

2494 **Chapter 10**
2495 **Strong Focusing Synchrotron**

2496 **Abstract** This Chapter introduces the strong focusing synchrotron, alternating gra-
2497 dient (AG) and separated focusing, and the theoretical material needed for the simula-
2498 tion exercises. It begins with a brief reminder of the historical context, and continues
2499 with beam optics, chromaticity, and acceleration. It relies on basic charged particle
2500 optics and acceleration concepts introduced in the previous Chapters, and further
2501 addresses the following aspects:

- 2502 - resonances and resonant extraction,
- 2503 - stochastic energy loss by synchrotron radiation.

2504 The simulation of a strong focusing synchrotron requires just two, possibly three,
2505 optical elements from zgoubi library: DIPOLE, BEND, or MULTIPOL to simu-
2506 late (possibly combined function) dipoles, DRIFT to simulate straight sections,
2507 and MULTIPOL to simulate lenses (which can be otherwise simulated using
2508 QUADRUPO, SEXTUPOL, OCTUPOLE, etc.). A fourth element, CAVITE, is re-
2509 quired for acceleration. Particle monitoring requires keywords introduced in the pre-
2510 vious Chapters, including FAISCEAU, FAISTORE, possibly PICKUPS, and some
2511 others. Spin motion computation and monitoring resort to SPNTRK, SPNPRT, FAI-
2512 STORE. Optics matching and optimization use FIT[2]. INCLUDE is used, mostly
2513 here in order to shorten the input data files. SYSTEM is used to, mostly, resort to
2514 gnuplot so as to end simulations with some specific graphs obtained by reading
2515 data from output files such as zgoubi.fai (resulting from the use of FAISTORE),
2516 zgoubi.plt (resulting from IL=2), or other zgoubi*.out files resulting from a PRINT
2517 command.

2518 **Notations used in the Text**

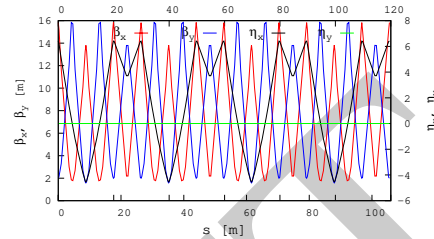
$B; \mathbf{B}, B_{x,y,s}$	field value; field vector, its components in the moving frame
$B\rho = p/q; B\rho_0$	particle rigidity; reference rigidity
$C; C_0$	orbit length, $C = 2\pi R + \left[\begin{array}{l} \text{straight} \\ \text{sections} \end{array} \right]$; reference, $C_0 = C(p = p_0)$
E	particle energy
EFB	Effective Field Boundary
$f_{\text{rev}}, f_{\text{rf}} = h f_{\text{rev}}$	revolution and RF accelerating voltage frequencies
G	gyromagnetic anomaly, $G = 1.792847$ for proton
$G; K = G/B\rho$	quadrupole gradient; focusing strength
h	RF harmonic number
$m; m_0; M$	mass, $m = \gamma m_0$; rest mass; in units of MeV/c^2
$\mathbf{p}; p; p_0$	momentum vector; its modulus; reference
P_i, P_f	beam polarization, initial, final
q	particle charge
r, R	orbital radius ; average radius, $R = C/2\pi$
s	path variable
v	particle velocity
$V(t); \hat{V}$	oscillating voltage; its peak value
2519 $x, x', y, y', l, \frac{dp}{p}$	horizontal, vertical, longitudinal coordinates in moving frame
α	depending on the context: momentum compaction or trajectory deviation
$\beta = v/c; \beta_0; \beta_s$	normalized particle velocity; reference; synchronous
β_u	betatron functions ($u : x, y, Y, Z$)
$\gamma = E/m_0$	Lorentz relativistic factor
δp	momentum offset or Dirac distribution
Δp	momentum offset
ε	wedge angle
ε_u	Courant-Snyder invariant ($u : x, r, y, l, Y, Z, s, \text{etc.}$)
ε_R	strength of a depolarizing resonance
μ_u	betatron phase advance, $\mu_u = \int_{\text{period}} ds/\beta_u(s)$ ($u : x, y, Y, Z$)
ν_u	wave numbers, horizontal, vertical, synchrotron ($u : x, y, Y, Z, l$)
ρ, ρ_0	curvature radius; reference
σ	beam matrix
$\phi; \phi_s$	particle phase at voltage gap; synchronous phase
ϕ_u	betatron phase advance, $\phi_u = \int ds/\beta_u$ ($u : x, y, Y, \text{or } Z$)
φ	spin angle to the vertical axis

2520 **10.1 Introduction**

2521 In the very manner that the 1930s-1940s cyclotron, betatron, microtron, weak fo-
 2522 cusing synchrotron, still in use today, have since essentially not changed in their

2523 concepts, design principles, magnet gap profile, today's gap profile, yoke and cur-
 2524 rent coil geometry of combined function alternating-gradient (AG) dipoles remain
 essentially as patented in 1950 (Fig. 10.1) [1].

Fig. 10.1 Bending magnet pole profiles for a *focusing system for ions and electrons* [1]. Assuming curvature center to the left, the right (respectively left) profile is defocusing (resp. focusing), the middle profile has zero index



2525
 2526 In 1952, in the context of studies relative to the Cosmotron, strong focusing was
 2527 devised at the Brookhaven National Laboratory (BNL): “*Strong focusing forces re-*
 2528 *sult from the alternation of large positive and negative n-values in successive sectors*
 2529 *of the magnetic guide field in a synchrotron. This sequence of alternately conver-*
 2530 *ing and diverging magnetic lenses [...] leads to significant reductions in oscillation*
 2531 *amplitude*” [2]. It led to the construction of the first two high-energy proton AG
 2532 synchrotrons, in the 30 GeV range, in the late 1950s: the proton-synchrotron (PS)
 2533 at CERN, and the AGS at BNL, major pieces 60 years later still, of the respective
 2534 injection chains of the two largest colliders in operation, the LHC and RHIC. Early
 2535 works at BNL provided theoretical formalism, still at work today, for the analysis of
 2536 beam dynamics in synchrotrons [3].

2537 The optical principle behind the AG concept is that a doublet of focusing and
 2538 defocusing lenses with proper strengths results in a short focal distance converging
 2539 system. The dramatic reduction of transverse beam size by AG focusing allows small
 2540 dipole gaps, thus small magnets: from lowest energies (medical synchrotrons in the
 2541 100 MeV range for instance) to the highest ones (particle physics and nuclear physics
 2542 colliders, hundreds of GeV to multi-TeV range), beams are essentially confined in a
 2543 centimeter scale transverse space, making a synchrotron a string of dipole magnets
 2544 containing the beam in a ring vacuum pipe of a few centimeters in diameter (hadrons)
 2545 or a few millimeters (electrons). The size of the ring is essentially determined by
 2546 its circumference, proportional to the magnetic rigidity. This revolutionized the race
 2547 to high energies, from the prior few GeV weak focusing synchrotrons and their
 2548 huge magnets, to today's 7 TeV at the LHC with magnets transverse size of a few
 2549 tens of centimeters, and with further plans for 100 TeV rings [5]. It fostered as well
 2550 the development of high energy synchrotron light sources around the world, with
 2551 electron beam energies up to 8 GeV.

