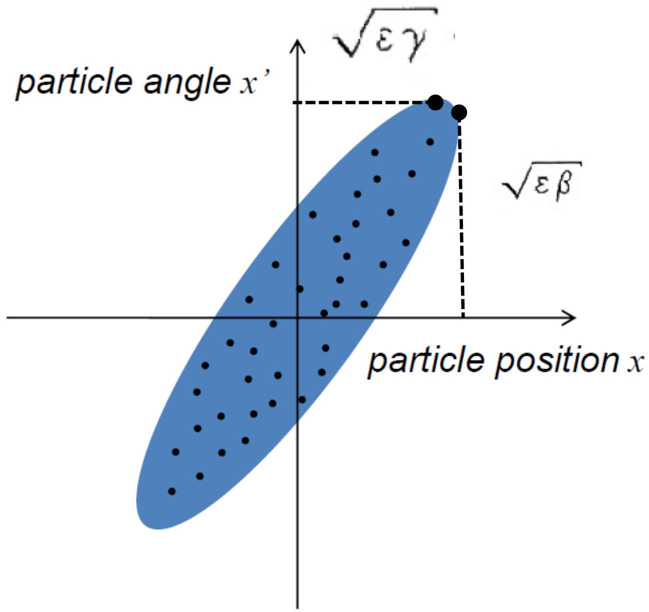
PHY542. Emittance Measurements

D. Kayran, M. Fedurin

March 20, 2023

PHY 542 Fundamentals of Accelerator Physics and Technology with Simulations and Measurements Lab

Emittance, what is it ?



ε = Area in *x, x'* plane occupied by beam particles divided by π

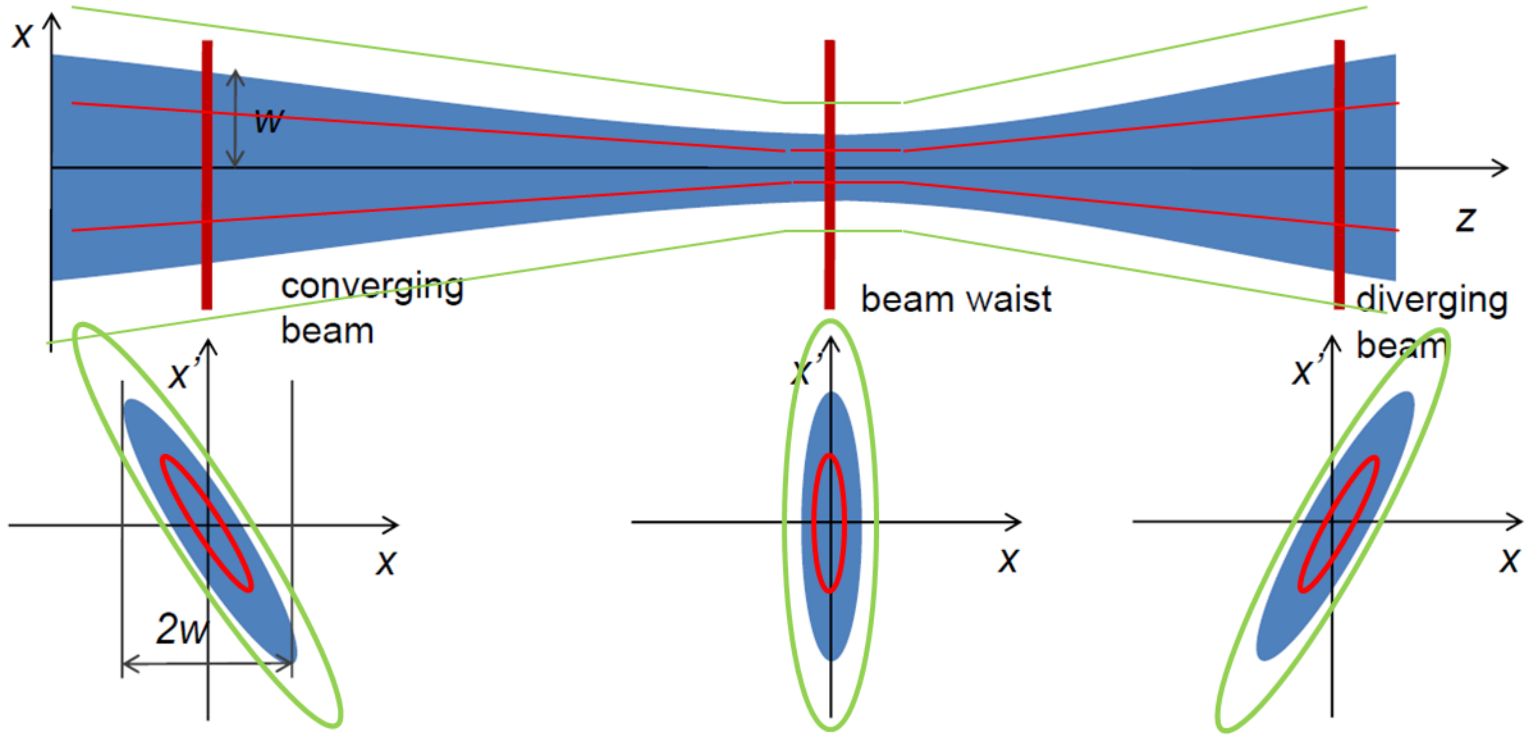
Beam ellipse and its orientation is described by 4 parameters

$$\varepsilon = \gamma x^2 + 2\,\alpha\, x\, x' + \beta\, x'^2$$

$\sqrt{\beta\varepsilon}$	Is the beam half width
$\sqrt{\gamma\varepsilon}$	Is the beam half divergence
α	Describes how strong x and x' are correlated a<0 beam diverging a>0 beam converging a=0 beam size is maximum or minimum(waist)

The three orientation parameters are connected by the relation $\gamma = \dfrac{1+\alpha^2}{\beta}$

Beam envelope along a beamline.

