

# Expansion of 4 – vector in accelerator coordinates

## A1 and A3

In[1]:= **n = 4;**

In[2]:= **A1 = Normal[Series[Integrate[ε\*Bs[ε\*x, ε\*y, ε\*t], y], {ε, 0, n}]] / 2**

$$\begin{aligned} \text{Out[2]}= & \frac{1}{2} \left( y \varepsilon \text{Bs}[0, 0, 0] + \varepsilon^2 \left( t y \text{Bs}^{(0,0,1)}[0, 0, 0] + \frac{1}{2} y^2 \text{Bs}^{(0,1,0)}[0, 0, 0] + x y \text{Bs}^{(1,0,0)}[0, 0, 0] \right) + \right. \\ & \frac{1}{6} \varepsilon^3 \left( 3 t^2 y \text{Bs}^{(0,0,2)}[0, 0, 0] + 3 t y^2 \text{Bs}^{(0,1,1)}[0, 0, 0] + y^3 \text{Bs}^{(0,2,0)}[0, 0, 0] + \right. \\ & 6 t x y \text{Bs}^{(1,0,1)}[0, 0, 0] + 3 x y^2 \text{Bs}^{(1,1,0)}[0, 0, 0] + 3 x^2 y \text{Bs}^{(2,0,0)}[0, 0, 0] \Big) + \frac{1}{24} \varepsilon^4 \\ & \left. \left( 4 t^3 y \text{Bs}^{(0,0,3)}[0, 0, 0] + 6 t^2 y^2 \text{Bs}^{(0,1,2)}[0, 0, 0] + 4 t y^3 \text{Bs}^{(0,2,1)}[0, 0, 0] + y^4 \text{Bs}^{(0,3,0)}[0, 0, 0] + \right. \right. \\ & 12 t^2 x y \text{Bs}^{(1,0,2)}[0, 0, 0] + 12 t x y^2 \text{Bs}^{(1,1,1)}[0, 0, 0] + 4 x y^3 \text{Bs}^{(1,2,0)}[0, 0, 0] + \\ & \left. \left. 12 t x^2 y \text{Bs}^{(2,0,1)}[0, 0, 0] + 6 x^2 y^2 \text{Bs}^{(2,1,0)}[0, 0, 0] + 4 x^3 y \text{Bs}^{(3,0,0)}[0, 0, 0] \right) \right) \end{aligned}$$

In[3]:= **A3 = -Normal[Series[Integrate[ε\*Bs[ε\*x, ε\*y, ε\*t], x], {ε, 0, n}]] / 2**

$$\begin{aligned} \text{Out[3]}= & \frac{1}{2} \left( -x \varepsilon \text{Bs}[0, 0, 0] - \varepsilon^2 \left( t x \text{Bs}^{(0,0,1)}[0, 0, 0] + x y \text{Bs}^{(0,1,0)}[0, 0, 0] + \frac{1}{2} x^2 \text{Bs}^{(1,0,0)}[0, 0, 0] \right) - \right. \\ & \frac{1}{6} \varepsilon^3 \left( 3 t^2 x \text{Bs}^{(0,0,2)}[0, 0, 0] + 6 t x y \text{Bs}^{(0,1,1)}[0, 0, 0] + 3 x y^2 \text{Bs}^{(0,2,0)}[0, 0, 0] + \right. \\ & 3 t x^2 \text{Bs}^{(1,0,1)}[0, 0, 0] + 3 x^2 y \text{Bs}^{(1,1,0)}[0, 0, 0] + x^3 \text{Bs}^{(2,0,0)}[0, 0, 0] \Big) - \\ & \frac{1}{24} \varepsilon^4 \left( 4 t^3 x \text{Bs}^{(0,0,3)}[0, 0, 0] + 12 t^2 x y \text{Bs}^{(0,1,2)}[0, 0, 0] + 12 t x y^2 \text{Bs}^{(0,2,1)}[0, 0, 0] + \right. \\ & 4 x y^3 \text{Bs}^{(0,3,0)}[0, 0, 0] + 6 t^2 x^2 \text{Bs}^{(1,0,2)}[0, 0, 0] + 12 t x^2 y \text{Bs}^{(1,1,1)}[0, 0, 0] + \\ & \left. \left. 6 x^2 y^2 \text{Bs}^{(1,2,0)}[0, 0, 0] + 4 t x^3 \text{Bs}^{(2,0,1)}[0, 0, 0] + 4 x^3 y \text{Bs}^{(2,1,0)}[0, 0, 0] + x^4 \text{Bs}^{(3,0,0)}[0, 0, 0] \right) \right) \\ & - \frac{1}{2} x \varepsilon \text{Bs}[0, 0, 0] \end{aligned}$$

In[4]:= **Normal[Series[ε\*Bs[ε\*x, ε\*y, ε\*t], {ε, 0, n-1}]]**

$$\begin{aligned} \text{Out[4]}= & \varepsilon \text{Bs}[0, 0, 0] + \varepsilon^2 \left( t \text{Bs}^{(0,0,1)}[0, 0, 0] + y \text{Bs}^{(0,1,0)}[0, 0, 0] + x \text{Bs}^{(1,0,0)}[0, 0, 0] \right) + \\ & \frac{1}{2} \varepsilon^3 \left( t^2 \text{Bs}^{(0,0,2)}[0, 0, 0] + 2 t y \text{Bs}^{(0,1,1)}[0, 0, 0] + y^2 \text{Bs}^{(0,2,0)}[0, 0, 0] + \right. \\ & \left. 2 t x \text{Bs}^{(1,0,1)}[0, 0, 0] + 2 x y \text{Bs}^{(1,1,0)}[0, 0, 0] + x^2 \text{Bs}^{(2,0,0)}[0, 0, 0] \right) \end{aligned}$$

In[5]:= -D[A3, x]

$$\begin{aligned} \text{Out}[5] = & \frac{1}{2} \left( \varepsilon \text{Bs}[0, 0, 0] + \varepsilon^2 (\text{t Bs}^{(0,0,1)}[0, 0, 0] + \text{y Bs}^{(0,1,0)}[0, 0, 0] + \text{x Bs}^{(1,0,0)}[0, 0, 0]) + \right. \\ & \frac{1}{6} \varepsilon^3 (3 \text{t}^2 \text{Bs}^{(0,0,2)}[0, 0, 0] + 6 \text{t y Bs}^{(0,1,1)}[0, 0, 0] + 3 \text{y}^2 \text{Bs}^{(0,2,0)}[0, 0, 0] + \\ & 6 \text{t x Bs}^{(1,0,1)}[0, 0, 0] + 6 \text{x y Bs}^{(1,1,0)}[0, 0, 0] + 3 \text{x}^2 \text{Bs}^{(2,0,0)}[0, 0, 0]) + \frac{1}{24} \varepsilon^4 \\ & (4 \text{t}^3 \text{Bs}^{(0,0,3)}[0, 0, 0] + 12 \text{t}^2 \text{y Bs}^{(0,1,2)}[0, 0, 0] + 12 \text{t y}^2 \text{Bs}^{(0,2,1)}[0, 0, 0] + 4 \text{y}^3 \text{Bs}^{(0,3,0)}[0, 0, 0] + \\ & 12 \text{t}^2 \text{x Bs}^{(1,0,2)}[0, 0, 0] + 24 \text{t x y Bs}^{(1,1,1)}[0, 0, 0] + 12 \text{x y}^2 \text{Bs}^{(1,2,0)}[0, 0, 0] + \\ & \left. 12 \text{t}^2 \text{x Bs}^{(2,0,1)}[0, 0, 0] + 12 \text{x}^2 \text{y Bs}^{(2,1,0)}[0, 0, 0] + 4 \text{x}^3 \text{Bs}^{(3,0,0)}[0, 0, 0] \right) \end{aligned}$$

In[6]:=

D[A1, y] - D[A3, x] - Normal[Series[\varepsilon \* Bs[\varepsilon \* x, \varepsilon \* y, \varepsilon \* t], {\varepsilon, 0, n}]]

$$\begin{aligned} \text{Out}[6] = & -\frac{1}{2} \varepsilon^3 (\text{t}^2 \text{Bs}^{(0,0,2)}[0, 0, 0] + 2 \text{t y Bs}^{(0,1,1)}[0, 0, 0] + \text{y}^2 \text{Bs}^{(0,2,0)}[0, 0, 0] + \\ & 2 \text{t x Bs}^{(1,0,1)}[0, 0, 0] + 2 \text{x y Bs}^{(1,1,0)}[0, 0, 0] + \text{x}^2 \text{Bs}^{(2,0,0)}[0, 0, 0]) + \\ & \frac{1}{6} \varepsilon^3 (3 \text{t}^2 \text{Bs}^{(0,0,2)}[0, 0, 0] + 6 \text{t y Bs}^{(0,1,1)}[0, 0, 0] + 3 \text{y}^2 \text{Bs}^{(0,2,0)}[0, 0, 0] + \\ & 6 \text{t x Bs}^{(1,0,1)}[0, 0, 0] + 6 \text{x y Bs}^{(1,1,0)}[0, 0, 0] + 3 \text{x}^2 \text{Bs}^{(2,0,0)}[0, 0, 0]) - \\ & \frac{1}{6} \varepsilon^4 (\text{t}^3 \text{Bs}^{(0,0,3)}[0, 0, 0] + 3 \text{t}^2 \text{y Bs}^{(0,1,2)}[0, 0, 0] + 3 \text{t y}^2 \text{Bs}^{(0,2,1)}[0, 0, 0] + \text{y}^3 \text{Bs}^{(0,3,0)}[0, 0, 0] + \\ & 3 \text{t}^2 \text{x Bs}^{(1,0,2)}[0, 0, 0] + 6 \text{t x y Bs}^{(1,1,1)}[0, 0, 0] + 3 \text{x y}^2 \text{Bs}^{(1,2,0)}[0, 0, 0] + \\ & 3 \text{t}^2 \text{x Bs}^{(2,0,1)}[0, 0, 0] + 3 \text{x}^2 \text{y Bs}^{(2,1,0)}[0, 0, 0] + \text{x}^3 \text{Bs}^{(3,0,0)}[0, 0, 0]) + \frac{1}{24} \varepsilon^4 \\ & (4 \text{t}^3 \text{Bs}^{(0,0,3)}[0, 0, 0] + 12 \text{t}^2 \text{y Bs}^{(0,1,2)}[0, 0, 0] + 12 \text{t y}^2 \text{Bs}^{(0,2,1)}[0, 0, 0] + 4 \text{y}^3 \text{Bs}^{(0,3,0)}[0, 0, 0] + \\ & 12 \text{t}^2 \text{x Bs}^{(1,0,2)}[0, 0, 0] + 24 \text{t x y Bs}^{(1,1,1)}[0, 0, 0] + 12 \text{x y}^2 \text{Bs}^{(1,2,0)}[0, 0, 0] + \\ & 12 \text{t}^2 \text{x Bs}^{(2,0,1)}[0, 0, 0] + 12 \text{x}^2 \text{y Bs}^{(2,1,0)}[0, 0, 0] + 4 \text{x}^3 \text{Bs}^{(3,0,0)}[0, 0, 0]) \end{aligned}$$

In[7]:= Coefficient[A1, \varepsilon]

$$\text{Out}[7] = \frac{1}{2} \text{y Bs}[0, 0, 0]$$

In[8]:= Coefficient[A3, \varepsilon]

$$\text{Out}[8] = -\frac{1}{2} \text{x Bs}[0, 0, 0]$$

# A2

```
In[9]:= m = 4;
A21 = Normal[
  Series[Integrate[\varepsilon * ((1 + K * \varepsilon * x) * By[\varepsilon * x, 0, \varepsilon * t] + \kappa * \varepsilon * x * Bs[\varepsilon * x, 0, \varepsilon * t]), x], {\varepsilon, 0, m}]]
```

$$\text{Out}[10]= x \varepsilon \text{By}[0, 0, 0] + \frac{1}{2} \varepsilon^2 \left( x^2 \times \text{Bs}[0, 0, 0] + K x^2 \text{By}[0, 0, 0] + 2 t x \text{By}^{(0,0,1)}[0, 0, 0] + x^2 \text{By}^{(1,0,0)}[0, 0, 0] \right) +$$

$$\frac{1}{6} \varepsilon^3 \left( 3 t x^2 \times \text{Bs}^{(0,0,1)}[0, 0, 0] + 3 K t x^2 \text{By}^{(0,0,1)}[0, 0, 0] + 3 t^2 x \text{By}^{(0,0,2)}[0, 0, 0] + \right.$$

$$2 x^3 \times \text{Bs}^{(1,0,0)}[0, 0, 0] + 2 K x^3 \text{By}^{(1,0,0)}[0, 0, 0] + 3 t x^2 \text{By}^{(1,0,1)}[0, 0, 0] + x^3 \text{By}^{(2,0,0)}[0, 0, 0] +$$

$$\frac{1}{24} \varepsilon^4 \left( 6 t^2 x^2 \times \text{Bs}^{(0,0,2)}[0, 0, 0] + 6 K t^2 x^2 \text{By}^{(0,0,2)}[0, 0, 0] + 4 t^3 x \text{By}^{(0,0,3)}[0, 0, 0] + \right.$$

$$8 t x^3 \times \text{Bs}^{(1,0,1)}[0, 0, 0] + 8 K t x^3 \text{By}^{(1,0,1)}[0, 0, 0] + 6 t^2 x^2 \text{By}^{(1,0,2)}[0, 0, 0] +$$