2552 Separated function focusing, whereby beam guiding is ensured by uniform field
 2553 dipoles while focusing is ensured separately by quadrupoles, followed from the
 2554 development of the latter (Fig.10.4), a spin-off of the strong index technology [9].
 2555 Separated function optics allows changing wave numbers at will (within the optical

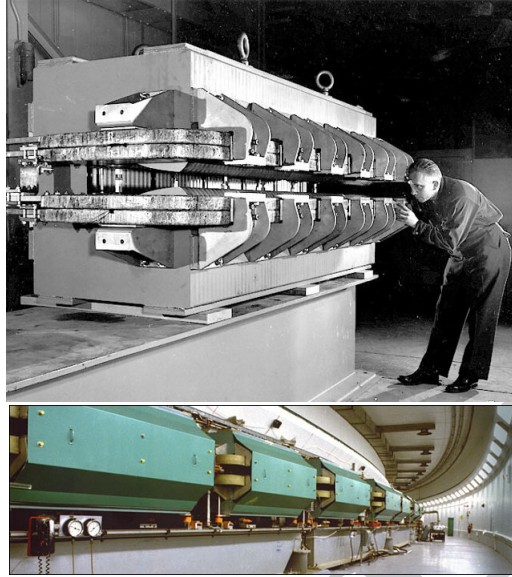


Fig. 10.2 Top: the AGS combined function main magnet - one of 240 steering the beam around the ring, bottom: the 809 m circumference AGS synchrotron [4]. The hyperbolic profile poles are visible on the top photo, partly hidden by the field coils

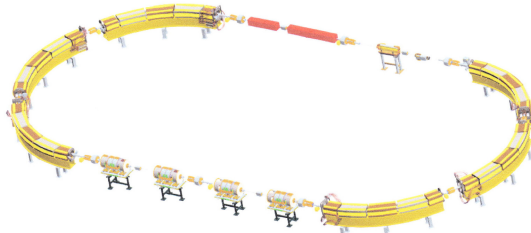


Fig. 10.3 The ion rapid cycling medical synchrotron (iRCMS) [6], an RCS aimed at providing ion beams for the treatment of cancer tumors

2556 system stability limits). It allows introducing modular functions in complex rings
 2557 such as dispersion suppression sections, low-beta or insertion device sections, long
 2558 straights, etc. Low-emittance, high-brightness light source lattices have complicated
 2559 focusing further, by introducing longitudinal field gradient bending systems, aimed
 2560 at minimizing the chromatic invariant [10]. AG focusing however has the benefit of
 2561 compactness and is still resorted to, today, in hadrontherapy applications (Fig. 10.3),
 2562 light source lattice [7], and other high energy collider design [8].

2563 Due to the necessary ramping of the field in order to maintain a constant orbit,
 2564 synchrotron accelerators are pulsed, storage rings in some cases as well, high energy
 2565 colliders in particular to bring beams to highest store energy. The acceleration is
 2566 cycled and the accelerating voltage frequency as well in ion accelerators, from
 2567 injection to top energy. If the ramping uses a constant electromotive force, then
 2568 (Eq. 9.3)

Fig. 10.4 A quadrupole magnet at LBL in 1957, used for beam lines at the 184-inch cyclotron. An early specimen here, obviously, being a spin-off of the early 1950s concept of strong focusing [11]



$$B(t) \approx \frac{t}{\tau} \quad (10.1)$$

2569 $\dot{B} = dB/dt$ does not exceed a few Tesla/second, thus the repetition rate of the
 2570 acceleration cycle is of the order of a Hertz. If instead the magnet winding is part of
 2571 a resonant circuit then the field oscillates,

$$B(t) = B_0 + \frac{\hat{B}}{2}(1 - \cos \omega t) \quad (10.2)$$

2572 so that, in the interval of half a voltage repetition period (*i.e.*, $t : 0 \rightarrow \pi/\omega$) the
 2573 field increases from an injection threshold value to a maximum value at highest
 2574 rigidity, $B(t) : B_0 \rightarrow B_0 + \hat{B}$. The latter determines the highest achievable energy:
 2575 $\hat{E} = pc/\beta = q\hat{B}\rho c/\beta$. The repetition rate with resonant magnet cycling can reach a
 2576 few tens of Hertz, a technique known as a rapid-cycling synchrotron (RCS). In both
 2577 cases anyway B imposes its law and the other parameters comprising the acceleration
 2578 cycle the RF frequency in particular, will follow $B(t)$.

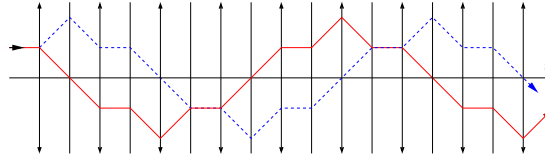
2579 Rapid cycling allows high intensity beams. Instances are the Cornell 12 GeV,
 2580 60 Hz, electron AG synchrotron, commissioned in 1967, still in use half a century
 2581 later as the injector of Cornell 5 GeV synchrotron light source (CHESS); Fermilab
 2582 8 GeV, 60 Hz, booster which provides protons for the production of neutrino beams;
 2583 the 30 GeV 500 kW beam J-PARC facility in Japan. Rapid cycling is also considered
 2584 in ion-therapy applications, Fig. 10.3.

2585 10.2 Basic Concepts and Formulæ

2586 Alternating gradient focusing is sketched in Fig. 10.5.

2587 The focusing index value can be estimated from the fields met in these structures:
 2588 say a maximum $B \sim 1$ Tesla in the dipole gap, and as well at pole tip in quadrupoles

Fig. 10.5 Horizontally focusing lenses (field index $n \gg 0$, the solid red trajectory) are vertically defocusing ($n \ll 0$, the dashed blue trajectory), and vice versa. This imposes alternating gradients in order for a sequence to be globally focusing.



2589 ~ 10 cm off axis. The latter results in $\frac{\Delta B}{\Delta x} \sim 10$ T/m, the former in meters to tens of
2590 meters dipole curvature radius. All in all,

$$n = \frac{\rho}{B} \frac{\partial B}{\partial x} \sim \frac{10^{0-2} \text{ [m]}}{1 \text{ [T]}} \times 10 \text{ [T/m]} \sim 10^{1-3} \gg 1 \quad (10.3)$$

2591 much greater than in a weak focusing structure, characterized by $0 < n < 1$.

2592 10.2.1 Components of the Strong Focusing Optics

2593 Combined function (AG) optics

2594 This is, typically, the BNL AGS and CERN PS optics, using dipoles that ensure both
2595 beam guiding and focusing (Fig. 10.2). Separate quadrupole and multipole lenses
2596 have later been introduced in these lattices as they provide knobs for the adjustment
2597 of optical functions and parameters.

2598 AG optics is still at work in modern designs, as in the iRCMS whose six 60 deg
2599 arcs are comprised of a sequence of five focusing and defocusing combined function
2600 dipoles [6], Fig. 10.3.

2601 Field

Referring to the normal conducting magnet technology, an hyperbolic pole profile (Fig. 10.1) is an equipotential of equation

$$V(x, y) = Axy$$

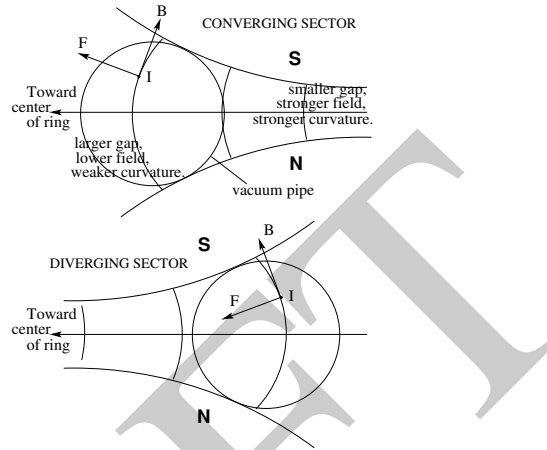
This pole profile therefore results in

$$B_y = \frac{\partial V}{\partial y} = Ax, \quad \text{i.e. a radial field index} \quad n = \frac{\rho}{B_y} \frac{\partial B_y}{\partial x} \Big|_{y=0} = \frac{\rho}{B_y} A$$

2602 responsible for the focusing. A is a constant, typically ~ 10 T/m in normal conducting
2603 magnets, cf. Eq. 10.3. (an order of magnitude greater with SC technology). The pole

2604 profile opens up either inward (toward the center of curvature, a horizontally focusing
2605 dipole, vertically defocusing) or outward (a vertically focusing dipole, horizontally
defocusing), Fig. 10.6.

