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COMPUTERLAB, EXERCISE 1.1-1, SOLUTION

Abstract

Exercise 1.1-1 builds a simulation of a cyclotron, a fixed-field ring accelerator. The cyclotron magnet here is modeled using a 180 deg two-dimensional map of the mid-plane field (in following exercises, a different way to model the magnet will be introduced, using instead a mathematical model of the vector field that the magnet produces). Ex. 1.1-1 computes the (circular) trajectories of ions around the cyclotron at fixed energies. Checks against theory are performed, including trajectory radius, revolution time of flight and else.

Contents

1 1.1-1.a - Make a cyclotron dipole field map

The fortran program page 3 constructs the required map of a field distribution in cylindrical coordinates, $B_y(r, \theta)$ (see p. 5).

It can be copy-pasted, compiled, and run. Get the executable doing: *gfortran -o geneSectorMap geneSectorMap.f*

Save its outcome file, geneSectorMap.out, to geneSectorMap_180deg.out. The field map will be used under that name in zgoubi input data files.

Note two things:

(i) the field map angle (now 180 deg) can be changed, this will happen in following exercises, to get a 60 deg sector field map instead.

(ii) the field is purely vertical as it is the mid-plane field of a mid-plane symmetry dipole magnet, and taken constant here, the same value $\forall r$, $\forall \theta$. In further exercises, a *focusing index* will be introduced, which will make $B_y(r, \theta)$ an r-dependent quantity. Beyond, Thomas focusing will make $B_y(r, \theta)$ both r- and θdependent a quantity.

The field can be plotted, Fig. 1, using gnuplot. An *ad hoc* gnuplot script is given page 3.

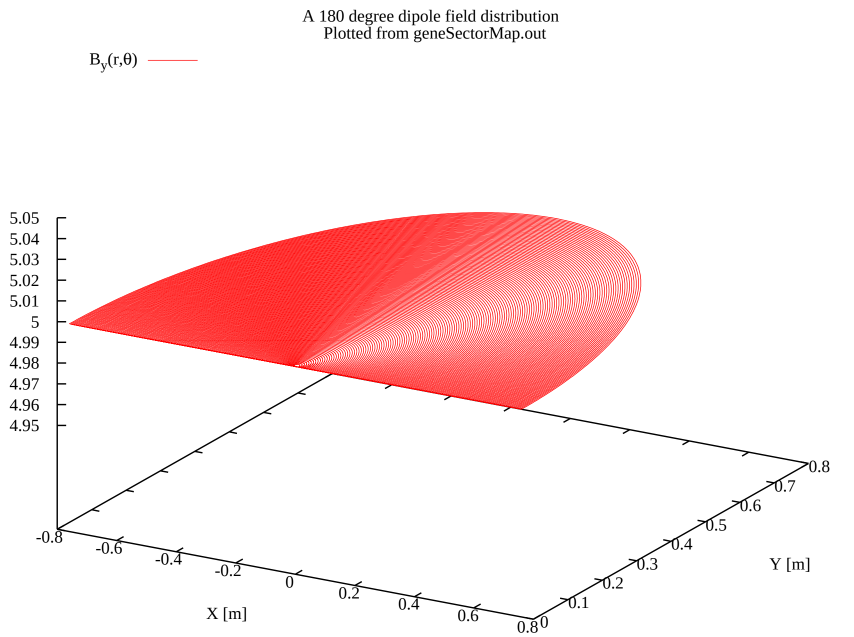

Figure 1: Constant magnetic field, over a 180 deg sector, modeled in the "geneSectorMap_180deg.out" field map.