$$3 x^4 \times \text{Bs}^{(2,0,0)}[0, 0, 0] + 3 K x^4 \text{By}^{(2,0,0)}[0, 0, 0] + 4 t x^3 \text{By}^{(2,0,1)}[0, 0, 0] + x^4 \text{By}^{(3,0,0)}[0, 0, 0] \Big)$$

```
In[11]:= A22 = -Normal[Series[Integrate[\varepsilon * (Bx[0, \varepsilon * y, \varepsilon * t] - \kappa * \varepsilon * y * Bs[\varepsilon * x, 0, \varepsilon * t]), y], {\varepsilon, 0, m}]]
```

$$\text{Out}[11]= -y \varepsilon \text{Bx}[0, 0, 0] - \varepsilon^2 \left( -\frac{1}{2} y^2 \times \text{Bs}[0, 0, 0] + t y \text{Bx}^{(0,0,1)}[0, 0, 0] + \frac{1}{2} y^2 \text{Bx}^{(0,1,0)}[0, 0, 0] \right) -$$

$$\frac{1}{6} \varepsilon^3 \left( -3 t y^2 \times \text{Bs}^{(0,0,1)}[0, 0, 0] + 3 t^2 y \text{Bx}^{(0,0,2)}[0, 0, 0] + \right.$$

$$3 t y^2 \text{Bx}^{(0,1,1)}[0, 0, 0] + y^3 \text{Bx}^{(0,2,0)}[0, 0, 0] - 3 x y^2 \times \text{Bs}^{(1,0,0)}[0, 0, 0] -$$

$$\frac{1}{24} \varepsilon^4 \left( -6 t^2 y^2 \times \text{Bs}^{(0,0,2)}[0, 0, 0] + 4 t^3 y \text{Bx}^{(0,0,3)}[0, 0, 0] + 6 t^2 y^2 \text{Bx}^{(0,1,2)}[0, 0, 0] + 4 t y^3 \right.$$

$$\text{Bx}^{(0,2,1)}[0, 0, 0] + y^4 \text{Bx}^{(0,3,0)}[0, 0, 0] - 12 t x y^2 \times \text{Bs}^{(1,0,1)}[0, 0, 0] - 6 x^2 y^2 \times \text{Bs}^{(2,0,0)}[0, 0, 0] \Big)$$

```
In[12]:= A23 = Integrate[
  Integrate[D[Normal[Series[(1 + K*ε*x)*By[ε*x, ε*y, ε*t] + κ*ε*x*Bs[ε*x, ε*y, ε*t], {ε, 0, m-1}], y], x], y]/2 -
  Integrate[Integrate[D[Normal[Series[(1 + K*ε*x)*Bx[ε*x, ε*y, ε*t] - κ*ε*x*y*Bs[ε*x, ε*y, ε*t], {ε, 0, m-1}], x], x], y]/2

Out[12]= 
$$\frac{1}{2} \left( \frac{1}{2} x^2 y \varepsilon^2 \kappa Bs^{(0,1,0)}[0, 0, 0] + xy \varepsilon By^{(0,1,0)}[0, 0, 0] + \frac{1}{2} K x^2 y \varepsilon^2 By^{(0,1,0)}[0, 0, 0] + \right. \\ \frac{1}{2} t x^2 y \varepsilon^3 \kappa Bs^{(0,1,1)}[0, 0, 0] + tx y \varepsilon^2 By^{(0,1,1)}[0, 0, 0] + \frac{1}{2} K t x^2 y \varepsilon^3 By^{(0,1,1)}[0, 0, 0] + \\ \frac{1}{2} t^2 x y \varepsilon^3 By^{(0,1,2)}[0, 0, 0] + \frac{1}{4} x^2 y^2 \varepsilon^3 \kappa Bs^{(0,2,0)}[0, 0, 0] + \frac{1}{2} x y^2 \varepsilon^2 By^{(0,2,0)}[0, 0, 0] + \\ \frac{1}{4} K x^2 y^2 \varepsilon^3 By^{(0,2,0)}[0, 0, 0] + \frac{1}{2} t x y^2 \varepsilon^3 By^{(0,2,1)}[0, 0, 0] + \frac{1}{6} x y^3 \varepsilon^3 By^{(0,3,0)}[0, 0, 0] + \\ \frac{1}{3} x^3 y \varepsilon^3 \kappa Bs^{(1,1,0)}[0, 0, 0] + \frac{1}{2} x^2 y \varepsilon^2 By^{(1,1,0)}[0, 0, 0] + \frac{1}{3} K x^3 y \varepsilon^3 By^{(1,1,0)}[0, 0, 0] + \\ \frac{1}{2} t x^2 y \varepsilon^3 By^{(1,1,1)}[0, 0, 0] + \frac{1}{4} x^2 y^2 \varepsilon^3 By^{(1,2,0)}[0, 0, 0] + \frac{1}{6} x^3 y \varepsilon^3 By^{(2,1,0)}[0, 0, 0] \Big) + \\ \frac{1}{2} \left( K x y \varepsilon Bx[0, 0, 0] + K t x y \varepsilon^2 Bx^{(0,0,1)}[0, 0, 0] + \frac{1}{2} K t^2 x y \varepsilon^3 Bx^{(0,0,2)}[0, 0, 0] + \right. \\ \frac{1}{2} K x y^2 \varepsilon^2 Bx^{(0,1,0)}[0, 0, 0] + \frac{1}{2} K t x y^2 \varepsilon^3 Bx^{(0,1,1)}[0, 0, 0] + \frac{1}{6} K x y^3 \varepsilon^3 Bx^{(0,2,0)}[0, 0, 0] - \\ \frac{1}{2} x y^2 \varepsilon^2 \kappa Bs^{(1,0,0)}[0, 0, 0] + x y \varepsilon Bx^{(1,0,0)}[0, 0, 0] + K x^2 y \varepsilon^2 Bx^{(1,0,0)}[0, 0, 0] - \\ \frac{1}{2} t x y^2 \varepsilon^3 \kappa Bs^{(1,0,1)}[0, 0, 0] + t x y \varepsilon^2 Bx^{(1,0,1)}[0, 0, 0] + K t x^2 y \varepsilon^3 Bx^{(1,0,1)}[0, 0, 0] + \\ \frac{1}{2} t^2 x y \varepsilon^3 Bx^{(1,0,2)}[0, 0, 0] - \frac{1}{3} x y^3 \varepsilon^3 \kappa Bs^{(1,1,0)}[0, 0, 0] + \frac{1}{2} x y^2 \varepsilon^2 Bx^{(1,1,0)}[0, 0, 0] + \\ \frac{1}{2} K x^2 y^2 \varepsilon^3 Bx^{(1,1,0)}[0, 0, 0] + \frac{1}{2} t x y^2 \varepsilon^3 Bx^{(1,1,1)}[0, 0, 0] + \frac{1}{6} x y^3 \varepsilon^3 Bx^{(1,2,0)}[0, 0, 0] - \\ \frac{1}{4} x^2 y^2 \varepsilon^3 \kappa Bs^{(2,0,0)}[0, 0, 0] + \frac{1}{2} x^2 y \varepsilon^2 Bx^{(2,0,0)}[0, 0, 0] + \frac{1}{2} K x^3 y \varepsilon^3 Bx^{(2,0,0)}[0, 0, 0] + \\ \left. \frac{1}{2} t x^2 y \varepsilon^3 Bx^{(2,0,1)}[0, 0, 0] + \frac{1}{4} x^2 y^2 \varepsilon^3 Bx^{(2,1,0)}[0, 0, 0] + \frac{1}{6} x^3 y \varepsilon^3 Bx^{(3,0,0)}[0, 0, 0] \right)$$

```

In[13]:= A2 = A21 + A22 + ε \* A23

Out[13]= 
$$\begin{aligned}
& -y \varepsilon Bx[0, 0, 0] + x \varepsilon By[0, 0, 0] - \\
& \varepsilon^2 \left( -\frac{1}{2} y^2 \times Bs[0, 0, 0] + t y Bx^{(0,0,1)}[0, 0, 0] + \frac{1}{2} y^2 Bx^{(0,1,0)}[0, 0, 0] \right) - \\
& \frac{1}{6} \varepsilon^3 \left( -3 t y^2 \times Bs^{(0,0,1)}[0, 0, 0] + 3 t^2 y Bx^{(0,0,2)}[0, 0, 0] + \right. \\
& \quad 3 t y^2 Bx^{(0,1,1)}[0, 0, 0] + y^3 Bx^{(0,2,0)}[0, 0, 0] - 3 x y^2 \times Bs^{(1,0,0)}[0, 0, 0] \Big) + \\
& \frac{1}{2} \varepsilon^2 \left( x^2 \times Bs[0, 0, 0] + K x^2 By[0, 0, 0] + 2 t x By^{(0,0,1)}[0, 0, 0] + x^2 By^{(1,0,0)}[0, 0, 0] \right) - \\
& \frac{1}{24} \varepsilon^4 \left( -6 t^2 y^2 \times Bs^{(0,0,2)}[0, 0, 0] + 4 t^3 y Bx^{(0,0,3)}[0, 0, 0] + 6 t^2 y^2 Bx^{(0,1,2)}[0, 0, 0] + 4 t y^3 \right. \\
& \quad Bx^{(0,2,1)}[0, 0, 0] + y^4 Bx^{(0,3,0)}[0, 0, 0] - 12 t x y^2 \times Bs^{(1,0,1)}[0, 0, 0] - 6 x^2 y^2 \times Bs^{(2,0,0)}[0, 0, 0] \Big) + \\
& \frac{1}{6} \varepsilon^3 \left( 3 t x^2 \times Bs^{(0,0,1)}[0, 0, 0] + 3 K t x^2 By^{(0,0,1)}[0, 0, 0] + 3 t^2 x By^{(0,0,2)}[0, 0, 0] + \right. \\
& \quad 2 x^3 \times Bs^{(1,0,0)}[0, 0, 0] + 2 K x^3 By^{(1,0,0)}[0, 0, 0] + 3 t x^2 By^{(1,0,1)}[0, 0, 0] + x^3 By^{(2,0,0)}[0, 0, 0] \Big) + \\
& \varepsilon \left( \frac{1}{2} \left( \frac{1}{2} x^2 y \varepsilon^2 \times Bs^{(0,1,0)}[0, 0, 0] + x y \varepsilon By^{(0,1,0)}[0, 0, 0] + \frac{1}{2} K x^2 y \varepsilon^2 By^{(0,1,0)}[0, 0, 0] + \right. \right. \\
& \quad \frac{1}{2} t x^2 y \varepsilon^3 \times Bs^{(0,1,1)}[0, 0, 0] + t x y \varepsilon^2 By^{(0,1,1)}[0, 0, 0] + \frac{1}{2} K t x^2 y \varepsilon^3 By^{(0,1,1)}[0, 0, 0] + \\
& \quad \frac{1}{2} t^2 x y \varepsilon^3 By^{(0,1,2)}[0, 0, 0] + \frac{1}{4} x^2 y^2 \varepsilon^3 \times Bs^{(0,2,0)}[0, 0, 0] + \frac{1}{2} x y^2 \varepsilon^2 By^{(0,2,0)}[0, 0, 0] + \\
& \quad \frac{1}{4} K x^2 y^2 \varepsilon^3 By^{(0,2,0)}[0, 0, 0] + \frac{1}{2} t x y^2 \varepsilon^3 By^{(0,2,1)}[0, 0, 0] + \frac{1}{6} x y^3 \varepsilon^3 By^{(0,3,0)}[0, 0, 0] + \\
& \quad \frac{1}{3} x^3 y \varepsilon^3 \times Bs^{(1,1,0)}[0, 0, 0] + \frac{1}{2} x^2 y \varepsilon^2 By^{(1,1,0)}[0, 0, 0] + \frac{1}{3} K x^3 y \varepsilon^3 By^{(1,1,0)}[0, 0, 0] + \\
& \quad \frac{1}{2} t x^2 y \varepsilon^3 By^{(1,1,1)}[0, 0, 0] + \frac{1}{4} x^2 y^2 \varepsilon^3 By^{(1,2,0)}[0, 0, 0] + \frac{1}{6} x^3 y \varepsilon^3 By^{(2,1,0)}[0, 0, 0] \Big) + \\
& \frac{1}{2} \left( K x y \varepsilon Bx[0, 0, 0] + K t x y \varepsilon^2 Bx^{(0,0,1)}[0, 0, 0] + \frac{1}{2} K t^2 x y \varepsilon^3 Bx^{(0,0,2)}[0, 0, 0] + \right. \\
& \quad \frac{1}{2} K x y^2 \varepsilon^2 Bx^{(0,1,0)}[0, 0, 0] + \frac{1}{2} K t x y^2 \varepsilon^3 Bx^{(0,1,1)}[0, 0, 0] + \frac{1}{6} K x y^3 \varepsilon^3 Bx^{(0,2,0)}[0, 0, 0] - \\
& \quad \frac{1}{2} x y^2 \varepsilon^2 \times Bs^{(1,0,0)}[0, 0, 0] + x y \varepsilon Bx^{(1,0,0)}[0, 0, 0] + K x^2 y \varepsilon^2 Bx^{(1,0,0)}[0, 0, 0] - \\
& \quad \frac{1}{2} t x y^2 \varepsilon^3 \times Bs^{(1,0,1)}[0, 0, 0] + t x y \varepsilon^2 Bx^{(1,0,1)}[0, 0, 0] + K t x^2 y \varepsilon^3 Bx^{(1,0,1)}[0, 0, 0] + \\
& \quad \frac{1}{2} t^2 x y \varepsilon^3 Bx^{(1,0,2)}[0, 0, 0] - \frac{1}{3} x y^3 \varepsilon^3 \times Bs^{(1,1,0)}[0, 0, 0] + \frac{1}{2} x y^2 \varepsilon^2 Bx^{(1,1,0)}[0, 0, 0] + \\
& \quad \frac{1}{2} K x^2 y^2 \varepsilon^3 Bx^{(1,1,0)}[0, 0, 0] + \frac{1}{2} t x y^2 \varepsilon^3 Bx^{(1,1,1)}[0, 0, 0] + \frac{1}{6} x y^3 \varepsilon^3 Bx^{(1,2,0)}[0, 0, 0] - \\
& \quad \frac{1}{4} x^2 y^2 \varepsilon^3 \times Bs^{(2,0,0)}[0, 0, 0] + \frac{1}{2} x^2 y \varepsilon^2 Bx^{(2,0,0)}[0, 0, 0] + \frac{1}{2} K x^3 y \varepsilon^3 Bx^{(2,0,0)}[0, 0, 0] + \\
& \quad \frac{1}{2} t x^2 y \varepsilon^3 Bx^{(2,0,1)}[0, 0, 0] + \frac{1}{4} x^2 y^2 \varepsilon^3 Bx^{(2,1,0)}[0, 0, 0] + \frac{1}{6} x^3 y \varepsilon^3 Bx^{(3,0,0)}[0, 0, 0] \Big) + \\
& \frac{1}{24} \varepsilon^4 \left( 6 t^2 x^2 \times Bs^{(0,0,2)}[0, 0, 0] + 6 K t^2 x^2 By^{(0,0,2)}[0, 0, 0] + 4 t^3 x By^{(0,0,3)}[0, 0, 0] + \right. \\
& \quad 8 t x^3 \times Bs^{(1,0,1)}[0, 0, 0] + 8 K t x^3 By^{(1,0,1)}[0, 0, 0] + 6 t^2 x^2 By^{(1,0,2)}[0, 0, 0] + \\
& \quad 3 x^4 \times Bs^{(2,0,0)}[0, 0, 0] + 3 K x^4 By^{(2,0,0)}[0, 0, 0] + 4 t x^3 By^{(2,0,1)}[0, 0, 0] + x^4 By^{(3,0,0)}[0, 0, 0] \Big)
\end{aligned}$$