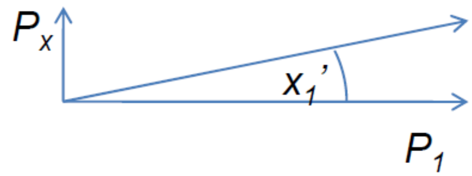


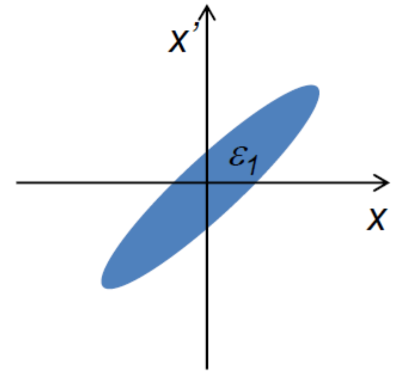
Along a beamline the orientation and aspect ratio of the beam ellipse in x, x' changes, but area (emittance) remains constant.
Alike initial beam distributions have similar phase space dynamics

Beam width along Z is described $$w(z) = \sqrt{\beta(z)\,\varepsilon}$$

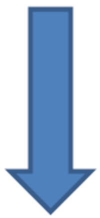
$\beta(z)$ describes the beam line, ε – describes beam quality

Geometrical emittance is only constant in beamlines without acceleration

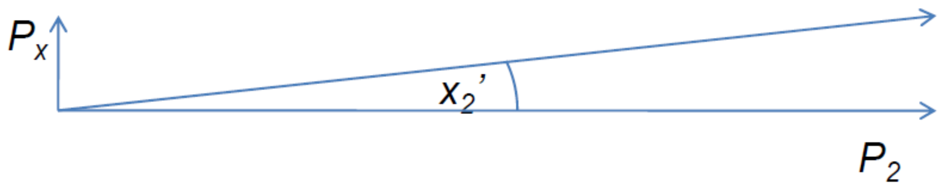


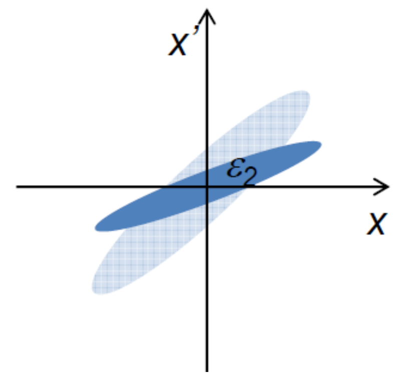


acceleration



$$\varepsilon_2 = \varepsilon_1 \frac{P_1}{P_2} \quad \text{with} \quad P = m\beta_{rel}\gamma_{rel}\, c$$



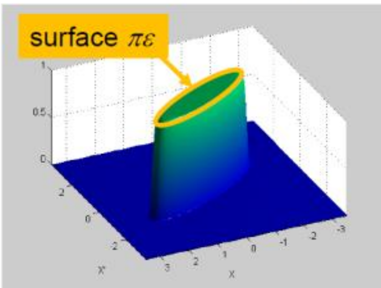


Normalized emittance preserved with acceleration

$$\varepsilon_N = \beta_{rel}\gamma_{rel}\ \varepsilon$$

R.m.s emittance

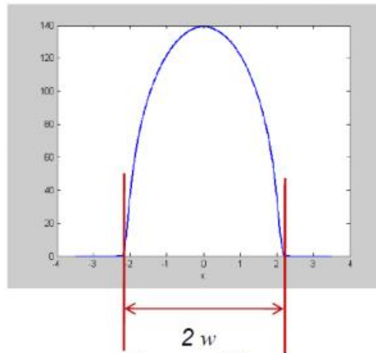
particle density in x, x' space



projection on x axis

$$f(x) = \int_{-\infty}^{\infty} \rho(x, x')\, dx'$$

transverse beam profile

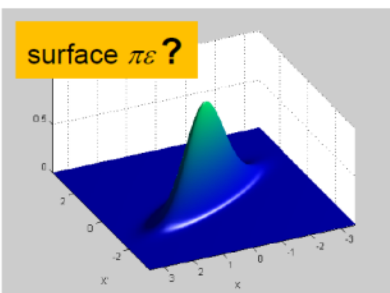


2 w
beamwidth

$$w = \sqrt{\beta\varepsilon}$$

In reality beam density in x, x' space is rarely a area with sharp boundary

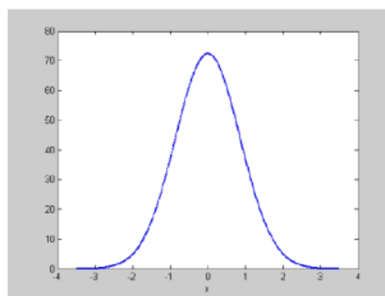
particle density in x, x' space



projection on x axis

$$f(x) = \int_{-\infty}^{\infty} \rho(x, x')\, dx'$$

transverse beam profile



beamwidth ?

