USPAS'23 | Hadron Beam Cooling

Optical Stochastic Cooling

Irina Petrushina, Vladimir N. Litvinenko, Yichao Jing, Gang Wang, Jun Ma

Lecture#3:

- Introduction to OSC: general idea & OSC basics
- Transit-time method of OSC
- Experimental realization of OSC

References:

- For more information, please refer to the following literature:

[1] Mikhailichenko, A. A., and M. S. Zolotorev. "Optical stochastic cooling." *Physical Review Letters* 71.25 (1993): 4146.
[2] Zolotorev, M. S., and A. A. Zholents. "Transit-time method of optical stochastic cooling." *Physical Review E* 50.4 (1994): 3087.
[3] Lebedev, V. A. "Optical stochastic cooling." *ICFA Beam Dyn. Newslett* 65 (2014): 100-116.
[4] Jarvis, J., Lebedev, V., Romanov, A. et al. Experimental demonstration of optical stochastic cooling. Nature 608, 287–292 (2022). https://doi.org/10.1038/s41586-022-04969-7

Why do we need Optical Stochastic Cooling?

Traditional SC is based on sampling of random fluctuations → the bandwidth of the integrated system and the particle density in the beam determine the cooling rate.

Goal: extend SC to optical frequencies with a subsequent increase of the bandwidth ($\sim 10^{13}$ Hz) to improve cooling rate by 3-4 orders of magnitude and enable cooling of high-density protons with energies of 0.25 - 4 TeV.

This goal can be achieved using two methods:

- ***Coherent electron Cooling (CeC)*** that uses electron beam as the pick-up, amplifier and kicker.
  You will learn about it tomorrow.

- ***Optical Stochastic Cooling (OSC)*** that uses free space EM waves as the signaling medium, undulators to couple the radiation to the circulating particle beam and optical amplifiers for signal amplification.

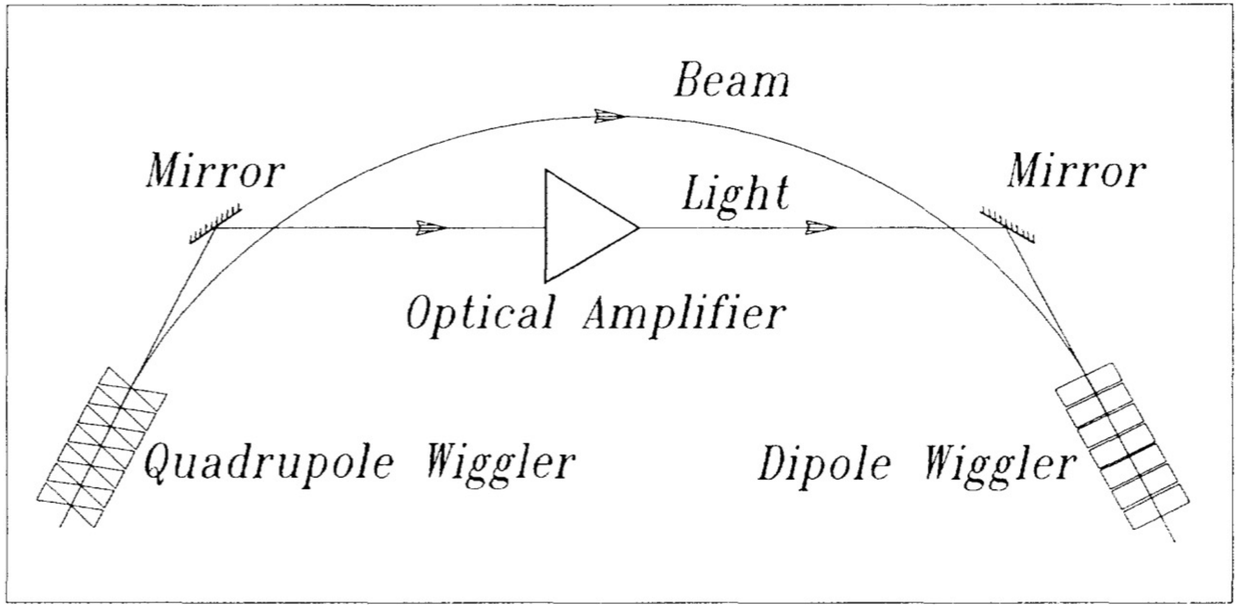
Brookhaven National Laboratory

First proposition of the idea of Optical Stochastic Cooling (OSC)

