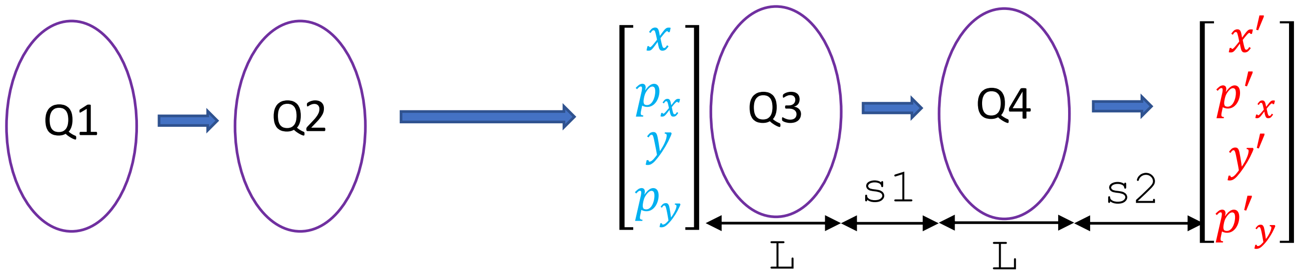
Slice Emittance Measurement

KS

01/8/2022



$$\therefore \begin{bmatrix} \begin{matrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{matrix} & 0 \\ 0 & \begin{matrix} \omega_{11} & \omega_{12} \\ \omega_{21} & \omega_{22} \end{matrix} \end{bmatrix} \begin{bmatrix} x \\ p_x \\ y \\ p_y \end{bmatrix} = \begin{bmatrix} x' \\ p'_x \\ y' \\ p'_y \end{bmatrix}$$

Machine Parameters

Vertical

$$\sqrt{\sigma_y'} = \frac{y'_{rms}}{\omega_{12}} \qquad \xi = \frac{\omega_{11}}{\omega_{12}}$$

$$\sigma_y' = a_y\xi^2 + b_y\xi + c_y$$

Horizontal

$$\sqrt{\sigma_x'} = \frac{x'_{rms}}{m_{12}} \qquad \upsilon = \frac{m_{11}}{m_{21}}$$

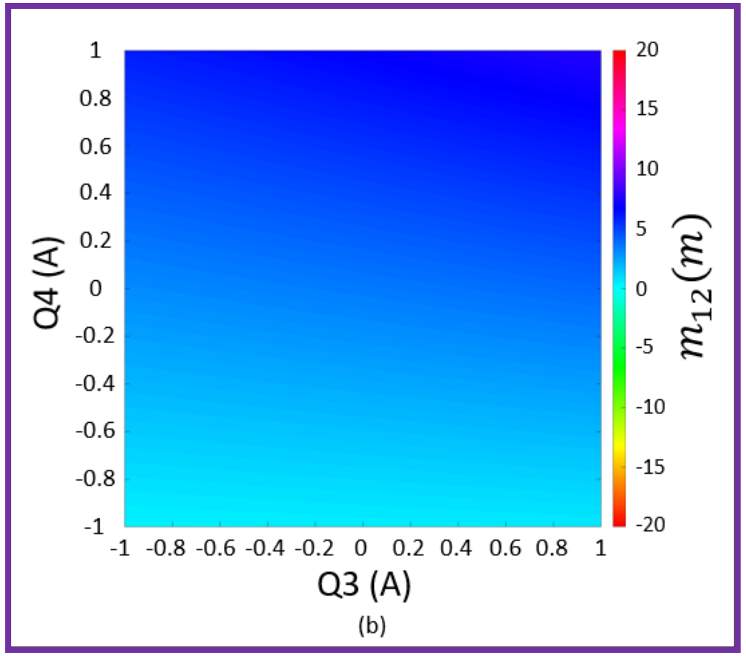
$$\sigma_x' = a_x\upsilon^2 + b_x\upsilon + c_x$$

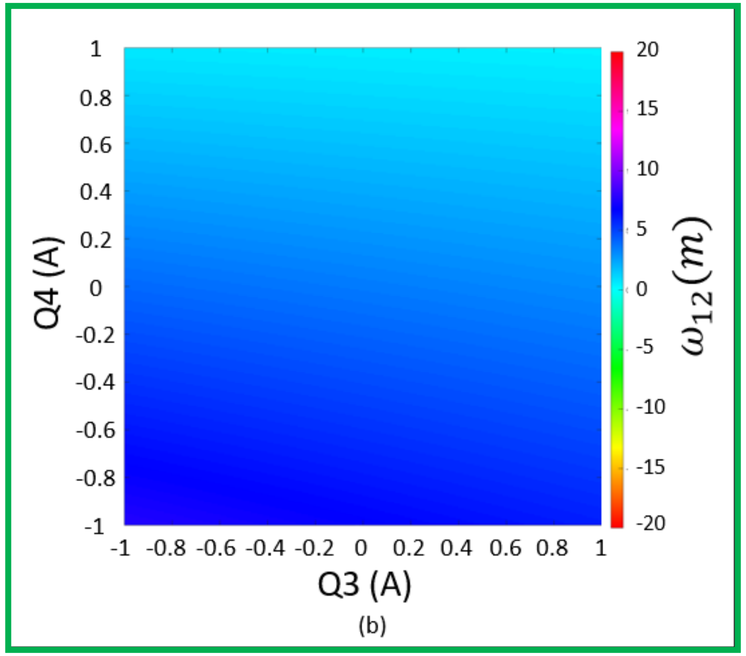
Beam Parameters

Vertical & Horizontal
Machine Parameters are related

$$\omega_{12}(Q3, Q4) = m_{12}(-Q3, -Q4) \qquad \xi(Q3, Q4) = \upsilon(-Q3, -Q4)$$