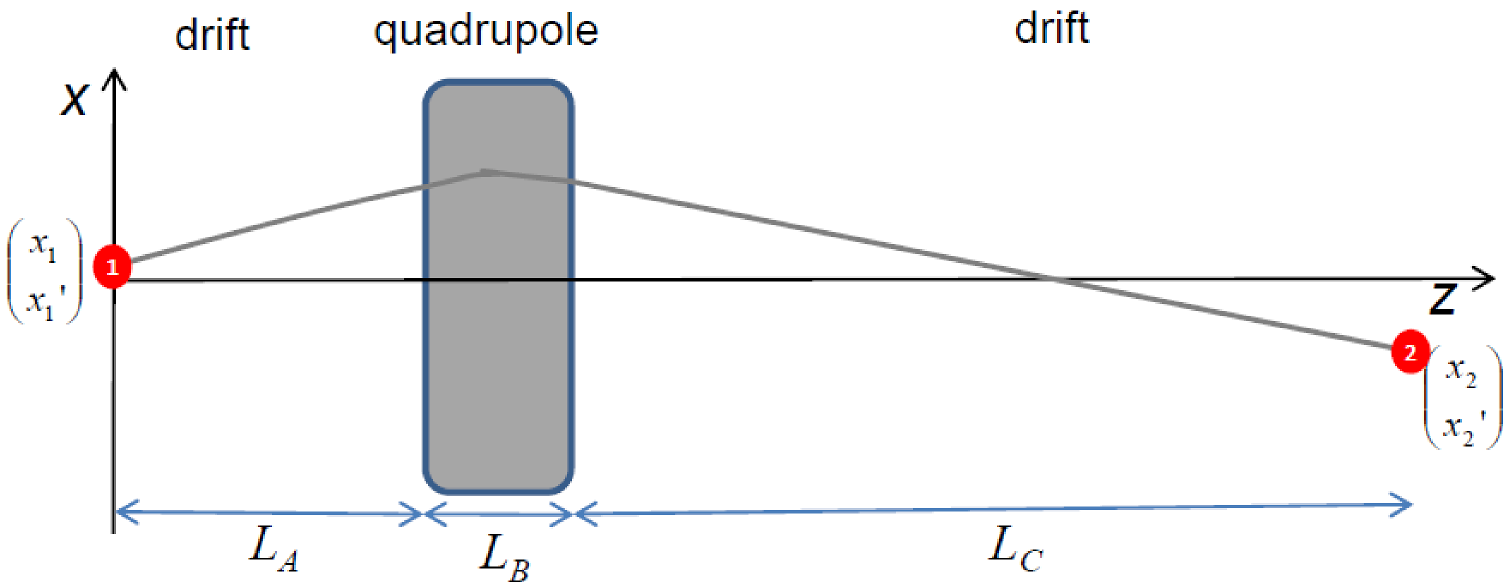
$$w_{RMS} = \sqrt{\frac{\int_{-\infty}^{+\infty} (x - x_{COG})^2 f(x)\, dx}{\int_{-\infty}^{+\infty} f(x)\, dx}}$$

For Gauss beam distribution

$$w_{rms} = \sigma_X$$

ε_{rms} , 1 sigma r.m.s. emittance $\Leftrightarrow$ $w_{RMS} = \sqrt{\beta\varepsilon}$

Transport of single particle described with matrix



$$M_{Drift} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \qquad M_{Quadrupole} = \begin{pmatrix} \cos\left(\sqrt{k}L\right) & 1/\sqrt{k}\sin\left(\sqrt{k}L\right) \\ -\sqrt{k}\sin\left(\sqrt{k}L\right) & \cos\left(\sqrt{k}L\right) \end{pmatrix}$$

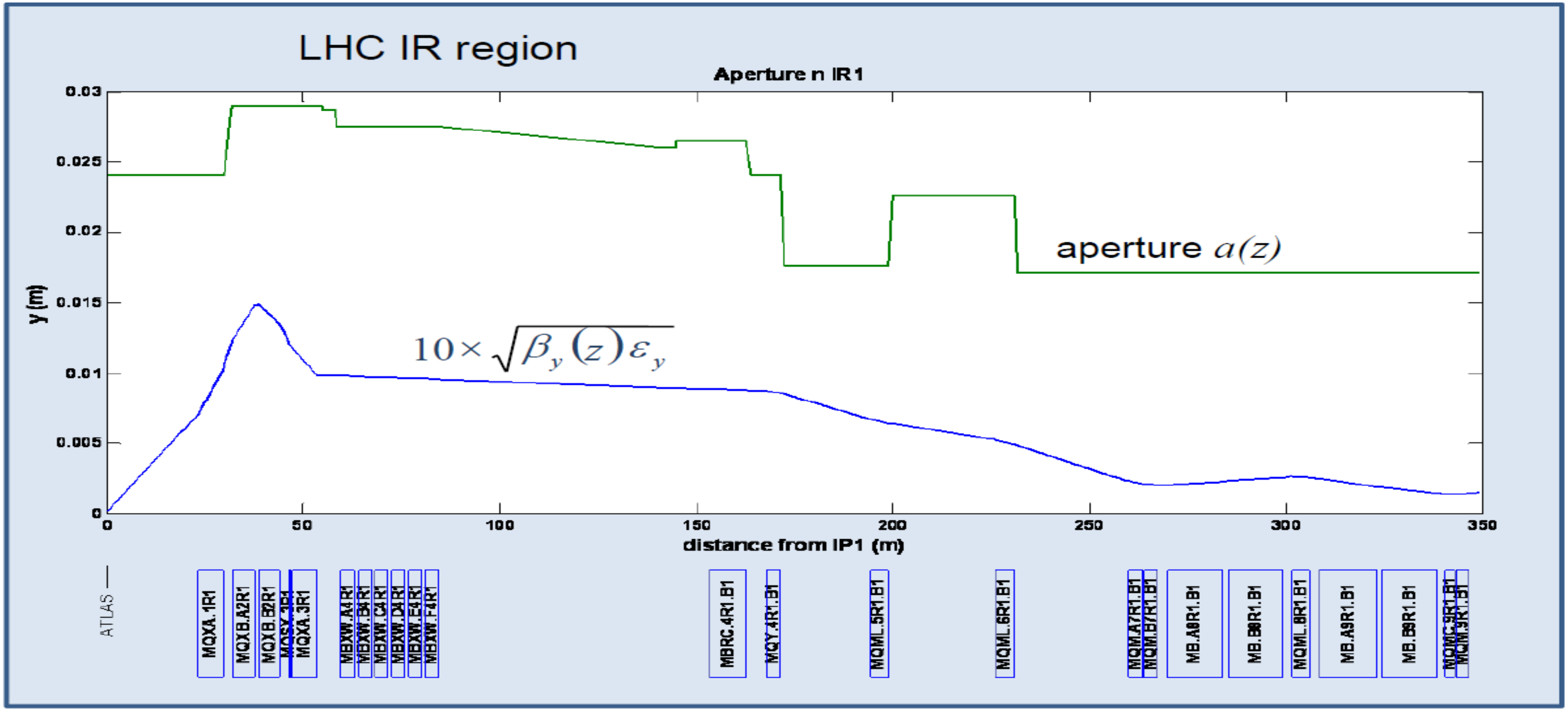
$$\begin{pmatrix} x_2 \\ x_2' \end{pmatrix} = M \cdot \begin{pmatrix} x_1 \\ x_1' \end{pmatrix}$$ There $$M = \begin{pmatrix} 1 & L_C \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos\left(\sqrt{k}L_B\right) & 1/\sqrt{k}\sin\left(\sqrt{k}L_B\right) \\ -\sqrt{k}\sin\left(\sqrt{k}L_B\right) & \cos\left(\sqrt{k}L_B\right) \end{pmatrix} \cdot \begin{pmatrix} 1 & L_A \\ 0 & 1 \end{pmatrix}$$

Transport line elements

- Dipoles
- Quadrupole
- Solenoid

Emittance, why measuring it ?