Fig. 10.6 Beam focusing in combined function dipoles. The center of curvature is to the left. The pole profile follows an equipotential $V = Axy$. Top: the pole profile opens up towards the center of curvature → the dipole is horizontally converging (vertically diverging: current I comes out of the page, force F results from field B). Bottom: pole profile closing toward the center of curvature → the dipole is horizontally diverging, vertically converging



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In a bent AG dipole a line of constant field is an arc of a circle; the field guides the reference particle along the arc in the median plane. The mid-plane field can be expressed as

$$B_y(r, \theta) = \mathcal{G}(r, \theta) B_0 \left(1 + n \frac{r - r_0}{r_0} + n_2 \left(\frac{r - r_0}{r_0} \right)^2 + n_3 \left(\frac{r - r_0}{r_0} \right)^3 + \dots \right) \quad (10.4)$$

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with r_0 the reference radius. Higher order indices, sextupole n_2 , octupole n_3 , ..., may be residual effects: fabrication tolerance, saturation, magnetic permeability, deformation of yoke with years, ..., as in the AGS dipoles, or included by design.

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In a straight AG dipole a line of constant field is a straight line; an instance is the AGS main magnet (Fig. 10.2). Another instance is the Fermilab recycler arcs permanent magnet dipole, which includes quadrupole and sextupole components [12, 13]. The modeling of the field in a straight combined function dipole can be derived from the scalar potential of Eq. 10.5.

2618 Separated function optics

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In a separated function lattice main bends have zero index and ensure beam guiding, quadrupole lenses, alternately focusing and defocusing, ensure the essential of the focusing. In smaller rings though, bending contributes horizontal focusing (see Sect. 9.2.1.2, Fig. 9.6), wedge angles in addition may be introduced and contribute some horizontal and vertical focusing/defocusing (Fig. 9.9).

2624 Higher order multipole lenses are used for the compensation of adverse effects:
 2625 coupling, aberrations, space charge, impedance, etc., and for beam manipulations:
 2626 coupling, resonant extraction, etc.

2627 The field in a multipole of order n ($n = 1, 2, 3, \dots$: dipole, quadrupole, sextupole,
 2628 ...) derives, via $\mathbf{B} = \mathbf{grad}V$, from the Laplace potential [15]

$$V_n = (n!)^2 \left\{ \sum_{q=0}^{\infty} (-)^q \alpha_{n,0}^{(2q)}(s) \frac{(x^2 + y^2)^q}{4^q q!(n+q)!} \right\} \left\{ \frac{x^{n-m} y^m}{m!(n-m)!} \sin m \frac{\pi}{2} \right\} \quad (10.5)$$

2629 wherein $\alpha_{n,0}^{(2q)} = d^{2q} \alpha_{n,0} / ds^{2q}$ accounts for the s -dependence of the field. Tech-
 2630 nologies for multipoles and combined multipoles include pole profiling, permanent
 2631 magnets [12, 17], superconducting $\cos \theta$ windings as in RHIC and LHC colliders,
 2632 and variants of all sorts.

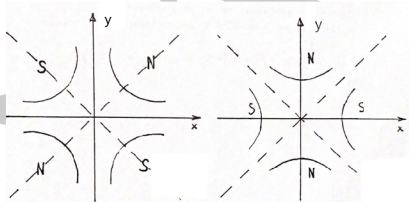
2633 In a hard-edge model the left sum in Eq. 10.5 is reduce to the $q = 0$ term, with
 2634 the following outcomes.

2635 *Quadrupole*

The equipotential (the pole profile) is an hyperbola: $Gxy = \text{constant}$, in an upright
 quadrupole (left), and $G(x^2 - y^2) = \text{constant}$ in a $\pi/4$ skewed quadrupole (right); the
 resulting field writes

$$B_x = \frac{\partial V}{\partial x} = Gy$$

$$B_y = \frac{\partial V}{\partial y} = Gx$$



$$B_x = Gx$$

$$B_y = -Gy$$

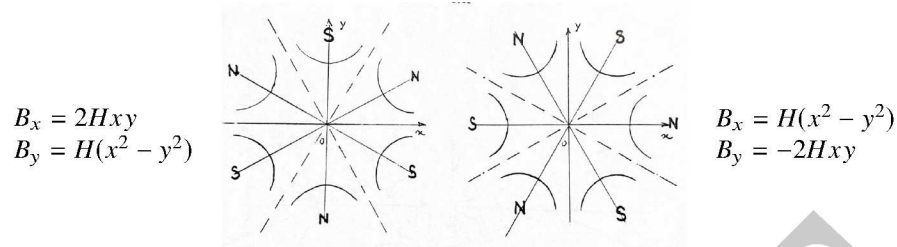
Upright quadrupoles are used for focusing, skew quadrupoles are used to compensate,
 or introduce, transverse coupling. Their focusing strength

$$K = \frac{1}{L} \frac{\int G(s) ds}{p/q}$$

2636 is momentum-dependent.

2637 *Sextupole*

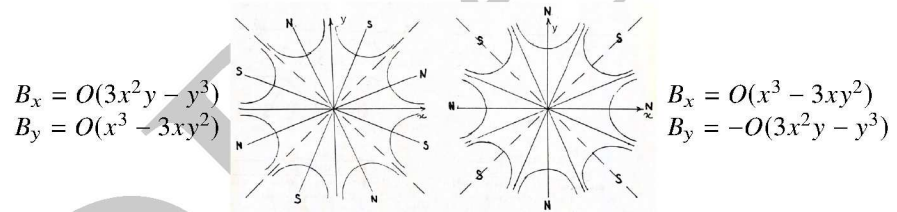
The equipotential satisfies $H(3x^2y - y^3) = \text{constant}$ in an upright sextupole (left),
 $H(x^3 - 3xy^2) = \text{constant}$ in a $\pi/6$ skewed sextupole (right), with resulting field



2638 Upright sextupoles introduce a vertical field component $B_y \propto x^2$, they are used
 2639 to correct optical aberrations, to modify the momentum dependence of the wave
 2640 numbers ν_x , ν_y , and in beam manipulations such as resonant extraction. Skew
 2641 sextupoles introduce a radial field component $B_x \propto y^2$, they are used to correct
 2642 optical aberrations.

2643 *Octupole*

The equipotential pole profile satisfies $O(x^3y - xy^3) = \text{constant}$ in an upright octupole
 (left), $O(x^4 - 6x^2y^2 + y^4) = \text{constant}$ in a $\pi/8$ skewed octupole (right), yielding the
 field



2644 Upright octupoles are used to introduce a vertical field component $B_y \propto x^3$; skew
 2645 octupoles introduce a vertical field component $B_y \propto y^3$. Octupoles are used to correct
 2646 aberrations, or to modify the amplitude dependence of wave numbers.

2647 **10.2.2 Transverse motion**

2648 The transverse motion of a particle in the periodic lattice of a cyclic accelerator
 2649 satisfies Hill's equations

$$\frac{d^2x}{ds^2} + K_x(s)x = \frac{1}{\rho_0} \frac{\Delta p}{p_0}, \quad \frac{d^2y}{ds^2} + K_y(s)y = 0 \quad (10.6)$$

2650 where $K_x(s)$, $K_y(s)$ have the periodicity of the lattice, and depend locally on the
 2651 nature of the optical elements:

$$\begin{aligned} \text{-- dipole :} & \quad \begin{cases} K_x = \frac{1-n}{\rho_0^2} \\ K_y = \frac{n}{\rho_0^2} \end{cases} \quad (n = -\frac{\rho_0}{B_0} \frac{\partial B_y}{\partial x}) \\ \text{-- a wedge at } s = s_w : & \quad \begin{cases} K_x \\ K_y \end{cases} = \pm \frac{\tan \varepsilon}{\rho_0} \delta(s - s_w) \quad (\text{with } \varepsilon \leq 0 \text{ for focusing defocusing}) \\ \text{-- quadrupole} & \quad (\text{gradient } G = \frac{\text{field at pole tip}}{\text{radius at pole tip}}) : K_x = \frac{\pm G}{B\rho}; \frac{1}{\rho_0} = 0 \\ \text{-- drift space :} & \quad K_x = K_y = 0; \frac{1}{\rho_0} = 0 \end{aligned} \quad (10.7)$$

2652 By contrast with the betatron and weak focusing technologies, strong focusing
 2653 with its independent focusing ($G > 0$) and defocusing ($G < 0$) families allows
 2654 separate adjustment of the horizontal and vertical focusing strengths, and wave
 2655 numbers as a consequence.