A fortran program which generates a 180 deg mid-plane field map

```
      implicit double precision (a-h,o-z)
      parameter (pi = 4.d0*atan(1.d0))

C------------ Hypothesis :
C Total angle extent of the field map. Can be changed, e.g., to 360, 0r 60 deg, or else.
      AT = 180.d0  /180.d0*pi
C Radial extent of the field map
      Rmi = 1.d0   ! cm
      Rma = 76.d0   ! cm
C Take RM=50 cm reference radius, as this (arbitray) value is found in other exercises
      RM = 50.d0
C dR is the radial distance between two nodes, a reasonable value is (by experience) dR = 0.5 cm
      dR = 0.5d0
      NR = NINT((Rma - Rmi) / dR) +1
C dX=RM*dA is the arc length between two nodes along R=RM arc, given angle increment dA
C A reasonable value is (by experience) is dX a few mm, say ~0.5 cm
      dX = 0.5d0             ! cm mesh step at RM, approximate: allows getting NX
      NX = NINT(RM*AT / dX)  +1
      dX = RM*AT / DBLE(NX - 1)  ! exact mesh step at RM, corresponding to NX
      dA = dX / RM               ! corresponding delta_angle
      A1 = 0.d0 ; A2 = AT
C-----------------------------------------------

      BY = 0.d0 ; BX = 0.d0 ; Z = 0.d0
      BZ = 5.d0  ! kG

      open(unit=2,file='geneSectorMap.out')
      write(2,*) Rmi,dR,dA/pi*180.d0,dZ,
     >'     !  Rmi/cm, dR/cm, dA/deg, dZ/cm'
      write(2,*) '# Field map generated using geneSectorMap.f '
      write(2,fmt='(a)') '# AT/rd,  AT/deg, Rmi/cm, Rma/cm, RM/cm,'
     >//' NR, dR/cm, NX, dX/cm, dA/rd : '
      write(2,fmt='(a,1p,5(e16.8,1x),2(i3,1x,e16.8,1x),e16.8)')
     >'# ',AT, AT/pi*180.d0,Rmi, Rma, RM, NR, dR, NX, dX, dA
      write(2,*) '# For TOSCA: ',NX,NR,' 1 22.1 1.  !IZ=1 -> 2D ; '
     >//'MOD=22 -> polar map ; .MOD2=.1 -> one map file'
      write(2,*) '# R*cosA (A:0->360), Z==0, R*sinA, BY, BZ, BX '
      write(2,*) '# cm                 cm    cm      kG  kG  kG '
      write(2,*) '# '

      do jr = 1, NR
        R = Rmi + dble(jr-1)*dR
        do ix = 1, NX
          A = A1 + dble(ix-1)*dA
C          write(2,fmt='(1p,6(e16.8),a)')  R, Z, A, BR, BZ, BA
          X = R * sin(A)
          Y = R * cos(A)
          write(2,fmt='(1p,6(e16.8),2(1x,i0))') Y,Z,X,BY,BZ,BX,ix,jr
        enddo
      enddo

      stop ' Job complete ! Field map stored in geneSectorMap.out.'
      end
```

Plot the field, using gnuplot

```
set title "A 180 degree dipole field distribution \n Plotted from geneSectorMap.out"

set key maxcol 1
set key t l

#set logscale y

set xtics mirror
set ytics mirror

set xlabel 'X [m]'
set ylabel 'Y [m]'

cm2m = 0.01
MeV2eV = 1e6
am = 938.27203
c = 2.99792458e8

splot  \
   'geneSectorMap.out' u ($1 *cm2m):($3 *cm2m):($5) w l lc rgb "red" tit "B_y(r,{/Symbol q})"

     set terminal postscript eps blacktext color  enh size 8.3cm,4cm "Times-Roman" 12
       set output "gnuplot_fieldMap.eps"
       replot
       set terminal X11
       unset output

 pause 8
exit
```

2 1.1-1.b - Track a few protons on circles

We proceed in two stages: a first stage uses the `FIT` procedure to find a series of closed orbits, a second stage tracks particles on these closed orbits, for plotting.

- in a first stage, closed circles for a series of different radii taken in $[10,80]$ cm are searched, using `FIT` to find the appropriate momenta. `REBELOTE` is used to repeat with a new value of R set in `OBJET`. Zgoubi input file in p. 5.
Particle coordinates *after the* `FIT` *procedure* are stored in initialRs.fai, using `FAISTORE`.

- in a second stage, these particles are all tracked concurrently, using `OBJET[KOBJ=3]` which reads initial coordinates from initialRs.fai. Zgoubi input file in p. 6.

The `SYSTEM` command is used to plot whatever is worth plotting, as part of the zgoubi run. gnuplot script for the plots is given in p. 6.