180 deg rotation

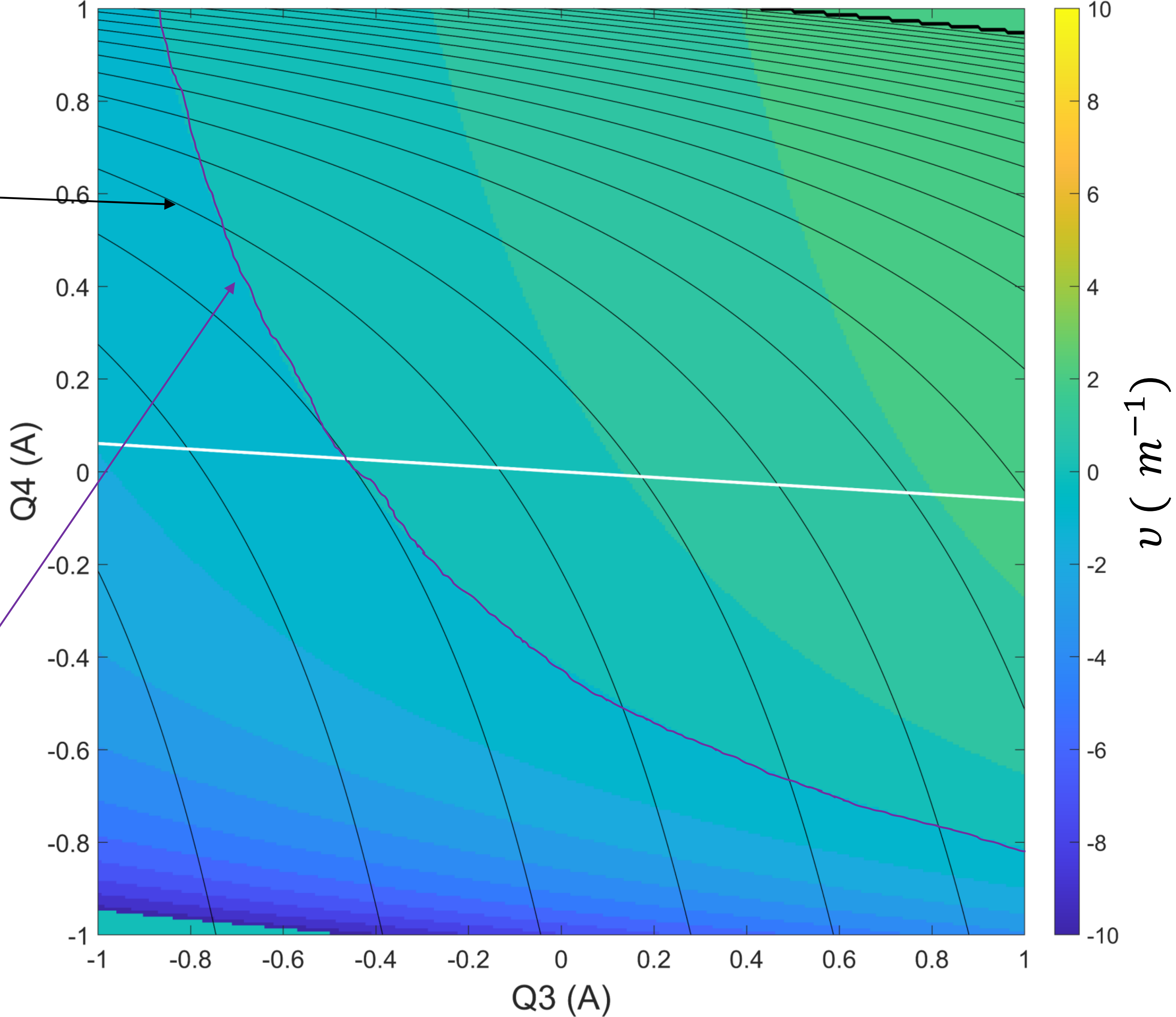




$$\sigma_y' = a_y \xi^2 + b_y \xi + c_y$$

ξ is constant

σ_y' is constant



$$\sigma_x' = a_x \upsilon^2 + b_x \upsilon + c_x$$

υ is constant

σ_x' is constant

ξ is constant

σ'_y is constant ⇒ $\omega_{12}(Q3, Q4)\big|_{\xi=const.} = \omega_{12}(Q4)$

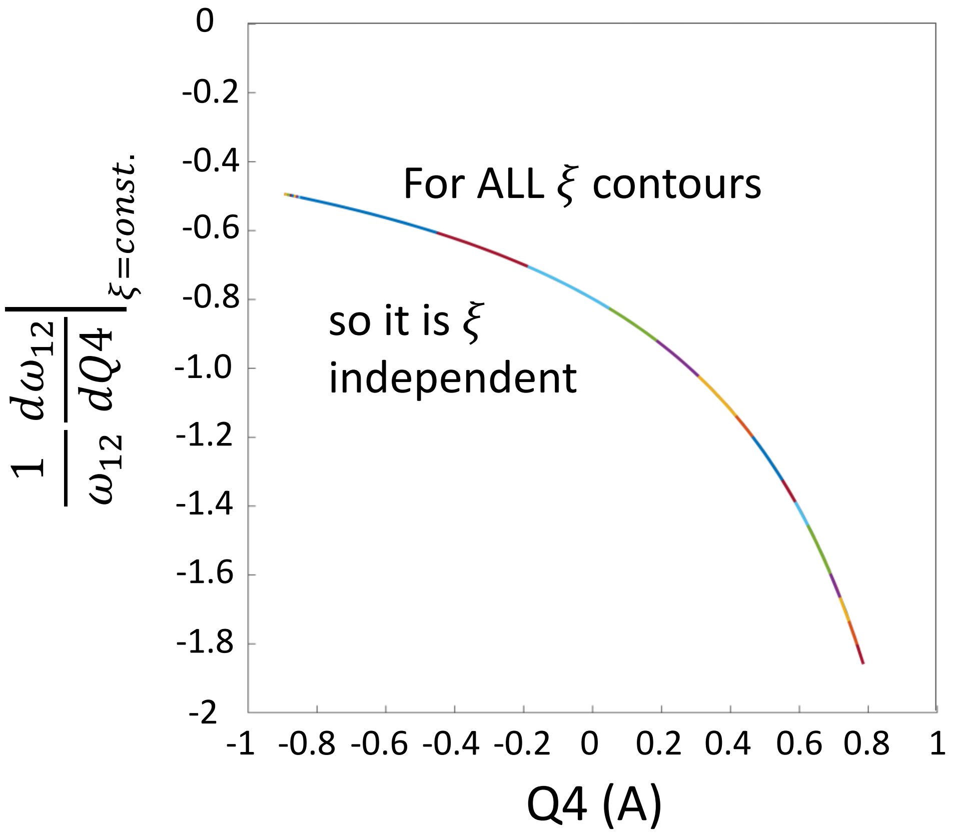
$$\left.\frac{d\,ln(y'_{rms})}{dQ4}\right|_{\xi=const.} = \frac{1}{\omega_{12}}\left.\frac{d\omega_{12}}{dQ4}\right|_{\xi=const.}$$

Only a machine property

$$\frac{1}{\omega_{12}}\left.\frac{d\omega_{12}}{dQ4}\right|_{\xi=const.} \approx \boxed{\frac{1}{aQ_4 + b}} + \boxed{c}$$

From thin lens

Second order term of ω_{12} O($Q4^2$)

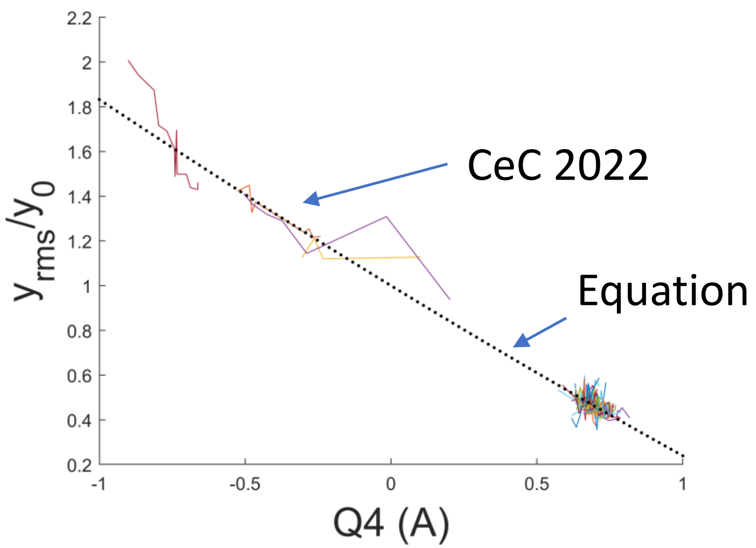


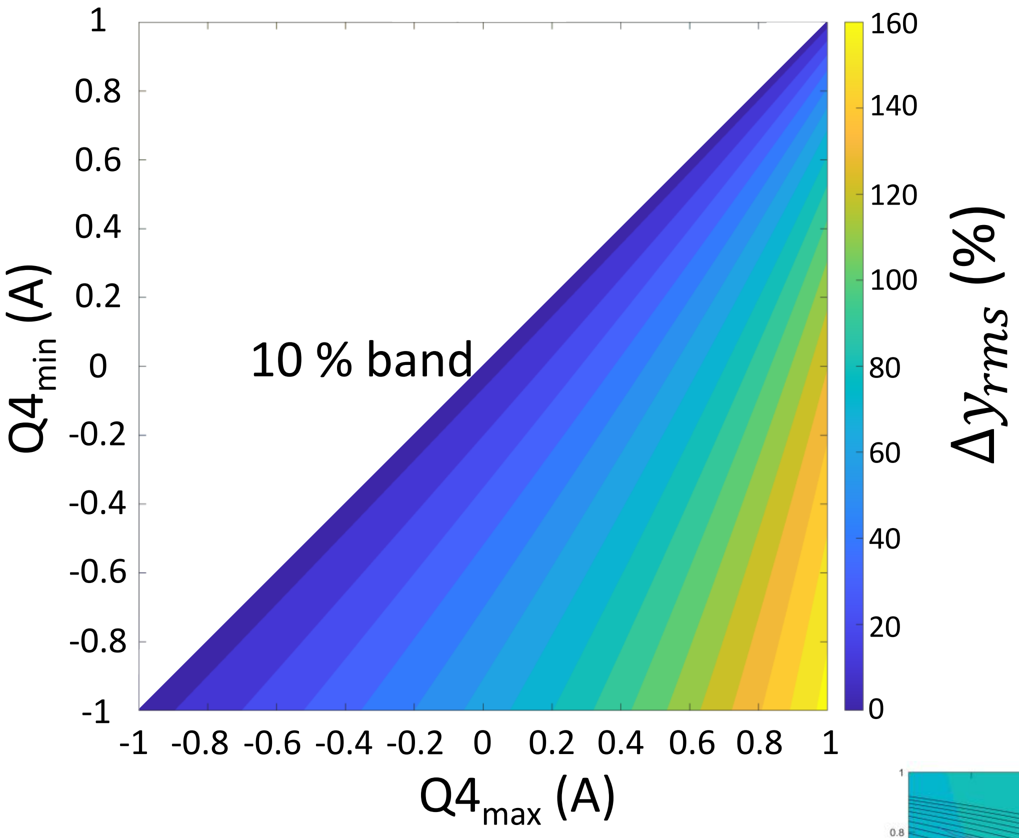
$$y_{rms}\big|_{\xi=const.} = y_0\left(\frac{a}{b}Q_4 + 1\right)^{\frac{1}{a}} e^{c\cdot Q_4}$$

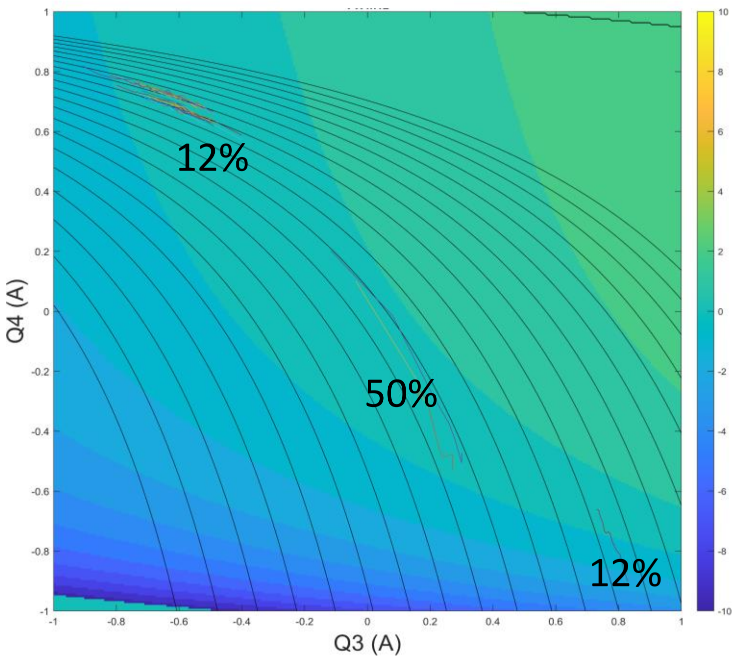
Where Q*Qk = k; $y_0 = y_{rms}(\xi, Q_4 = 0)$

Thin lens	Thick lens
a = 1;	a = 0.9679;
b = -1.2534[A];	b = -1.3080[A];
c = 0[A^{-1}];	c = -0.0333[A^{-1}];
* b =(s1+s2)/(s1*s2*L*Qk)	

$$y_{rms}\Big|_{\xi=const.} = y_0\left(\frac{a}{b}Q_4+1\right)^{\frac{1}{a}} e^{c\cdot Q_4}$$



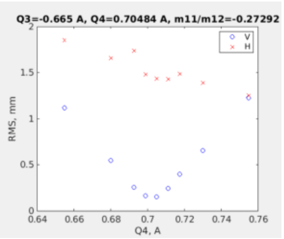




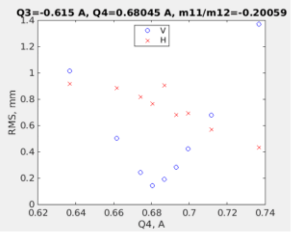
So Q4 range will defines time resolve resolution

In the search of vertical focusing

Pick a Q3 then scan Q4 for minimum y .
Next, change Q3 and scan Q4 for minimum y again.



