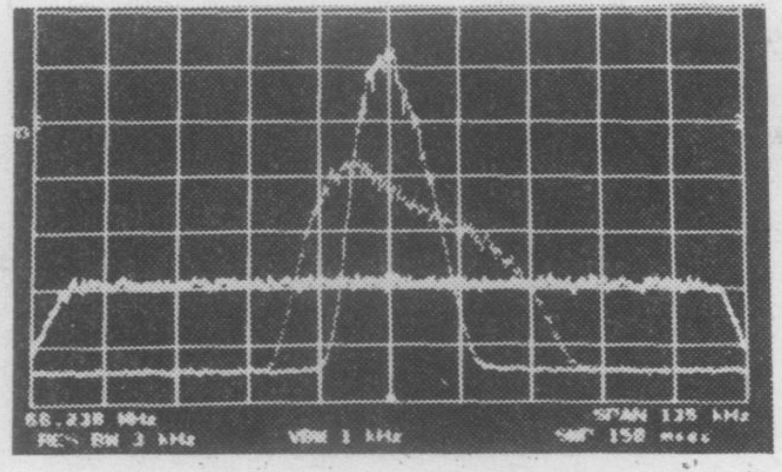
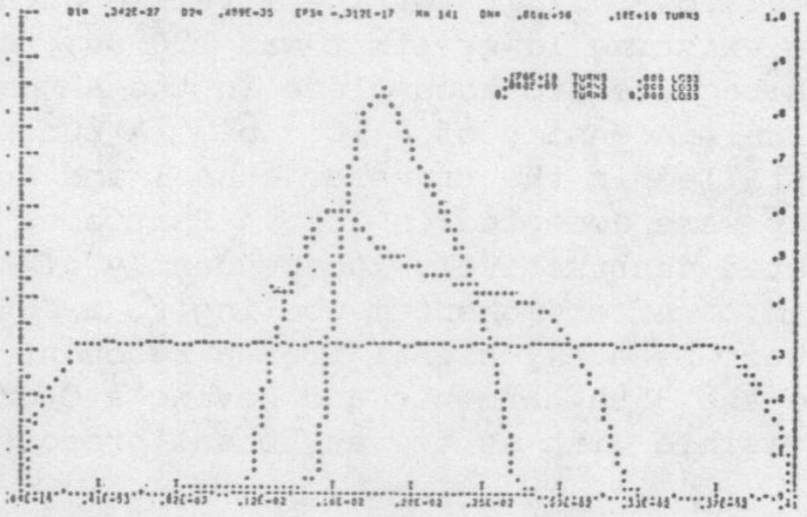
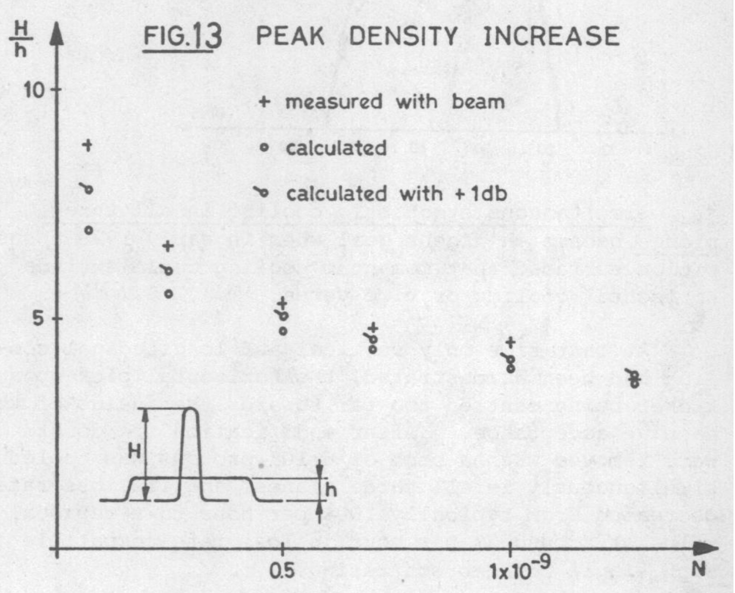


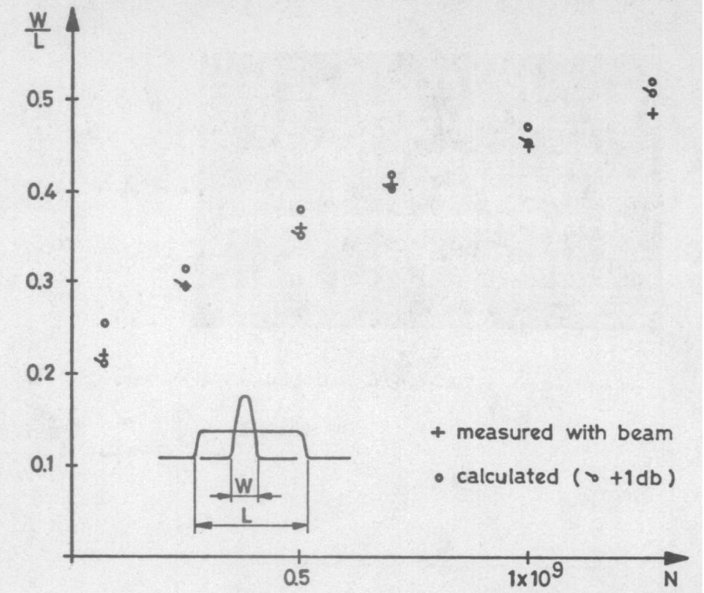
Fig. 8. 60 min, factor 28 in peak density, $2 \cdot 10^7$ protons.

The e-folding time for the peak density was further lowered to 15 s.









Experiments on SC in ICE: 3D cooling

1977:

- Longitudinal and transverse cooling systems with ~1kW power and 100-180 MHz bandwidth were installed.
- With $7\text{x}10^7$ protons of 1 GeV energy, a longitudinal mean cooling time of 15 seconds was achieved.
- With $3.9\text{x}10^8$ protons, horizontal and vertical cooling time of 4 mins was achieved.
- Later on, a beam of $6\text{x}10^9$ protons was cooled simultaneously in all 3 planes. Loss rate decreased from 100% per hour to 2% per hour.
- The ICE firmly established the SC technique.

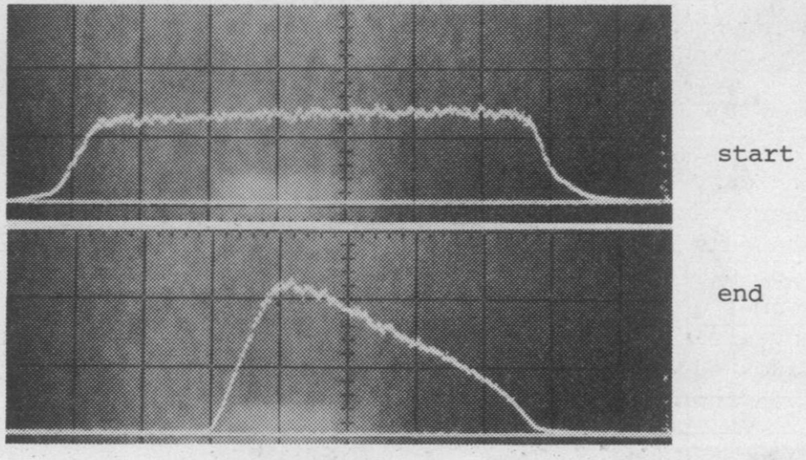


Fig. 17. Longitudinal Schottky scans before and after cooling in three planes for 30 min.