In[14]:= **Simplify[%]**

$$\begin{aligned} \text{Out[14]} = & \frac{1}{24} \varepsilon \\ & (-y (4 (6 Bx[0, 0, 0] + t \varepsilon (6 Bx^{(0,0,1)}[0, 0, 0] + t \varepsilon (3 Bx^{(0,0,2)}[0, 0, 0] + t \varepsilon Bx^{(0,0,3)}[0, 0, 0])))) - 6 \\ & y \varepsilon (\kappa (2 Bs[0, 0, 0] + t \varepsilon (2 Bs^{(0,0,1)}[0, 0, 0] + t \varepsilon Bs^{(0,0,2)}[0, 0, 0])) - \\ & 2 Bx^{(0,1,0)}[0, 0, 0] - t \varepsilon (2 Bx^{(0,1,1)}[0, 0, 0] + t \varepsilon Bx^{(0,1,2)}[0, 0, 0])) + \\ & 4 y^2 \varepsilon^2 (Bx^{(0,2,0)}[0, 0, 0] + t \varepsilon Bx^{(0,2,1)}[0, 0, 0]) + y^3 \varepsilon^3 Bx^{(0,3,0)}[0, 0, 0]) + \\ & 2 x (12 By[0, 0, 0] + K y \varepsilon (6 Bx[0, 0, 0] + \varepsilon (6 t Bx^{(0,0,1)}[0, 0, 0] + 3 t^2 \varepsilon Bx^{(0,0,2)}[0, 0, 0] + \\ & 3 y Bx^{(0,1,0)}[0, 0, 0] + 3 t y \varepsilon Bx^{(0,1,1)}[0, 0, 0] + y^2 \varepsilon Bx^{(0,2,0)}[0, 0, 0])) + \varepsilon (2 t^3 \varepsilon^2 \\ & By^{(0,0,3)}[0, 0, 0] + 3 t^2 \varepsilon (2 By^{(0,0,2)}[0, 0, 0] + y \varepsilon (By^{(0,1,2)}[0, 0, 0] + Bx^{(1,0,2)}[0, 0, 0])) + \\ & 3 t (4 By^{(0,0,1)}[0, 0, 0] + y \varepsilon (2 By^{(0,1,1)}[0, 0, 0] + 2 Bx^{(1,0,1)}[0, 0, 0] + y \varepsilon (By^{(0,2,1)}[0, 0, 0] + \\ & \kappa Bs^{(1,0,1)}[0, 0, 0] + Bx^{(1,1,1)}[0, 0, 0]))) + y (6 By^{(0,1,0)}[0, 0, 0] + \\ & 6 Bx^{(1,0,0)}[0, 0, 0] + 3 y \varepsilon (By^{(0,2,0)}[0, 0, 0] + \kappa Bs^{(1,0,0)}[0, 0, 0] + Bx^{(1,1,0)}[0, 0, 0]) + \\ & y^2 \varepsilon^2 (By^{(0,3,0)}[0, 0, 0] - 2 \kappa Bs^{(1,1,0)}[0, 0, 0] + Bx^{(1,2,0)}[0, 0, 0])))) + \\ & 3 x^2 \varepsilon (4 By^{(1,0,0)}[0, 0, 0] + 4 t \varepsilon By^{(1,0,1)}[0, 0, 0] + 2 t^2 \varepsilon^2 By^{(1,0,2)}[0, 0, 0] + \\ & K (4 By[0, 0, 0] + \varepsilon (2 t^2 \varepsilon By^{(0,0,2)}[0, 0, 0] + \\ & 2 t (2 By^{(0,0,1)}[0, 0, 0] + y \varepsilon (By^{(0,1,1)}[0, 0, 0] + 2 Bx^{(1,0,1)}[0, 0, 0])) + y (2 By^{(0,1,0)}[0, \\ & 0, 0] + 4 Bx^{(1,0,0)}[0, 0, 0] + y \varepsilon (By^{(0,2,0)}[0, 0, 0] + 2 Bx^{(1,1,0)}[0, 0, 0])))) + \\ & 2 y \varepsilon By^{(1,1,0)}[0, 0, 0] + 2 t y \varepsilon^2 By^{(1,1,1)}[0, 0, 0] + y^2 \varepsilon^2 By^{(1,2,0)}[0, 0, 0] + \\ & \kappa (4 Bs[0, 0, 0] + \varepsilon (2 t^2 \varepsilon Bs^{(0,0,2)}[0, 0, 0] + 2 t (2 Bs^{(0,0,1)}[0, 0, 0] + y \varepsilon Bs^{(0,1,1)}[0, 0, 0]) + \\ & y (2 Bs^{(0,1,0)}[0, 0, 0] + y \varepsilon (Bs^{(0,2,0)}[0, 0, 0] + Bs^{(2,0,0)}[0, 0, 0])))) + \\ & 2 y \varepsilon Bx^{(2,0,0)}[0, 0, 0] + 2 t y \varepsilon^2 Bx^{(2,0,1)}[0, 0, 0] + y^2 \varepsilon^2 Bx^{(2,1,0)}[0, 0, 0]) + \\ & 2 x^3 \varepsilon^2 (2 \kappa (2 Bs^{(1,0,0)}[0, 0, 0] + 2 t \varepsilon Bs^{(1,0,1)}[0, 0, 0] + y \varepsilon Bs^{(1,1,0)}[0, 0, 0]) + \\ & K (4 By^{(1,0,0)}[0, 0, 0] + 4 t \varepsilon By^{(1,0,1)}[0, 0, 0] + 2 y \varepsilon By^{(1,1,0)}[0, 0, 0] + 3 y \varepsilon Bx^{(2,0,0)}[0, 0, 0]) + \\ & 2 By^{(2,0,0)}[0, 0, 0] + 2 t \varepsilon By^{(2,0,1)}[0, 0, 0] + \\ & y \varepsilon By^{(2,1,0)}[0, 0, 0] + y \varepsilon Bx^{(3,0,0)}[0, 0, 0]) + \\ & x^4 \varepsilon^3 (3 \kappa Bs^{(2,0,0)}[0, 0, 0] + 3 K By^{(2,0,0)}[0, 0, 0] + By^{(3,0,0)}[0, 0, 0])) \end{aligned}$$

In[15]:= **Coefficient[A2, ε]**

$$\text{Out[15]} = -y Bx[0, 0, 0] + x By[0, 0, 0]$$

# Electric field

```

In[16]:= k = 4;
φ1 = -Normal[Series[Integrate[ε*Ex[ε*x, 0, ε*t], x], {ε, 0, k}]] +
      -Normal[Series[Integrate[ε*Ey[0, ε*y, ε*t], y], {ε, 0, k}]]
```

$$\text{Out}[17]= -x \varepsilon \text{Ex}[0, 0, 0] - y \varepsilon \text{Ey}[0, 0, 0] - \varepsilon^2 \left( t y \text{Ey}^{(0,0,1)}[0, 0, 0] + \frac{1}{2} y^2 \text{Ey}^{(0,1,0)}[0, 0, 0] \right) -$$

$$\frac{1}{6} \varepsilon^3 \left( 3 t^2 y \text{Ey}^{(0,0,2)}[0, 0, 0] + 3 t y^2 \text{Ey}^{(0,1,1)}[0, 0, 0] + y^3 \text{Ey}^{(0,2,0)}[0, 0, 0] \right) - \frac{1}{24} \varepsilon^4$$

$$(4 t^3 y \text{Ey}^{(0,0,3)}[0, 0, 0] + 6 t^2 y^2 \text{Ey}^{(0,1,2)}[0, 0, 0] + 4 t y^3 \text{Ey}^{(0,2,1)}[0, 0, 0] + y^4 \text{Ey}^{(0,3,0)}[0, 0, 0]) -$$

$$\varepsilon^2 \left( t x \text{Ex}^{(0,0,1)}[0, 0, 0] + \frac{1}{2} x^2 \text{Ex}^{(1,0,0)}[0, 0, 0] \right) -$$

$$\frac{1}{6} \varepsilon^3 \left( 3 t^2 x \text{Ex}^{(0,0,2)}[0, 0, 0] + 3 t x^2 \text{Ex}^{(1,0,1)}[0, 0, 0] + x^3 \text{Ex}^{(2,0,0)}[0, 0, 0] \right) - \frac{1}{24} \varepsilon^4$$

$$(4 t^3 x \text{Ex}^{(0,0,3)}[0, 0, 0] + 6 t^2 x^2 \text{Ex}^{(1,0,2)}[0, 0, 0] + 4 t x^3 \text{Ex}^{(2,0,1)}[0, 0, 0] + x^4 \text{Ex}^{(3,0,0)}[0, 0, 0])$$
  

```

In[18]:= φ2 = -Integrate[Integrate[D[Normal[Series[Ex[ε*x, ε*y, ε*t], {ε, 0, m-1}]], y], x], y] / 2 -
          Integrate[Integrate[D[Normal[Series[Ey[ε*x, ε*y, ε*t], {ε, 0, m-1}]], x], x], y] / 2
```

$$\text{Out}[18]= \frac{1}{2} \left( -x y \varepsilon \text{Ex}^{(0,1,0)}[0, 0, 0] - t x y \varepsilon^2 \text{Ex}^{(0,1,1)}[0, 0, 0] - \right.$$

$$\frac{1}{2} t^2 x y \varepsilon^3 \text{Ex}^{(0,1,2)}[0, 0, 0] - \frac{1}{2} x y^2 \varepsilon^2 \text{Ex}^{(0,2,0)}[0, 0, 0] - \frac{1}{2} t x y^2 \varepsilon^3 \text{Ex}^{(0,2,1)}[0, 0, 0] -$$

$$\frac{1}{6} x y^3 \varepsilon^3 \text{Ex}^{(0,3,0)}[0, 0, 0] - \frac{1}{2} x^2 y \varepsilon^2 \text{Ex}^{(1,1,0)}[0, 0, 0] - \frac{1}{2} t x^2 y \varepsilon^3 \text{Ex}^{(1,1,1)}[0, 0, 0] -$$

$$\frac{1}{4} x^2 y^2 \varepsilon^3 \text{Ex}^{(1,2,0)}[0, 0, 0] - \frac{1}{6} x^3 y \varepsilon^3 \text{Ex}^{(2,1,0)}[0, 0, 0] \right) +$$

$$\frac{1}{2} \left( -x y \varepsilon \text{Ey}^{(1,0,0)}[0, 0, 0] - t x y \varepsilon^2 \text{Ey}^{(1,0,1)}[0, 0, 0] - \frac{1}{2} t^2 x y \varepsilon^3 \text{Ey}^{(1,0,2)}[0, 0, 0] - \right.$$

$$\frac{1}{2} x y^2 \varepsilon^2 \text{Ey}^{(1,1,0)}[0, 0, 0] - \frac{1}{2} t x y^2 \varepsilon^3 \text{Ey}^{(1,1,1)}[0, 0, 0] -$$

$$\frac{1}{6} x y^3 \varepsilon^3 \text{Ey}^{(1,2,0)}[0, 0, 0] - \frac{1}{2} x^2 y \varepsilon^2 \text{Ey}^{(2,0,0)}[0, 0, 0] - \frac{1}{2} t x^2 y \varepsilon^3 \text{Ey}^{(2,0,1)}[0, 0, 0] -$$

$$\left. \frac{1}{4} x^2 y^2 \varepsilon^3 \text{Ey}^{(2,1,0)}[0, 0, 0] - \frac{1}{6} x^3 y \varepsilon^3 \text{Ey}^{(3,0,0)}[0, 0, 0] \right)$$