The emittance tells if a beam **fits** in the vacuum chamber or not $w(z) = \sqrt{\beta(z)\,\varepsilon} < a(z)$



Emittance is one of key parameters for overall performance of an accelerator:

- *Luminosity of colliders for particle physics*
- *Brightness of synchrotron radiation sources*
- *Wavelength range of free electron lasers*
- *Resolution of fixed target experiments*

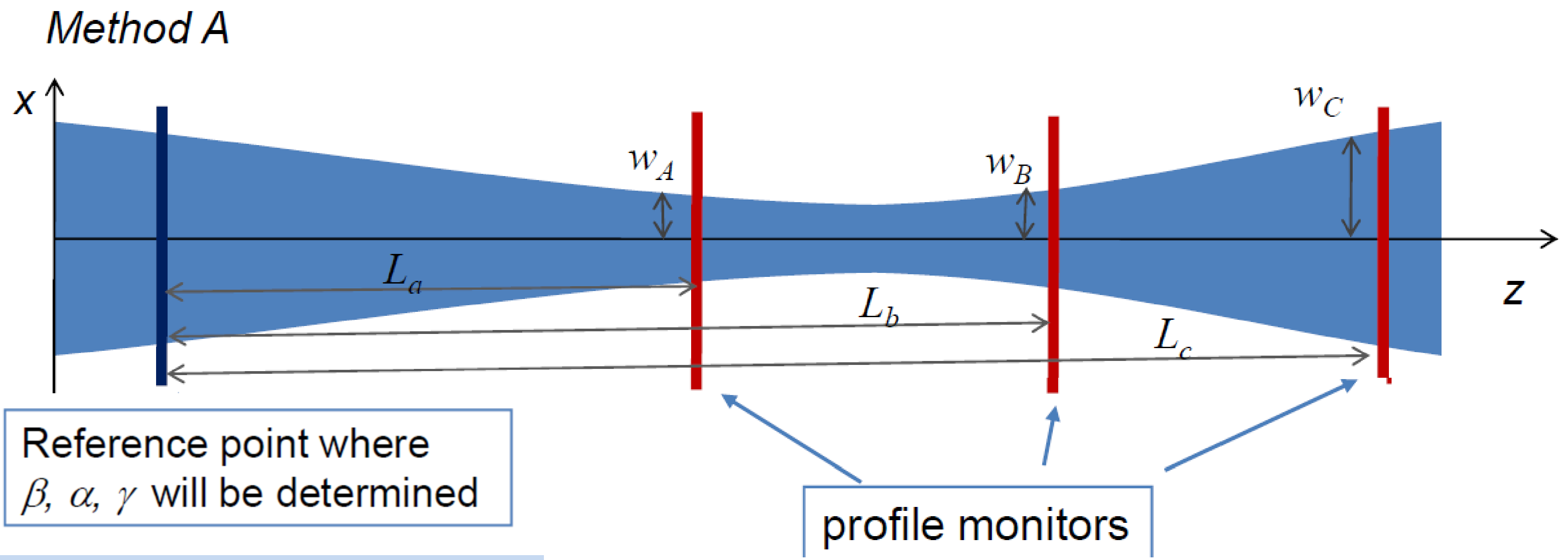
Emittance, how to measure it ?

- Methods based on transverse beam profile measurements
  - Different location beam profile measurements
  - Quadrupole scan

  Slit and pepper pot(multi slit) methods

Emittance measurement in transfer line or linac

Twiss parameters α,β,γ are a priori not known, they have to be determined together with emittance ε