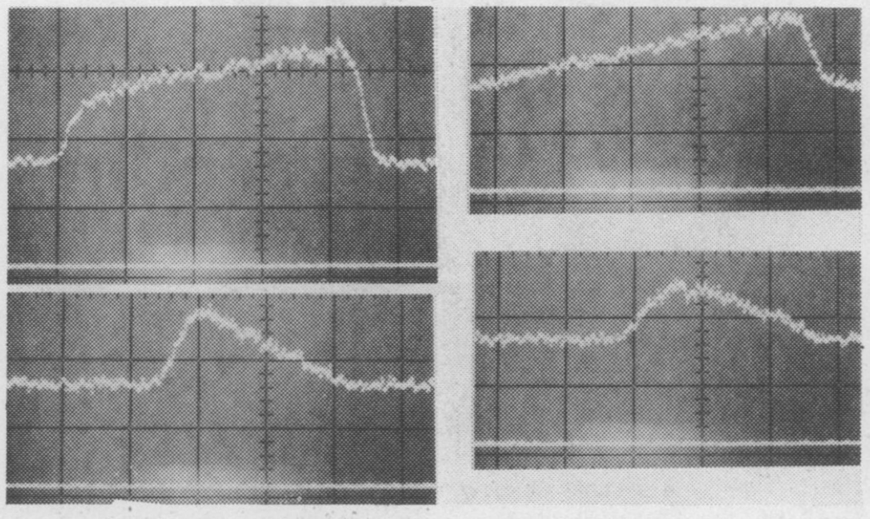


Fig. 18. Vertical and horizontal Schottky signals before and after 30 min of cooling. The rms betatron oscillation amplitude is ∿ Y, E ∿ x.

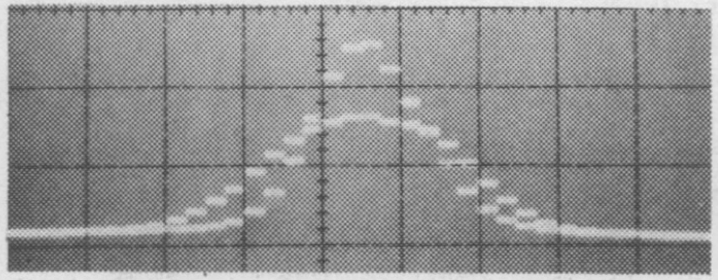


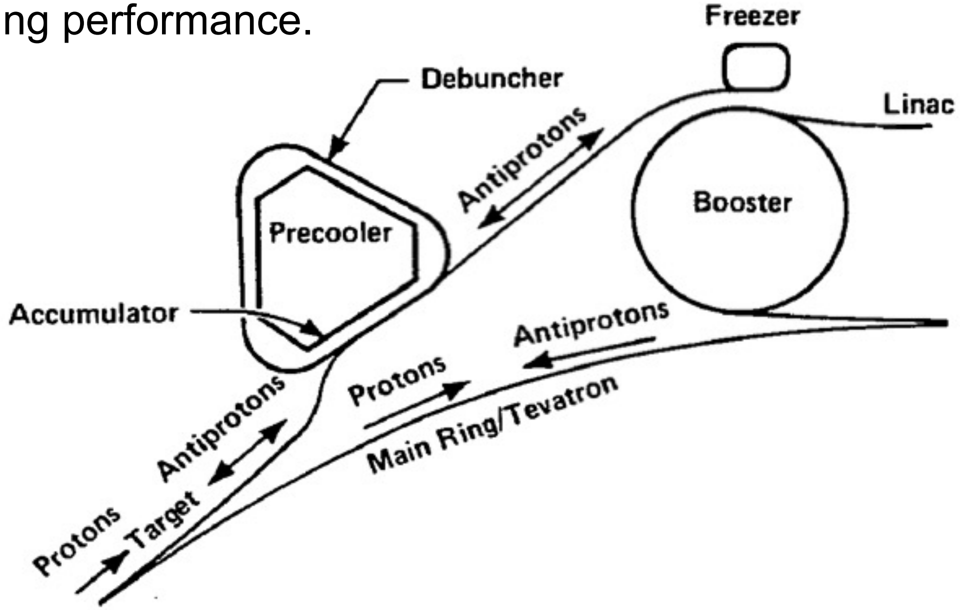
Fig. 19. Horizontal beam profile before and after cooling, as seen by a monitor based on beam-induced ionization electrons from the residual gas.

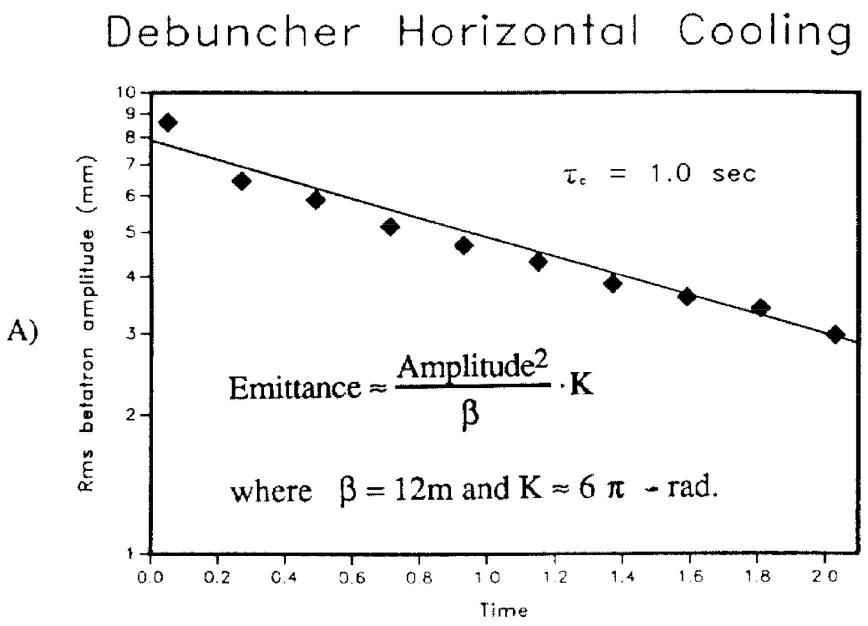
SC at FNAL

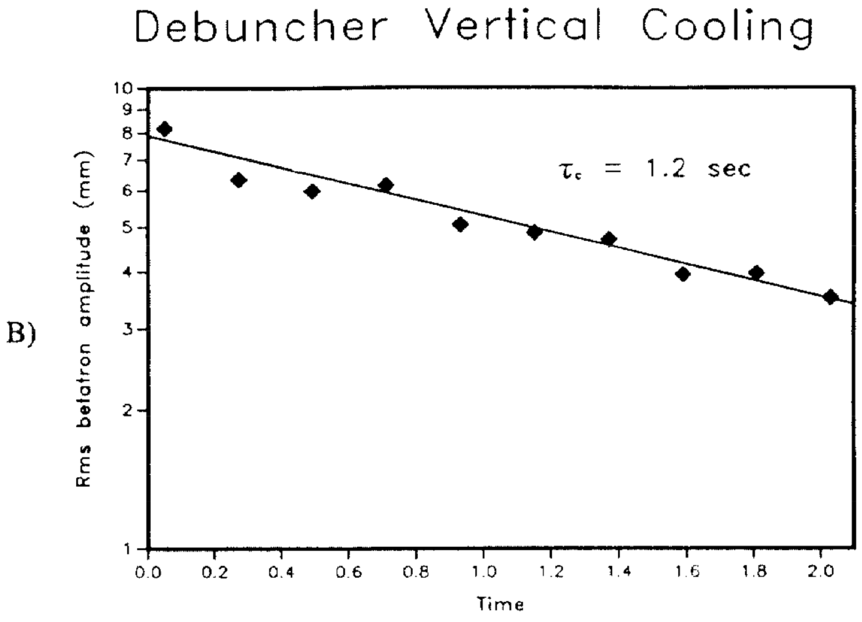
1983: the Tevatron-I project at Fermilab was approved with the aim to collide 1 TeV antiprotons with 1 TeV protons in the superconducting Tevatron.