In[19]:=  $\varphi t = \varphi 1 + \varepsilon * \varphi 2$ 

$$\begin{aligned} \text{Out}[19] = & -x \varepsilon \text{Ex}[0, 0, 0] - y \varepsilon \text{Ey}[0, 0, 0] - \varepsilon^2 \left( t y \text{Ey}^{(0,0,1)}[0, 0, 0] + \frac{1}{2} y^2 \text{Ey}^{(0,1,0)}[0, 0, 0] \right) - \\ & \frac{1}{6} \varepsilon^3 \left( 3 t^2 y \text{Ey}^{(0,0,2)}[0, 0, 0] + 3 t y^2 \text{Ey}^{(0,1,1)}[0, 0, 0] + y^3 \text{Ey}^{(0,2,0)}[0, 0, 0] \right) - \frac{1}{24} \varepsilon^4 \\ & \left( 4 t^3 y \text{Ey}^{(0,0,3)}[0, 0, 0] + 6 t^2 y^2 \text{Ey}^{(0,1,2)}[0, 0, 0] + 4 t y^3 \text{Ey}^{(0,2,1)}[0, 0, 0] + y^4 \text{Ey}^{(0,3,0)}[0, 0, 0] \right) - \\ & \varepsilon^2 \left( t x \text{Ex}^{(0,0,1)}[0, 0, 0] + \frac{1}{2} x^2 \text{Ex}^{(1,0,0)}[0, 0, 0] \right) - \\ & \frac{1}{6} \varepsilon^3 \left( 3 t^2 x \text{Ex}^{(0,0,2)}[0, 0, 0] + 3 t x^2 \text{Ex}^{(1,0,1)}[0, 0, 0] + x^3 \text{Ex}^{(2,0,0)}[0, 0, 0] \right) - \frac{1}{24} \varepsilon^4 \\ & \left( 4 t^3 x \text{Ex}^{(0,0,3)}[0, 0, 0] + 6 t^2 x^2 \text{Ex}^{(1,0,2)}[0, 0, 0] + 4 t x^3 \text{Ex}^{(2,0,1)}[0, 0, 0] + x^4 \text{Ex}^{(3,0,0)}[0, 0, 0] \right) + \\ & \varepsilon \left( \frac{1}{2} \left( -x y \varepsilon \text{Ex}^{(0,1,0)}[0, 0, 0] - t x y \varepsilon^2 \text{Ex}^{(0,1,1)}[0, 0, 0] - \frac{1}{2} t^2 x y \varepsilon^3 \text{Ex}^{(0,1,2)}[0, 0, 0] - \right. \right. \\ & \frac{1}{2} x y^2 \varepsilon^2 \text{Ex}^{(0,2,0)}[0, 0, 0] - \frac{1}{2} t x y^2 \varepsilon^3 \text{Ex}^{(0,2,1)}[0, 0, 0] - \frac{1}{6} x y^3 \varepsilon^3 \text{Ex}^{(0,3,0)}[0, 0, 0] - \\ & \frac{1}{2} x^2 y \varepsilon^2 \text{Ex}^{(1,1,0)}[0, 0, 0] - \frac{1}{2} t x^2 y \varepsilon^3 \text{Ex}^{(1,1,1)}[0, 0, 0] - \frac{1}{4} x^2 y^2 \varepsilon^3 \text{Ex}^{(1,2,0)}[0, 0, 0] - \\ & \left. \left. \frac{1}{6} x^3 y \varepsilon^3 \text{Ex}^{(2,1,0)}[0, 0, 0] \right) + \frac{1}{2} \left( -x y \varepsilon \text{Ey}^{(1,0,0)}[0, 0, 0] - \right. \right. \\ & t x y \varepsilon^2 \text{Ey}^{(1,0,1)}[0, 0, 0] - \frac{1}{2} t^2 x y \varepsilon^3 \text{Ey}^{(1,0,2)}[0, 0, 0] - \frac{1}{2} x y^2 \varepsilon^2 \text{Ey}^{(1,1,0)}[0, 0, 0] - \\ & \frac{1}{2} t x y^2 \varepsilon^3 \text{Ey}^{(1,1,1)}[0, 0, 0] - \frac{1}{6} x y^3 \varepsilon^3 \text{Ey}^{(1,2,0)}[0, 0, 0] - \frac{1}{2} x^2 y \varepsilon^2 \text{Ey}^{(2,0,0)}[0, 0, 0] - \\ & \left. \left. \frac{1}{2} t x^2 y \varepsilon^3 \text{Ey}^{(2,0,1)}[0, 0, 0] - \frac{1}{4} x^2 y^2 \varepsilon^3 \text{Ey}^{(2,1,0)}[0, 0, 0] - \frac{1}{6} x^3 y \varepsilon^3 \text{Ey}^{(3,0,0)}[0, 0, 0] \right) \right) \end{aligned}$$

In[20]:=  $\varphi 1 = \text{Normal}[\text{Series}[\text{Integrate}[\varepsilon * (\text{Es}[0, 0, \varepsilon * t] - \text{Es}[0, 0, 0]), t], \{\varepsilon, 0, k\}]]$ 

$$\text{Out}[20] = \frac{1}{2} t^2 \varepsilon^2 \text{Es}^{(0,0,1)}[0, 0, 0] + \frac{1}{6} t^3 \varepsilon^3 \text{Es}^{(0,0,2)}[0, 0, 0] + \frac{1}{24} t^4 \varepsilon^4 \text{Es}^{(0,0,3)}[0, 0, 0]$$

In[21]:=  $t = -\tau / c$ 

$$\text{Out}[21] = -\frac{\tau}{c}$$

# Hamiltonian

$$\begin{aligned} \text{In}[22] = & H = -(1 + K * \varepsilon * x) * p0 * \\ & \text{Sqrt}[1 + 2 * (\varepsilon * \delta - e * \varphi t / c) / p0 / \beta 0 + (\varepsilon * \delta - e * \varphi t / c)^2 / p0^2 - (\varepsilon * p1 - e * a1 / c)^2 / p0^2 - \\ & (\varepsilon * p3 - e * a3 / c)^2 / p0^2] - e * a2 / c + x * \varepsilon * x * (\varepsilon * p3 - e * a3 / c) - \\ & x * \varepsilon * y * (\varepsilon * p1 - e * a1 / c) + \varepsilon * \delta / \beta 0 - e * \varphi 1 \end{aligned}$$