2656 The on-momentum ($p = p_0$) closed orbit coincides with the reference axis of
 2657 the optical structure. The betatron motion for an on-momentum particle satisfies
 2658 Eq. 10.6 with $\Delta p = 0$. Solving the latter (see Sect. 9.2) requires introducing two
 2659 independent solutions $u_{\frac{1}{2}}(s)$ (Eq. 9.12), the linear combination of which yields the
 2660 pseudo harmonic motion (Eq. 9.15)

$$\begin{cases} u(s) = \sqrt{\beta(s)\varepsilon/\pi} \cos\left(\int \frac{ds}{\beta(s)} + \varphi\right) \\ u'(s) = -\sqrt{\frac{\varepsilon/\pi}{\beta(s)}} \sin\left(\int \frac{ds}{\beta(s)} + \varphi\right) + \alpha(s) \cos\left(\int \frac{ds}{\beta(s)} + \varphi\right) \end{cases} \quad (10.8)$$

2661 The motion satisfies the Courant-Snyder invariant, namely (Fig. 9.10)

$$\gamma_u(s)u^2 + 2\alpha_u(s)uu' + \beta_u(s)u'^2 = \frac{\varepsilon_u}{\pi} \quad (10.9)$$

2662 The form and the orientation of this phase space ellipse change along the period,
 2663 more than in a weak focusing lattice as a consequence of the strong modulation of
 2664 the betatron functions (Fig. 10.7), a consequence of the strong focusing. The surface
 2665 of the ellipse does not change.

2666 Beam envelopes are given by the extrema,

$$\hat{x}_{\text{env}}(s) = \pm \sqrt{\beta_x(s) \frac{\varepsilon_x}{\pi}}, \quad \hat{y}_{\text{env}}(s) = \pm \sqrt{\beta_y(s) \frac{\varepsilon_y}{\pi}} \quad (10.10)$$

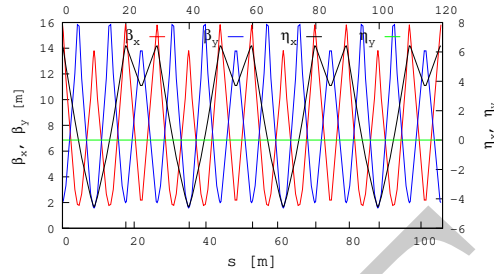


Fig. 10.7 Optical functions in SATURNE II synchrotron (see exercise 10.1)

2667 *Phase space motion*

2668 Write the two independent solutions $u_1(s)$ (Eq. 9.12) under the form

$$u_1(s) = \underbrace{F(s)}_{S\text{-periodic}} \times \underbrace{e^{i\mu \frac{s}{S}}}_{\frac{2\pi S}{\mu}\text{-periodic}} \quad \text{and} \quad u_2(s) = u_1^*(s) = F^*(s) e^{-i\mu \frac{s}{S}} \quad (10.11)$$

2669 wherein $F(s) = \sqrt{\beta(s)} e^{i \left(\int_0^s \frac{ds}{\beta(s)} - \mu \frac{s}{S} \right)}$. Introduce $\psi(s) = \int_0^s \frac{ds}{\beta(s)} - \mu \frac{s}{S}$ so that

2670 $F(s) = \sqrt{\beta(s)} e^{i\psi(s)}$, Eq. 10.8 thus takes the form

$$\begin{cases} u(s) = \sqrt{\beta(s)\varepsilon/\pi} \cos \left[\underbrace{\nu \frac{s}{R}}_{S\text{-periodic}} + \underbrace{\psi(s) + \varphi}_{\frac{2\pi S}{\mu}\text{-periodic}} \right] \\ u'(s) = -\sqrt{\frac{\varepsilon/\pi}{\beta(s)}} \sin \left[\nu \frac{s}{R} + \psi(s) + \varphi \right] + \alpha(s) \cos \left[\nu \frac{s}{R} + \psi(s) + \varphi \right] \end{cases} \quad (10.12)$$

2671 wherein $\nu = \frac{N\mu}{2\pi}$. Thus, as the betatron function $\beta(s)$ and phase $\psi(s)$ are S -periodic,
 2672 the turn-by-turn motion observed at a given azimuth s (i.e., $u(s)$, $u(s+S)$, $u(s+2S)$,
 2673 ...) is sinusoidal and its frequency is $\nu = N\mu/2\pi$. Successive particle positions
 2674 ($u(s)$, $u'(s)$) in phase space lie on the Courant-Snyder invariant (Eq. 10.9).

2675 The wave numbers ν_x and ν_y can be adjusted independently in a separated function
 2676 lattice, by means of two independent quadrupole families. The working point (ν_x, ν_y)
 2677 fully characterizes the first order optical setting of the ring.

2678 *Off-momentum motion*

2679 The motion of an off-momentum particle satisfies the inhomogeneous Hill's hori-
2680 zontal differential Eq. 10.6. The chromatic closed orbit

$$x_{\text{ch}}(s) = D_x(s) \frac{\delta p}{p} \quad (10.13)$$

2681 is a particular solution of the equation, its periodicity is that of the cell.

2682 By contrast with the weak focusing configuration, where the on-momentum closed
2683 orbit and chromatic closed orbits are parallel (Eq. 9.26: $D_x = \text{constant}$, independent
2684 of s), chromatic closed orbits in a strong focusing optical structure are distorted
2685 (Fig. 10.7), their excursion depends on the distribution along the cell of (i) the
2686 dispersive elements which are the dipoles, and (ii) the focusing.

2687 The horizontal motion of an off-momentum particle is a superposition of the par-
2688 ticular solution (Eq. 10.13) and of the betatron motion, solution of the homogeneous
2689 Hill's equation (Eq. 10.6 with $\delta p/p = 0$), namely

$$x(s) = x_\beta(s) + x_{\text{ch}}(s) = \sqrt{\beta_x(s) \frac{\varepsilon_x}{\pi}} \cos \left(\int \frac{ds}{\beta_x} + \varphi \right) + D_x(s) \frac{\Delta p}{p_0} \quad (10.14)$$

2690 whereas the vertical motion is unchanged (Eq. 10.12 taken for $u(s) \equiv y(s)$).

2691 **10.2.3 Resonances**

2692 Consider the excitation of transverse beam motion by a generator of frequency Ω
2693 located at some azimuth along the ring [18]. The action of the excitation $S \times \sin \Omega t$
2694 on the oscillating motion $u(t)$ can be written under the form

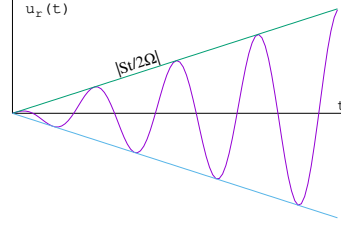
$$\frac{d^2 u}{dt^2} + \omega^2 u = S \sin \Omega t \quad (10.15)$$

2695 Assume harmonic motion for simplicity (as in a weak focusing lattice). Take S
2696 constant, the solution (superposition of the solution of the homogeneous differential
2697 equation and of a particular solution of the inhomogeneous differential equation)
2698 writes

$$u(t) = U \cos(\omega t + \varphi) + \frac{S}{\omega^2 - \Omega^2} \sin \Omega t \quad (10.16)$$

If betatron motion and excitation are in synchronism, *i.e.* on the resonance, $\omega = \Omega$,
a particular solution of Eq. 10.15 is

$$u_r(t) = -\frac{St}{2\Omega} \cos \Omega t$$



2699 the amplitude of the oscillatory motion grows rapidly with time, at a rate $|St/2\Omega|$.

Assume S periodic instead, take its Fourier expansion $S(t) = \sum_{p=0}^{\infty} a_p \cos(p\omega't + \varphi_p)$, the equation of motion thus writes

$$\frac{d^2u}{dt^2} + \omega^2 u = \sum_{p=0}^{\infty} a_p \cos(p\omega't + \varphi_p) \sin \Omega t =$$

$$\sum_{p=0}^{\infty} \frac{a_p}{2} \left[\sin[(\Omega - p\omega')t + \varphi_p] + \sin[(\Omega + p\omega')t + \varphi_p] \right]$$

2700 Resonance may occur at oscillator frequencies $\omega = \Omega \pm p\omega'$, their strength depends
2701 on the amplitude a_p of the excitation harmonics. If the generator is located at one
2702 point in the ring, it excites all harmonics.