1987: The original design of the stochastic cooling systems in the Debuncher included only transverse cooling operating from 2-4 GHz. Was designed to cool the beam emittance from 20π mm-mrad to 7π mm-mrad within 2 second cycle time.

1998:
- Increased bandwidth to 4-8 GHz.
- 8 500 MHz wide bands for the pickups combined to four 1 GHz bands for the kickers utilizing slotted waveguide structures.
- Liquid helium cooled pickups, amplifiers, and components are responsible for a front-end effective noise temperature ranging from 10K to 30K—dramatic improvement in cooling performance.

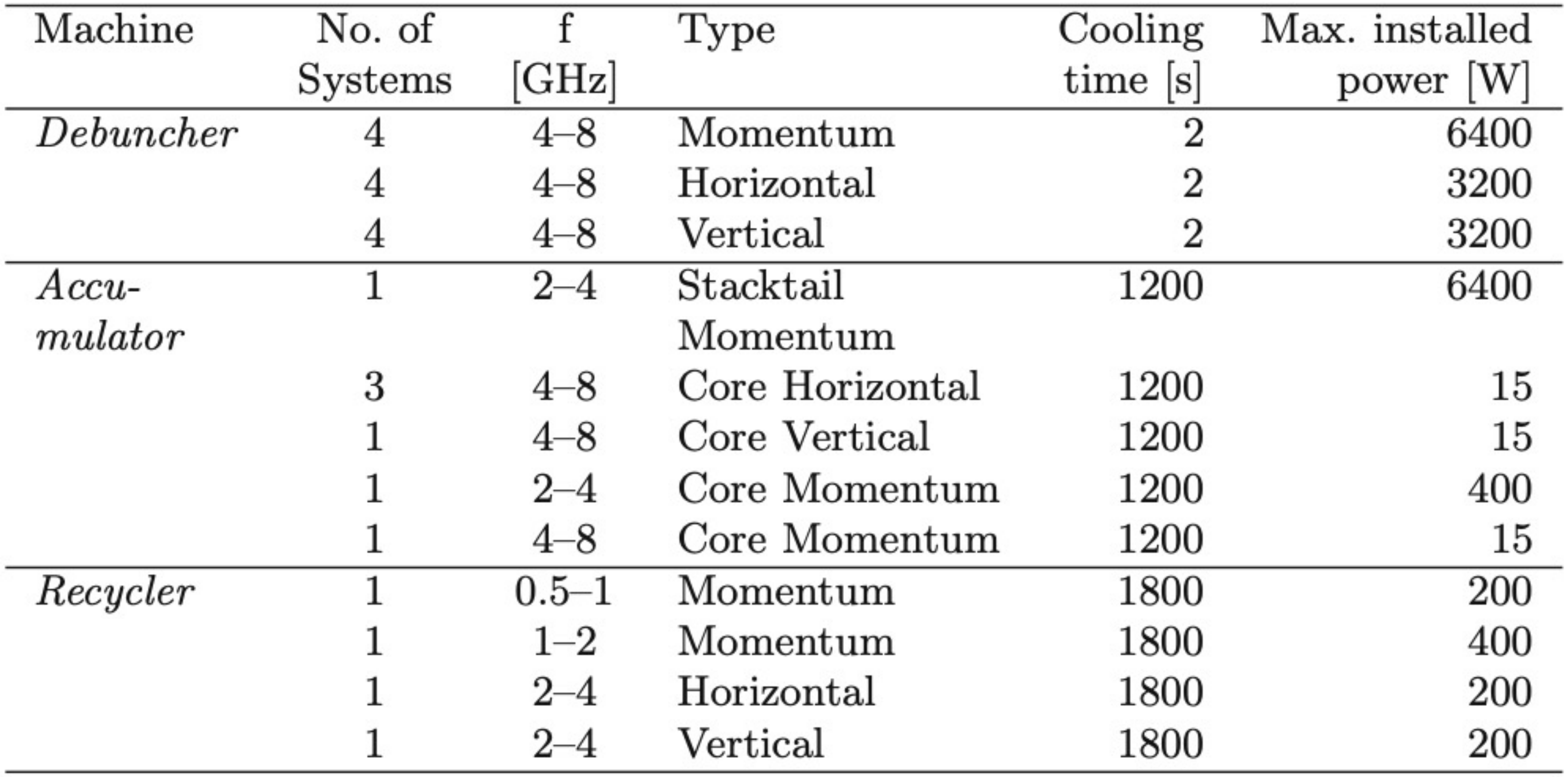






SC at FNAL

Throughout the years, the improvements demonstrated an increase in performance:
peak luminosities in the Tevatron reached $4.3\mathrm{x}10^{32}\,\mathrm{cm}^{-2}\mathrm{s}^{-1}$ – 430 times the original design value!



Machine	No. of Systems	f [GHz]	Type	Cooling time [s]	Max. installed power [W]
Debuncher	4	4–8	Momentum	2	6400
	4	4–8	Horizontal	2	3200
	4	4–8	Vertical	2	3200
Accumulator	1	2–4	Stacktail Momentum	1200	6400
	3	4–8	Core Horizontal	1200	15
	1	4–8	Core Vertical	1200	15
	1	2–4	Core Momentum	1200	400
	1	4–8	Core Momentum	1200	15
Recycler	1	0.5–1	Momentum	1800	200
	1	1–2	Momentum	1800	400
	1	2–4	Horizontal	1800	200
	1	2–4	Vertical	1800	200

SC at COoler SYnchrotron (COSY)

- SC for COSY I designed for proton momenta 1.5-3.4 GeV/c.
- 1 pickup tank of 4 m + 1 kicker of 2 m for each, horizontal and vertical, plane.
- Frequency range covered 1-3 GHz.
- The longitudinal cooling uses horizontal band I (1-1.8 GHz) cooling hardware in the sum mode.