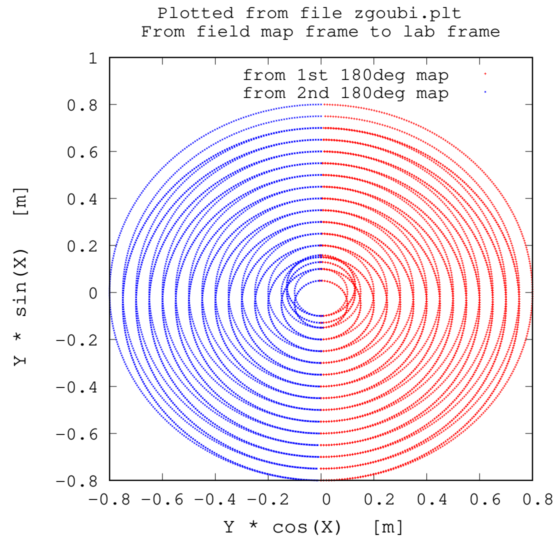

Figure 2: Stage 1: circular trajectories go by pair: before `FIT` (R has been fixed by `REBELOTE`, but the momentum in `OBJET` is still that of the previous particle), and after `FIT` (the proper momentum value has been found by `FIT`, consistent with R).

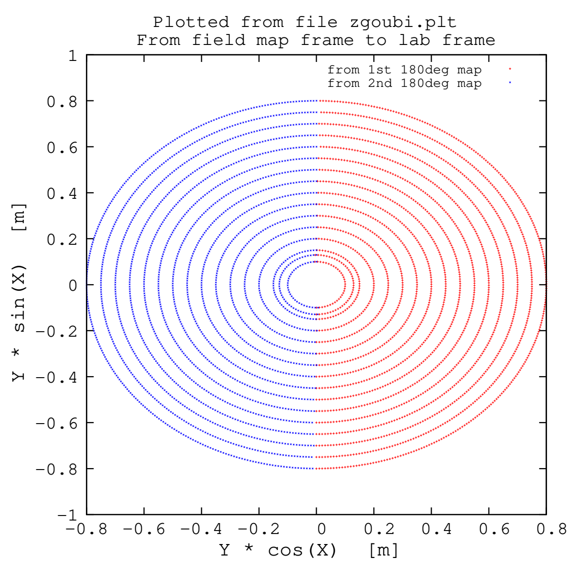

Figure 3: Stage 2: proper circular trajectories, centered on the field map center.

Optical sequence in zgoubi, phase 1

```
Uniform field sector
 'OBJET'                                                                                                                   1
64.62444403717985                           ! 200keV proton
2
1 1
12.9248888074 0.  0.   0.   0.  1. 'm'      ! This initial radius yields BR=64.6244440372 kG.cm
1

 'FAISCEAU'                ! Print out particle coordinates (to zgoubi.res), here.                                        2
 'FAISTORE'                                                                                                                3
zgoubi.fai  afterFIT       ! Store particle coordinates at label 'afterFIT' (below).
1
 'TOSCA'                                                                                                                   4
0  2
1.  1. 1. 1.
HEADER_8 Field B=5kG
315 151 1 22.1 1.          !  IZ=1 -> 2D ; MOD=22 -> polar map ; .MOD2=.1 -> one map file
geneSectorMap_180deg.out
0 0 0 0
2
1.
2
0. 0. 0. 0.
 'FAISCEAU'                                                                                                                5
 'TOSCA'                                                                                                                   6
0  2
1.  1. 1. 1.
HEADER_8 Field B=5kG
315 151 1 22.1 1.      !  IZ=1 -> 2D ; MOD=22 -> polar map ; .MOD2=.1 -> one map file
geneSectorMap_180deg.out
0 0 0 0
2
1.
2
0. 0. 0. 0.

 'FAISCEAU'  #End                                                                                                          7

 'FIT'                                                                                                                     8
1
1  35  0  6.         ! Vary momentum.
1
3.1 1 2 4 0. 1. 0    ! Request same radius after 180deg rotation (at keyword #4, i.e. TOSCA)
                     ! ensures orbit centering on center of map.
 'MARKER'  afterFIT                                                                                                        9
 'REBELOTE'                                                                                                               10
15  0.1  0 1
1
OBJET 30  10:80

 'SYSTEM'                                                                                                                 11
4
cp zgoubi.fai initialRs.fai
gnuplot <./gnuplot_zgoubi.plt.cmd
cp gnuplot_zgoubi.plt_XYLab.eps gnuplot_zgoubi.plt_XYLab_stage1.eps
okular gnuplot_zgoubi.plt_XYLab_stage1.eps &
 'END'                                                                                                                    12
```