$$\begin{aligned}
\text{Out}[22] = & \frac{\delta \varepsilon}{\beta 0} - e \left( \frac{\varepsilon^2 \tau^2 Es^{(0,0,1)}[0, 0, 0]}{2 c^2} - \frac{\varepsilon^3 \tau^3 Es^{(0,0,2)}[0, 0, 0]}{6 c^3} + \frac{\varepsilon^4 \tau^4 Es^{(0,0,3)}[0, 0, 0]}{24 c^4} \right) + x \varepsilon \kappa \left( P3 \varepsilon - \frac{1}{2 c} \right. \\
& e \left( -x \varepsilon Bs[0, 0, 0] - \varepsilon^2 \left( -\frac{x \tau Bs^{(0,0,1)}[0, 0, 0]}{c} + x y Bs^{(0,1,0)}[0, 0, 0] + \frac{1}{2} x^2 Bs^{(1,0,0)}[0, 0, 0] \right) - \right. \\
& \frac{1}{6} \varepsilon^3 \left( \frac{3 x \tau^2 Bs^{(0,0,2)}[0, 0, 0]}{c^2} - \frac{6 x y \tau Bs^{(0,1,1)}[0, 0, 0]}{c} + 3 x y^2 Bs^{(0,2,0)}[0, 0, 0] - \right. \\
& \left. \left. \frac{3 x^2 \tau Bs^{(1,0,1)}[0, 0, 0]}{c} + 3 x^2 y Bs^{(1,1,0)}[0, 0, 0] + x^3 Bs^{(2,0,0)}[0, 0, 0] \right) - \right. \\
& \frac{1}{24} \varepsilon^4 \left( -\frac{4 x \tau^3 Bs^{(0,0,3)}[0, 0, 0]}{c^3} + \frac{12 x y \tau^2 Bs^{(0,1,2)}[0, 0, 0]}{c^2} - \frac{12 x y^2 \tau Bs^{(0,2,1)}[0, 0, 0]}{c} + \right. \\
& 4 x y^3 Bs^{(0,3,0)}[0, 0, 0] + \frac{6 x^2 \tau^2 Bs^{(1,0,2)}[0, 0, 0]}{c^2} - \\
& \left. \frac{12 x^2 y \tau Bs^{(1,1,1)}[0, 0, 0]}{c} + 6 x^2 y^2 Bs^{(1,2,0)}[0, 0, 0] - \frac{4 x^3 \tau Bs^{(2,0,1)}[0, 0, 0]}{c} + \right. \\
& \left. \left. 4 x^3 y Bs^{(2,1,0)}[0, 0, 0] + x^4 Bs^{(3,0,0)}[0, 0, 0] \right) \right) - y \varepsilon \kappa \left( P1 \varepsilon - \frac{1}{2 c} \right. \\
& e \left( y \varepsilon Bs[0, 0, 0] + \varepsilon^2 \left( -\frac{y \tau Bs^{(0,0,1)}[0, 0, 0]}{c} + \frac{1}{2} y^2 Bs^{(0,1,0)}[0, 0, 0] + x y Bs^{(1,0,0)}[0, 0, 0] \right) + \right. \\
& \frac{1}{6} \varepsilon^3 \left( \frac{3 y \tau^2 Bs^{(0,0,2)}[0, 0, 0]}{c^2} - \frac{3 y^2 \tau Bs^{(0,1,1)}[0, 0, 0]}{c} + y^3 Bs^{(0,2,0)}[0, 0, 0] - \right. \\
& \left. \left. \frac{6 x y \tau Bs^{(1,0,1)}[0, 0, 0]}{c} + 3 x y^2 Bs^{(1,1,0)}[0, 0, 0] + 3 x^2 y Bs^{(2,0,0)}[0, 0, 0] \right) + \right. \\
& \frac{1}{24} \varepsilon^4 \left( -\frac{4 y \tau^3 Bs^{(0,0,3)}[0, 0, 0]}{c^3} + \frac{6 y^2 \tau^2 Bs^{(0,1,2)}[0, 0, 0]}{c^2} - \frac{4 y^3 \tau Bs^{(0,2,1)}[0, 0, 0]}{c} + \right. \\
& y^4 Bs^{(0,3,0)}[0, 0, 0] + \frac{12 x y \tau^2 Bs^{(1,0,2)}[0, 0, 0]}{c^2} - \frac{12 x y^2 \tau Bs^{(1,1,1)}[0, 0, 0]}{c} + \\
& 4 x y^3 Bs^{(1,2,0)}[0, 0, 0] - \frac{12 x^2 y \tau Bs^{(2,0,1)}[0, 0, 0]}{c} + \\
& \left. \left. 6 x^2 y^2 Bs^{(2,1,0)}[0, 0, 0] + 4 x^3 y Bs^{(3,0,0)}[0, 0, 0] \right) \right) - \frac{1}{c} \\
& e \left( -y \varepsilon Bx[0, 0, 0] + x \varepsilon By[0, 0, 0] - \varepsilon^2 \left( -\frac{1}{2} y^2 \kappa Bs[0, 0, 0] - \frac{y \tau Bx^{(0,0,1)}[0, 0, 0]}{c} + \right. \right. \\
& \frac{1}{2} y^2 Bx^{(0,1,0)}[0, 0, 0] - \\
& \frac{1}{6} \varepsilon^3 \left( \frac{3 y^2 \kappa \tau Bs^{(0,0,1)}[0, 0, 0]}{c} + \frac{3 y \tau^2 Bx^{(0,0,2)}[0, 0, 0]}{c^2} - \frac{3 y^2 \tau Bx^{(0,1,1)}[0, 0, 0]}{c} + \right. \\
& y^3 Bx^{(0,2,0)}[0, 0, 0] - 3 x y^2 \kappa Bs^{(1,0,0)}[0, 0, 0] \left. \right) + \\
& \frac{1}{2} \varepsilon^2 \left( x^2 \kappa Bs[0, 0, 0] + K x^2 By[0, 0, 0] - \frac{2 x \tau By^{(0,0,1)}[0, 0, 0]}{c} + x^2 By^{(1,0,0)}[0, 0, 0] \right) - \\
& \frac{1}{24} \varepsilon^4 \left( -\frac{6 y^2 \kappa \tau^2 Bs^{(0,0,2)}[0, 0, 0]}{c^2} - \frac{4 y \tau^3 Bx^{(0,0,3)}[0, 0, 0]}{c^3} + \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{6 y^2 \tau^2 Bx^{(0,1,2)}[0, 0, 0]}{c^2} - \frac{4 y^3 \tau Bx^{(0,2,1)}[0, 0, 0]}{c} + y^4 Bx^{(0,3,0)}[0, 0, 0] + \\
& \left. \frac{12 xy^2 \kappa \tau Bs^{(1,0,1)}[0, 0, 0]}{c} - 6 x^2 y^2 \kappa Bs^{(2,0,0)}[0, 0, 0] \right) + \\
& \frac{1}{6} \varepsilon^3 \left( - \frac{3 x^2 \kappa \tau Bs^{(0,0,1)}[0, 0, 0]}{c} - \frac{3 K x^2 \tau By^{(0,0,1)}[0, 0, 0]}{c} + \frac{3 x \tau^2 By^{(0,0,2)}[0, 0, 0]}{c^2} + 2 x^3 \right. \\
& \times Bs^{(1,0,0)}[0, 0, 0] + 2 K x^3 By^{(1,0,0)}[0, 0, 0] - \frac{3 x^2 \tau By^{(1,0,1)}[0, 0, 0]}{c} + x^3 By^{(2,0,0)}[0, 0, 0] \Big) + \\
& \varepsilon \left( \frac{1}{2} \left( \frac{1}{2} x^2 y \varepsilon^2 \kappa Bs^{(0,1,0)}[0, 0, 0] + x y \varepsilon By^{(0,1,0)}[0, 0, 0] + \frac{1}{2} K x^2 y \varepsilon^2 By^{(0,1,0)}[0, 0, 0] - \right. \right. \\
& \frac{x^2 y \varepsilon^3 \kappa \tau Bs^{(0,1,1)}[0, 0, 0]}{2 c} - \frac{x y \varepsilon^2 \tau By^{(0,1,1)}[0, 0, 0]}{c} - \frac{K x^2 y \varepsilon^3 \tau By^{(0,1,1)}[0, 0, 0]}{2 c} + \\
& \frac{x y \varepsilon^3 \tau^2 By^{(0,1,2)}[0, 0, 0]}{2 c^2} + \frac{1}{4} x^2 y^2 \varepsilon^3 \kappa Bs^{(0,2,0)}[0, 0, 0] + \frac{1}{2} x y^2 \varepsilon^2 By^{(0,2,0)}[0, 0, 0] + \\
& \frac{1}{4} K x^2 y^2 \varepsilon^3 By^{(0,2,0)}[0, 0, 0] - \frac{x y^2 \varepsilon^3 \tau By^{(0,2,1)}[0, 0, 0]}{2 c} + \frac{1}{6} x y^3 \varepsilon^3 By^{(0,3,0)}[0, 0, 0] + \\
& \frac{1}{3} x^3 y \varepsilon^3 \kappa Bs^{(1,1,0)}[0, 0, 0] + \frac{1}{2} x^2 y \varepsilon^2 By^{(1,1,0)}[0, 0, 0] + \frac{1}{3} K x^3 y \varepsilon^3 By^{(1,1,0)}[0, 0, 0] - \\
& \left. \frac{x^2 y \varepsilon^3 \tau By^{(1,1,1)}[0, 0, 0]}{2 c} + \frac{1}{4} x^2 y^2 \varepsilon^3 By^{(1,2,0)}[0, 0, 0] + \frac{1}{6} x^3 y \varepsilon^3 By^{(2,1,0)}[0, 0, 0] \right) + \\
& \frac{1}{2} \left( K x y \varepsilon Bx[0, 0, 0] - \frac{K x y \varepsilon^2 \tau Bx^{(0,0,1)}[0, 0, 0]}{c} + \frac{K x y \varepsilon^3 \tau^2 Bx^{(0,0,2)}[0, 0, 0]}{2 c^2} + \right. \\
& \frac{1}{2} K x y^2 \varepsilon^2 Bx^{(0,1,0)}[0, 0, 0] - \frac{K x y^2 \varepsilon^3 \tau Bx^{(0,1,1)}[0, 0, 0]}{2 c} + \frac{1}{6} K x y^3 \varepsilon^3 Bx^{(0,2,0)}[0, 0, 0] - \\
& \frac{1}{2} x y^2 \varepsilon^2 \kappa Bs^{(1,0,0)}[0, 0, 0] + x y \varepsilon Bx^{(1,0,0)}[0, 0, 0] + K x^2 y \varepsilon^2 Bx^{(1,0,0)}[0, 0, 0] + \\
& \frac{x y^2 \varepsilon^3 \kappa \tau Bs^{(1,0,1)}[0, 0, 0]}{2 c} - \frac{x y \varepsilon^2 \tau Bx^{(1,0,1)}[0, 0, 0]}{c} - \frac{K x^2 y \varepsilon^3 \tau Bx^{(1,0,1)}[0, 0, 0]}{c} + \\
& \frac{x y \varepsilon^3 \tau^2 Bx^{(1,0,2)}[0, 0, 0]}{2 c^2} - \frac{1}{3} x y^3 \varepsilon^3 \kappa Bs^{(1,1,0)}[0, 0, 0] + \frac{1}{2} x y^2 \varepsilon^2 Bx^{(1,1,0)}[0, 0, 0] + \\
& \frac{1}{2} K x^2 y^2 \varepsilon^3 Bx^{(1,1,0)}[0, 0, 0] - \frac{x y^2 \varepsilon^3 \tau Bx^{(1,1,1)}[0, 0, 0]}{2 c} + \frac{1}{6} x y^3 \varepsilon^3 Bx^{(1,2,0)}[0, 0, 0] - \\
& \frac{1}{4} x^2 y^2 \varepsilon^3 \kappa Bs^{(2,0,0)}[0, 0, 0] + \frac{1}{2} x^2 y \varepsilon^2 Bx^{(2,0,0)}[0, 0, 0] + \frac{1}{2} K x^3 y \varepsilon^3 Bx^{(2,0,0)}[0, 0, 0] - \\
& \left. \frac{x^2 y \varepsilon^3 \tau Bx^{(2,0,1)}[0, 0, 0]}{2 c} + \frac{1}{4} x^2 y^2 \varepsilon^3 Bx^{(2,1,0)}[0, 0, 0] + \frac{1}{6} x^3 y \varepsilon^3 Bx^{(3,0,0)}[0, 0, 0] \right) + \\
& \frac{1}{24} \varepsilon^4 \left( \frac{6 x^2 \kappa \tau^2 Bs^{(0,0,2)}[0, 0, 0]}{c^2} + \frac{6 K x^2 \tau^2 By^{(0,0,2)}[0, 0, 0]}{c^2} - \frac{4 x \tau^3 By^{(0,0,3)}[0, 0, 0]}{c^3} - \right. \\
& \frac{8 x^3 \kappa \tau Bs^{(1,0,1)}[0, 0, 0]}{c} - \frac{8 K x^3 \tau By^{(1,0,1)}[0, 0, 0]}{c} + \\
& \frac{6 x^2 \tau^2 By^{(1,0,2)}[0, 0, 0]}{c^2} + 3 x^4 \kappa Bs^{(2,0,0)}[0, 0, 0] + \\
& \left. 3 K x^4 By^{(2,0,0)}[0, 0, 0] - \frac{4 x^3 \tau By^{(2,0,1)}[0, 0, 0]}{c} + x^4 By^{(3,0,0)}[0, 0, 0] \right) +
\end{aligned}$$

$$\begin{aligned}
& p0 (-1 - K x \varepsilon) \sqrt{\left(1 - \frac{1}{p0^2} \left(P3 \varepsilon - \frac{1}{2c} e \left(-x \varepsilon B s [0, 0, 0] - \varepsilon^2 \left(-\frac{x \tau B s^{(0,0,1)} [0, 0, 0]}{c} + x y\right.\right.\right.\right.} \\
& \left.\left.\left.\left. B s^{(0,1,0)} [0, 0, 0] + \frac{1}{2} x^2 B s^{(1,0,0)} [0, 0, 0]\right) - \frac{1}{6} \varepsilon^3 \left(\frac{3 x \tau^2 B s^{(0,0,2)} [0, 0, 0]}{c^2} - \right.\right.\right.} \\
& \left.\left.\left.\frac{6 x y \tau B s^{(0,1,1)} [0, 0, 0]}{c} + 3 x y^2 B s^{(0,2,0)} [0, 0, 0] - \frac{3 x^2 \tau B s^{(1,0,1)} [0, 0, 0]}{c} + 3\right.\right.\right.} \\
& \left.\left.\left.x^2 y B s^{(1,1,0)} [0, 0, 0] + x^3 B s^{(2,0,0)} [0, 0, 0]\right) - \frac{1}{24} \varepsilon^4 \left(-\frac{4 x \tau^3 B s^{(0,0,3)} [0, 0, 0]}{c^3} + \right.\right.\right.} \\
& \left.\left.\left.\frac{12 x y \tau^2 B s^{(0,1,2)} [0, 0, 0]}{c^2} - \frac{12 x y^2 \tau B s^{(0,2,1)} [0, 0, 0]}{c} + 4 x y^3 B s^{(0,3,0)} [0, 0, 0] + \right.\right.\right.} \\
& \left.\left.\left.\frac{6 x^2 \tau^2 B s^{(1,0,2)} [0, 0, 0]}{c^2} - \frac{12 x^2 y \tau B s^{(1,1,1)} [0, 0, 0]}{c} + 6 x^2 y^2 B s^{(1,2,0)} [0, 0, 0] - \right.\right.\right.} \\
& \left.\left.\left.\frac{4 x^3 \tau B s^{(2,0,1)} [0, 0, 0]}{c} + 4 x^3 y B s^{(2,1,0)} [0, 0, 0] + x^4 B s^{(3,0,0)} [0, 0, 0]\right)\right)^2 - \frac{1}{p0^2}\right) \\
& \left(P1 \varepsilon - \frac{1}{2c} e \left(y \varepsilon B s [0, 0, 0] + \varepsilon^2 \left(-\frac{y \tau B s^{(0,0,1)} [0, 0, 0]}{c} + \frac{1}{2} y^2 B s^{(0,1,0)} [0, 0, 0] + x y B s^{(1,0,0)} [0, 0, 0]\right)\right.\right.} \\
& \left.\left.\left.+ \frac{1}{6} \varepsilon^3 \left(\frac{3 y \tau^2 B s^{(0,0,2)} [0, 0, 0]}{c^2} - \frac{3 y^2 \tau B s^{(0,1,1)} [0, 0, 0]}{c} + y^3 B s^{(0,2,0)} [0, 0, 0]\right) - \right.\right.\right.} \\
& \left.\left.\left.\frac{6 x y \tau B s^{(1,0,1)} [0, 0, 0]}{c} + 3 x y^2 B s^{(1,1,0)} [0, 0, 0] + 3 x^2 y B s^{(2,0,0)} [0, 0, 0]\right) + \right.\right.\right.} \\
& \left.\frac{1}{24} \varepsilon^4 \left(-\frac{4 y \tau^3 B s^{(0,0,3)} [0, 0, 0]}{c^3} + \frac{6 y^2 \tau^2 B s^{(0,1,2)} [0, 0, 0]}{c^2} - \frac{4 y^3 \tau B s^{(0,2,1)} [0, 0, 0]}{c} + \right.\right.\right.} \\
& \left.y^4 B s^{(0,3,0)} [0, 0, 0] + \frac{12 x y \tau^2 B s^{(1,0,2)} [0, 0, 0]}{c^2} - \frac{12 x y^2 \tau B s^{(1,1,1)} [0, 0, 0]}{c} + 4\right.\right.\right.} \\
& \left.x y^3 B s^{(1,2,0)} [0, 0, 0] - \frac{12 x^2 y \tau B s^{(2,0,1)} [0, 0, 0]}{c} + 6 x^2 y^2\right.\right.\right.} \\
& \left.B s^{(2,1,0)} [0, 0, 0] + 4 x^3 y B s^{(3,0,0)} [0, 0, 0]\right)\right)^2 + \frac{1}{p0 \beta 0} 2 \left(\delta \varepsilon - \frac{1}{c}\right) \\
& e \left(-x \varepsilon E x [0, 0, 0] - y \varepsilon E y [0, 0, 0] - \varepsilon^2 \left(-\frac{y \tau E y^{(0,0,1)} [0, 0, 0]}{c} + \frac{1}{2} y^2 E y^{(0,1,0)} [0, 0, 0]\right) - \right. \\
& \left.\frac{1}{6} \varepsilon^3 \left(\frac{3 y \tau^2 E y^{(0,0,2)} [0, 0, 0]}{c^2} - \frac{3 y^2 \tau E y^{(0,1,1)} [0, 0, 0]}{c} + y^3 E y^{(0,2,0)} [0, 0, 0]\right) - \right. \\
& \left.\frac{1}{24} \varepsilon^4 \left(-\frac{4 y \tau^3 E y^{(0,0,3)} [0, 0, 0]}{c^3} + \frac{6 y^2 \tau^2 E y^{(0,1,2)} [0, 0, 0]}{c^2} - \frac{4 y^3 \tau E y^{(0,2,1)} [0, 0, 0]}{c} + \right.\right.\right.} \\
& \left.y^4 E y^{(0,3,0)} [0, 0, 0]\right) - \varepsilon^2 \left(-\frac{x \tau E x^{(0,0,1)} [0, 0, 0]}{c} + \frac{1}{2} x^2 E x^{(1,0,0)} [0, 0, 0]\right) - \right. \\
& \left.\frac{1}{6} \varepsilon^3 \left(\frac{3 x \tau^2 E x^{(0,0,2)} [0, 0, 0]}{c^2} - \frac{3 x^2 \tau E x^{(1,0,1)} [0, 0, 0]}{c} + x^3 E x^{(2,0,0)} [0, 0, 0]\right) - \right. \\
& \left.\frac{1}{24} \varepsilon^4 \left(-\frac{4 x \tau^3 E x^{(0,0,3)} [0, 0, 0]}{c^3} + \frac{6 x^2 \tau^2 E x^{(1,0,2)} [0, 0, 0]}{c^2} - \right.\right.\right.} \\
& \left.\frac{4 x^3 \tau E x^{(2,0,1)} [0, 0, 0]}{c} + x^4 E x^{(3,0,0)} [0, 0, 0]\right) +
\end{aligned}$$