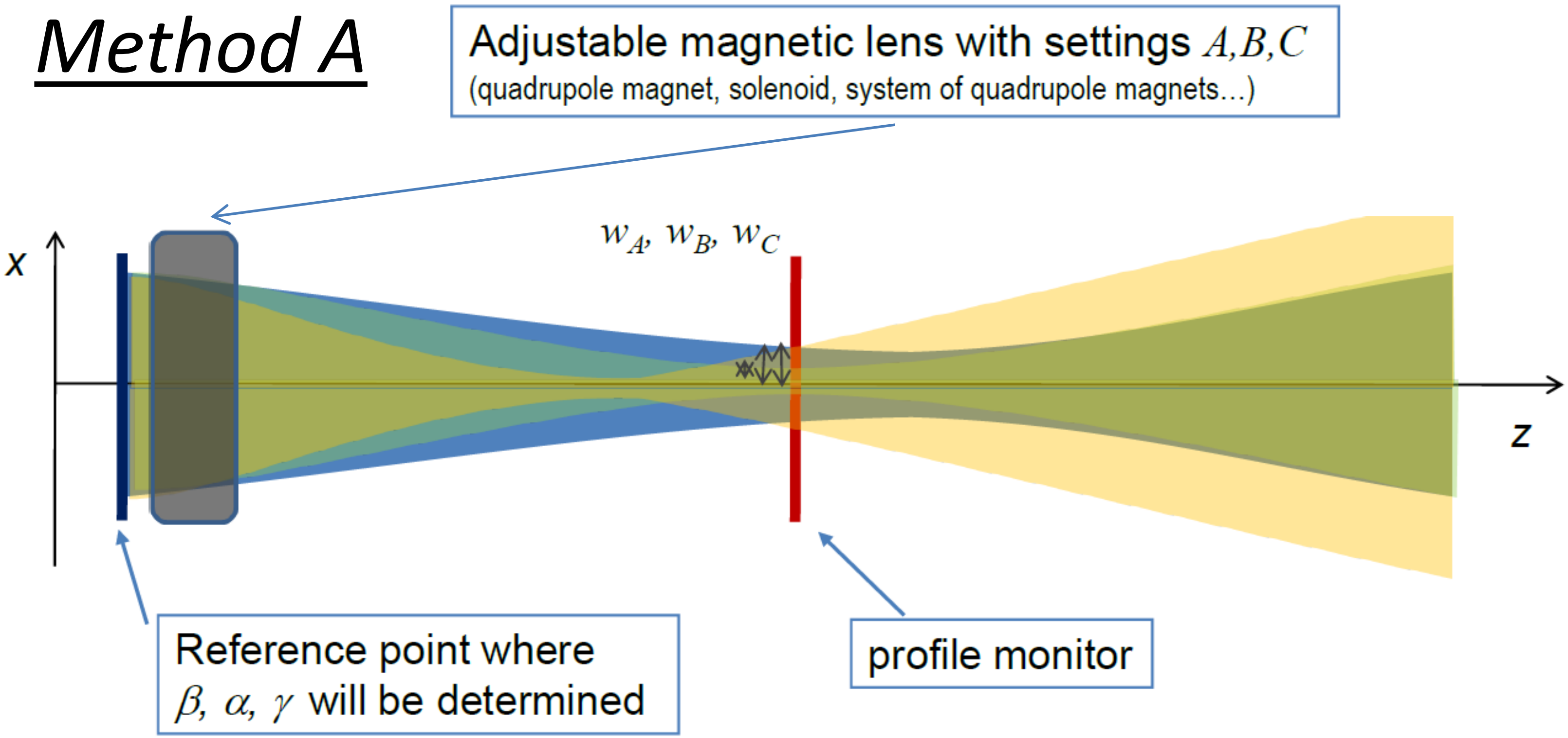
$$w_A^2 = \beta\,\varepsilon - 2\,L_A\,\alpha\,\varepsilon + L_A^2\,\gamma\,\varepsilon$$
$$w_B^2 = \beta\,\varepsilon - 2\,L_B\,\alpha\,\varepsilon + L_B^2\,\gamma\,\varepsilon$$
$$w_C^2 = \beta\,\varepsilon - 2\,L_C\,\alpha\,\varepsilon + L_C^2\,\gamma\,\varepsilon$$

3 linear equations, 3 independent variable
Solved by inverting matrix.

$$\beta\varepsilon\cdot\gamma\varepsilon - (\alpha\varepsilon)^2 = \varepsilon^2\left(\beta\cdot\gamma - \alpha^2\right) = \varepsilon^2 \quad \Rightarrow \quad \sqrt{\beta\varepsilon\cdot\gamma\varepsilon - (\alpha\varepsilon)^2} = \varepsilon, \quad \beta = \frac{\beta\varepsilon}{\varepsilon}, \quad \alpha = \frac{\alpha\varepsilon}{\varepsilon}$$

Emittancemeasurement in transfer line or linac, (count.)

- ***Method A***



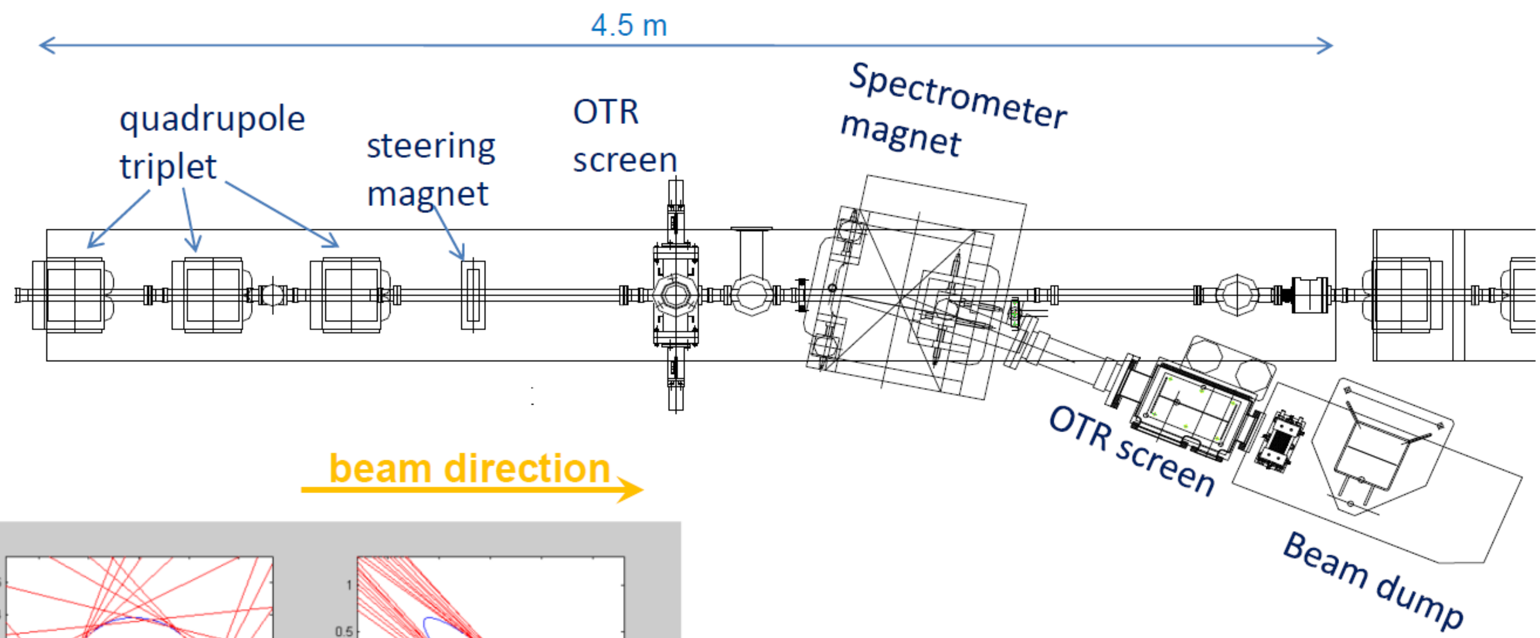
$$w^2 = c^2\,\beta\varepsilon - 2cs\,\alpha\,\varepsilon + s^2\,\gamma\,\varepsilon, \qquad \begin{pmatrix} c & s \\ c' & s' \end{pmatrix} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} m_{11}(I_{mag}) & m_{12}(I_{mag}) \\ m_{21}(I_{mag}) & m_{22}(I_{mag}) \end{pmatrix}$$