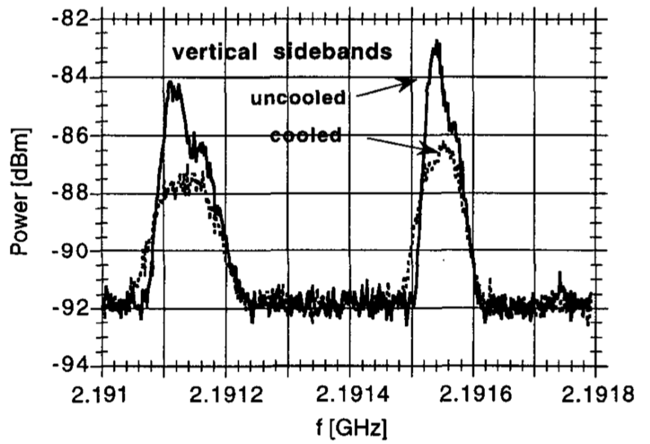
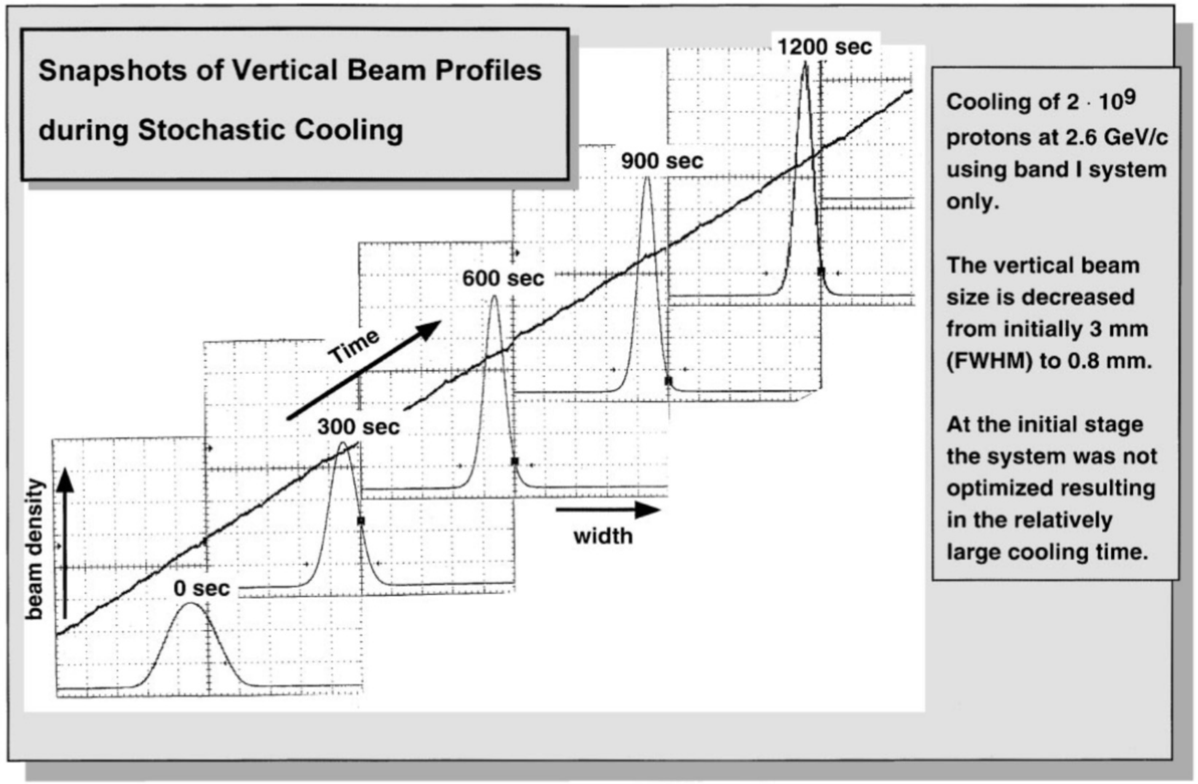


Figure 4: First stochastic cooling of the vertical beam emittance. The emittance is cooled by a factor of two.

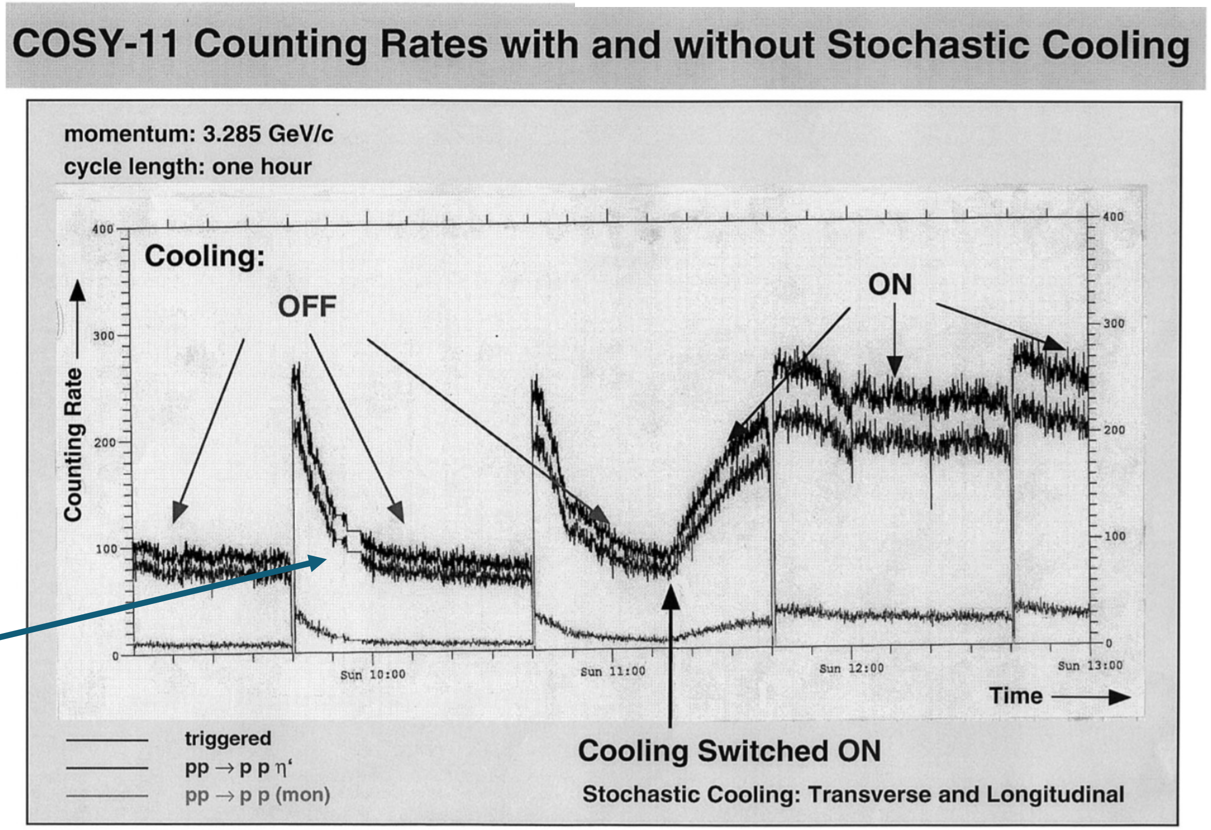


Snapshots of Vertical Beam Profiles during Stochastic Cooling

Cooling of $2 \cdot 10^9$ protons at 2.6 GeV/c using band I system only.

The vertical beam size is decreased from initially 3 mm (FWHM) to 0.8 mm.

At the initial stage the system was not optimized resulting in the relatively large cooling time.