$Q3_1 \rightarrow \{\ldots Q4_1\ Q4_2\ \boxed{Q4_3} \ldots\} \rightarrow y_{*1} = y(Q3_1,\ Q4_3)$

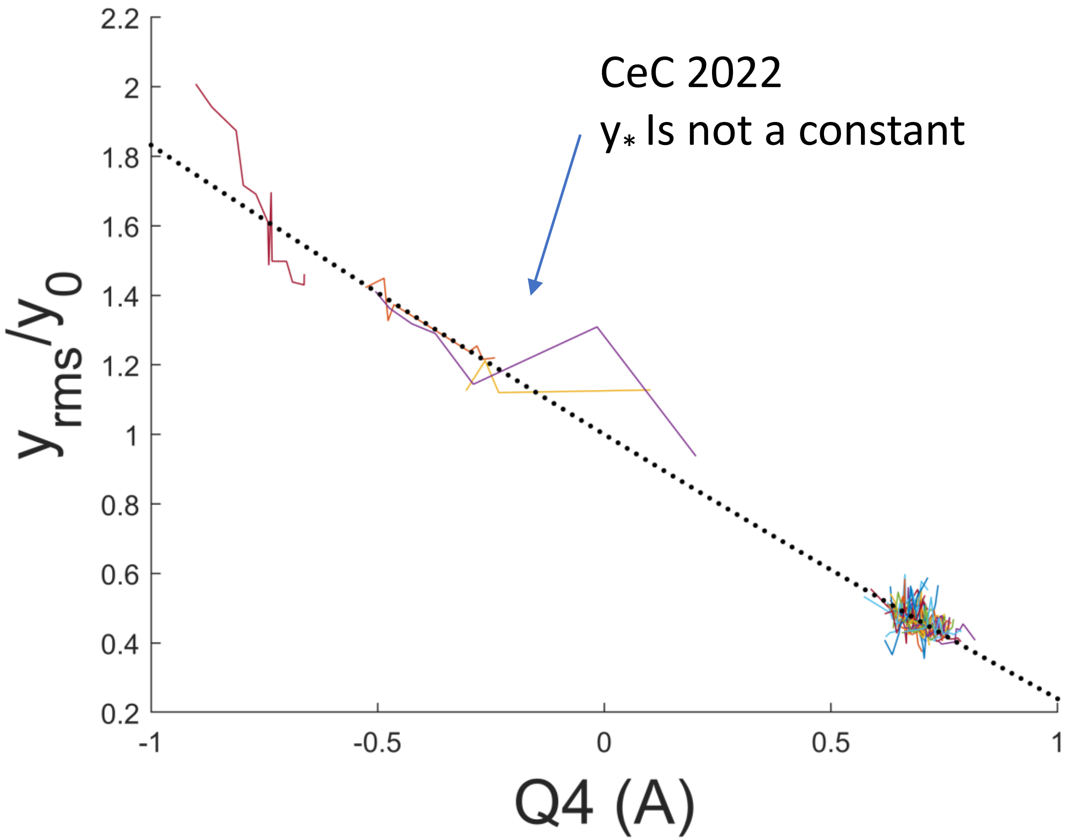


$Q3_2 \rightarrow \{\ldots \boxed{Q4_1}\ Q4_2\ Q4_3 \ldots\} \rightarrow y_{*2} = y(Q3_2,\ Q4_1)$

$\vdots$

y_{*1} is the minimum of $y(\text{Q31},\ \{Q_4\})$

y_{*2} is the minimum of $y(\text{Q32},\ \{Q_4\})$



But why $y_{*1} = y_{*2} = \cdots ?$

No! They Don’t for [Drift][t-Quad] system

$$y_* = \frac{\varepsilon}{\beta^2} s^2$$

Define vertical beam focal point $y = y_*$, when :

$$\left.\frac{\partial y(Q_3,\ Q_4)}{\partial Q_3}\right|_{Q_4=const.} = 0 \quad \text{or} \quad \left.\frac{\partial y(Q_3,\ Q_4)}{\partial Q_4}\right|_{Q_3=const.} = 0$$

So y_* is also a function of $(Q_3,\ Q_4)$

$$\because\ y_{rms}\Big|_{\xi=const.} = y_0\left(\frac{a}{b}Q_4+1\right)^{\frac{1}{a}} e^{c\cdot Q_4}$$

In here:

ξ and Q_4 is independent.

Since Changing Q_4, Q_3 will also change to keep ξ constant

$$\therefore\ y(\xi,\ Q_4) = y_0(\xi)L(Q_4)$$

$$\Rightarrow \left.\frac{\partial y}{\partial Q_3}\right|_{Q_4=const.} = 0$$

$$\left.\frac{\partial y}{\partial \xi}\frac{\partial \xi}{\partial Q_3}\right|_{Q_4=const.} = 0$$

$$\because\ \frac{\partial \xi}{\partial Q_3} \neq 0 \quad \therefore\ \left.\frac{\partial y}{\partial \xi}\right|_{Q_4=const.} = 0$$

$$\frac{\partial y}{\partial \xi}(Q_4,\xi) = L(Q_4) \times \frac{\partial y_0(\xi)}{\partial \xi}$$

$$\because\ L(Q_4) \neq 0$$

$$\therefore\ \frac{\partial y_0(\xi)}{\partial \xi} = 0 \Rightarrow \left.\frac{\partial y}{\partial \xi}\right|_{Q_4=const.} = 0 \quad \text{focal point } y = y_*$$

$$\xi = \xi_* \Rightarrow y = y_* \quad \text{Independent of } Q_4$$

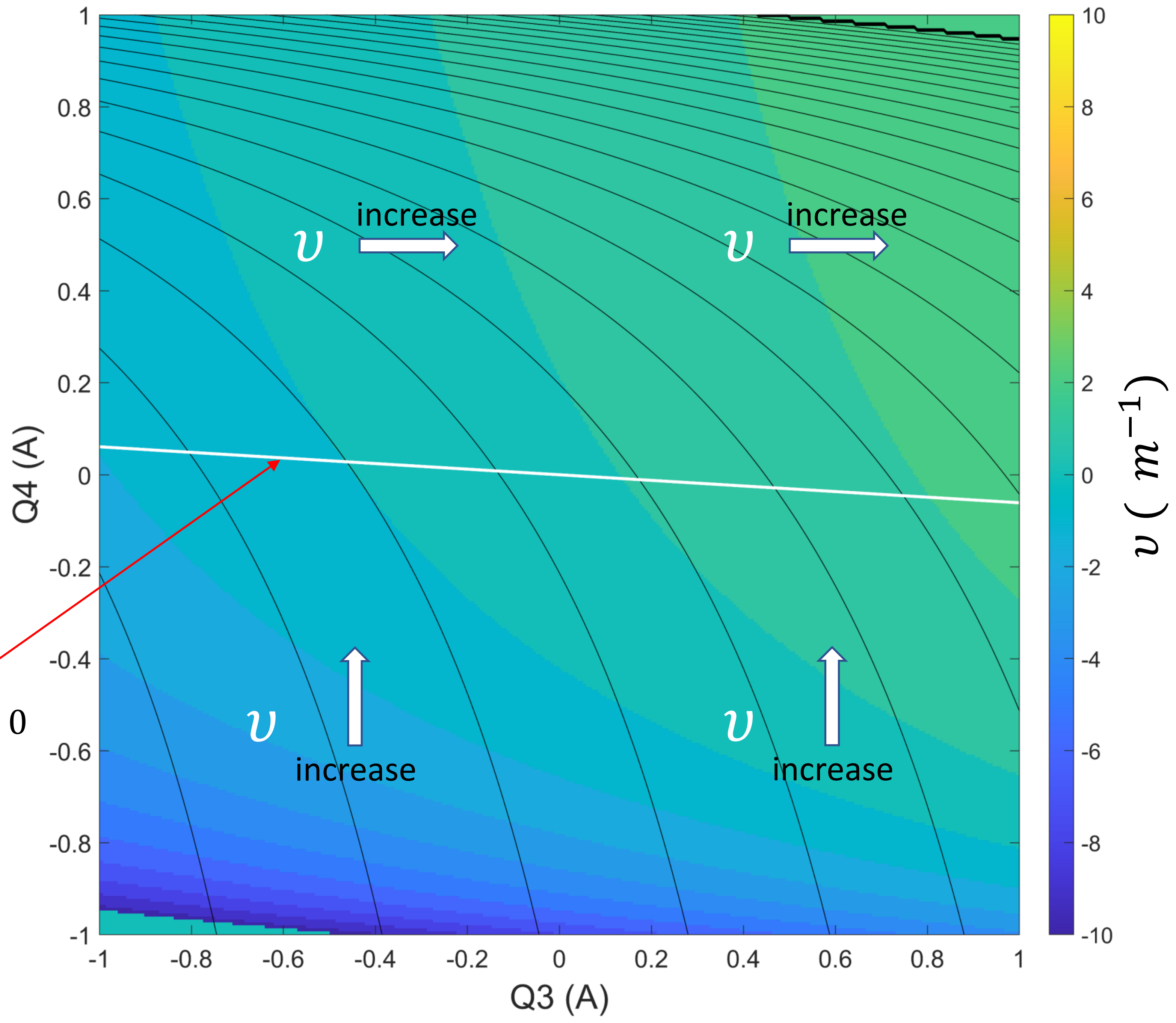
At ξ_* contour ⇒ $y_{rms}(Q_3, Q_4)$ in focus BUT $y_{rms} = y_0(\xi_*)L(Q_4)$