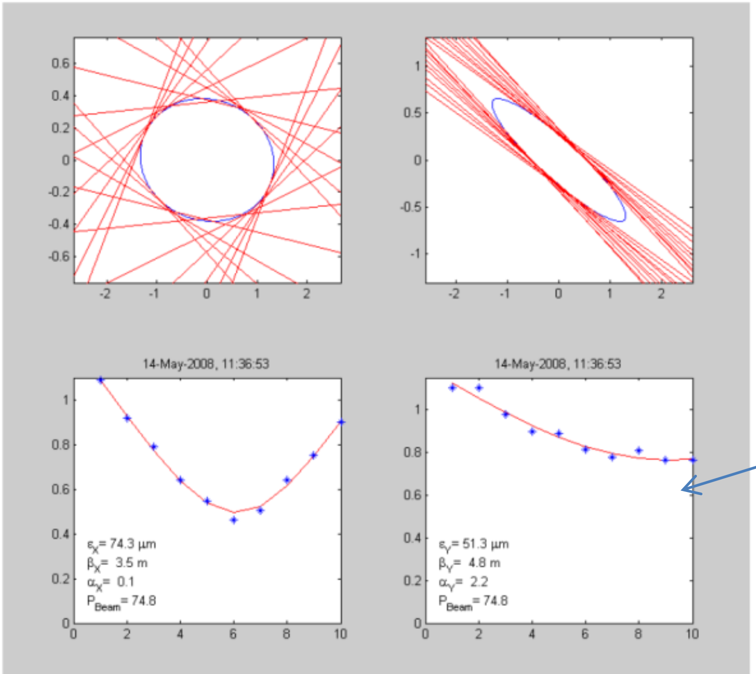
$$w_A^2 = c_A^2\beta\varepsilon - 2c_A s_A\,\alpha\,\varepsilon + s_A^2\,\gamma\,\varepsilon$$
$$w_B^2 = c_B^2\beta\varepsilon - 2c_B s_B\,\alpha\,\varepsilon + s_B^2\,\gamma\,\varepsilon$$
$$w_C^2 = c_C^2\beta\varepsilon - 2c_C s_C\,\alpha\,\varepsilon + s_C^2\,\gamma\,\varepsilon$$

3 linear equations, 3 independent variable
Solved by inverting matrix.

$$\beta\varepsilon\cdot\gamma\varepsilon - (\alpha\varepsilon)^2 = \varepsilon^2\left(\beta\cdot\gamma - \alpha^2\right) = \varepsilon^2 \quad \Rightarrow \quad \sqrt{\beta\varepsilon\cdot\gamma\varepsilon - (\alpha\varepsilon)^2} = \varepsilon, \quad \beta = \frac{\beta\varepsilon}{\varepsilon}, \quad \alpha = \frac{\alpha\varepsilon}{\varepsilon}$$

Emittance measurement example:



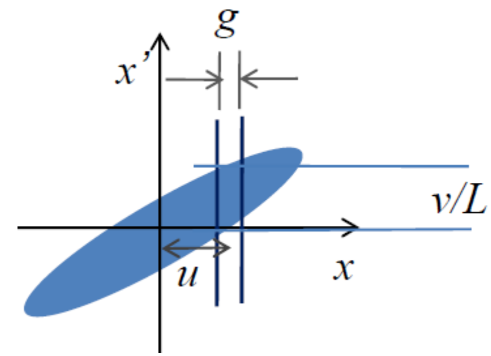


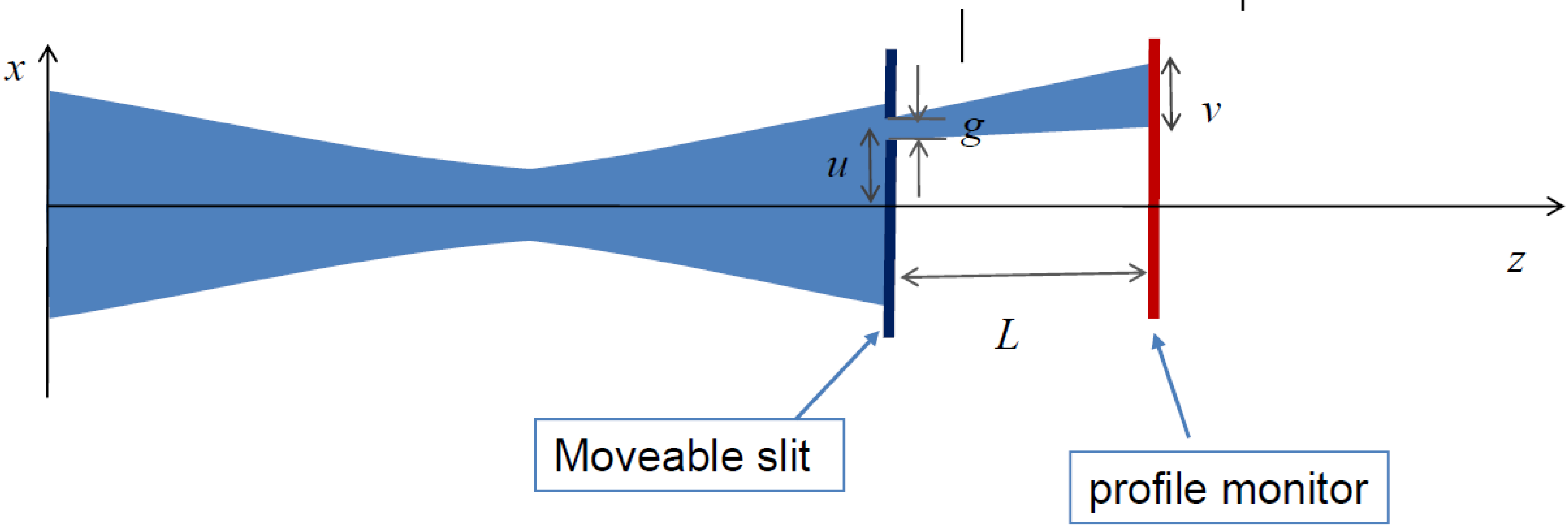
Parabola fit for quadrupole scan

Summary beam profile technics

- To determine ε, β, α at a reference point in a beamline one needs at least three *w* measurements with different transfer matrices between the reference point and the *w* measurements location.
- Different transfer matrices can be achieved with different profile monitor locations, different focusing magnet settings or combinations of both.
- Once β, α at one reference point is determined the values of β, α at every point in the beamline can be calculated.