$$\begin{aligned}
& \varepsilon \left( \frac{1}{2} \left( -x y \varepsilon \text{Ex}^{(0,1,0)}[0, 0, 0] + \frac{x y \varepsilon^2 \tau \text{Ex}^{(0,1,1)}[0, 0, 0]}{c} - \frac{x y \varepsilon^3 \tau^2 \text{Ex}^{(0,1,2)}[0, 0, 0]}{2 c^2} - \right. \right. \\
& \quad \frac{1}{2} \frac{x y^2 \varepsilon^2 \text{Ex}^{(0,2,0)}[0, 0, 0]}{c} + \frac{x y^2 \varepsilon^3 \tau \text{Ex}^{(0,2,1)}[0, 0, 0]}{2 c} - \frac{1}{6} x y^3 \varepsilon^3 \text{Ex}^{(0,3,0)}[0, 0, 0] - \\
& \quad \frac{1}{2} \frac{x^2 y \varepsilon^2 \text{Ex}^{(1,1,0)}[0, 0, 0]}{c} + \frac{x^2 y \varepsilon^3 \tau \text{Ex}^{(1,1,1)}[0, 0, 0]}{2 c} - \frac{1}{4} x^2 y^2 \varepsilon^3 \text{Ex}^{(1,2,0)}[0, 0, 0] - \\
& \quad \left. \left. \frac{1}{6} x^3 y \varepsilon^3 \text{Ex}^{(2,1,0)}[0, 0, 0] \right) + \frac{1}{2} \left( -x y \varepsilon \text{Ey}^{(1,0,0)}[0, 0, 0] + \frac{x y \varepsilon^2 \tau \text{Ey}^{(1,0,1)}[0, 0, 0]}{c} - \right. \right. \\
& \quad \frac{x y \varepsilon^3 \tau^2 \text{Ey}^{(1,0,2)}[0, 0, 0]}{2 c^2} - \frac{1}{2} x y^2 \varepsilon^2 \text{Ey}^{(1,1,0)}[0, 0, 0] + \frac{x y^2 \varepsilon^3 \tau \text{Ey}^{(1,1,1)}[0, 0, 0]}{2 c} - \\
& \quad \frac{1}{6} x y^3 \varepsilon^3 \text{Ey}^{(1,2,0)}[0, 0, 0] - \frac{1}{2} x^2 y \varepsilon^2 \text{Ey}^{(2,0,0)}[0, 0, 0] + \frac{x^2 y \varepsilon^3 \tau \text{Ey}^{(2,0,1)}[0, 0, 0]}{2 c} - \\
& \quad \left. \left. \frac{1}{4} x^2 y^2 \varepsilon^3 \text{Ey}^{(2,1,0)}[0, 0, 0] - \frac{1}{6} x^3 y \varepsilon^3 \text{Ey}^{(3,0,0)}[0, 0, 0] \right) \right) \Bigg) + \frac{1}{p_0^2} \\
& \left( \delta \varepsilon - \frac{1}{c} e \left( -x \varepsilon \text{Ex}[0, 0, 0] - y \varepsilon \text{Ey}[0, 0, 0] - \varepsilon^2 \left( -\frac{y \tau \text{Ey}^{(0,0,1)}[0, 0, 0]}{c} + \frac{1}{2} y^2 \text{Ey}^{(0,1,0)}[0, 0, 0] \right) - \right. \right. \\
& \quad \frac{1}{6} \varepsilon^3 \left( \frac{3 y \tau^2 \text{Ey}^{(0,0,2)}[0, 0, 0]}{c^2} - \frac{3 y^2 \tau \text{Ey}^{(0,1,1)}[0, 0, 0]}{c} + y^3 \text{Ey}^{(0,2,0)}[0, 0, 0] \right) - \\
& \quad \frac{1}{24} \varepsilon^4 \left( -\frac{4 y \tau^3 \text{Ey}^{(0,0,3)}[0, 0, 0]}{c^3} + \frac{6 y^2 \tau^2 \text{Ey}^{(0,1,2)}[0, 0, 0]}{c^2} - \frac{4 y^3 \tau \text{Ey}^{(0,2,1)}[0, 0, 0]}{c} + y^4 \right. \\
& \quad \left. \text{Ey}^{(0,3,0)}[0, 0, 0] \right) - \varepsilon^2 \left( -\frac{x \tau \text{Ex}^{(0,0,1)}[0, 0, 0]}{c} + \frac{1}{2} x^2 \text{Ex}^{(1,0,0)}[0, 0, 0] \right) - \\
& \quad \frac{1}{6} \varepsilon^3 \left( \frac{3 x \tau^2 \text{Ex}^{(0,0,2)}[0, 0, 0]}{c^2} - \frac{3 x^2 \tau \text{Ex}^{(1,0,1)}[0, 0, 0]}{c} + x^3 \text{Ex}^{(2,0,0)}[0, 0, 0] \right) - \\
& \quad \frac{1}{24} \varepsilon^4 \left( -\frac{4 x \tau^3 \text{Ex}^{(0,0,3)}[0, 0, 0]}{c^3} + \frac{6 x^2 \tau^2 \text{Ex}^{(1,0,2)}[0, 0, 0]}{c^2} - \frac{4 x^3 \tau \text{Ex}^{(2,0,1)}[0, 0, 0]}{c} + x^4 \right. \\
& \quad \left. \text{Ex}^{(3,0,0)}[0, 0, 0] \right) + \varepsilon \left( \frac{1}{2} \left( -x y \varepsilon \text{Ex}^{(0,1,0)}[0, 0, 0] + \frac{x y \varepsilon^2 \tau \text{Ex}^{(0,1,1)}[0, 0, 0]}{c} - \right. \right. \\
& \quad \left. \left. \frac{x y \varepsilon^3 \tau^2 \text{Ex}^{(0,1,2)}[0, 0, 0]}{2 c^2} - \frac{1}{2} x y^2 \varepsilon^2 \text{Ex}^{(0,2,0)}[0, 0, 0] + \frac{x y^2 \varepsilon^3 \tau \text{Ex}^{(0,2,1)}[0, 0, 0]}{2 c} - \right. \right. \\
& \quad \left. \left. \frac{1}{6} x y^3 \varepsilon^3 \text{Ex}^{(0,3,0)}[0, 0, 0] - \frac{1}{2} x^2 y \varepsilon^2 \text{Ex}^{(1,1,0)}[0, 0, 0] + \frac{x^2 y \varepsilon^3 \tau \text{Ex}^{(1,1,1)}[0, 0, 0]}{2 c} - \right. \right. \\
& \quad \left. \left. \frac{1}{4} x^2 y^2 \varepsilon^3 \text{Ex}^{(1,2,0)}[0, 0, 0] - \frac{1}{6} x^3 y \varepsilon^3 \text{Ex}^{(2,1,0)}[0, 0, 0] \right) + \frac{1}{2} \right. \\
& \quad \left( -x y \varepsilon \text{Ey}^{(1,0,0)}[0, 0, 0] + \frac{x y \varepsilon^2 \tau \text{Ey}^{(1,0,1)}[0, 0, 0]}{c} - \frac{x y \varepsilon^3 \tau^2 \text{Ey}^{(1,0,2)}[0, 0, 0]}{2 c^2} - \right. \\
& \quad \left. \frac{1}{2} x y^2 \varepsilon^2 \text{Ey}^{(1,1,0)}[0, 0, 0] + \frac{x y^2 \varepsilon^3 \tau \text{Ey}^{(1,1,1)}[0, 0, 0]}{2 c} - \frac{1}{6} x y^3 \varepsilon^3 \text{Ey}^{(1,2,0)}[0, 0, 0] - \right. \\
& \quad \left. \frac{1}{2} x^2 y \varepsilon^2 \text{Ey}^{(2,0,0)}[0, 0, 0] + \frac{x^2 y \varepsilon^3 \tau \text{Ey}^{(2,0,1)}[0, 0, 0]}{2 c} - \right. \\
& \quad \left. \left. \frac{1}{4} x^2 y^2 \varepsilon^3 \text{Ey}^{(2,1,0)}[0, 0, 0] - \frac{1}{6} x^3 y \varepsilon^3 \text{Ey}^{(3,0,0)}[0, 0, 0] \right) \right)^2 \right)
\end{aligned}$$

# First order terms

In[23]:= **H1 = Normal[Series[H, {ε, 0, 1}]]**

$$\text{Out}[23]= -\mathbf{p}_0 + \frac{\varepsilon (-c K p_0 x \beta_0 + e y \beta_0 Bx[0, 0, 0] - e x \beta_0 By[0, 0, 0] - e x Ex[0, 0, 0] - e y Ey[0, 0, 0])}{c \beta_0}$$

In[24]:= **Cx = Coefficient[H1, ε x]**

$$\text{Out}[24]= \frac{-c K p_0 \beta_0 - e \beta_0 By[0, 0, 0] - e Ex[0, 0, 0]}{c \beta_0}$$

In[25]:= **Cy = Coefficient[H1, ε y]**

$$\text{Out}[25]= \frac{e \beta_0 Bx[0, 0, 0] - e Ey[0, 0, 0]}{c \beta_0}$$

# Second order terms

In[26]:= **H2 = Normal[Series[H, {ε, 0, 2}]]**

$$\begin{aligned} \text{Out[26]}= & -\mathbf{p0} + \frac{\varepsilon (-c K p0 x \beta0 + e y \beta0 Bx[0, 0, 0] - e x \beta0 By[0, 0, 0] - e x Ex[0, 0, 0] - e y Ey[0, 0, 0])}{c \beta0} + \\ & \varepsilon^2 \left( P3 x \kappa - P1 y \kappa - \frac{e K x y Bx[0, 0, 0]}{2 c} - \frac{e K x^2 By[0, 0, 0]}{2 c} - \right. \\ & \frac{e y \tau Bx^{(0,0,1)}[0, 0, 0]}{c^2} + \frac{e x \tau By^{(0,0,1)}[0, 0, 0]}{c^2} - \frac{e \tau^2 Es^{(0,0,1)}[0, 0, 0]}{2 c^2} + \\ & \frac{e y^2 Bx^{(0,1,0)}[0, 0, 0]}{2 c} - \frac{e x y By^{(0,1,0)}[0, 0, 0]}{2 c} - \frac{e x y Bx^{(1,0,0)}[0, 0, 0]}{2 c} - \\ & \frac{e x^2 By^{(1,0,0)}[0, 0, 0]}{2 c} + \mathbf{p0} \left( -\frac{1}{2} K x \left( \frac{2 \delta}{p0 \beta0} + \frac{2 e x Ex[0, 0, 0]}{c p0 \beta0} + \frac{2 e y Ey[0, 0, 0]}{c p0 \beta0} \right) + \right. \\ & \frac{1}{2} \left( \frac{\left( P3 + \frac{e x Bs[0, 0, 0]}{2 c} \right)^2}{p0^2} + \frac{\left( P1 - \frac{e y Bs[0, 0, 0]}{2 c} \right)^2}{p0^2} - \frac{\left( \delta + \frac{e x Ex[0, 0, 0]}{c} + \frac{e y Ey[0, 0, 0]}{c} \right)^2}{p0^2} + \right. \\ & \frac{1}{4} \left( \frac{2 \delta}{p0 \beta0} + \frac{2 e x Ex[0, 0, 0]}{c p0 \beta0} + \frac{2 e y Ey[0, 0, 0]}{c p0 \beta0} \right)^2 + \\ & \frac{2 e x \tau Ex^{(0,0,1)}[0, 0, 0]}{c^2 p0 \beta0} + \frac{2 e y \tau Ey^{(0,0,1)}[0, 0, 0]}{c^2 p0 \beta0} - \frac{e x y Ex^{(0,1,0)}[0, 0, 0]}{c p0 \beta0} - \\ & \left. \frac{e y^2 Ey^{(0,1,0)}[0, 0, 0]}{c p0 \beta0} - \frac{e x^2 Ex^{(1,0,0)}[0, 0, 0]}{c p0 \beta0} - \frac{e x y Ey^{(1,0,0)}[0, 0, 0]}{c p0 \beta0} \right) \left. \right) \end{aligned}$$

In[27]:= **Cxx = Coefficient[H2, ε^2 x^2]**

$$\begin{aligned} \text{Out[27]}= & \frac{1}{2} \left( -\frac{e K By[0, 0, 0]}{c} - \frac{e By^{(1,0,0)}[0, 0, 0]}{c} + \mathbf{p0} \left( -\frac{2 e K Ex[0, 0, 0]}{c p0 \beta0} + \right. \right. \\ & \frac{1}{2} \left( \frac{e^2 Bs[0, 0, 0]^2}{2 c^2 p0^2} - \frac{2 e^2 Ex[0, 0, 0]^2}{c^2 p0^2} + \frac{2 e^2 Ex[0, 0, 0]^2}{c^2 p0^2 \beta0^2} - \frac{2 e Ex^{(1,0,0)}[0, 0, 0]}{c p0 \beta0} \right) \left. \right) \end{aligned}$$