At y_* contour ⇒ $y_{rms}(Q_3, Q_4)$ Out of focus (not a minimum) BUT $y_{rms} = y_*(Q_{3*}, Q_{4*})$ is constant

- focuses beam vertically with Q3 Q4
- Test 3 points σ_x' along the ξ contour
- Set Q3 Q4 according to the changes of υ that requires
- Change Q1 Q2 to re-focuses beam vertically
- Essentially, we shifted ξ contour to where contains the minimum σ_x'

$$\frac{\langle x^2 \rangle}{m_{12}^2} = \sigma_h = \varepsilon\beta \cdot \Delta\nu^2 + \frac{\varepsilon}{\beta}$$

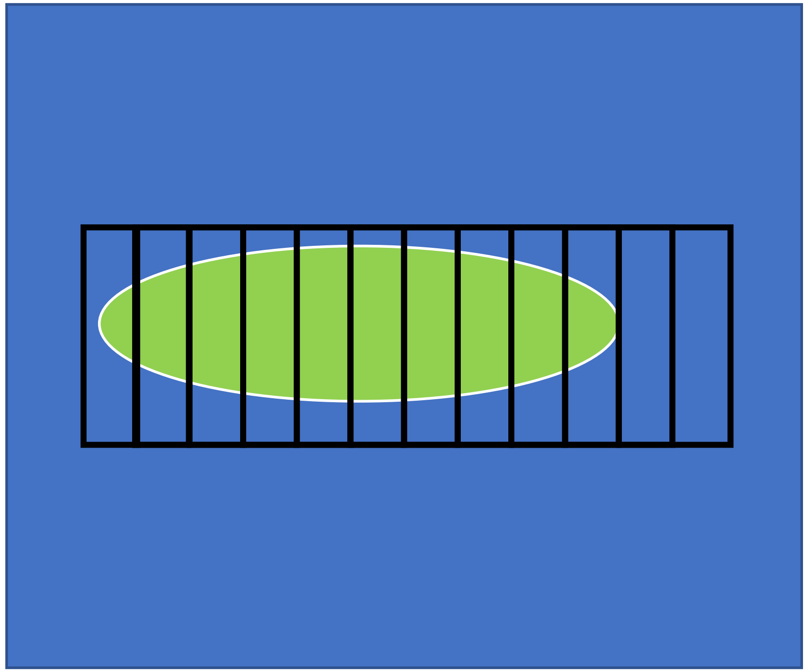
$$\left.\frac{d\upsilon}{dQ4}\right|_{\xi=const.} = 0$$