COSY-11 Counting Rates with and without Stochastic Cooling

momentum: 3.285 GeV/c
cycle length: one hour

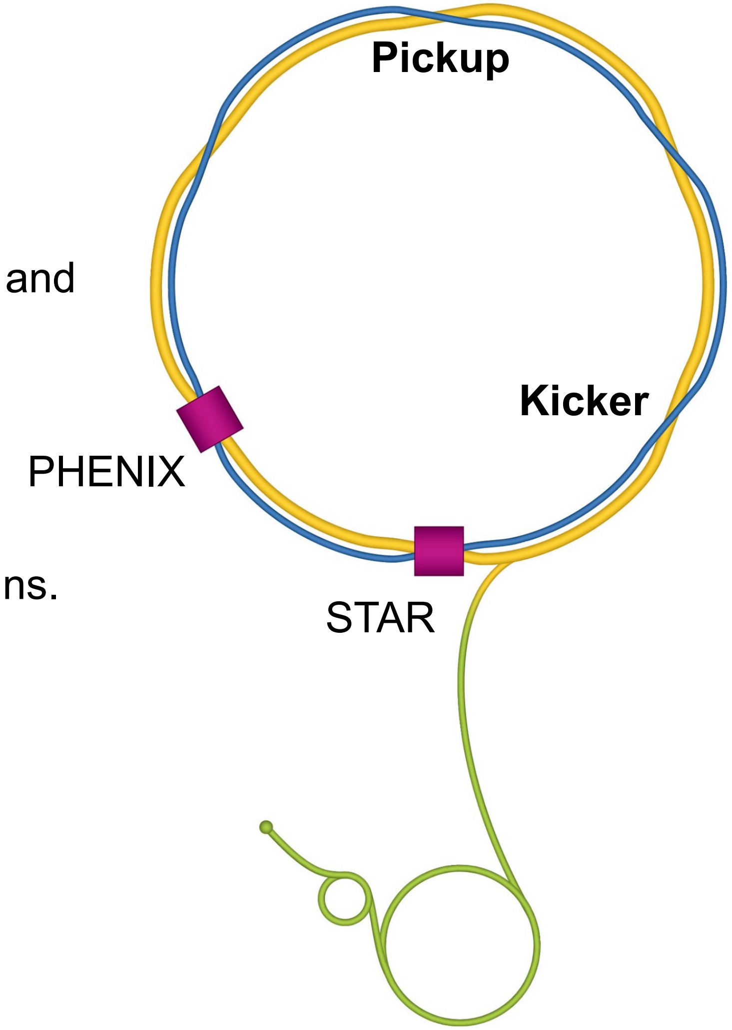
Stochastic Cooling: Transverse and Longitudinal

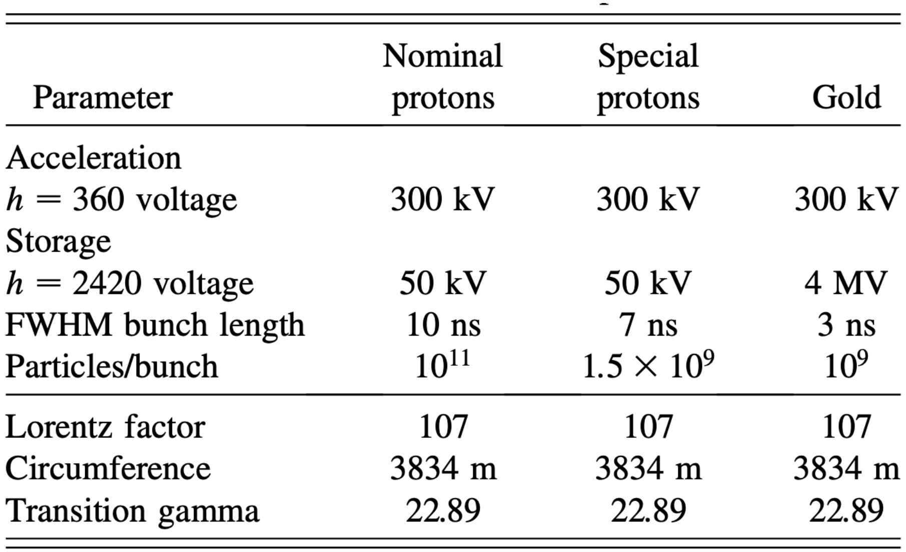
OFF: counting rate decreases rapidly due to the emittance growth of the beam and the smaller overlap between beam and target

SC at RHIC

The **Relativistic Heavy-Ion Collider (RHIC)** consists of two synchrotrons called **blue** and **yellow**, which share a common tunnel.

- A longitudinal, filter cooling system was installed in the yellow RHIC ring at first (2007).
- Intrabeam scattering rates for gold beams in RHIC are of the order of one hour.
- Optical fibers transmit the signal from the pickup to the kicker. Fibers are in the tunnel and travel against the beam.
- The cooling system operates between 5 and 8 GHz and an rms kicker voltage ~1 kV is needed for optimal cooling .
- Heavy ions are bunched by a 197 MHz rf system, and the bunches are spaced by 106 ns.