Emittance measurement with moveable slit





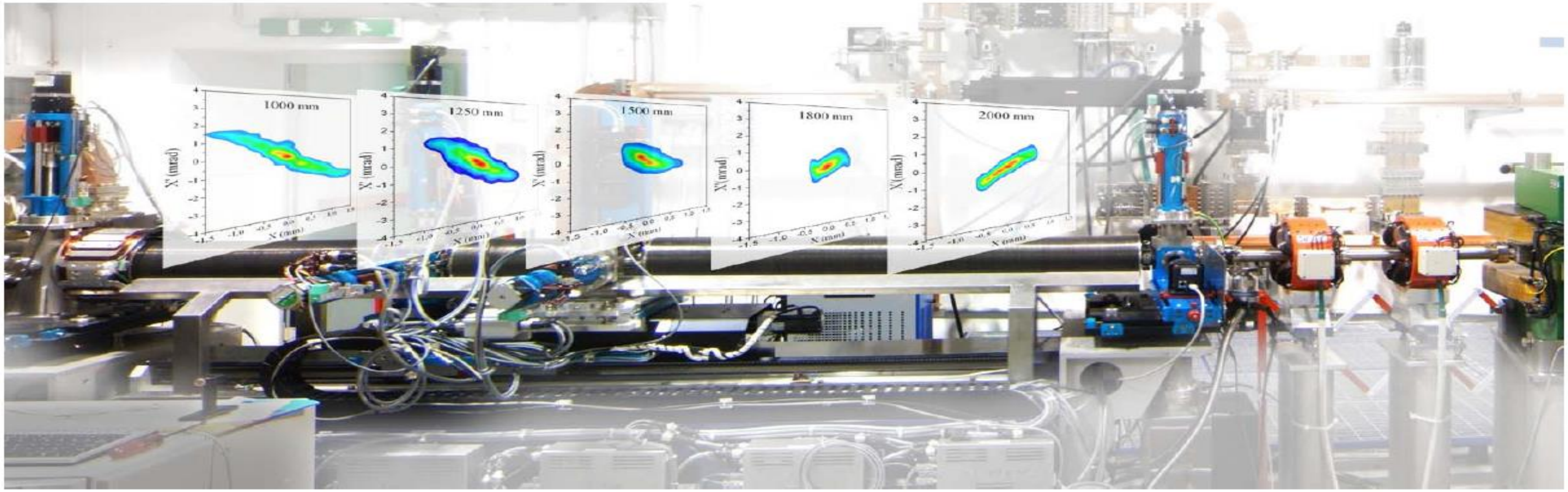
From width and position of slit image mean beam angle and divergence of slice at position *u* is readily computed.

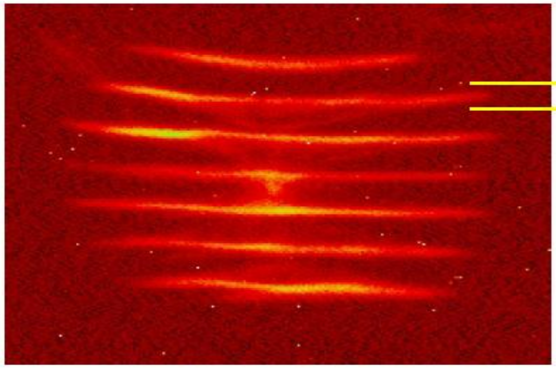
By moving slit across the beam complete distribution in *x, x'* space is reconstructed.

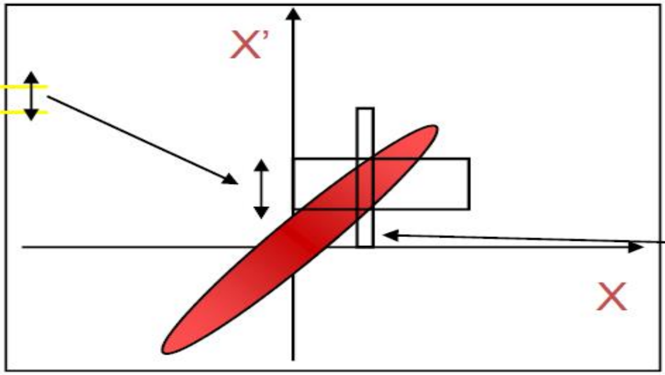
Conditions for good resolution: $v >> s$

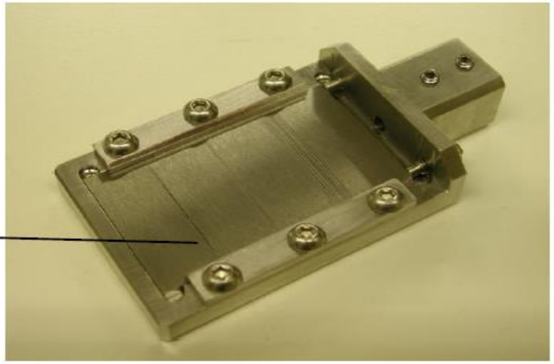
Required multiple shots to reconstruct the phase space.