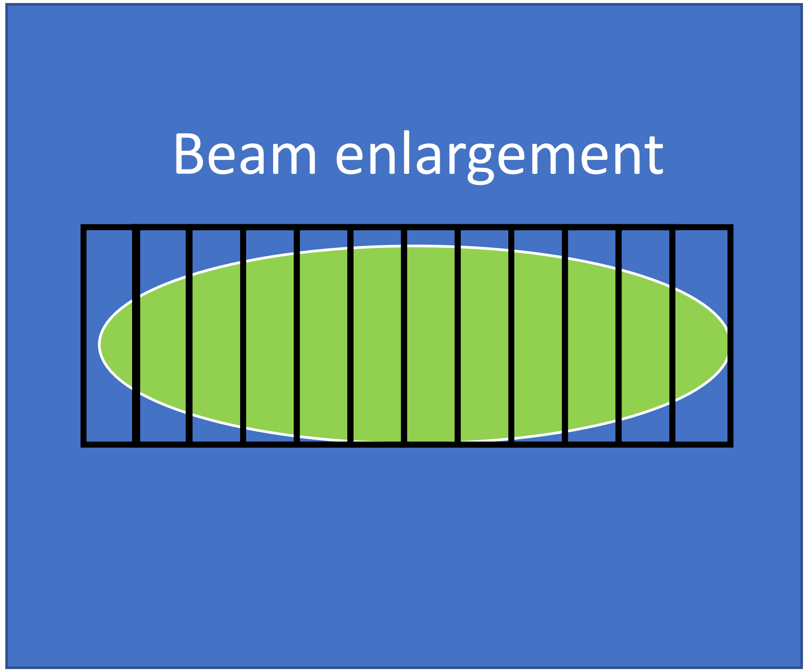


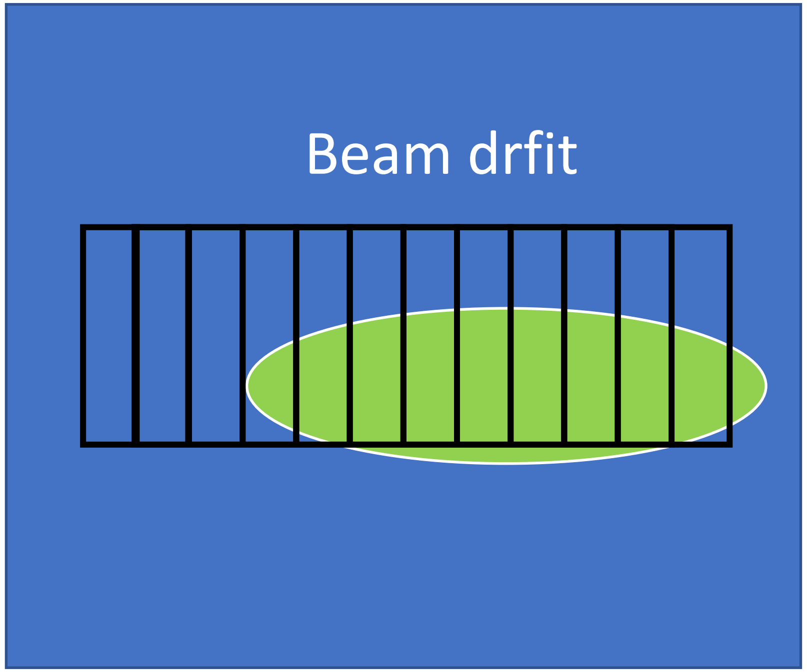
Beam Slicing

Digital Grid

Mesh in a fixed YAG location

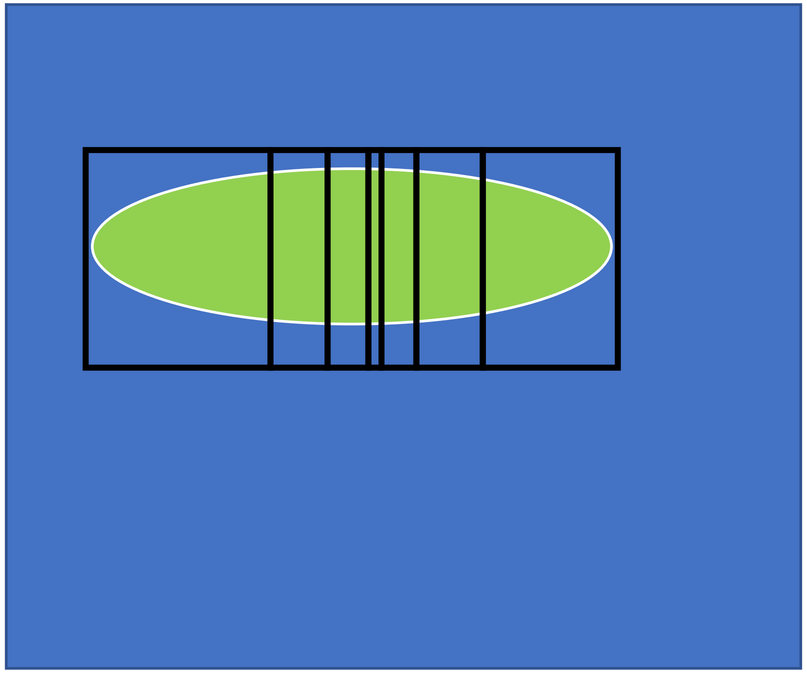


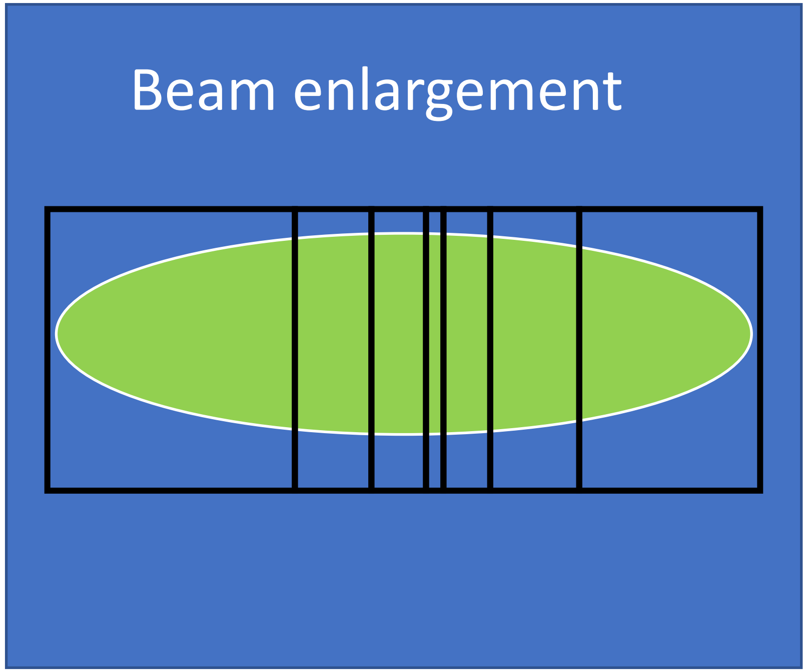


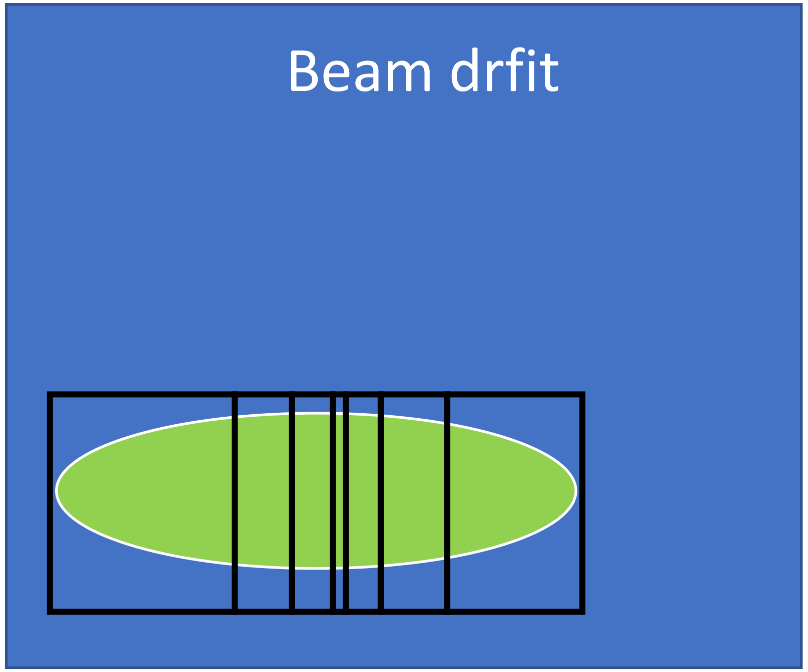


Constant Charge

Mesh in a fixed portion of beam charge



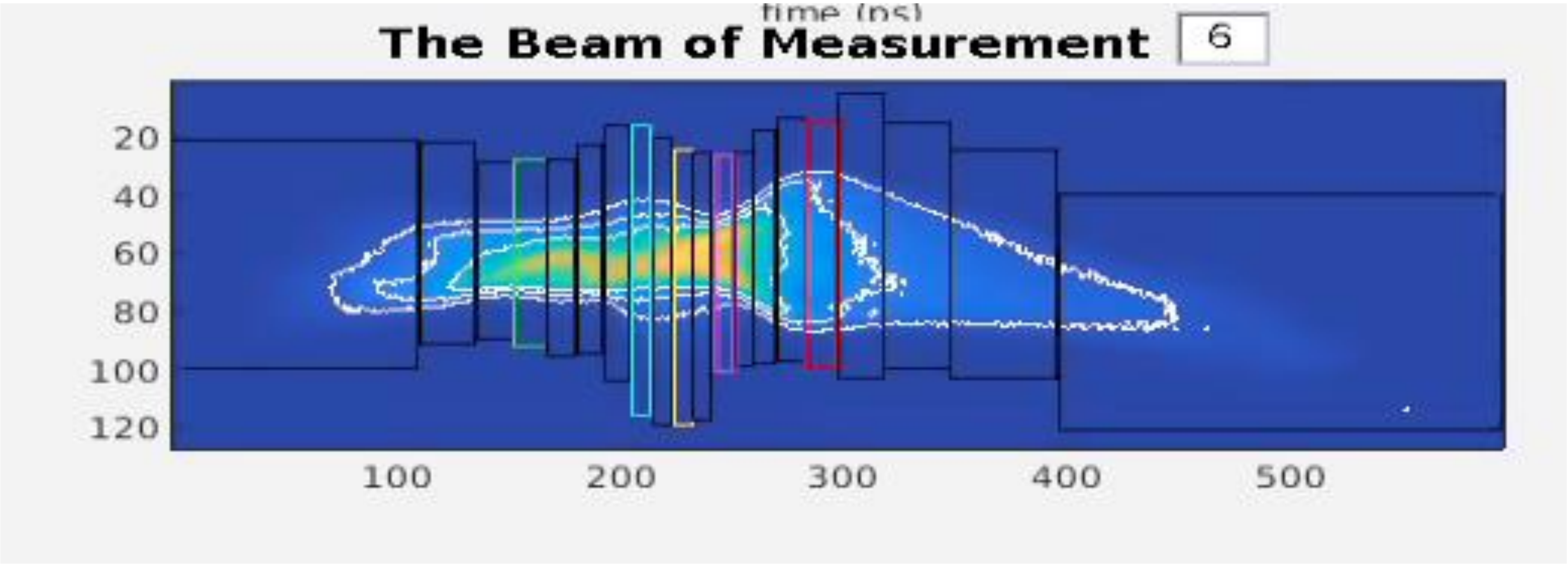




Pervious

Constant Charge Method

beam charge in each slices are equal → slice widths are different → resolution problem