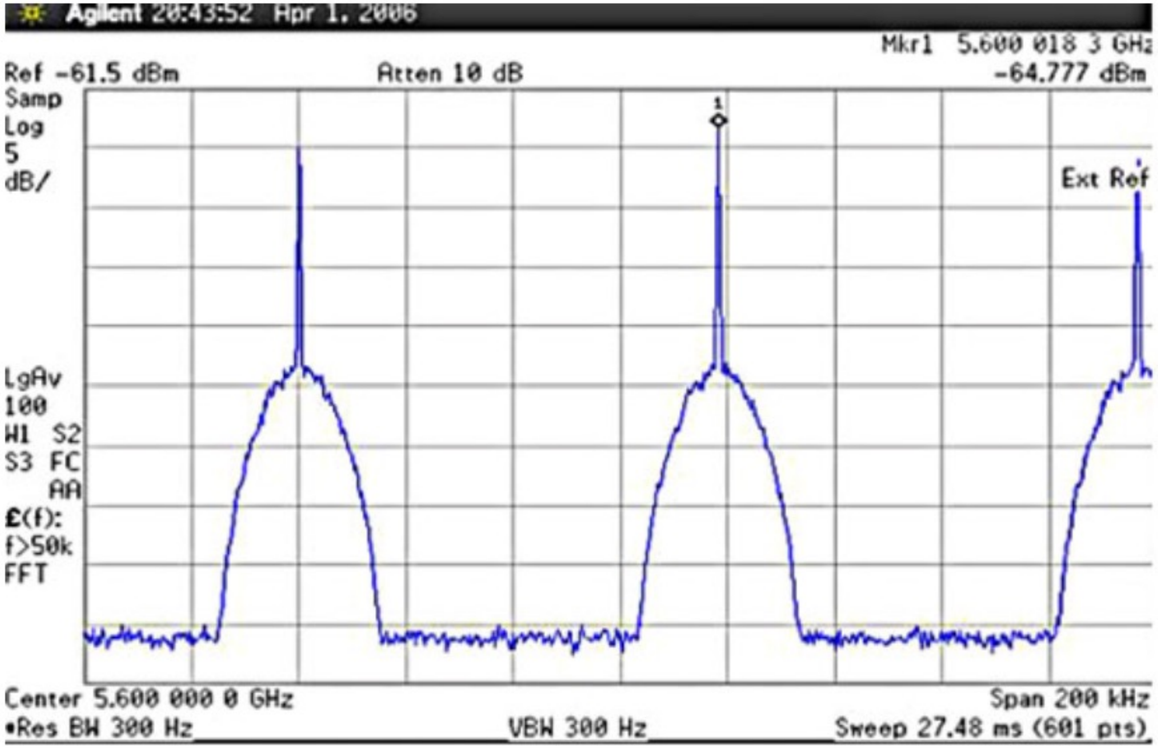


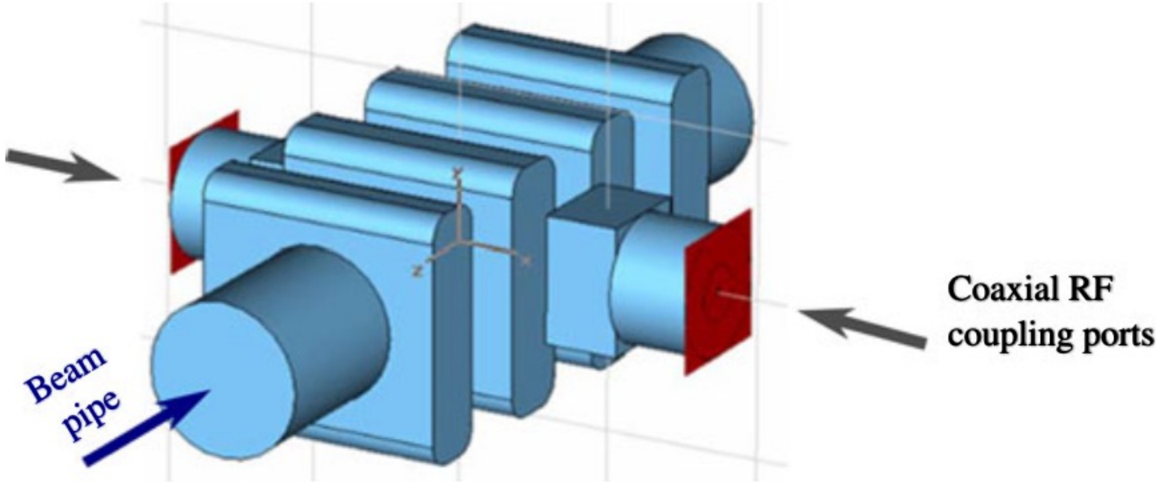


Parameter	Nominal protons	Special protons	Gold
Acceleration			
$h = 360$ voltage	300 kV	300 kV	300 kV
Storage			
$h = 2420$ voltage	50 kV	50 kV	4 MV
FWHM bunch length	10 ns	7 ns	3 ns
Particles/bunch	10^{11}	1.5×10^9	10^9
Lorentz factor	107	107	107
Circumference	3834 m	3834 m	3834 m
Transition gamma	22.89	22.89	22.89

SC at RHIC

- The parabolic shaped signal distribution represents the incoherent response from individual particles (the Schottky signal).
- The narrow peaks on top of the parabolae, situated at each revolution harmonic show the coherent contribution of particles due to a residual bunching effect.
- **2004:** the notion of using narrow-band cavities with a “Fourier series implementation” was introduced that allowed for multi-kilovolt kicks with affordable equipment.





SC at RHIC

- The Schottky signal from the bunch is processed giving the required kicker voltage.

- The red trace:

$$V(t) = \sum_{k=n_-}^{n_+} a_k(t)\cos(2\pi kt/\tau_0) + b_k(t)\sin(2\pi kt/\tau_0),$$

The longitudinal motion, neglecting cooling and multiparticle effects, is well modeled by:

$$\frac{d\tau}{dn} = T_0\eta\epsilon/E_0\beta^2 \qquad (3)$$

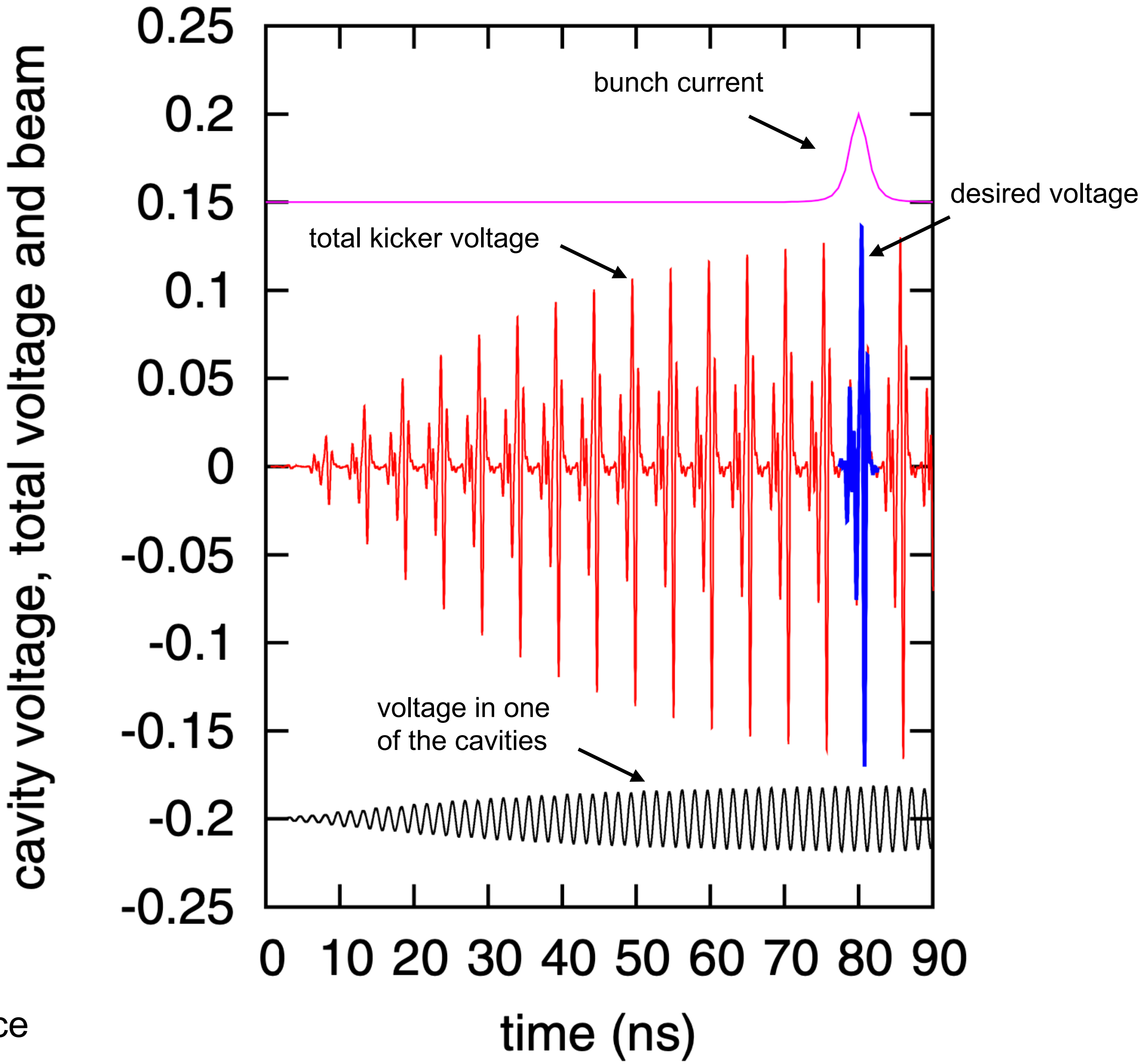
$$\frac{d\epsilon}{dn} = -q\hat{V}_1\sin(h_1\omega_0\tau) - q\hat{V}_2\sin(h_2\omega_0\tau), \qquad (4)$$