VOLUME 71, NUMBER 25 PHYSICAL REVIEW LETTERS 20 DECEMBER 1993

Optical Stochastic Cooling

A. A. Mikhailichenko* and M. S. Zolotorev



- **Pick-Up:**
  - Use a sequence of quadrupoles with changing polarity (Quadrupole Wiggler - QW).
  - Radiation in a quadrupole is defined by fluctuation of the current density in a cross-section.
  - The number of FODO periods will define the bandwidth of the radiation.
  - Polarization of radiation allows to distinguish between the transverse directions.
- **Amplifier:**
  - Optical amplifiers deliver bandwidth $\Delta f/f$ in the range of 10% to 20% and normally operate at central wavelength $\lambda \simeq$ 0.3-1 um.
- **Kicker:**
  - A dipole wiggler with corresponding period can be used as a longitudinal kicker, if it's installed at a point of non-zero dispersion with the correct betatron phase shift relative to the QW.

Brookhaven National Laboratory

Radiation from a quadrupole wiggler: part 1

For a steady current of particles passing through a quadrupole field wiggler, for each particle with a given transverse position, there exists another particle which is accelerated in the opposite direction. This process yields destructive radiation interference in the forward direction. Hence, radiation in a quadrupole field is defined by the fluctuation of current density in a cross section.

The undulatory factor of a QW with period of *2L*: $K = \dfrac{eH_{\perp}2L}{2\pi mc^2}$

The number of photons radiated by one particle: $\Delta N_{\gamma} \approxeq -\dfrac{4e^2}{\hbar c}\dfrac{K^2}{1+K^2}$

The energy of a quanta:

$$E_{\gamma} \approxeq \frac{2\pi\hbar c\gamma^2}{L(1+K^2+\gamma^2\theta^2)}$$

angle of observation

The number of photons radiated by a particle with amplitude *A* in the relative bandwidth $\Delta f/f \simeq 1/M$:

amplitude after cooling

$$\Delta N_{\gamma} \simeq \alpha\,(A/A_0)^2\,,\ \alpha = -e^2/\hbar c$$

amplitude before cooling

Brookhaven National Laboratory

Radiation from a quadrupole wiggler: part 2

For a bunch with *N* particles and length l_b, the number of particles in a bandwidth: $\Delta N_u \simeq MN(\lambda_u/l_b) \simeq MN(L/l_b)/\gamma^2$

The number of photons in the bandwidth is defined by fluctuations of the centroid of charge from these particles:

emittance after cooling

$$\Delta N_\gamma^u \simeq \Delta N_\gamma N_u \simeq \frac{1}{3}\alpha N_u \frac{\varepsilon}{\varepsilon_0}$$

emittance before cooling

The total number of radiated photons in the bandwidth:

$$\Delta N^t \simeq \frac{1}{3}\alpha N \frac{\varepsilon}{\varepsilon_0}$$

Note:

- During damping, factor K decreases → wavelength shifts $\Delta\lambda \simeq 1/(1+K^2)$
- Beam envelope in the wiggler. Can be dynamically changed to restore K.

Necessary energy change and amplification

The energy carried by the photons produced by a beam whose center if off by $\Delta x \simeq A/(N_u)^{1/2}$:

$$\Delta E_\gamma \simeq E_\gamma N^t \simeq 2\left(\frac{e^2}{L}\right) N\gamma^2 (A/A_0)^2$$

We need to damp this amplitude with a kicker. Use longitudinal kicker—a transverse displacement provided by an energy change at a lattice position with nonzero dispersion. We must provide a kick:

$$\Delta E/E \simeq \Delta x/\eta$$

Assume a kicker undulator with undulatory factor, *P*, and the same number of periods as QW, *M*:

$$\eta \frac{eE_\perp PL}{E\gamma} M = A/\sqrt{N_u}$$

The energy that must be contained in the photon radiation:

cross section of the emitted radiation at the IP

$$\mathcal{E} = \frac{c}{4\pi} E_\perp^2 \mathcal{L} \frac{l_b}{c} \simeq \frac{c}{4\pi} \frac{A^2}{N_u} \left(\frac{E\gamma}{ePL\eta M}\right)^2 \mathcal{L} \frac{l_b}{c}$$