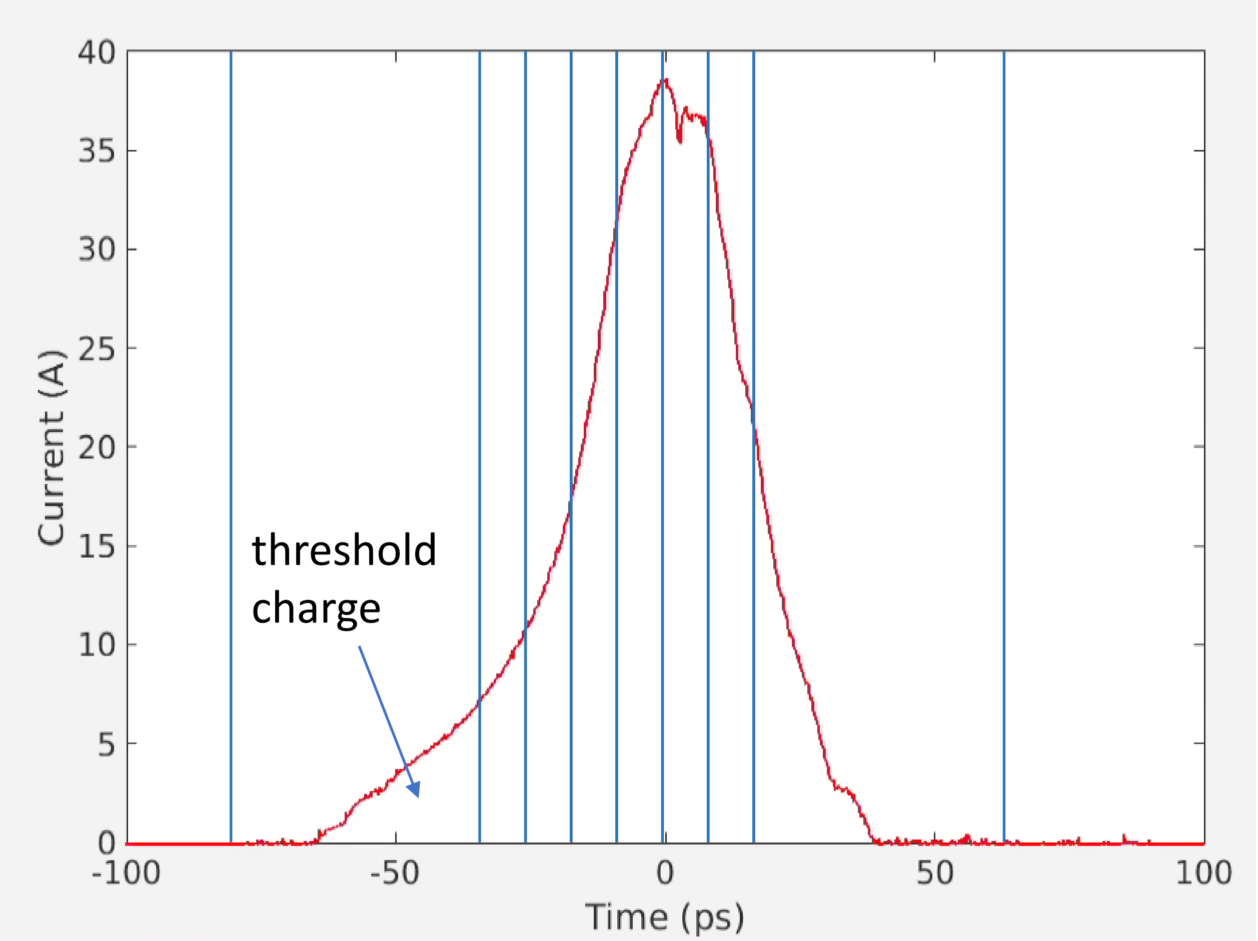


New
Constant Charge Method

Bin the reference current by the width of multipoles of y_{0rms}

Using this reference slice charge profile to slice up other beams

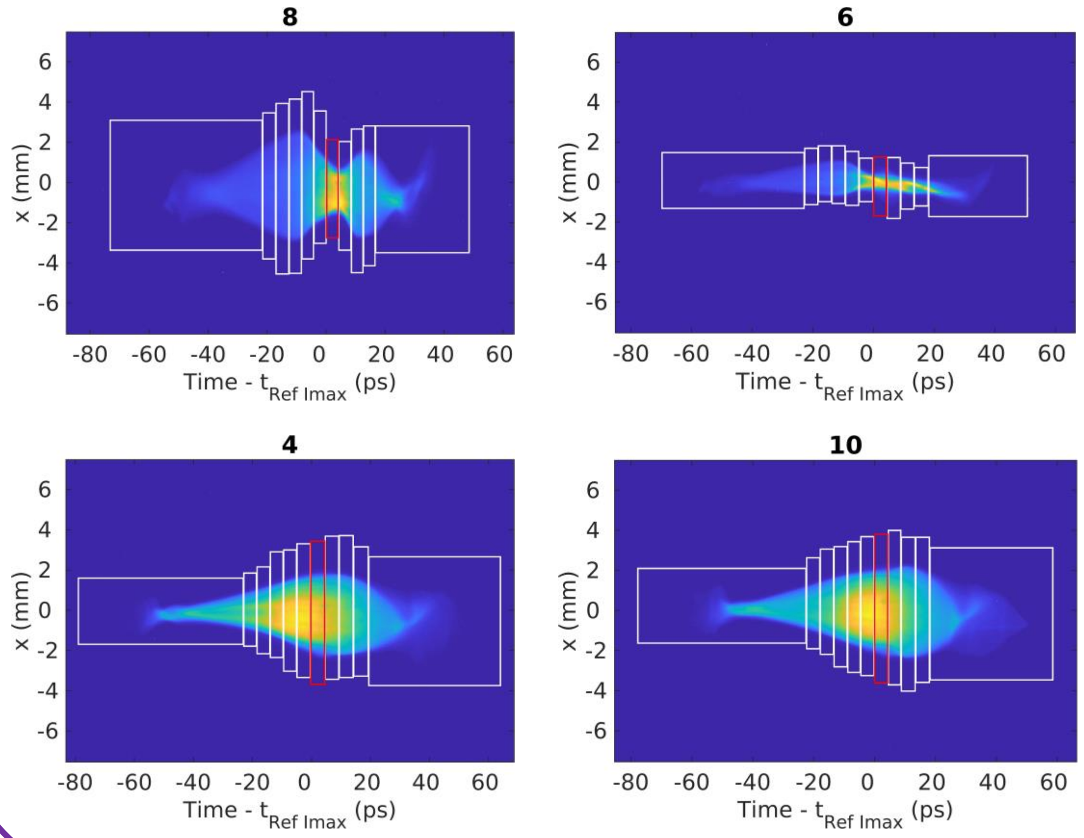
Also, we set a threshold charge for the edge slice to Increase slice accuracy