For small the motion is well approximated by:

$$\tau(n) = \hat{\tau}\sin[\omega_s(\hat{\tau})nT_0 + \psi_0], \qquad (5)$$

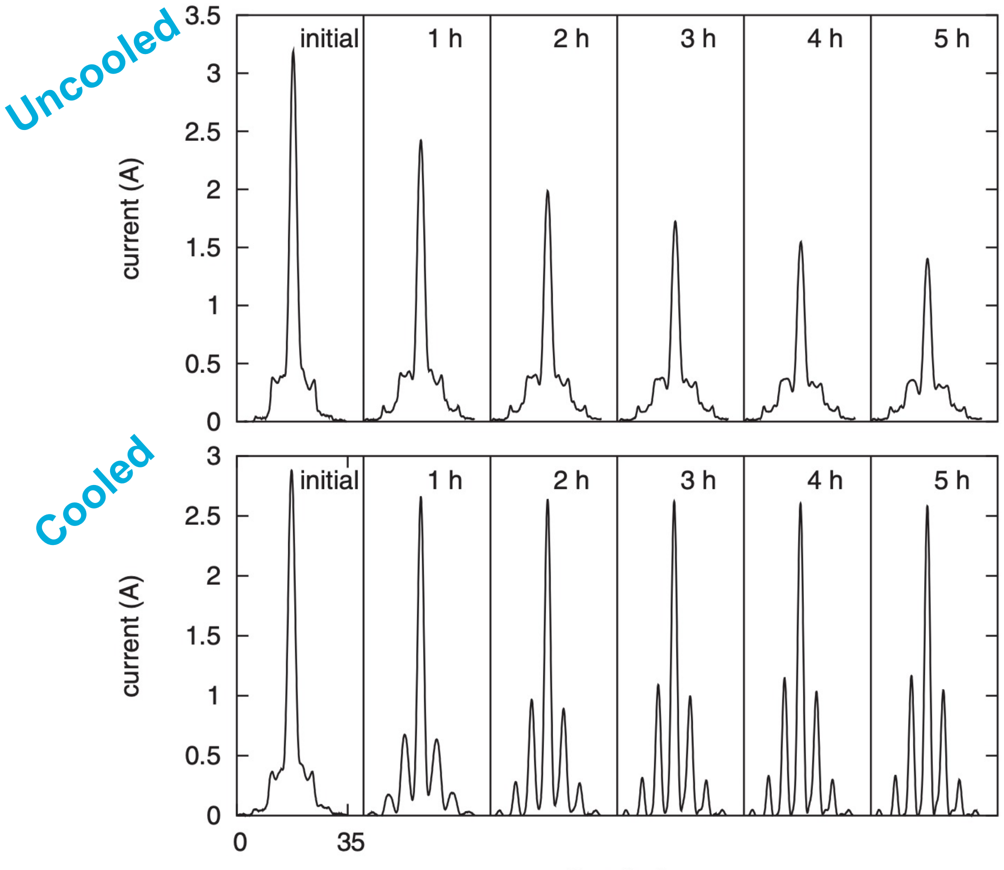
where ψ_0 is the initial phase, $\hat{\tau}$ is the amplitude, and $\omega_s(\hat{\tau})$ is the amplitude dependent synchrotron frequency. In this

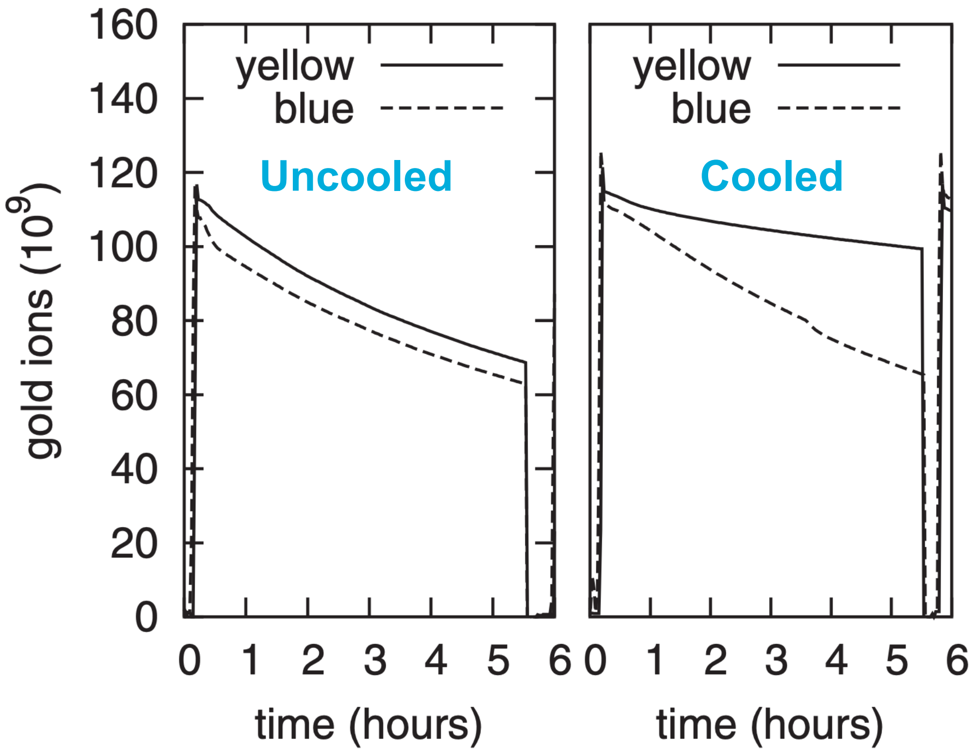
The pickup is a planar loop array consisting of 2 arrays with 16 planar loops. The signals from the two arrays are combined in sum mod e resulting in a net transfer impedance of 50 Ohm at 4 GHz.

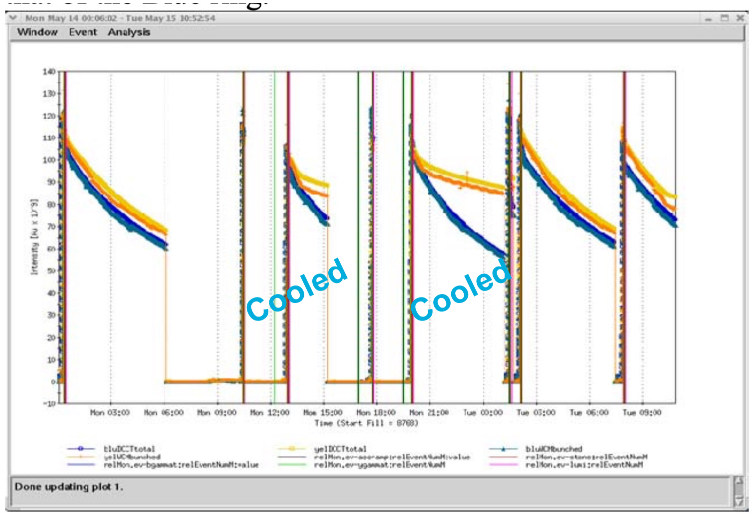


SC at RHIC

- *Uncooled:* IBS causes significant longitudinal diffusion, reducing both the peak current and the total charge in the bunch.
- RF kick has 7 zero crossings within a single 28 MHz bucket → the cooling system causes the beam to coalesce around each of these stable fixed points.
- The peak current is slightly reduced over the 5 h, but losses are far smaller than in the uncooled beam. After the first hour, the total charge outside the central 5 ns of the profile was constant to within 5%.