Phase space reconstruction at SPARC (LNF) Multi slit









Measurements could be performed for single bunch in one shot at the single multi slit location

Things to do in UED control room

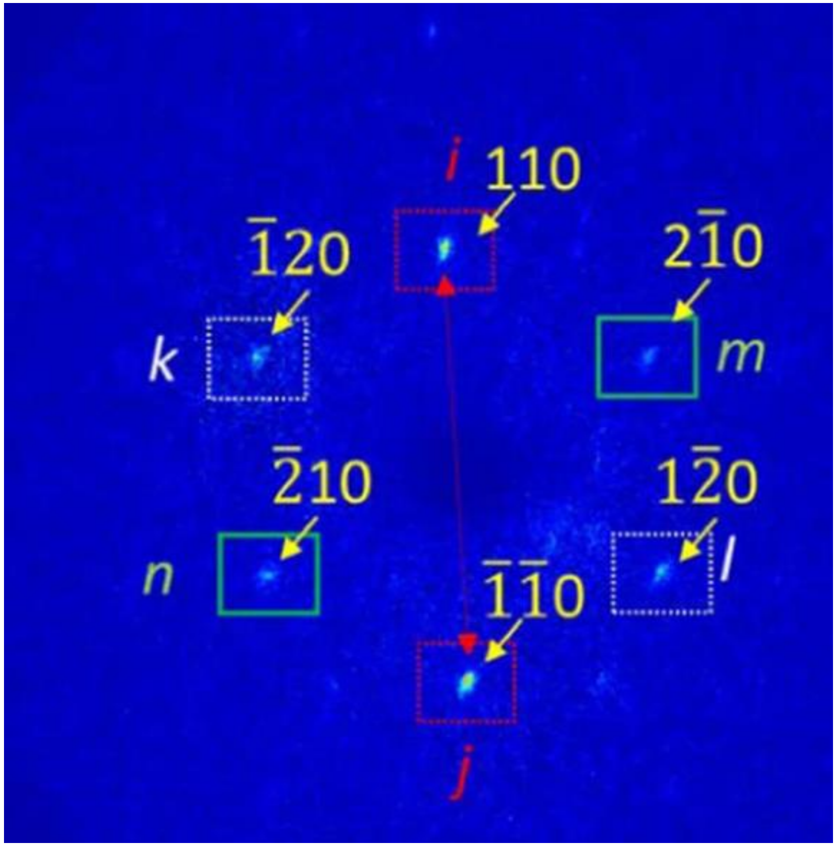
Today (demonstration)

1. Measuring Beam Energy using electron diffraction images
2. Measure cathode QE
3. Learn how to measure emittance using solenoid scan

Possible Lab experiment at one of the future ATF/UED classes :

1. Collect minimum 10 diffraction images.
   - ***HW1 Analyze them to calculate energy (see next slide) and shot-to-shot energy stability***
   - ***HW2 Plot Histogram***
2. Collect and analyze data for QE efficiency measurements
   - ***HW1 plot bunch charge vs laser intensity find the slope and conduct error analysis***
3. Measure emittance for different RF gun solenoid current settings Collect several data for each solenoid settings (minimum 10 solenoid settings).
   - ***HW1: Plot these data. Find the optimum solenoid settings and minimum emittance.***
   - ***Conduct error analysis.***
   - ***HW2: Plot bunch size vs solenoid current. Calculate emittance (see next slide) for each solenoid settings (units?).***
   - ***HW3: Based on quadrupole scan find quadrupole calibration coefficient K[1/(m*A)] to convert quadrupole current to quadrupole focus strength (1/f[m]=K*I[A]). For given energy***

Ultrafast Electron Diffraction Example



Single-shot Bragg diffraction image on the detector. Miller indexes of Bragg peaks used in data analysis are labeled (yellow)

https://www.nature.com/articles/s41598-019-53824-9.pdf

https://en.wikipedia.org/wiki/Electron_diffraction

The information of the electron beam energy, energy fluctuation and spatial-pointing jitter is intrinsically encoded to the shot-to-shot diffraction image.

The Bragg peaks of a diffraction image are formed through the summation of the intensity distribution of all diffracted electrons

The diffraction pattern of a single electron is determined by the constructive interference governed by Bragg's law $2d \sin\vartheta = n\lambda$, where ϑ is the incident angle, d is the crystal interplanar distance, λ is the deBroglie wavelength, n is the order of Bragg reflections

The separation between the peak pair ij (D_{ij}) is determined by the interplane distance d, the distance between the sample and the detector L_{S2D} and the electron beam energy E,

$$
\begin{aligned}
D_{ij}(E, d) &= L_{S2D} \cdot \left\{\tan[2\theta_i(E, d)] - \tan[2\theta_j(E, d)]\right\} \\
&= L_{S2D} \cdot 2 \cdot \tan\left[2sin^{-1}\left(\frac{n \cdot \lambda(E)}{2d}\right)\right] \\
&\approx L_{S2D} \cdot \frac{2n \cdot \lambda(E)}{d}
\end{aligned}
$$

The wavelength of the electrons λ in vacuum is

$$\lambda = 1/k = \frac{h}{\sqrt{2Em^*}} = \frac{hc}{\sqrt{E(2m_0c^2 + E)}},$$

Quadrupole scan

The quadrupole scan technique is a standard technique used in accelerator facilities to measure the transverse emittance. It is based on the fact that the squared rms beam radius (x_{rms}^2) is proportional to the quadrupole "strength" or inverse focal-length f squared, so

$$x_{rms}^2 = < x^2 > = A\left(\frac{1}{f^2}\right) - 2AB\left(\frac{1}{f}\right) + (C + AB^2) \tag{1}$$

where A, B, C are constants and f is the focal length defined as

$$\frac{1}{f} = \kappa l, \tag{2}$$

$$k\left[\frac{1}{cm^2}\right] = \frac{\mathrm{G}[\frac{\mathrm{Gauss}}{\mathrm{cm}}]}{\mathrm{Brho}[\mathrm{Gauss} \cdot \mathrm{cm}]}$$

here κ is the magnet focusing strength in units of 1 over length squared and l is the effective length of the magnet.

The emittance can be estimated according to

$$\varepsilon = \frac{\sqrt{AC}}{d^2} \tag{3}$$

where d is the distance from the magnet you scan to the point you calculate the beam rms radius.