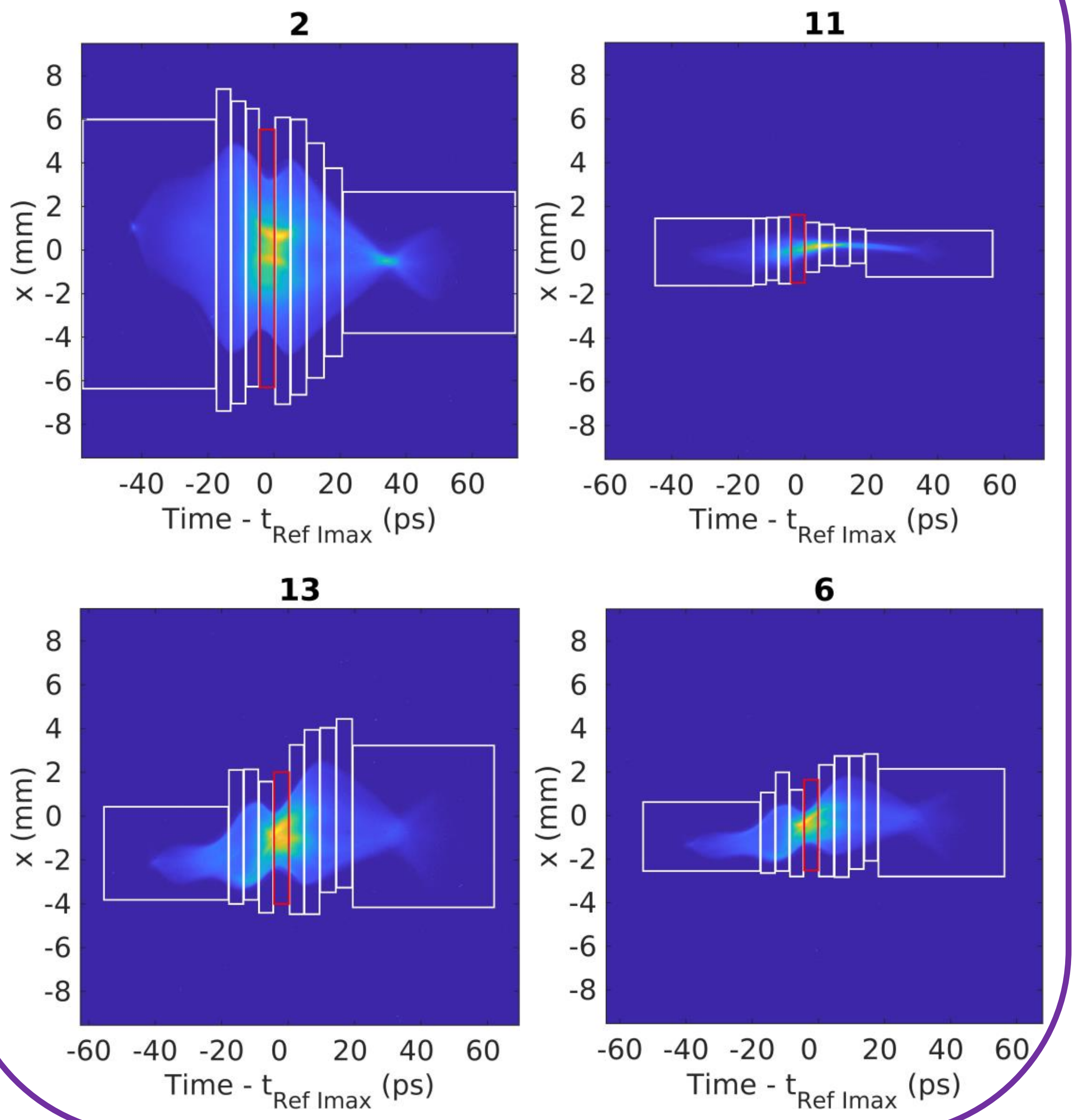
10% threshold

$4y_{0rms}$ slice width

03/20/2022 CeC YAG3



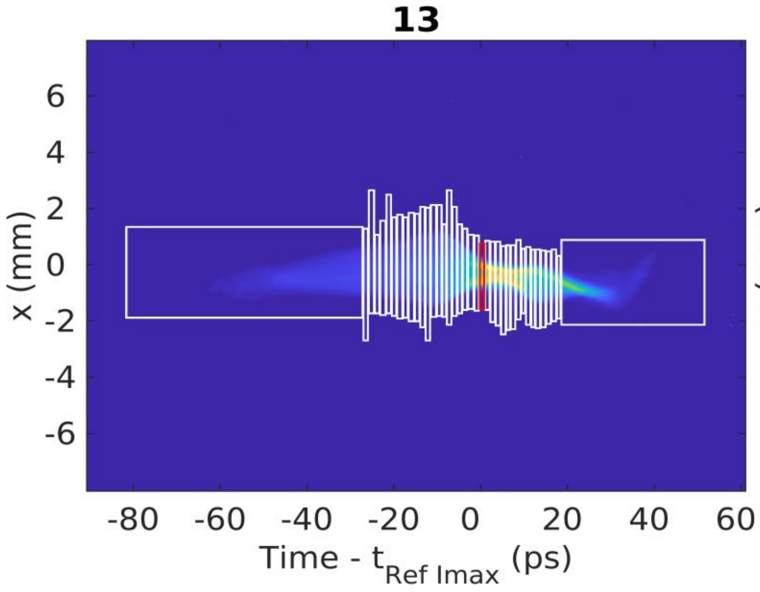
04/09/2022 CeC YAG3



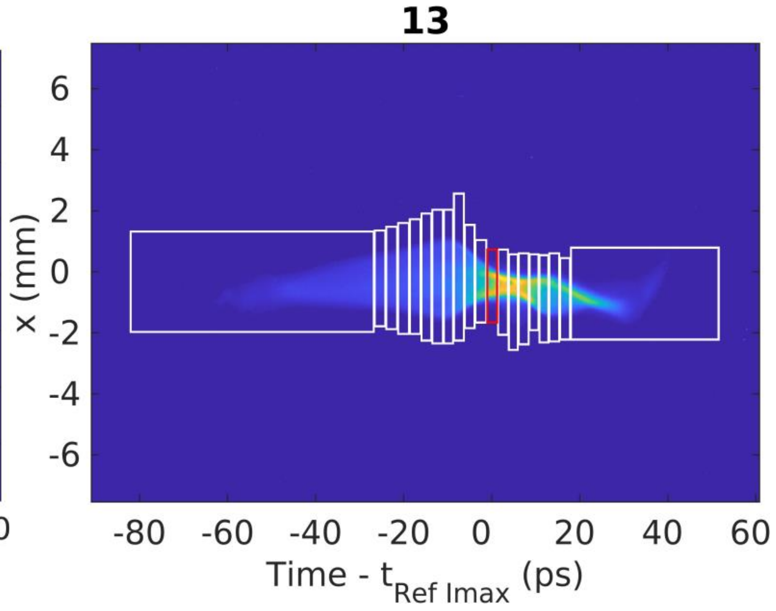
Max Ref current slice

10% threshold

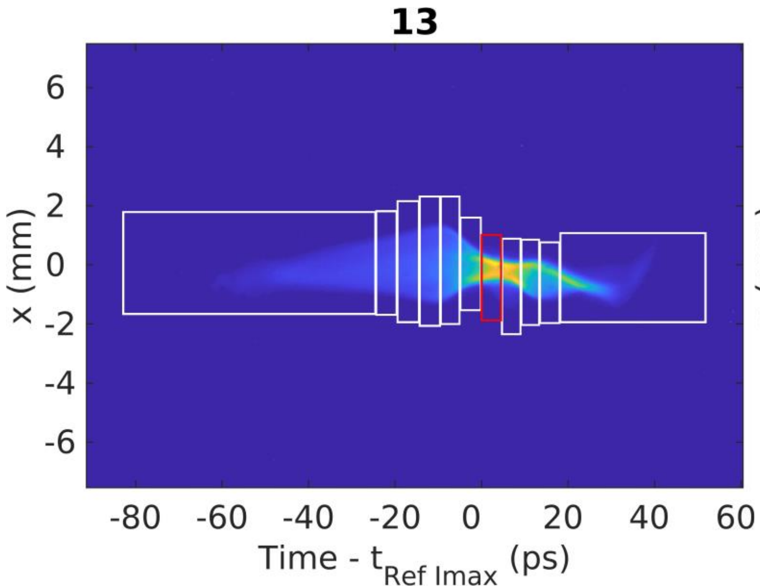
$1y_{0rms}$ slice width



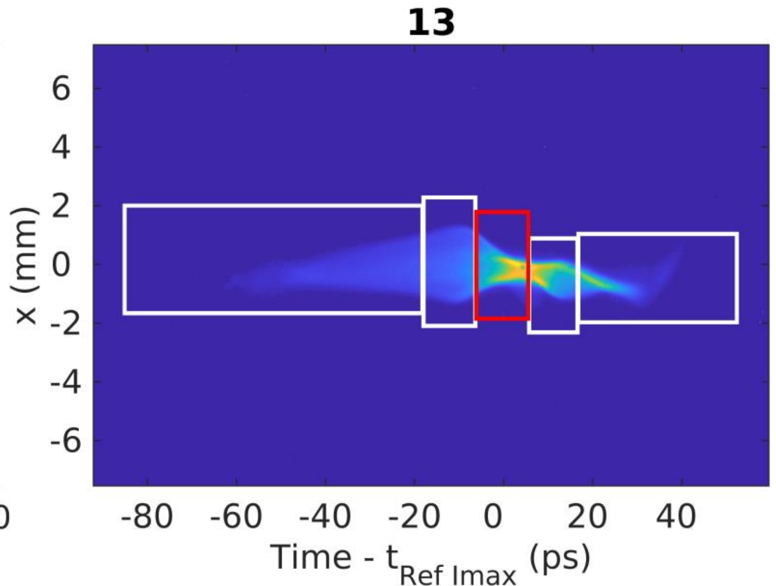
$2y_{0rms}$ slice width



$4y_{0rms}$ slice width



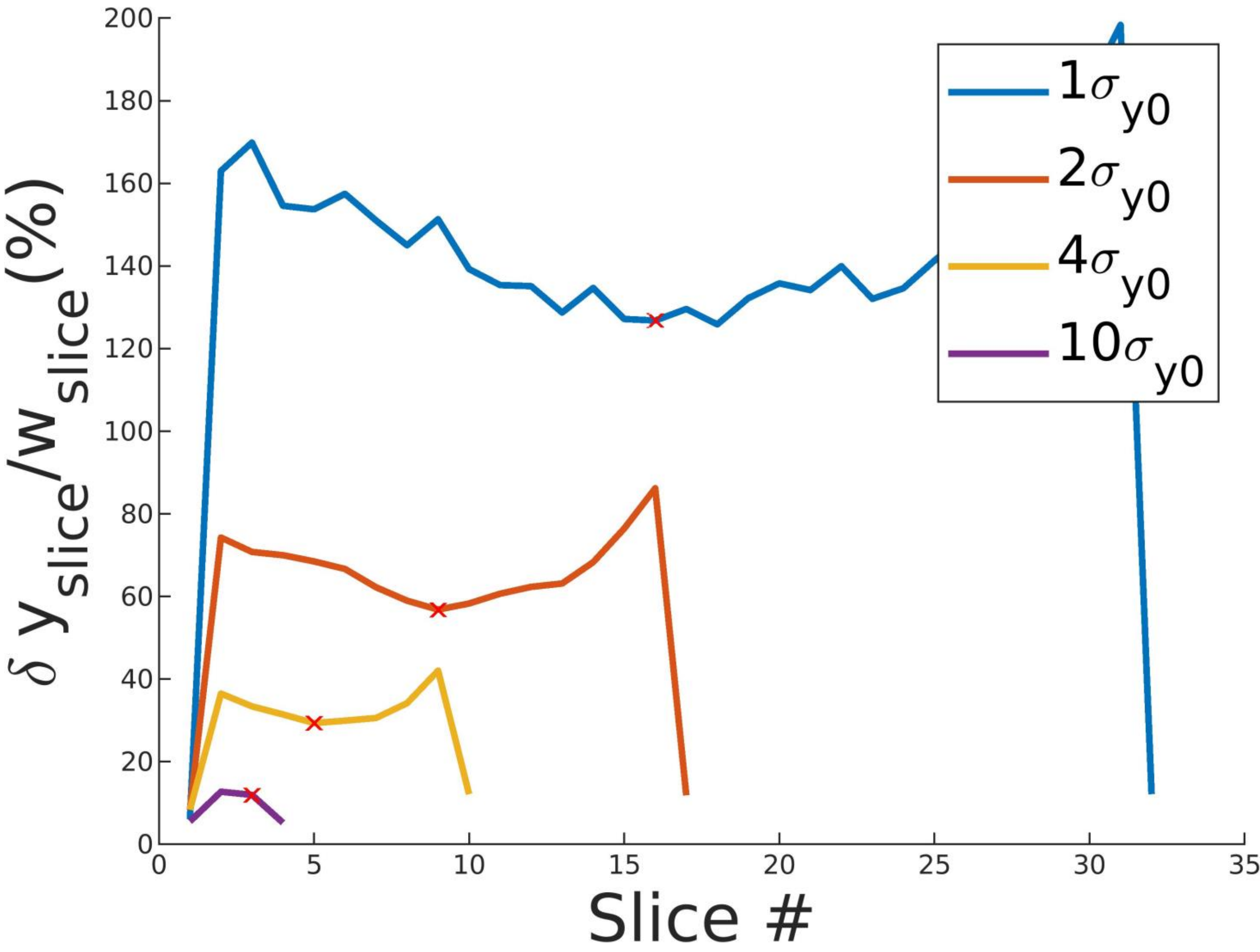
$10y_{0rms}$ slice width



$\boldsymbol{\delta} y_{slice}$ RMS slice center drift on YAG during scan

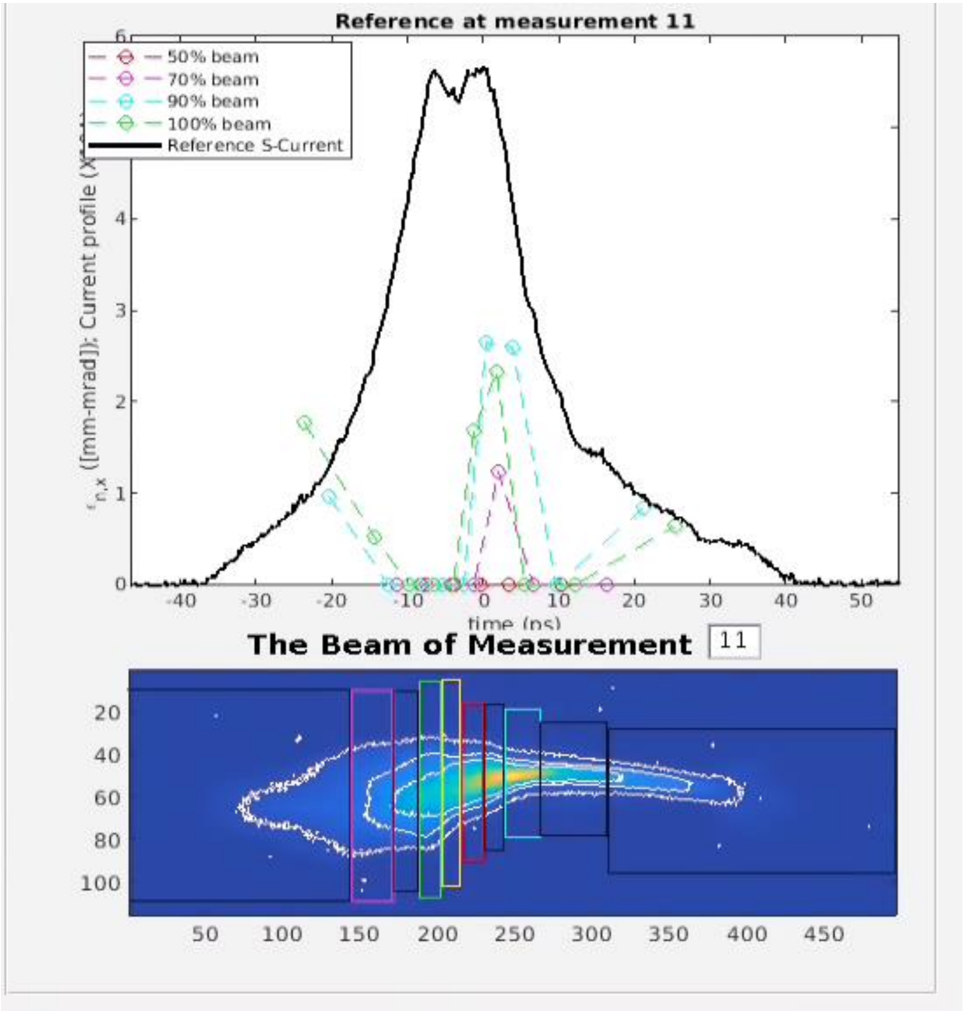
W_{slice} is the slice width

- In these data, we already try our best to put every beam on the YAG center.
- Therefore, Digital Grid slicing is highly affected by resolution. And should not be used.