Optical sequence in zgoubi, phase 2

```
Uniform field sector
 'OBJET'                                                                                                              1
64.62444403717985                          ! 200keV proton
3
1 999 1
1 999 1
1. 1. 1. 1. 1. 1. 1. '*'
0.  0.  0.  0.  0.  0.  0.
0
initialRs.fai

 'FAISCEAU'                ! Print out particle coordinates (to zgoubi.res), here.                                    2
 'FAISTORE'                                                                                                           3
zgoubi.fai  afterFIT       ! Store particle coordinates at label 'afterFIT' (below).
1
 'TOSCA'                                                                                                              4
0  2
1.  1. 1. 1.
HEADER_8 Field B=5kG
315 151 1 22.1 1.      !  IZ=1 -> 2D ; MOD=22 -> polar map ; .MOD2=.1 -> one map file
geneSectorMap_180deg.out
0 0 0 0
2
1.
2
0. 0. 0. 0.
 'FAISCEAU'                                                                                                           5
 'TOSCA'                                                                                                              6
0  2
1.  1. 1. 1.
HEADER_8 Field B=5kG
315 151 1 22.1 1.      !  IZ=1 -> 2D ; MOD=22 -> polar map ; .MOD2=.1 -> one map file
geneSectorMap_180deg.out
0 0 0 0
2
1.
2
0. 0. 0. 0.

 'FAISCEAU'  #End                                                                                                     7
 'SYSTEM'                                                                                                             8
3
gnuplot <./gnuplot_zgoubi.plt.cmd
cp gnuplot_zgoubi.plt_XYLab.eps gnuplot_zgoubi.plt_XYLab_stage2.eps
okular gnuplot_zgoubi.plt_XYLab_stage2.eps &
 'END'                                                                                                                9
```

Plot trajectories, using gnuplot

```
set title "Plotted from file zgoubi.plt  \n From field map frame to lab frame " font "sans, 16"

set key font "sana 16"
set key maxcol 1
set key t r

#set logscale y

set xtics mirror font  "sans, 16"
set ytics mirror font  "sans, 16"

set size ratio 1

set xlabel "Y * cos(X)  [m] \n" font  "sans, 18"
set ylabel "Y * sin(X)  [m] \n"    font  "sans, 18"

cm2m = 0.01
MeV2eV = 1e6
am = 938.27203
c = 2.99792458e8
pi = 4. *atan(1.)

set xrange []
set x2range []

plot  \
"zgoubi.plt" u ($42==4 ? $10 *cm2m *cos($22)    :1/0):($10 *cm2m *sin($22)   ) w lp ps .3 lc rgb "red"  tit "from 1st 180deg map" ,\
"zgoubi.plt" u ($42==6 ? $10 *cm2m *cos($22+pi) :1/0):($10 *cm2m *sin($22+pi)) w lp ps .2 lc rgb "blue" tit "from 2nd 180deg map"

#     set terminal postscript eps blacktext color  enh size 8cm,8cm "Times-Sans" 12
     set terminal postscript eps blacktext color  enh  "Times-Sans" 12
     set output "gnuplot_zgoubi.plt_XYLab.eps"
     replot
     set terminal X11
     unset output


pause 1
exit
```

3 1.1-1.c - Time of flight, energy, compare with theory

This part of the exercise compares numerical outcomes and theoretical data (Eqs. 1-3). Zgoubi input data file in p. 8. gnuplot file in p. 9.