In[28]:= **Expand[Cxx]**

$$\begin{aligned} \text{Out[28]}= & \frac{e^2 Bs[0, 0, 0]^2}{8 c^2 p0} - \frac{e K By[0, 0, 0]}{2 c} - \frac{e K Ex[0, 0, 0]}{c \beta0} - \\ & \frac{e^2 Ex[0, 0, 0]^2}{2 c^2 p0} + \frac{e^2 Ex[0, 0, 0]^2}{2 c^2 p0 \beta0^2} - \frac{e By^{(1,0,0)}[0, 0, 0]}{2 c} - \frac{e Ex^{(1,0,0)}[0, 0, 0]}{2 c \beta0} \end{aligned}$$

In[29]:= **Cyy = Coefficient[H2, ε^2 y^2]**

$$\text{Out}[29]= \frac{1}{2} \left( \frac{e Bx^{(0,1,0)} [0, 0, 0]}{c} + \frac{1}{2} p_0 \left( \frac{e^2 Bs [0, 0, 0]^2}{2 c^2 p_0^2} - \frac{2 e^2 Ey [0, 0, 0]^2}{c^2 p_0^2} + \frac{2 e^2 Ey [0, 0, 0]^2}{c^2 p_0^2 β_0^2} - \frac{2 e Ey^{(0,1,0)} [0, 0, 0]}{c p_0 β_0} \right) \right)$$

In[30]:= **Expand[Cyy]**

$$\text{Out}[30]= \frac{e^2 Bs [0, 0, 0]^2}{8 c^2 p_0} - \frac{e^2 Ey [0, 0, 0]^2}{2 c^2 p_0} + \frac{e^2 Ey [0, 0, 0]^2}{2 c^2 p_0 β_0^2} + \frac{e Bx^{(0,1,0)} [0, 0, 0]}{2 c} - \frac{e Ey^{(0,1,0)} [0, 0, 0]}{2 c β_0}$$

In[31]:= **Cxy = Coefficient[H2, ε^2 x y]**

$$\text{Out}[31]= -\frac{e K Bx [0, 0, 0]}{2 c} - \frac{e By^{(0,1,0)} [0, 0, 0]}{2 c} - \frac{e Bx^{(1,0,0)} [0, 0, 0]}{2 c} + \\ p_0 \left( -\frac{e K Ey [0, 0, 0]}{c p_0 β_0} + \frac{1}{2} \left( -\frac{2 e^2 Ex [0, 0, 0] Ey [0, 0, 0]}{c^2 p_0^2} + \frac{2 e^2 Ex [0, 0, 0] Ey [0, 0, 0]}{c^2 p_0^2 β_0^2} - \frac{e Ex^{(0,1,0)} [0, 0, 0]}{c p_0 β_0} - \frac{e Ey^{(1,0,0)} [0, 0, 0]}{c p_0 β_0} \right) \right)$$

In[32]:= **Expand[Cxy]**

$$\text{Out}[32]= -\frac{e K Bx [0, 0, 0]}{2 c} - \frac{e K Ey [0, 0, 0]}{c β_0} - \frac{e^2 Ex [0, 0, 0] Ey [0, 0, 0]}{c^2 p_0} + \frac{e^2 Ex [0, 0, 0] Ey [0, 0, 0]}{c^2 p_0 β_0^2} - \\ \frac{e By^{(0,1,0)} [0, 0, 0]}{2 c} - \frac{e Ex^{(0,1,0)} [0, 0, 0]}{2 c β_0} - \frac{e Bx^{(1,0,0)} [0, 0, 0]}{2 c} - \frac{e Ey^{(1,0,0)} [0, 0, 0]}{2 c β_0}$$

In[33]:= **CxP1 = Coefficient[H2, ε^2 x P1]**

$$\text{Out}[33]= 0$$

In[34]:= **CxP3 = Expand[Coeficient[H2, ε^2 x P3]]**

$$\text{Out}[34]= K + \frac{e Bs [0, 0, 0]}{2 c p_0}$$

In[35]:= **CyP1 = Expand[Coeficient[H2, ε^2 y P1]]**

$$\text{Out}[35]= -K - \frac{e Bs [0, 0, 0]}{2 c p_0}$$

In[36]:= **CyP3 = Coeficient[H2, ε^2 y P3]**

$$\text{Out}[36]= 0$$

In[37]:= **Cxδ = Expand[Coeficient[H2, ε^2 x δ]]**

$$\text{Out}[37]= -\frac{K}{β_0} - \frac{e Ex [0, 0, 0]}{c p_0} + \frac{e Ex [0, 0, 0]}{c p_0 β_0^2}$$

In[38]:= **Cyδ = Expand[Coeficient[H2, ε^2 y δ]]**

$$\text{Out}[38]= -\frac{e Ey [0, 0, 0]}{c p_0} + \frac{e Ey [0, 0, 0]}{c p_0 β_0^2}$$

```
In[39]:= CP1δ = Expand[Coefficient[H2, ε^2 P1 δ]]
```

```
Out[39]= 0
```

```
In[40]:= CP3δ = Expand[Coefficient[H2, ε^2 P3 δ]]
```

```
Out[40]= 0
```

```
In[41]:= CPδδ = Expand[Coefficient[H2, ε^2 δ^2]]
```

$$\text{Out}[41] = -\frac{1}{2 p_0} + \frac{1}{2 p_0 \beta_0^2}$$

```
In[42]:= CPtt = Expand[Coefficient[H2, ε^2 τ^2]]
```

$$\text{Out}[42] = -\frac{e Es^{(0,0,1)} [0, 0, 0]}{2 c^2}$$

```
In[43]:= Cxt = Expand[Coefficient[H2, ε^2 x τ]]
```

$$\text{Out}[43] = \frac{e By^{(0,0,1)} [0, 0, 0]}{c^2} + \frac{e Ex^{(0,0,1)} [0, 0, 0]}{c^2 \beta_0}$$

```
In[44]:= Cyt = Expand[Coefficient[H2, ε^2 y τ]]
```

$$\text{Out}[44] = -\frac{e Bx^{(0,0,1)} [0, 0, 0]}{c^2} + \frac{e Ey^{(0,0,1)} [0, 0, 0]}{c^2 \beta_0}$$

```
In[45]:= CP1t = Expand[Coefficient[H2, ε^2 P1 τ]]
```

```
Out[45]= 0
```

```
In[46]:= CP3t = Expand[Coefficient[H2, ε^2 P3 τ]]
```

```
Out[46]= 0
```

```
In[47]:= Cδt = Expand[Coefficient[H2, ε^2 δ τ]]
```

```
Out[47]= 0
```

# Third order terms

```
In[48]:= H3 = Normal[Series[H, {ε, 0, 3}]]
```

$$\text{Out}[48] = -p_0 + \frac{\varepsilon (-c K p_0 x \beta_0 + e y \beta_0 Bx [0, 0, 0] - e x \beta_0 By [0, 0, 0] - e x Ex [0, 0, 0] - e y Ey [0, 0, 0])}{c \beta_0} +$$

$$\varepsilon^2 \left( P3 x \kappa - P1 y \kappa + \frac{e x^2 \times Bs [0, 0, 0]}{2 c} + \frac{e y^2 \times Bs [0, 0, 0]}{2 c} - \frac{e \tau^2 Es^{(0,0,1)} [0, 0, 0]}{2 c^2} + \frac{1}{c} \right)$$

$$e \left( -\frac{1}{2} x^2 \times Bs [0, 0, 0] - \frac{1}{2} y^2 \times Bs [0, 0, 0] - \frac{1}{2} K x^2 By [0, 0, 0] - \frac{y \tau Bx^{(0,0,1)} [0, 0, 0]}{c} + \right)$$



$$\begin{aligned}
& \frac{\left(\delta + \frac{e(xEx[0,0,0]+yEy[0,0,0])}{c}\right)^2}{p0^2 \beta 0^2} + \frac{1}{c p0 \beta 0} \\
& 2 e \left( -\frac{x \tau Ex^{(0,0,1)}[0,0,0]}{c} - \frac{y \tau Ey^{(0,0,1)}[0,0,0]}{c} + \frac{1}{2} x y Ex^{(0,1,0)}[0,0,0] + \right. \\
& \left. \frac{1}{2} y^2 Ey^{(0,1,0)}[0,0,0] + \frac{1}{2} x^2 Ex^{(1,0,0)}[0,0,0] + \frac{1}{2} x y Ey^{(1,0,0)}[0,0,0] \right) + \\
& \frac{1}{3} \left( \frac{1}{2 p0 \beta 0} 3 \left( \delta + \frac{e(xEx[0,0,0]+yEy[0,0,0])}{c} \right) \left( -\frac{(P3 + \frac{e x Bs[0,0,0]}{2 c})^2}{p0^2} - \right. \right. \\
& \left. \left. \frac{(P1 - \frac{e y Bs[0,0,0]}{2 c})^2}{p0^2} + \frac{\left(\delta + \frac{e(xEx[0,0,0]+yEy[0,0,0])}{c}\right)^2}{p0^2} - \frac{\left(\delta + \frac{e(xEx[0,0,0]+yEy[0,0,0])}{c}\right)^2}{p0^2 \beta 0^2} + \right. \right. \\
& \left. \left. \frac{1}{c p0 \beta 0} 2 e \left( -\frac{x \tau Ex^{(0,0,1)}[0,0,0]}{c} - \frac{y \tau Ey^{(0,0,1)}[0,0,0]}{c} + \frac{1}{2} x y Ex^{(0,1,0)}[0,0,0] + \right. \right. \right. \\
& \left. \left. \left. \frac{1}{2} y^2 Ey^{(0,1,0)}[0,0,0] + \frac{1}{2} x^2 Ex^{(1,0,0)}[0,0,0] + \frac{1}{2} x y Ey^{(1,0,0)}[0,0,0] \right) \right) - \right. \\
& \left. \frac{3}{2} \left( -\frac{1}{p0^2} 2 \left( P3 + \frac{e x Bs[0,0,0]}{2 c} \right) \left( -\frac{e x \tau Bs^{(0,0,1)}[0,0,0]}{2 c^2} + \frac{e x y Bs^{(0,1,0)}[0,0,0]}{2 c} + \right. \right. \right. \\
& \left. \left. \left. \frac{e x^2 Bs^{(1,0,0)}[0,0,0]}{4 c} \right) - \frac{1}{p0^2} 2 \left( P1 - \frac{e y Bs[0,0,0]}{2 c} \right) \right. \\
& \left. \left. \left( \frac{e y \tau Bs^{(0,0,1)}[0,0,0]}{2 c^2} - \frac{e y^2 Bs^{(0,1,0)}[0,0,0]}{4 c} - \frac{e x y Bs^{(1,0,0)}[0,0,0]}{2 c} \right) + \right. \\
& \left. \left. \left. \frac{1}{c p0^2} 2 e \left( \delta + \frac{e(xEx[0,0,0]+yEy[0,0,0])}{c} \right) \left( -\frac{x \tau Ex^{(0,0,1)}[0,0,0]}{c} - \right. \right. \right. \\
& \left. \left. \left. \frac{y \tau Ey^{(0,0,1)}[0,0,0]}{c} + \frac{1}{2} x y Ex^{(0,1,0)}[0,0,0] + \frac{1}{2} y^2 Ey^{(0,1,0)}[0,0,0] + \right. \right. \right. \\
& \left. \left. \left. \frac{1}{2} x^2 Ex^{(1,0,0)}[0,0,0] + \frac{1}{2} x y Ey^{(1,0,0)}[0,0,0] \right) + \frac{1}{c p0 \beta 0} \right. \\
& \left. \left. 2 e \left( \frac{x \tau^2 Ex^{(0,0,2)}[0,0,0]}{2 c^2} + \frac{y \tau^2 Ey^{(0,0,2)}[0,0,0]}{2 c^2} - \frac{x y \tau Ex^{(0,1,1)}[0,0,0]}{2 c} - \right. \right. \right. \\
& \left. \left. \left. \frac{y^2 \tau Ey^{(0,1,1)}[0,0,0]}{2 c} + \frac{1}{4} x y^2 Ex^{(0,2,0)}[0,0,0] + \frac{1}{6} y^3 Ey^{(0,2,0)}[0,0,0] - \right. \right. \right. \\
& \left. \left. \left. \frac{x^2 \tau Ex^{(1,0,1)}[0,0,0]}{2 c} - \frac{x y \tau Ey^{(1,0,1)}[0,0,0]}{2 c} + \frac{1}{4} x^2 y Ex^{(1,1,0)}[0,0,0] + \right. \right. \right. \\
& \left. \left. \left. \frac{1}{4} x y^2 Ey^{(1,1,0)}[0,0,0] + \frac{1}{6} x^3 Ex^{(2,0,0)}[0,0,0] + \frac{1}{4} x^2 y Ey^{(2,0,0)}[0,0,0] \right) \right) \right) \right)
\end{aligned}$$