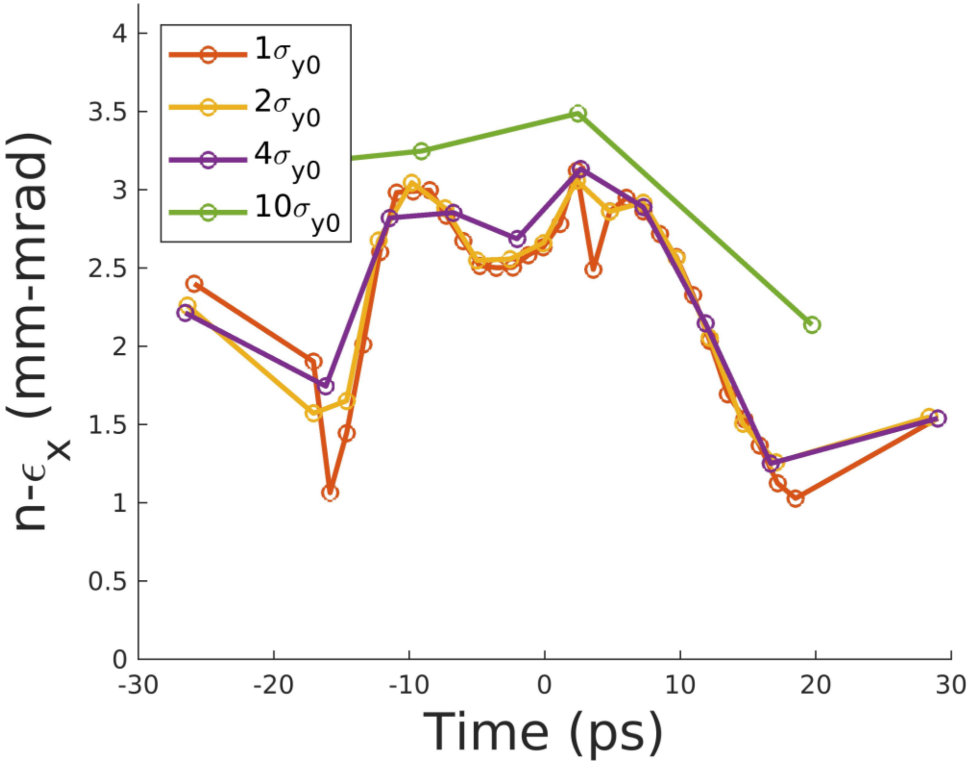
Pervious

Constant Charge Method



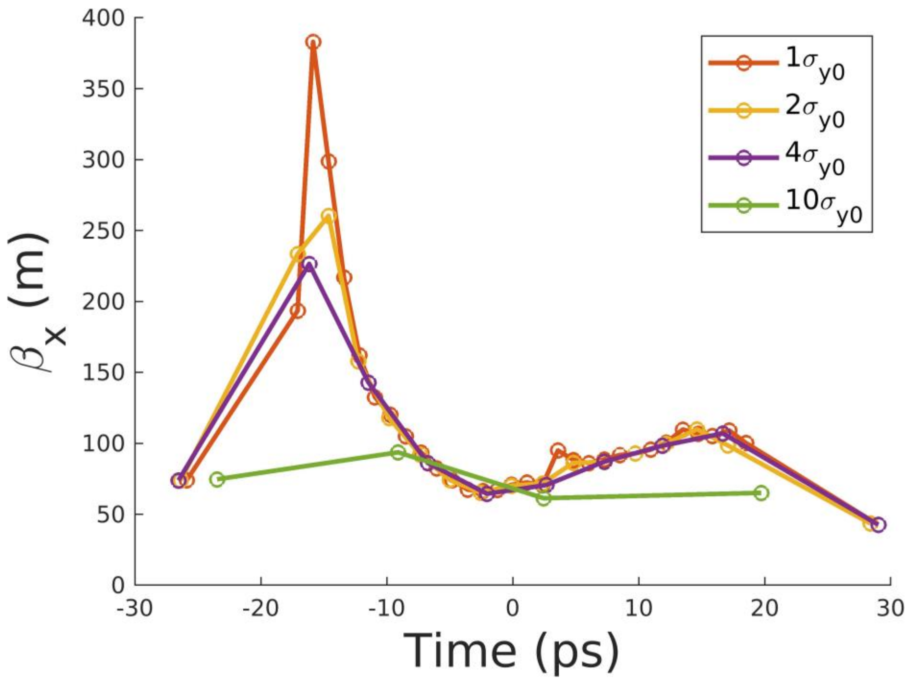
New

Constant Charge Method



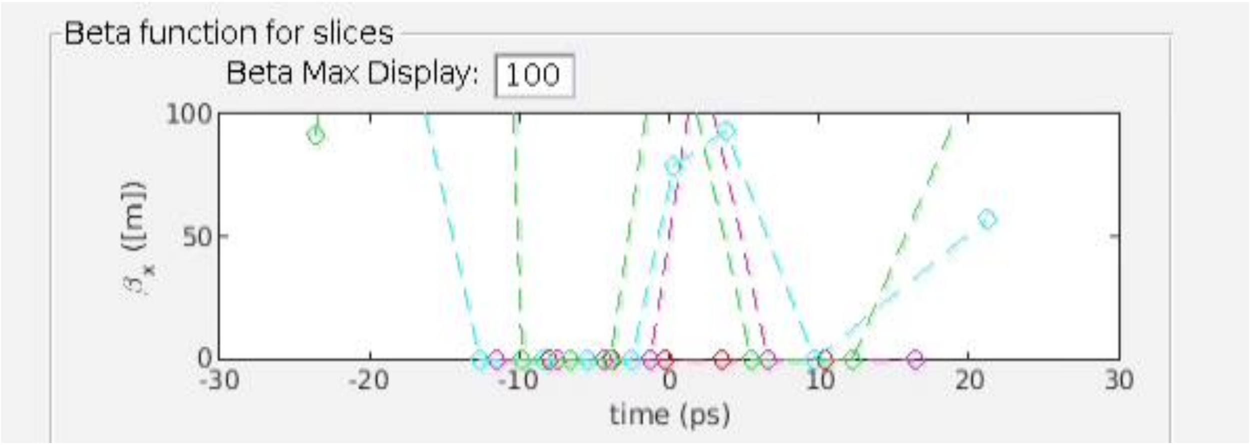
New

Constant Charge Method



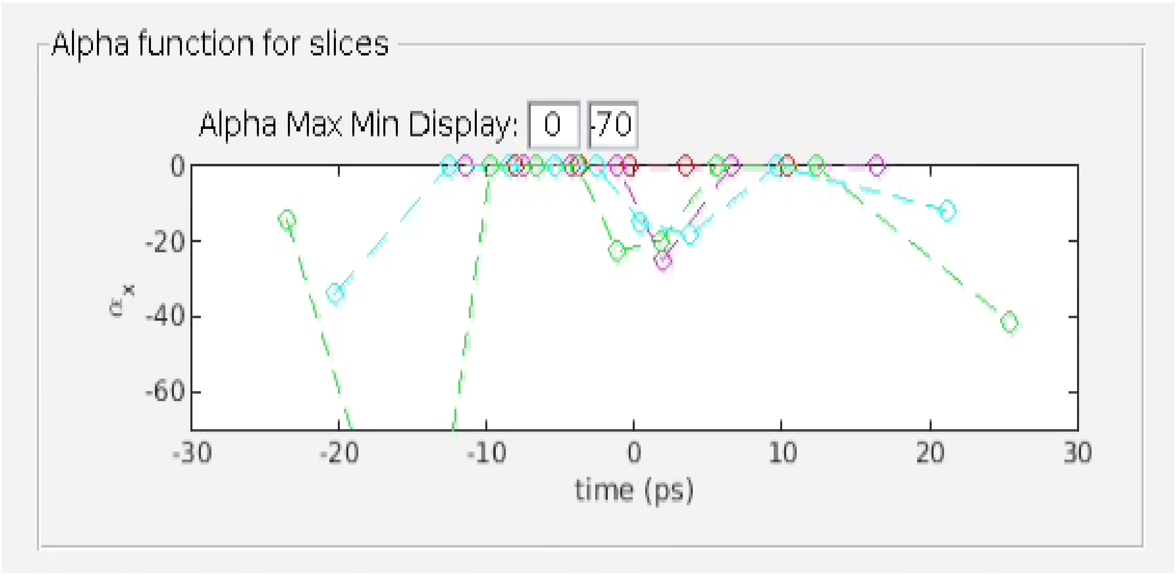
Pervious

Constant Charge Method



Pervious

Constant Charge Method



New

Constant Charge Method