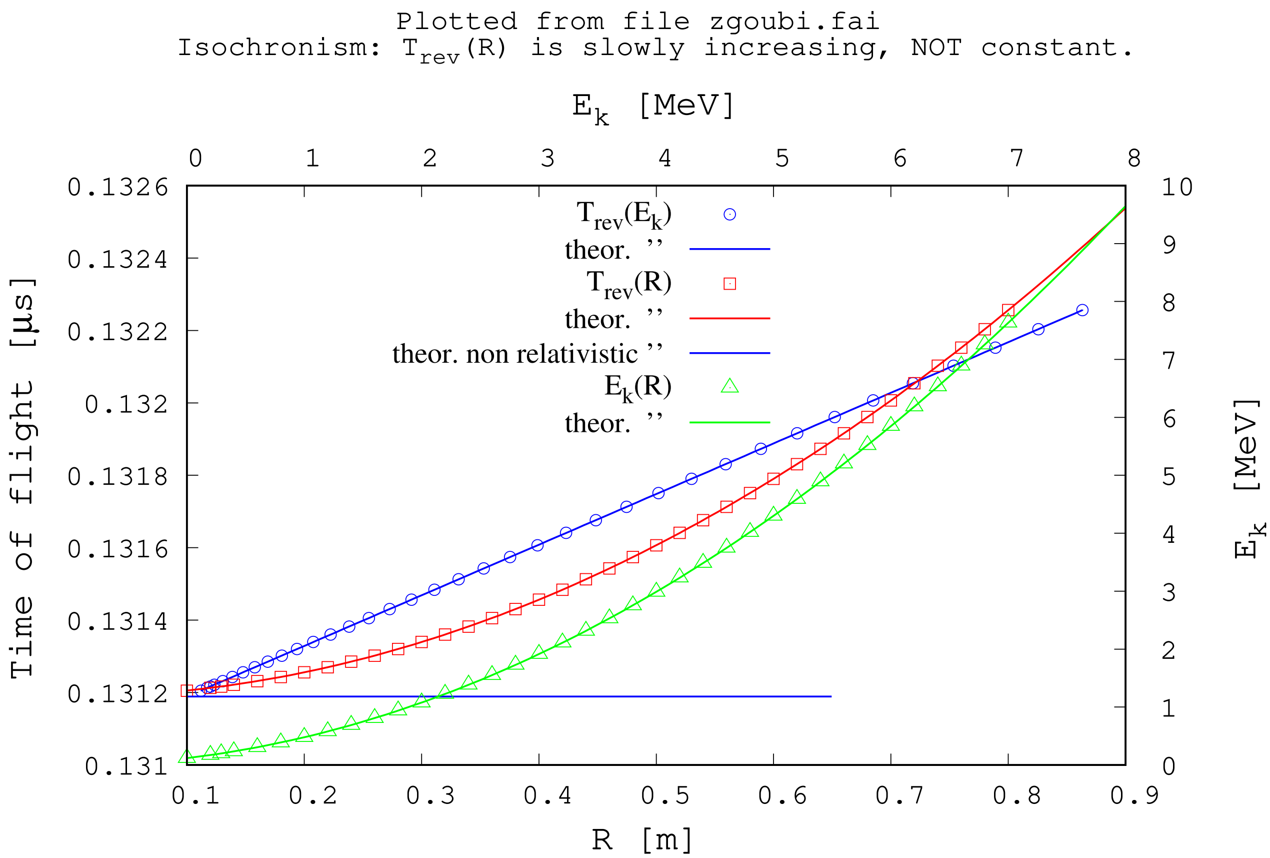

Figure 4: Particle dynamics in a cyclotron, Numerical versus theoretical data.

Kinetic energy versus orb it radius, computed using data available in zgoubi.plt (taking M in MeV/c^2, p in MeV/c):

$$\mathrm{E_k} = \sqrt{\mathrm{p^2 + M^2}} - \mathrm{M} = \sqrt{\mathrm{(cBR)^2 + M^2}} - \mathrm{M} \tag{1}$$

with $\mathrm{p = cBR}$ (from $\mathrm{p/q = BR}$ in MKSA units, with q=e for proton assumed here).

Revolution time, function of radius:

$$\mathrm{T_{rev}(R)} = \frac{2\pi\mathrm{R}}{\mathrm{v}} = 2\pi\frac{\sqrt{\mathrm{(cBR)^2 + M^2}}}{\mathrm{c^2B}} \qquad \text{(this uses } \beta = \mathrm{v/c} = \frac{\mathrm{p}}{\sqrt{\mathrm{p^2+M^2}}}\text{)} \tag{2}$$

Note that $\mathrm{T_{rev}}$ depends on R, whereas in the non-relativistic hypothesis $\omega_{\mathrm{rev}} = \mathrm{v/R = (p/m)/R = qBR\,/\,mR = qB\,/\,m}$, R-independent.

Revolution time, function of kinetic energy:

$$\mathrm{T(E_k)} = 2\pi\frac{\mathrm{E_k + M}}{\mathrm{Bc^2}} \tag{3}$$

Optical sequence in zgoubi

```
Uniform field sector
 'OBJET'                                                                                                       1
64.62444403717985                          ! 200keV proton
2
1 1
12.9248888074 0.  0.   0.   0.  1. 'm'      ! This initial radius yields BR=64.6244440372 kG.cm
1
 'PARTICUL'               ! Recommended form as it sets proton data to                                         2
PROTON                    ! zgoubi's hadr-coded values (see block.f).
 'FAISTORE'                                                                                                    3
zgoubi.fai  afterFIT       ! Store particle coordinates at label 'afterFIT' (below).
1
 'TOSCA'                                                                                                       4
0  2
1.  1. 1. 1.
HEADER_8 Field B=5kG
315 151 1 22.1 1.      !  IZ=1 -> 2D ; MOD=22 -> polar map ; .MOD2=.1 -> one map file
geneSectorMap_180deg.out
0 0 0 0
2
1.
2
0. 0. 0. 0.
 'TOSCA'                                                                                                       6
0  2
1.  1. 1. 1.
HEADER_8 Field B=5kG
315 151 1 22.1 1.      !  IZ=1 -> 2D ; MOD=22 -> polar map ; .MOD2=.1 -> one map file
geneSectorMap_180deg.out
0 0 0 0
2
1.
2
0. 0. 0. 0.
 'FAISCEAU'  #End                                                                                              7
 'FIT'                                                                                                         8
1
1  35  0  6.         ! Vary momentum.
1
3.1 1 2 4 0. 1. 0    ! Request same radius after 180deg rotation (at keyword #4, i.e. TOSCA)
                     ! ensures orbit centering on center of map.
 'MARKER'  afterFIT                                                                                            9
 'REBELOTE'                                                                                                   10
36  0.1  0 1
1
OBJET 30  10:80
 'SYSTEM'                                                                                                     11
2
gnuplot < ./gnuplot_zgoubi.fai.cmd
okular  gnuplot_zgoubi.fai_T.vs.R.eps &
 'END'                                                                                                        12
```

Plot dynamical parameters ($\mathrm{T_{rev}(R),\ E_k(R)}$, etc.), using gnuplot

```
set title "Plotted from file zgoubi.fai  \n Isochronism: T_{rev}(R) is slowly increasing, NOT constant. \n" font "sans, 16 \n ~ "

set key maxcol 1
set key spac 1.2
set key t l  font "Roman, 16"

#set logscale y

set xtics nomirror font  "sans, 16"
set x2tics nomirror font  "sans, 16"
set ytics nomirror font  "sans, 16"
set y2tics nomirror font  "sans, 16"

# set size ratio 1.

set xlabel "R [m] \n" font  "sans, 20"
set x2label "E_k [MeV]" font  "sans, 20"
set ylabel "Time of flight [{/Symbol m}s]" font  "sans, 20"
set y2label "E_k [MeV]" font  "sans, 20"

cm2m = 0.01
s2mus = 1e6
MeV2eV = 1e6
am = 938.27203
c = 2.99792458e8
pi = 4. * atan(1.)
B=0.5  # dipole field

set xrange [.1:.9]        # m
#set x2range [0.02:6.12]                   # MeV

# Ek = sqrt(p^2 +M^2)-M and m bta c*2/q =bta*M = c B R
Ek(x) = sqrt((c * B * x/ MeV2eV)**2 + am**2) - am
# T=C/v and  R=mv/qB = M[eV] v / c^2 B
Tclass(x) = 2.*pi * am / (c**2 *B) * MeV2eV * s2mus
# Trel = C/v and v=bta*c  and bta = p/sqrt(p^2 + M^2)  and bta*M=cBR
Trel(x) = 2.*pi*x /c *1e6 / ((c * B * x/ MeV2eV) / sqrt((c * B * x/ MeV2eV)**2 + am**2))
# T(Ek) = 2piR/v = 2pim/qB = 2pi*(Ek+M)/ Bc^2
Tw(x) = 2.*pi * (x+am) / (c**2* B) * MeV2eV * s2mus

plot  \
   "zgoubi.fai" u ($24):($15) axes x2y1 w p pt 6 lc rgb "blue" tit "T_{rev}(E_k)" ,\
    Tw(x) axes x2y1  w l lt 1 lw 2 lc rgb "blue" tit "theor.  '' "  ,\
   "zgoubi.fai" u ($10 *cm2m):($15) axes x1y1 w p pt 4 lc rgb "red" tit "T_{rev}(R)" ,\
    Trel(x) axes x1y1  w l lt 1 lw 2 lc rgb "red" tit "theor.  '' "  ,\
    Tclass(x < 0.65 ? x :1/0) axes x1y1  w l lt 2 lw 2 lc rgb "blue" tit "theor. non relativistic '' "  ,\
   "zgoubi.fai" u ($10 *cm2m):($24) axes x1y2 w p pt 8 ps 1.5 lc rgb "green" tit "E_k(R)"  ,\
    Ek(x) axes x1y2  w l lt 1 lw 2 lc rgb "green" tit "theor.  '' "

     set terminal postscript eps blacktext color  enh  "Times-Sans" 16
     set output "gnuplot_zgoubi.fai_T.vs.R.eps"
     replot
     set terminal X11
     unset output

pause 1
exit
```