2703 *Sextupole and octupole resonances*

2704 The horizontal motion in the presence of a sextupole component ($B_y(\theta)|_{y=0} = S(\theta)x^2$,
2705 see *Sextupole*, above) as part of the ring optical lattice satisfies

$$\frac{d^2x}{d\theta^2} + \nu_x^2 x = S(\theta)x^2 \quad (10.17)$$

Assume weak perturbation of the motion, so that $x(\theta) \approx \hat{x} \cos(\nu_x \theta + \varphi)$, the solution for unperturbed motion, and $S(\theta)$ 2π -periodic. Substitute its Fourier series expansion $S(\theta) = \sum_{p=0}^{\infty} a_p \cos(p\omega'\theta + \varphi_p)$ in Eq. 10.17, develop to get

$$\frac{d^2x}{d\theta^2} + \nu_x^2 x = \frac{\hat{x}^2}{2} \sum_{p=0}^{\infty} a_p \left[\cos(p\theta + \varphi_p) +$$

$$\cos[(p - 2\nu_x)\theta + \varphi_p - 2\varphi] + \cos[(p + 2\nu_x)\theta + \varphi_p + 2\varphi] \right]$$

Thus resonance may occur at betatron frequency families $\nu_x = \pm p$, $\nu_x = \pm(p - 2\nu_x)$, and $\nu_x = \pm(p + 2\nu_x)$, i.e.,

$$\begin{cases} \nu_x = \text{integer} \\ 3\nu_x = \text{integer} \end{cases}$$

2706 In the case of a single sextupole in the ring, all the harmonics p are excited with the
2707 same amplitude a_p .

An octupole perturbation introduces a field component $B_y(\theta)|_{y=0} = O(\theta)x^3$ (see *Octupole*, above) in the optical lattice. In a similar way, assume weak perturbation so that $x(\theta) \approx \hat{x} \cos(\nu_x \theta + \varphi)$, and to $O(\theta)$ substitute its Fourier expansion. This yields the resonant betatron frequencies

$$\begin{cases} \nu_x = \text{integer} \\ 2\nu_x = \text{integer} \\ 4\nu_x = \text{integer} \end{cases}$$

Resonances in a general manner occur at betatron frequencies satisfying

$$m\nu_x + n\nu_y = \text{integer}$$

with the property that

$$\frac{\varepsilon_x}{m} - \frac{\varepsilon_y}{n} = \text{constant}, \quad \text{an invariant of the motion}$$

2708 with the following consequences:

2709 - if m and n have opposite signs the resonance causes energy exchange between
2710 the horizontal and vertical motions: $\frac{\varepsilon_x}{|m|} + \frac{\varepsilon_y}{|n|} = \text{constant}$, an increase of ε_x correlates
2711 with a decrease of ε_y and vice-versa; in the presence of linear coupling for instance,
2712 $\nu_x - \nu_y = \text{integer}$, $\varepsilon_x + \varepsilon_y = \text{constant}$. An increase in motion amplitude anyway
2713 may cause particle loss, an issue in cyclotrons for instance where the Walkinshaw
2714 resonance $\nu_x = 2\nu_y$ causes vertical beam loss due to the increase of ε_y ;

2715 - if m and n have the same sign the resonance induces motion instability: $\frac{\varepsilon_x}{m} - \frac{\varepsilon_y}{n} =$
2716 constant, ε_x and ε_y may both increase with no limit.

2717 10.2.4 Synchrotron Motion

Particle motion in the longitudinal phase space (phase, momentum) is determined by the lattice and by the acceleration parameters. The synchrotron acceleration technique has been discussed in Sect. 9.2.2: acceleration parameters include RF voltage \hat{V} , frequency $f_{\text{rf}} = \omega_{\text{rf}}/2\pi = hf_{\text{rev}}$, the synchronous phase ϕ_s . The synchrotron angular frequency is

$$\Omega_s = (\omega_{\text{rev}}^2 |\eta| h_{\text{RF}} e \hat{V} \cos \phi_s / 2\pi E_s)^{1/2}$$

2718 with $\eta = 1/\gamma^2 - \alpha$ the phase slip factor (Eq. 9.33) and $\alpha = 1/\gamma_{\text{tr}}^2 = \frac{\Delta C}{C} \bigg/ \frac{\Delta p}{p_0}$ the
2719 momentum compaction. The transition gamma γ_{tr} is a property of the lattice and
2720 determines the proper synchronous phase region for acceleration, either $[0, \pi/2]$ or
2721 $[\pi/2, \pi]$ (Fig. 9.15).

2722 The bucket height, “momentum acceptance”, satisfies

$$\pm \frac{\Delta p}{p} = \pm \frac{1}{\beta} \sqrt{\frac{q\hat{V}}{\pi h \eta E_s} [-(\pi - 2\varphi_s) \sin \varphi_s + 2 \cos \varphi_s]} \quad (10.18)$$

2723 The maximum extent in phase for small amplitude oscillations satisfies

$$\pm \Delta\varphi_{\max} = \frac{h\eta E_s}{p_s R_s \Omega_s} \times \max\left(\frac{\Delta E}{E_s}\right) \quad (10.19)$$

2724 ***** separatrix *****

2725 The motion of a particle with energy offset $\delta E = E - E_s$ satisfies the longitudinal
2726 invariants

$$\epsilon_l = \frac{\alpha E_s}{2\Omega_s} \left[\left(\frac{\delta E}{E_s}\right)^2 + \frac{1}{\Omega_s^2} \left(\frac{d}{dt} \frac{\delta E}{E_s}\right)^2 \right] \quad (10.20)$$

2727

$$(\widehat{\delta E})^2 = (\delta E)^2 + \frac{1}{\Omega_s^2} \left(\frac{d\delta E}{dt}\right)^2 \quad (10.21)$$

2728 Introducing the squared *rms* relative synchrotron amplitude $\sigma_{\delta E/E}^2 \equiv (\widehat{\delta E}/E_s)^2$ this
2729 yields in addition

$$\epsilon_l = \frac{\alpha E_s}{2\Omega_s} \sigma_{\delta E/E}^2 \quad (10.22)$$

2730 10.2.5 Radiative Energy Loss

2731 A particle of rest mass m_0 and charge e traveling in a magnetic field is subject
2732 to stochastic photon emission, which causes energy loss [21]. The phenomenon
2733 involves two random processes:

2734 - the probability of photon emission over a trajectory arc δs , a Poisson law,

$$p(k) = \frac{\Lambda^k}{k!} e^{-\Lambda} \quad \text{with} \quad \Lambda = \langle k \rangle = \langle k^2 \rangle \quad (10.23)$$

2735 wherein k is the number of photons emitted over δs , $\Lambda = \frac{5er_0}{2h\sqrt{3}} B\rho \frac{\delta s}{\rho}$ is its average
2736 value, $r_0 = e^2 / 4\pi\epsilon_0 m_0 c^2$ is the classical radius of the particle, $\epsilon_0 = 1 / 36\pi 10^9$, \hbar is
2737 the Plank constant,

2738 - the energy ϵ of the photon(s), following the probability law

$$\mathcal{P}\left(\frac{\epsilon}{\epsilon_c}\right) = \frac{3}{5\pi} \int_0^{\epsilon/\epsilon_c} \frac{d\epsilon}{\epsilon_c} \int_{\epsilon/\epsilon_c}^{\infty} K_{5/3}(x) dx \quad (10.24)$$

2739 with $K_{5/3}$ the modified Bessel function, $\gamma = E/E_0$ with $E_0 = m_0 c^2$ the rest energy,
2740 and ϵ_c the critical energy of the radiation,

$$\epsilon_c = \frac{3\hbar\gamma^3 c}{2\rho} \quad (10.25)$$

2741 The average energy loss over δs is, assuming ultra-relativistic particles: $\beta = v/c \approx 1$,

$$\delta E = \frac{2}{3} r_0 E_0 \gamma^4 \frac{\delta s}{\rho^2} = \frac{2}{3} r_0 e c \gamma^3 B \frac{\delta s}{\rho} \approx \underbrace{1.88 \cdot 10^{-15}}_{\text{for electrons}} \gamma^3 \frac{\delta s}{\rho^2} \quad (10.26)$$