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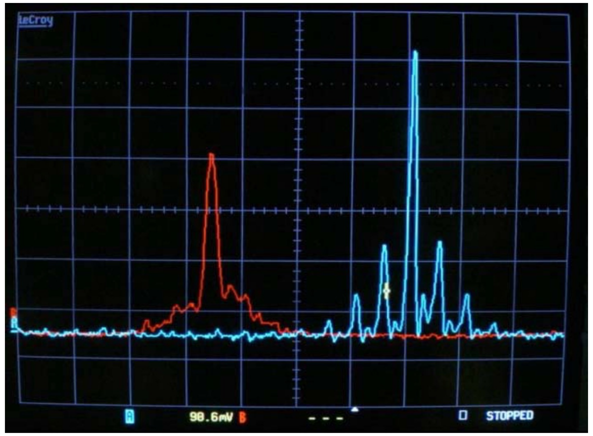
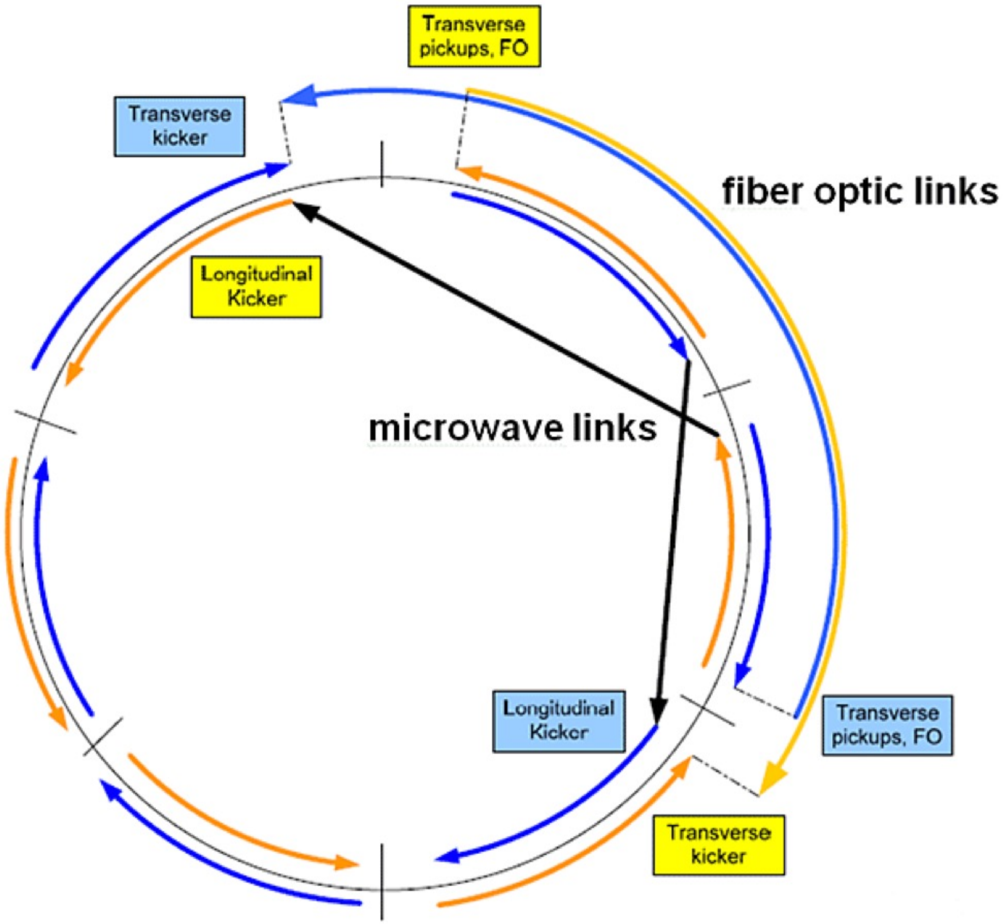
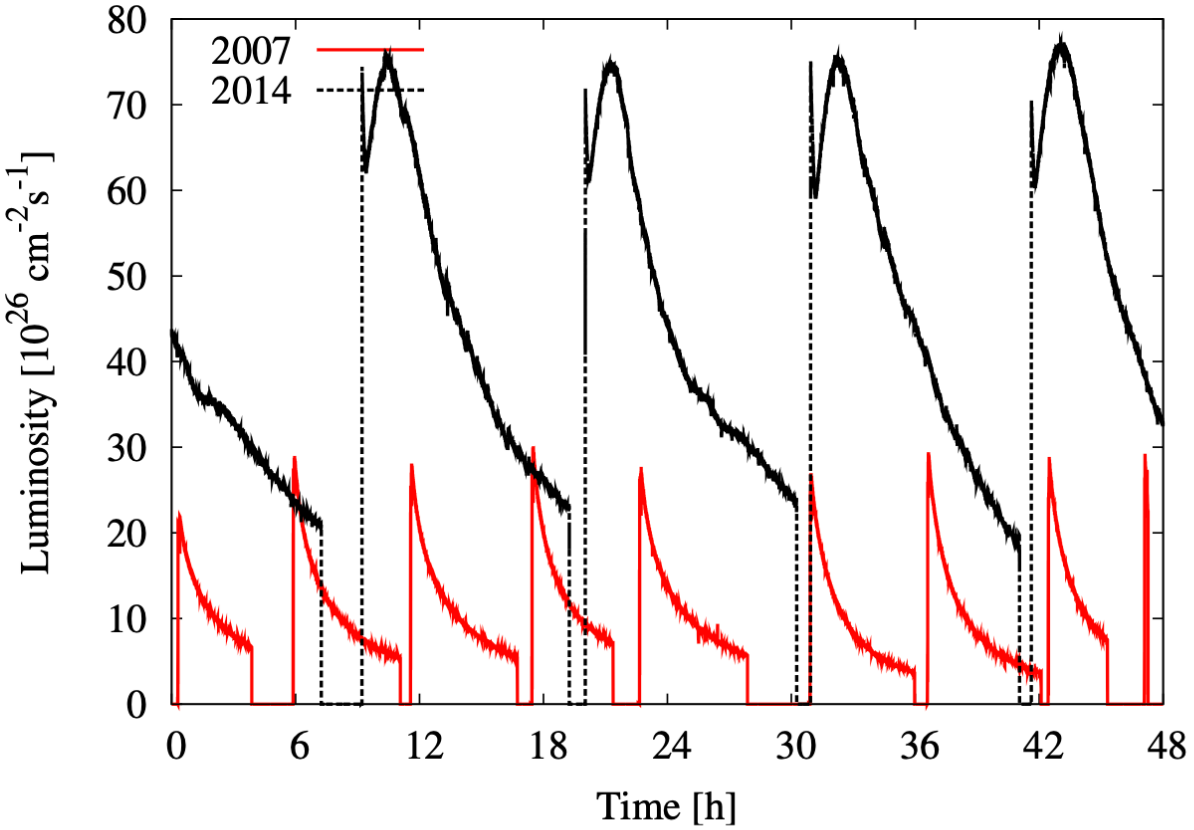


SC at RHIC: 3D cooling

- Each plane (horizontal, vertical, longitudinal) has a pick-up and a set of kickers.
- Information travels from the pick-up to the kickers via fiber-optic links for the transverse planes, and via a microwave link in the longitudinal planes.





Cooling led to first ever increase of instantaneous luminosity and smallest emittance in a hadron collider

Conclusion

- SC has been explored throughout the years and resulted in spectacular achievements including the experimental observation of the W and Z bosons, the top quark and anti-hydrogen atoms.

- The technique has grown from a curiosity in 1972 to one of the most powerful tools of accelerator technology.

- The technique of stochastic cooling of the bunched beam in a high energy collider has been proven and considerably advanced by the outstanding developments at BNL.