Homework 16 **Due Nov. 17, 2021**

1. (5 points) Show that for $\hat{C} << 1$, the eigenvalue of the growing mode for the 1-D FEL (cold beam) can be approximated as

$$\lambda = a_0 + a_1\hat{C} + a_2\hat{C}^2$$

with

$$a_0 = \frac{\sqrt{3}}{2} + i\frac{1}{2} \ ,$$

$$a_1 = -i\frac{2}{3} \ ,$$

and

$$a_2 = -\frac{1}{9}\left(\frac{\sqrt{3}}{2} - i\frac{1}{2}\right) \ .$$

Hint: Insert the expansion, $\lambda = a_0 + a_1\hat{C} + a_2\hat{C}^2$, into the polynomial equation for the eigenvalues in a cold beam (lecture slide # 10) and request coefficients of $\hat{C}^0$, $\hat{C}^1$ and $\hat{C}^2$ to vanish.

2. (5 points) Assuming the saturation of a FEL takes place at the condition (slides 32)

$$\Omega_{p,sat} L_G \approx 1 ,$$

where $\Omega_{p,sat} = \sqrt{\dfrac{eE_{sat}\theta_s\omega}{\gamma_z^2 c\mathcal{E}_0}}$ is the small-amplitude angular frequency of an electron oscillating in the radiation fields, E_{sat} is the amplitude of the radiation field at saturation and $L_G = \dfrac{1}{\sqrt{3}\Gamma}$ is the 1-D gain length of the radiation power, show that the radiation power at saturation is given by

$$P_{sat} = \varepsilon_0 c E_{sat}{}^2 A = \chi \cdot \rho \cdot \frac{\mathcal{E}_0}{e} I_e ,$$

where A is the cross-section of the radiation fields (which is equal to the cross-section of the electron beam for 1-D model) and I_e is the peak current of the electron beam, find the numerical coefficient χ.