In[49]:= **Cx3 = Coefficient[H3, ε^3 x^3]; Expand[Cx3]**

$$\begin{aligned}
& \frac{e^2 K Bs[0, 0, 0]^2}{8 c^2 p0} - \frac{e^3 Bs[0, 0, 0]^2 Ex[0, 0, 0]}{8 c^3 p0^2 \beta 0} - \frac{e^2 K Ex[0, 0, 0]^2}{2 c^2 p0} + \frac{e^2 K Ex[0, 0, 0]^2}{2 c^2 p0 \beta 0^2} - \\
& \frac{e^3 Ex[0, 0, 0]^3}{2 c^3 p0^2 \beta 0^3} + \frac{e^3 Ex[0, 0, 0]^3}{2 c^3 p0^2 \beta 0} - \frac{e K Bs^{(1,0,0)}[0, 0, 0]}{12 c} + \frac{e^2 Bs[0, 0, 0] Bs^{(1,0,0)}[0, 0, 0]}{8 c^2 p0} - \\
& \frac{e K By^{(1,0,0)}[0, 0, 0]}{3 c} - \frac{e K Ex^{(1,0,0)}[0, 0, 0]}{2 c \beta 0} - \frac{e^2 Ex[0, 0, 0] Ex^{(1,0,0)}[0, 0, 0]}{2 c^2 p0} + \\
& \frac{e^2 Ex[0, 0, 0] Ex^{(1,0,0)}[0, 0, 0]}{2 c^2 p0 \beta 0^2} - \frac{e By^{(2,0,0)}[0, 0, 0]}{6 c} - \frac{e Ex^{(2,0,0)}[0, 0, 0]}{6 c \beta 0} \\
& \frac{e^2 K Bs[0, 0, 0]^2}{8 c^2 p0} - \frac{e^3 Bs[0, 0, 0]^2 Ex[0, 0, 0]}{8 c^3 p0^2 \beta 0} - \frac{e^2 K Ex[0, 0, 0]^2}{2 c^2 p0} + \frac{e^2 K Ex[0, 0, 0]^2}{2 c^2 p0 \beta 0^2} - \\
& \frac{e^3 Ex[0, 0, 0]^3}{2 c^3 p0^2 \beta 0^3} + \frac{e^3 Ex[0, 0, 0]^3}{2 c^3 p0^2 \beta 0} - \frac{e K Bs^{(1,0,0)}[0, 0, 0]}{12 c} + \frac{e^2 Bs[0, 0, 0] Bs^{(1,0,0)}[0, 0, 0]}{8 c^2 p0} - \\
& \frac{e K By^{(1,0,0)}[0, 0, 0]}{3 c} - \frac{e K Ex^{(1,0,0)}[0, 0, 0]}{2 c \beta 0} - \frac{e^2 Ex[0, 0, 0] Ex^{(1,0,0)}[0, 0, 0]}{2 c^2 p0} + \\
& \frac{e^2 Ex[0, 0, 0] Ex^{(1,0,0)}[0, 0, 0]}{2 c^2 p0 \beta 0^2} - \frac{e By^{(2,0,0)}[0, 0, 0]}{6 c} - \frac{e Ex^{(2,0,0)}[0, 0, 0]}{6 c \beta 0}
\end{aligned}$$

In[50]:= **Cy3 = Coefficient[H3, ε^3 y^3]; Expand[Cy3]**

$$\begin{aligned}
& \frac{e^3 Bs[0, 0, 0]^2 Ey[0, 0, 0]}{8 c^3 p0^2 \beta 0} - \frac{e^3 Ey[0, 0, 0]^3}{2 c^3 p0^2 \beta 0^3} + \frac{e^3 Ey[0, 0, 0]^3}{2 c^3 p0^2 \beta 0} + \\
& \frac{e K Bs^{(0,1,0)}[0, 0, 0]}{4 c} + \frac{e^2 Bs[0, 0, 0] Bs^{(0,1,0)}[0, 0, 0]}{8 c^2 p0} - \frac{e^2 Ey[0, 0, 0] Ey^{(0,1,0)}[0, 0, 0]}{2 c^2 p0} + \\
& \frac{e^2 Ey[0, 0, 0] Ey^{(0,1,0)}[0, 0, 0]}{2 c^2 p0 \beta 0^2} + \frac{e Bx^{(0,2,0)}[0, 0, 0]}{6 c} - \frac{e Ey^{(0,2,0)}[0, 0, 0]}{6 c \beta 0}
\end{aligned}$$

In[51]:= **Cx2y = Coefficient[H3, ε^3 x^2 y]; Expand[Cx2y]**

$$\begin{aligned}
& \frac{e^3 Bs[0, 0, 0]^2 Ey[0, 0, 0]}{8 c^3 p0^2 \beta 0} - \frac{e^2 K Ex[0, 0, 0] Ey[0, 0, 0]}{c^2 p0} + \frac{e^2 K Ex[0, 0, 0] Ey[0, 0, 0]}{c^2 p0 \beta 0^2} - \\
& \frac{3 e^3 Ex[0, 0, 0]^2 Ey[0, 0, 0]}{2 c^3 p0^2 \beta 0^3} + \frac{3 e^3 Ex[0, 0, 0]^2 Ey[0, 0, 0]}{2 c^3 p0^2 \beta 0} + \frac{e K Bs^{(0,1,0)}[0, 0, 0]}{4 c} + \\
& \frac{e^2 Bs[0, 0, 0] Bs^{(0,1,0)}[0, 0, 0]}{4 c^2 p0} - \frac{e K By^{(0,1,0)}[0, 0, 0]}{4 c} - \frac{e K Ex^{(0,1,0)}[0, 0, 0]}{2 c \beta 0} - \\
& \frac{e^2 Ex[0, 0, 0] Ex^{(0,1,0)}[0, 0, 0]}{2 c^2 p0} + \frac{e^2 Ex[0, 0, 0] Ex^{(0,1,0)}[0, 0, 0]}{2 c^2 p0 \beta 0^2} - \\
& \frac{e K Bx^{(1,0,0)}[0, 0, 0]}{2 c} - \frac{e^2 Ey[0, 0, 0] Ex^{(1,0,0)}[0, 0, 0]}{2 c^2 p0} + \frac{e^2 Ey[0, 0, 0] Ex^{(1,0,0)}[0, 0, 0]}{2 c^2 p0 \beta 0^2} - \\
& \frac{e K Ey^{(1,0,0)}[0, 0, 0]}{2 c \beta 0} - \frac{e^2 Ex[0, 0, 0] Ey^{(1,0,0)}[0, 0, 0]}{2 c^2 p0} + \frac{e^2 Ex[0, 0, 0] Ey^{(1,0,0)}[0, 0, 0]}{2 c^2 p0 \beta 0^2} - \\
& \frac{e By^{(1,1,0)}[0, 0, 0]}{4 c} - \frac{e Ex^{(1,1,0)}[0, 0, 0]}{4 c \beta 0} - \frac{e Bx^{(2,0,0)}[0, 0, 0]}{4 c} - \frac{e Ey^{(2,0,0)}[0, 0, 0]}{4 c \beta 0}
\end{aligned}$$

In[52]:= **Cxy2 = Coefficient[H3,  $\varepsilon^3 x y^2$ ]; Expand[Cxy2]**

$$\text{Out}[52]= \frac{e^2 K B s[0, 0, 0]^2}{8 c^2 p0} - \frac{e^3 B s[0, 0, 0]^2 E x[0, 0, 0]}{8 c^3 p0^2 \beta 0} - \frac{e^2 K E y[0, 0, 0]^2}{2 c^2 p0} +$$

$$\frac{e^2 K E y[0, 0, 0]^2}{2 c^2 p0 \beta 0^2} - \frac{3 e^3 E x[0, 0, 0] E y[0, 0, 0]^2}{2 c^3 p0^2 \beta 0^3} + \frac{3 e^3 E x[0, 0, 0] E y[0, 0, 0]^2}{2 c^3 p0^2 \beta 0} -$$

$$\frac{e K B x^{(0,1,0)}[0, 0, 0]}{4 c} - \frac{e^2 E y[0, 0, 0] E x^{(0,1,0)}[0, 0, 0]}{2 c^2 p0} + \frac{e^2 E y[0, 0, 0] E x^{(0,1,0)}[0, 0, 0]}{2 c^2 p0 \beta 0^2} -$$

$$\frac{e K E y^{(0,1,0)}[0, 0, 0]}{2 c \beta 0} - \frac{e^2 E x[0, 0, 0] E y^{(0,1,0)}[0, 0, 0]}{2 c^2 p0} + \frac{e^2 E x[0, 0, 0] E y^{(0,1,0)}[0, 0, 0]}{2 c^2 p0 \beta 0^2} -$$

$$\frac{e B y^{(0,2,0)}[0, 0, 0]}{4 c} - \frac{e E x^{(0,2,0)}[0, 0, 0]}{4 c \beta 0} + \frac{e \kappa B s^{(1,0,0)}[0, 0, 0]}{4 c} +$$

$$\frac{e^2 B s[0, 0, 0] B s^{(1,0,0)}[0, 0, 0]}{4 c^2 p0} - \frac{e^2 E y[0, 0, 0] E y^{(1,0,0)}[0, 0, 0]}{2 c^2 p0} +$$

$$\frac{e^2 E y[0, 0, 0] E y^{(1,0,0)}[0, 0, 0]}{2 c^2 p0 \beta 0^2} - \frac{e B x^{(1,1,0)}[0, 0, 0]}{4 c} - \frac{e E y^{(1,1,0)}[0, 0, 0]}{4 c \beta 0}$$

In[53]:= **CP1p3 = Coefficient[H3,  $\varepsilon^3 P1^3$ ]; Expand[CP1p3]**

Out[53]= 0

In[54]:= **CP3p3 = Coefficient[H3,  $\varepsilon^3 P3^3$ ]; Expand[CP3p3]**

Out[54]= 0

In[55]:= **Cδ3 = Coefficient[H3,  $\varepsilon^3 \delta^3$ ]; Expand[Cδ3]**

Out[55]=  $-\frac{1}{2 p0^2 \beta 0^3} + \frac{1}{2 p0^2 \beta 0}$

In[56]:= **Ct3 = Coefficient[H3,  $\varepsilon^3 \tau^3$ ]; Expand[Ct3]**

Out[56]=  $\frac{e E s^{(0,0,2)}[0, 0, 0]}{6 c^3}$

# Forth order terms

In[57]:= **H4 = Normal[Series[H, {ε, 0, 4}]] ; Expand[H4]**

A very large output was generated. Here is a sample of it:

$$\begin{aligned} \text{Out[57]}= & -\frac{P1^2 \varepsilon^2}{2 p0} + \frac{P3^2 \varepsilon^2}{2 p0} - \frac{K x \delta \varepsilon^2}{\beta 0} - \frac{\delta^2 \varepsilon^2}{2 p0} + \frac{\delta^2 \varepsilon^2}{2 p0 \beta 0^2} + \frac{K P1^2 x \varepsilon^3}{2 p0} + \text{O}(1156) + \frac{e x^2 y \varepsilon^4 \tau EY^{(2,0,1)}[0,0,0]}{4 c^2 \beta 0} - \\ & \frac{e x^2 y^2 \varepsilon^4 Bx^{(2,1,0)}[0,0,0]}{8 c} - \frac{e x^3 y \varepsilon^4 By^{(2,1,0)}[0,0,0]}{12 c} - \frac{e x^3 y \varepsilon^4 Ex^{(2,1,0)}[0,0,0]}{12 c \beta 0} - \frac{e x^2 y^2 \varepsilon^4 Ey^{(2,1,0)}[0,0,0]}{8 c \beta 0} - \\ & \frac{e x^3 y \varepsilon^4 Bx^{(3,0,0)}[0,0,0]}{12 c} - \frac{e x^4 \varepsilon^4 By^{(3,0,0)}[0,0,0]}{24 c} - \frac{e x^4 \varepsilon^4 Ex^{(3,0,0)}[0,0,0]}{24 c \beta 0} - \frac{e x^3 y \varepsilon^4 Ey^{(3,0,0)}[0,0,0]}{12 c \beta 0} \end{aligned}$$

[Show Less](#) [Show More](#) [Show Full Output](#) [Set Size Limit...](#)

In[58]:= **CP1p4 = Coefficient[H4, ε^4 P1^4] ; Expand[CP1p4]**

$$\text{Out[58]}= \frac{1}{8 p0^3}$$

In[59]:= **CP3p4 = Coefficient[H4, ε^4 P3^4] ; Expand[CP3p4]**

$$\text{Out[59]}= \frac{1}{8 p0^3}$$

In[60]:= **CP1p2P3p2 = Coefficient[H4, ε^4 P1^2 P3^2] ; Expand[CP1p2P3p2]**

$$\text{Out[60]}= \frac{1}{4 p0^3}$$

In[61]:= **CP1p1P3p3 = Coefficient[H4, ε^4 P1 P3^3] ; Expand[CP1p1P3p3]**

$$\text{Out[61]}= 0$$

In[62]:=

## Higher orders : just for fun

In[63]:= **H6 = Normal[Series[H, {ε, 0, 6}]] ;**

In[64]:= **CP1p6 = Coefficient[H6, ε^6 P1^6] ; Expand[CP1p6]**

$$\text{Out[64]}= \frac{1}{16 p0^5}$$

In[65]:= **H14 = Normal[Series[H, {ε, 0, 14}]] ;**

**CP1p14 = Coefficient[H14, ε^14 P1^14] ; Expand[CP1p14]**