2742 The energy spread resulting from the stochastic emission is

$$\sigma_{\delta E/E} = \frac{\sqrt{110\sqrt{3}\hbar c / \pi\epsilon_0}}{24E_0/e} \gamma^{5/2} \frac{\sqrt{\delta s}}{\rho^{3/2}} \approx \underbrace{3.80 \cdot 10^{-14}}_{\text{for electrons}} \gamma^{5/2} \frac{\sqrt{\delta s}}{\rho} \quad (10.27)$$

2743 In a storage ring the RF system restores on average the energy lost by SR. Useful
2744 formulas are given in Tab. 10.1, in particular, assuming a flat ring the partition
2745 of energy between radial and longitudinal motions is determined by the partition
2746 numbers

$$J_x = 1 - \mathcal{D}, \quad J_y = 1, \quad J_l = 2 + \mathcal{D}, \quad \text{with } \mathcal{D} = \frac{\overline{D_x(1-2n)}/\rho^3}{\rho^2} \quad (10.28)$$

2747 where $\overline{(*)}$ denotes an average over the ring circumference.

2748 *Damping of accelerated motion*

2749 In an accelerator (a light source injector for instance), the RF voltage increases
2750 during acceleration in order to compensate the increasing energy loss. To first order
2751 in the invariant ϵ_u (with u standing for x or y) transverse damping in the presence
2752 of acceleration satisfies [16]

$$\frac{d\epsilon_u}{dt} = -\frac{2}{\tau_u(t)} \epsilon_u + C_u(t) - \frac{1}{p} \frac{dp}{dt} \epsilon_u, \quad \text{where } \tau_u^{-1} = J_u \frac{\overline{P}}{2E}, \quad \begin{cases} C_x = \frac{\overline{\mathcal{H} \dot{N} \langle \epsilon^2 \rangle}}{E^2} \\ C_y = \frac{B_y}{2\gamma^2} \frac{\overline{\dot{N} \langle \epsilon^2 \rangle}}{E^2} \end{cases} \quad (10.29)$$

2753 Longitudinal damping satisfies

$$\frac{d(\widehat{\delta E})^2}{dt} = -\frac{2(\widehat{\delta E})^2}{\tau_l(t)} + (\dot{N} \langle \epsilon^2 \rangle)(t) + \frac{(\widehat{\delta E})^2}{2E} \frac{dE_s}{dt} \quad \text{with } \tau_l^{-1} = J_l \frac{\overline{U_s}}{2E_s} \quad (10.30)$$

2754 ***** Figures ??, ?? display the evolution of horizontal and vertical emittance
2755 with time, respectively

Table 10.1 Radiation parameters^(a), energy loss and equilibrium quantities at the synchronous energy, E_s , in an isomagnetic ring

Critical photon energy, ϵ_c	keV	$\frac{3\hbar\gamma^3 c}{2\rho}$
Average photon energy, $\bar{\epsilon}$	keV	$\frac{8}{15\sqrt{3}} \epsilon_c$
rms energy spread, $\sqrt{(\epsilon - \bar{\epsilon})^2}$	keV	$\frac{\sqrt{211}}{15\sqrt{3}} \epsilon_c$
Energy loss, U_s	MeV / turn	$C_\gamma \frac{E_s^4}{\rho}$
Nb. of average photons	/turn/particle	$U_s / \bar{\epsilon}$
Longitudinal:		
equil. emittance, $\epsilon_{l,eq}$	$\mu\text{eV}\cdot\text{s}$	$\frac{\alpha E_s}{\Omega_s} \frac{C_q \gamma^2}{J_l \rho}$
rms energy spread, $\sigma_{\delta E/E}$		$\frac{1}{\sqrt{2}} \sigma_{\delta E/E} = \sqrt{\frac{C_q}{J_l \rho}} \gamma$
rms bunch length, σ_l	mm	$\frac{\alpha c}{\Omega_s} \sigma_{\frac{\delta E}{E}}$
Radial:		
equil. emittance, $\epsilon_{x,eq}$	nm	$= \frac{C_q \gamma^2}{J_x \rho} q \tilde{H}$
rms width, $\sigma_x(s)^{(b)}$	m	$\left(\beta_x(s) \epsilon_{x,eq} + D_x^2(s) \sigma_{\frac{\delta E}{E}}^2 \right)^{1/2}$
Damping times, $\tau_{x,y,l}$	ms	$\frac{T_{rev} E_s}{U_s J_{x,y,l}}$

(a) Units are, c: m/s; ρ : m; E_s : GeV

$$C_\gamma = \frac{4\pi}{3} \frac{r_0}{(m_0 c^2)^3} (= 8.846276 \cdot 10^{-5} \text{ m/GeV}^3 \text{ for electrons}).$$

$$C_q = \frac{55}{32\sqrt{3}} \frac{\hbar}{m_0 c} (= 3.8319386 \times 10^{-13} \text{ m for electrons}).$$

(b) With $\epsilon_{x,eq}, \beta(s)$ and dispersion $D_x(s)$ in meter.

$$\bar{\epsilon}_x(t) = \epsilon_{x,0} \left(e^{t/|\tau_x|} - 1 \right), \quad \bar{\epsilon}_y(t) = \epsilon_{y,i} e^{-t/\tau_y} \quad (10.31)$$

with $\epsilon_{x,0}$ a constant and $\epsilon_{y,i}$ an initial value.

10.2.6 Depolarizing resonances

By contrast with weak focusing optics where depolarizing resonances are weak because horizontal field components are weak (Sect. 9.2.3), the use of strong focusing field gradients in the combined function magnets and/or focusing lenses of strong focusing optics results in strong radial field components and therefore strong depolarizing resonances.

Spin precession and resonant spin motion in the magnetic components of a cyclic accelerator have been introduced in Sects. 4.2.5, 5.2.5. The general conditions for depolarizing resonance to occur have been introduced in Sect. 9.2.3. In a strong

2766 focusing synchrotron they essentially result from the radial field components in the
2767 focusing magnets and their strength is determined by the lattice optics, as follows.

2768 *Strength of imperfection resonances*

2769 Imperfection, or integer, depolarizing resonances are driven by a non-vanishing
2770 vertical closed orbit $y_{co}(\theta)$ which causes spins to experience periodic radial fields in
2771 focusing magnets, dipoles in combined function lattices and quadrupoles in separated
2772 function lattices, namely,

$$B_x(\theta) = G y(\theta) = K(\theta) \times B_0 \rho_0 \times y_{co}(\theta) \quad (10.32)$$

with θ the orbital angle, $B_0 \rho_0$ the lattice rigidity and $y_{co}(\theta)$ the closed orbit excursion.
Resonance occurs if the spin undergoes an integer number of precessions over a turn
(it then undergoes 1-turn-periodic torques), so that spin tilts at field perturbations
along the closed orbit add up coherently. Thus resonances occur at integer values

$$G\gamma_n = n$$

A Fourier development of these perturbative fields yields the strength of the $G\gamma_n$
harmonic [23, Sect. 2.3.5.1]

$$\epsilon_n^{\text{imp}} = (1 + G\gamma) \frac{R}{2\pi} \oint K(\theta) y_{co}(\theta) e^{-jG\gamma(\theta - \alpha)} e^{jn\theta} d\theta$$

2773 In the thin-lens approximation, near the resonance where $G\gamma - n \rightarrow 0$, this simplifies
2774 into a series over the quadrupole fields,

$$\epsilon_n^{\text{imp}} = \frac{1 + G\gamma_n}{2\pi} \sum_{\text{Qpoles}} [\cos G\gamma_n \alpha_i + \sin G\gamma_n \alpha_i] (KL)_i y_{co}(\theta_i) \quad (10.33)$$

2775 with θ_i the quadrupole location, $(KL)_i$ the integrated strength (slice the dipoles as
2776 necessary in an AG lattice for this series to converge) and α_i the cumulated orbit
2777 deviation.

