Solutions for HW # 6

1. The longitudinal impedance is defined as

$$Z_{//}(\omega) = \frac{1}{c}\int_0^\infty w_{//}(s)e^{i\omega s/c}ds \quad . \tag{1}$$

Taking the complex conjugate of eq. (1) and noticing that

$$w_{//}^*(s) = w_{//}(s) \quad , \tag{2}$$

it follows that

$$Z_{//}^*(\omega) = \frac{1}{c}\int_0^\infty w_{//}(s)e^{-i\omega s/c}ds = \frac{1}{c}\int_0^\infty w_{//}(s)e^{i(-\omega)s/c}ds = Z_{//}(-\omega) \quad . \tag{3}$$

The transverse impedance is defined as

$$Z_\perp(\omega) = -\frac{i}{c}\int_0^\infty w_\perp(s)e^{i\omega s/c}ds \quad . \tag{4}$$

Taking the complex conjugate of eq. (4) and noticing that

$$w_\perp^*(s) = w_\perp(s) \tag{5}$$

yield

$$Z_\perp^*(\omega) = \frac{i}{c}\int_0^\infty w_{//}(s)e^{-i\omega s/c}ds = -\left[-i\frac{1}{c}\int_0^\infty w_{//}(s)e^{i(-\omega)s/c}ds\right] = -Z_\perp(-\omega) \quad . \tag{6}$$

2. Using the relation

$$F(z) = \frac{1}{2\pi}\int_{-\infty}^\infty e^{ikz}\tilde{F}(k)dk \quad ,$$

we obtain

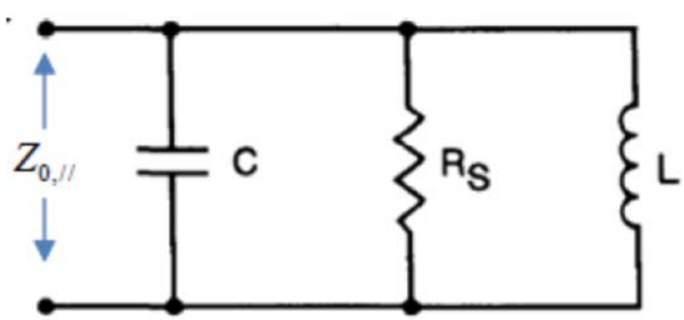
$$\sum_{l=-\infty}^{\infty} F(lC) = \frac{1}{2\pi}\sum_{l=-\infty}^{\infty}\int_{-\infty}^{\infty} e^{iklC}\tilde{F}(k)\,dk$$
$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} dk\tilde{F}(k)\sum_{l=-\infty}^{\infty} e^{i2\pi l\cdot\frac{kC}{2\pi}}$$
$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} dk\tilde{F}(k)\sum_{p=-\infty}^{\infty}\delta\left(\frac{kC}{2\pi}-p\right) \qquad ,$$
$$= \frac{1}{2\pi}\sum_{p=-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{2\pi\delta\left(k-\frac{2\pi p}{C}\right)}{C}\tilde{F}(k)\,dk$$
$$= \frac{1}{C}\sum_{p=-\infty}^{\infty}\tilde{F}\left(\frac{2\pi p}{C}\right)$$

where we also used

$$\delta(g(x)) = \sum_i \frac{\delta(x-x_i)}{|g'(x_i)|}$$

with x_i being the roots of $g(x)$.

3.



The impedance is determined by

$$
\begin{aligned}
\frac{1}{Z_{0,//}} &= \frac{1}{Z_R} + \frac{1}{Z_L} + \frac{1}{Z_C} \\
&= \frac{1}{R_s} + \frac{1}{j\omega L} + j\omega C \\
&= \frac{1 + jR_s\sqrt{\frac{C}{L}}\left(\omega\sqrt{LC} - \frac{1}{\omega\sqrt{LC}}\right)}{R_s} \,, \\
&= \frac{1 + jQ\left(\frac{\omega}{\omega_R} - \frac{\omega_R}{\omega}\right)}{R_s} \\
&= \frac{1 + iQ\left(\frac{\omega_R}{\omega} - \frac{\omega}{\omega_R}\right)}{R_s}
\end{aligned}
\tag{7}
$$

i.e.

$$
Z_{0,//} = \frac{R_s}{1 + iQ\left(\frac{\omega_R}{\omega} - \frac{\omega}{\omega_R}\right)}
$$

where $j = -i$, $Q \equiv R_s\sqrt{\frac{C}{L}}$ and $\omega_R \equiv \frac{1}{\sqrt{LC}}$.