2778 Orbit harmonics near the betatron tune ($n = G\gamma_n \approx \nu_y$) excite strong resonances.
2779 Imperfection resonance strength is further amplified in P-superperiodic rings, with
2780 m-cell superperiods, if the betatron tune $\nu_y \approx \text{integer} \times m \times P$ [24, Chap.3-I].

2781 *Strength of intrinsic resonances*

2782 Intrinsic depolarizing resonances are driven by betatron motion, which causes spins
2783 to experience strong radial field components in quadrupoles, namely

$$B_x(\theta) = G y(\theta) = K(\theta) \times B_0 \rho_0 \times y_\beta(\theta) \quad (10.34)$$

The effect of resonances on spin depends upon betatron amplitude and phase, their effect on beam polarization depends on beam emittance. Longitudinal fields from dipole ends are usually weak by comparison and ignored. The location of intrinsic resonances depends on betatron tune, it is given in an M-periodic structure by

$$G\gamma_n = nM \pm \nu_y$$

A Fourier development of the perturbative fields yields the two families of strengths [23, Sect. 2.3.5.2]

$$\epsilon_n^{\text{intr}\pm} = \frac{\lambda_x \rho_0}{4\pi} \int_0^{2\pi} K(\theta) \sqrt{\beta_y(\theta)} \frac{\epsilon_y}{\pi} e^{\pm j \left(\int_0^{s(\theta)} \frac{ds}{\beta_y} - \nu_y \theta \right)} e^{-jG\gamma(\theta - \alpha(\theta))} e^{jn\theta} d\theta$$

2784 In the thin-lens approximation, near the resonance where $G\gamma \pm \nu_y - n \rightarrow 0$, this
2785 simplifies into a series over the quadrupole fields,

$$\begin{Bmatrix} \mathcal{R}e(\epsilon_n^{\text{intr}\pm}) \\ j \mathcal{I}m(\epsilon_n^{\text{intr}\pm}) \end{Bmatrix} = \frac{1 + G\gamma_n}{4\pi} \sum_{\text{Qpoles}} \begin{Bmatrix} \cos(G\gamma_n \alpha_i \pm \varphi_i) \\ j \sin(G\gamma_n \alpha_i \pm \varphi_i) \end{Bmatrix} (KL)_i \sqrt{\beta_{y,i}} \frac{\epsilon_y}{\pi} \quad (10.35)$$

2786 10.3 Exercises

2787 In complement to the present exercises, an extensive tutorial on depolarizing res-
 2788 onances in a strong focusing synchrotron, considering proton, helion, or electron
 2789 beams, using the lattice of the AGS Booster at BNL, can be found in Ref. [23,
 2790 Chap. 14]. The simulations include the use of tune-jump quadrupoles, a solenoid,
 2791 Siberian snakes, spin rotators in an electron ring and their effect on polarization life
 2792 time.

2793 10.1 Construct SATURNE II synchrotron. Spin Dynamics With Snakes

2794 Solution: page 361

2795 Over the years 1978-1997 the 3 GeV synchrotron SATURNE II at Saclay
 2796 (Fig. 10.8) delivered ion beams up to 1.1 GeV/nucleon, including polarized proton,
 2797 deuteron and ${}^6\text{Li}$ beams, for intermediate energy nuclear physics research, including
 2798 meson production [19, 20]. The separated function synchrotron was designed *ab*
 2799 *initio* for the acceleration of polarized beams [22], and the first strong focusing syn-
 2800 chrotron to do so - ZGS, first to accelerate polarized beams, protons and deuterons,
 2801 was a weak focusing synchrotron (see Chap. 9).

2802 SATURNE II is a FODO lattice with missing dipole. Its parameters are given in
 2803 Tab. 10.2.

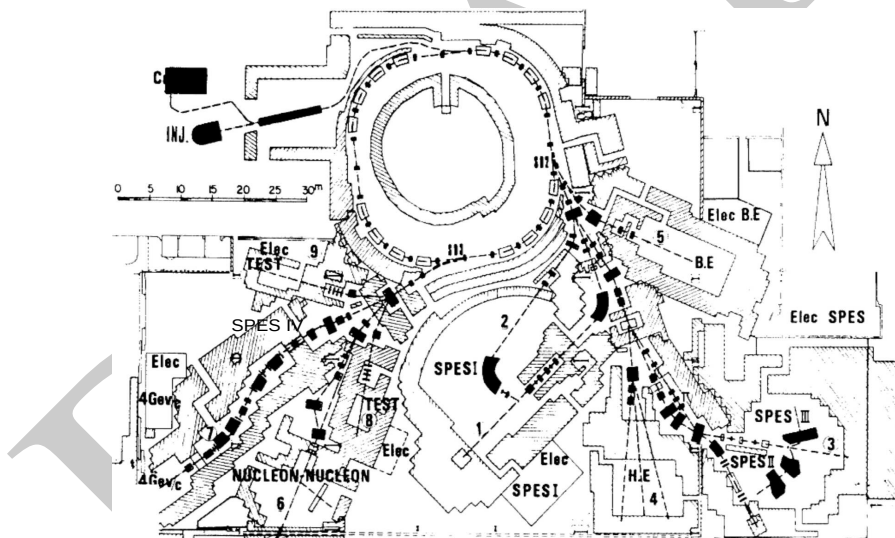


Fig. 10.8 SATURNE II synchrotron and its experimental areas [25], including mass spectrometers SPES I to SPES IV, a typical 1960-80s nuclear physics accelerator facility. Polarized ion sources are on the top left, followed by a 20 MeV linac

(a) Simulate the main dipole using BEND, include fringe fields assuming $\lambda = 8$ cm extent and the following Enge coefficient values (Eq. 15.13, Sect. 15.2.6):

Table 10.2 Parameters of SATURNE II separated function FODO lattice. ρ_0 denotes the reference bending radius in the main dipole; the reference orbit, wave numbers, etc., are taken along that radius

Orbit length, C	m	105.5556
Average radius, $R = C/2\pi$	m	16.8
Length of long straight section	m	
Wave numbers, $\nu_x; \nu_y$		3.64; 3.60
Chromaticities, $\xi_x; \xi_y$		negative, a few units
Momentum compaction α		0.015
Injection energy (proton)	MeV	20
Top energy	GeV	3
\dot{B}	T/s	4.2
Synchronous energy gain	keV/turn	1.160
RF harmonic		2
Dipole:		
- bend angle, α	deg	$\pi/8$
- magnetic length, $\rho\alpha$	m	2.489
- magnetic radius, ρ	m	6.3381
- wedge angle, ε	deg	2.45
Quadrupole:		
- gradient	T/m	0.5 - 10.56
- magnetic length F/D	m	0.46723 / 0.486273

$$C_0 = 0.2401, C_1 = 1.8639, C_2 = -0.5572, C_3 = 0.3904, C_4 = C_5 = 0$$

2804 Produce a graph of the field across the dipole along the reference orbit, in the median
2805 plane and at 5 cm vertical distance. Produce the transport matrix, check against
2806 theory. Compare with the matrix of the hard edge model.

2807 Simulate the F and D quadrupoles, using respectively QUADRUPOLE and MUL-
2808 TIPOL. Compare matrices with theory.

2809 Construct the cell. Produce machine parameters (tunes, chromaticities), check
2810 against data, Tab. 10.2.

2811 Construct the 4-cell ring. Produce a graph of the optical functions.

2812 (b) Accelerate a bunch with Gaussian densities comprised of a few tens of particles
2813 (it can be defined using MCOBJET), from injection to top energy; use harmonic 3
2814 RF frequency, and (unrealistic, for a reduced number of turns) peak RF voltage
2815 $\hat{V} = 1$ MV.

2816 Produce a graph of the three phase spaces. Check the transverse betatron damping.

2817 (c) Simulate multiturn injection in the ring. Take the injection point at the center
2818 of a long straight section.

2819 (d) Simulate resonant extraction from the ring, on $\nu_x = 11/3$. Take the extraction
2820 point at the center of a long straight section.

2821 10.2 Depolarizing Resonances In SATURNE II

2822 The input data file to simulate the ring is given in Tab. 17.73, an outcome of
2823 exercise 10.1.

2824 (a) Calculate the strength of the intrinsic depolarizing resonances (systematic and
2825 non-systematic) over 0.5-3 GeV, using Eq. 10.35.

2826 (b) $G_{\gamma} = 7 - \nu_y$ was found to be a potentially harmful depolarizing resonance
2827 - unexpectedly as this is not a systematic resonance. Produce a crossing of that
2828 resonance, for a 100-particle bunch. Get its strength from this simulation, compare
2829 with (a).

2830 (c) Multiple resonance xing - ref to Phys. Rev. article ***

2831 **10.3 Cornell electron RCS. Radiative Energy Loss**

2832 Short intro energy loss by synchrotron radiation [26]

2833 Tab.: RCS parameter list

2834 (a) Cornell RCS parameters are given in Tab. ???. Construct the ring, produce its
2835 optical parameters. Produce a graph of the optical functions.

2836 (b) Raytrace a few tens of particles over 3000 turns in Cornell RCS, from ***
2837 to *** GeV. Assume emittances $\epsilon_x = \epsilon_y = \epsilon_z$, Gaussian densities, initial *rms*
2838 $\delta p/p = 10^{-4}$. Produce a graph of the three phase spaces. produce graphs of horizontal
2839 and vertical transverse excursions versus turn number.

2840 (c) Re-do (b) with synchrotron radiation energy loss.

2841 (d) Produce the average beam polarization obtained in (c).

2842 (c) Multiple resonance crossing.

2843 **References**

- 2844 1. Christofilos, Nicholas: Focussing system for ions and electrons. US Patent Office Application
2845 filed March 10, 1950, Serial No. 148,920.
2846 <https://patentimages.storage.googleapis.com/fa/bb/52/0ce28e28b492a6/US2736799.pdf>
2847 2. Courant, Ernest D., Livingston, M. Stanley, and Snyder, Hartland S.: The Strong-Focusing
2848 Synchrotron - A New High Energy Accelerator. Phys. Rev. 88, 1190 - December 1952
2849 3. Courant, E.D., and Snyder, H.S.: Theory of the Alternating-Gradient Synchrotron. Annals of
2850 Physics, No. 3 (1958), 1-48
2851 4. Credit: Brookhaven National Laboratory.
2852 <https://www.flickr.com/photos/brookhavenlab/8495311598/in/album-72157611796003039/>
2853 5. Agapov, I., et al.: Future Circular Lepton Collider FCC-ee: Overview and Status. Submitted
2854 on 15 Mar 2022; arXiv:2203.08310 [physics.acc-ph].
2855 <https://doi.org/10.48550/arXiv.2203.08310>
2856 6. Méot, F., et al.: Progress on the optics modeling of BMITs ion rapid-cycling medical
2857 synchrotron at BNL. THPMP050, 10th Int. Particle Accelerator Conf. IPAC2019, Melbourne,
2858 Australia. <https://accelconf.web.cern.ch/ipac2019/papers/thmp050.pdf>
2859 Copyrights under license CC-BY-3.0, <https://creativecommons.org/licenses/by/3.0/>; no change
2860 to the material
2861 7. Nishimori, N.: A new compact 3 GeV light source in Japan. 13th Int. Particle Acc. Conf.
2862 IPAC2022, Bangkok, Thailand.
2863 <https://accelconf.web.cern.ch/ipac2022/papers/thixsp1.pdf>
2864 8. Méot, F.: eRHIC ERL modeling in Zgoubi. BNL-111832-2016-TECH; EIC/49;BNL-111832-
2865 2016-IR.
2866 <https://technotes.bnl.gov/PDF?publicationId=38865>
2867 9. Radial Focusing in the Linear Accelerator. Phys. Rev. Vol. 88, Num. 5, Dec. 1, 1952
2868 10. Benabderrahmane, C.: Status of the ESRF-EBS magnets. WEPMK009, 9th International
2869 Particle Accelerator Conference, IPAC2018, Vancouver, BC, Canada.
2870 <https://accelconf.web.cern.ch/ipac2018/papers/wepmk009.pdf>
2871 11. Credit: Lawrence Berkeley National Laboratory. The Regents of the University of California,
2872 Lawrence Berkeley Laboratory."
2873 12. Jackson, G., Editor: Fermilab recycler ring technical design report. Rev. 1.1. FERMILAB-
2874 TM-1981 (July 1996).
2875 <http://inspirehep.net/record/424541/files/fermilab-tm-1981.PDF>
2876 13. Méot, F.: On the Effects of Fringe Fields in the Recycler Ring. FERMILAB-TM-2016
2877 (Aug. 1997).
2878 <http://inspirehep.net/record/448603/files/fermilab-tm-2016.PDF>
2879 14. Ahrens, L., et al.: Development of a stepwise ray-tracing based on-line model at the AGS.
2880 WEP141, Proceedings of 2011 Particle Accelerator Conference, New York, NY, USA.
2881 <https://accelconf.web.cern.ch/PAC2011/papers/wep141.pdf>
2882 15. Leleux, G.: Compléments sur la Physique des Accélérateurs. DEA "Physique et Technologie
2883 des Grands Instruments", Université Paris VI. Rapport interne LNS//86-101, CEA Saclay
2884 (1986)
2885 16. Leleux, G.: Synchrotron radiation. DEA "Physique et Technologie des Grands Instruments",
2886 Université Paris VI. Rapport interne LNS, CEA Saclay (1991)
2887 17. F. Méot, et al.: Beam dynamics validation of the Halbach Technology FFAG Cell for Cornell-
2888 BNL Energy Recovery Linac. Nuclear Inst. and Methods in Physics Research, A 896 (2018)
2889 60-67
2890 18. Leleux, G.: Accélérateurs Circulaires. INSTN lectures, internal report CEA Saclay (1978),
2891 unpublished
2892 19. The 20 Years of the Synchrotron SATURNE-2. In: Proceedings of the Colloquium, Paris,
2893 France, 04 - 05 May 1998, A. Boudard and P.-A. Chamouard Editors. Edited By CEA -
2894 Laboratoire National SATURNE & CEN Saclay, France.
2895 <https://doi.org/10.1142/3965>

- 2896 20. Plus d'anneaux autour de SATURNE (pp. 33-34) Published in: Courrier CERN Volume 39,
2897 N° 2, Mars 1999.
2898 <https://cds.cern.ch/record/1740121>
- 2899 21. Hofmann, A.: The Physics of Synchrotron Radiation. Cambridge Monographs on Particle
2900 Physics, Nuclear Physics and Cosmology (20), Cambridge University Press (2004)
- 2901 22. E. Grorud, J.L. Laclare and G. Leleux: Crossing of Depolarization Resonances in Strongly
2902 Modulated Structures. IEEE Transactions on Nuclear Science, Vol. NS-26, NO. 3, June 1979.
2903 https://accelconf.web.cern.ch/p79/PDF/PAC1979_3209.PDF
- 2904 23. Méot, F.: Polarized Beam Dynamics and Instrumentation in Particle Accelerators, USPAS
2905 Summer 2021 Spin Class Lectures, Springer Nature, Open Access (2023).
2906 <https://link.springer.com/book/10.1007/978-3-031-16715-7>
- 2907 24. Lee, S. Y.: Spin Dynamics and Snakes in Synchrotrons. World Scientific (1997)
- 2908 25. Akin, J.P., et al.: Status report on rejuvenating SATURNE and future aspects. PAC 1979
2909 Conference. IEEE Tans. Nucl. Sci., Vol. NS 26, No. 3, June 1979.
2910 https://accelconf.web.cern.ch/p79/PDF/PAC1979_3138.PDF
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2913 has been added on the picture
- 2914 26. D. L. Rubin, et al.: Upgrade of the Cornell electron storage ring as a synchrotron light source.
2915 WEPOB36, Proceedings of NAPAC2016, Chicago, IL, USA.
2916 <https://accelconf.web.cern.ch/napac2016/papers/wepob36.pdf>