Comparing with energy radiated in QW, we obtain amplification factor:

$$\kappa \simeq \sqrt{\mathcal{E}/\Delta E_\gamma} \simeq \frac{1}{4} \frac{\varepsilon_{||}}{r_0} \frac{1}{N} \frac{\Delta f}{f}, \;\; r_0 = e^2/mc^2, \;\; \varepsilon_{||} = \gamma l_b \Delta E/E$$

Brookhaven National Laboratory

Necessary energy change and amplification: example

Consider $N \approxeq 1 \times 10^{10},\ l_b \approxeq 15\ \mathrm{cm},\ M = 5,\ \Delta E/E \approxeq 10^{-3},\ \gamma \approxeq 10^3$

$$\kappa \simeq \sqrt{\mathcal{E}/\Delta E_\gamma} \simeq \frac{1}{4}\frac{\varepsilon_{||}}{r_0}\frac{1}{N}\frac{\Delta f}{f},\ r_0 = e^2/mc^2,\ \varepsilon_{||} = \gamma l_b \Delta E/E$$

We obtain $\kappa \approxeq 3 \times 10^2$ for an electron damping, which is well below $\kappa \approxeq 10^5$ for microwave SC

The required pulsed power is about 5 kW at an average power of 25 W, assuming a repetition rate of 10 MHz.
→ no apparent limitation for the design of an amplifier to effectively cool positron/electron beams.

Note: the amplification factor for protons with the same γ will increase as r_p/r_0

Damping time and temperature

Change of emittance per turn: $$\frac{\mathrm{d}\varepsilon}{\mathrm{d}n} + \frac{\varepsilon}{N_u} = 0$$

So the number of revolutions is approximately N_u.

- The process of cooling will stop when the number of radiated quanta is of the order unity.
- The theoretical limit for the noise of the amplifier is of the same order, i.e., one photon in the coherence volume.
- This corresponds to one photon per sample which has N_u electrons in it.
- This gives a decrease in emittance equal to $\varepsilon_f/\varepsilon_0 \approx 1/(\alpha N_u)$

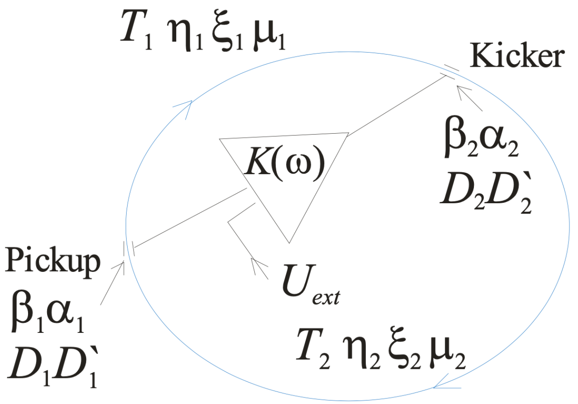
For our earlier example, the reduction is 10^3.

- The damping time: $\tau \approx N_u T = \dfrac{N}{(\Delta f/f)}(\lambda_u/l_b)T$ 10 ms for 1/T = 10 MHz.

Note:
- For cooling of the energy spread, the QW must be installed in a place where the beam size is mostly defined by the energy spread.
- For both transverse emittances and energy spread cooling in the same time it is necessary to arrange the bend between the pickup and the kicker so that the betatron phase shift must be more than $\pi/2$.
- This makes the bend not isochronous and yields the existence of a threshold emittance!

OSC basics: Transfer Matrix

In the proposed system, the efficiency of the transverse kick is lower compared to the longitudinal kick→ we will rely on **longitudinal kick** and **plane-to-plane coupling**.



Let's first discuss **coupled horizontal and longitudinal motion**.

Consider that vertical motion is uncoupled and can be omitted for this example.

Transfer matrix between pick-up and kicker:

$$\mathbf{M} = \begin{bmatrix} M_{11} & M_{12} & 0 & M_{16} \\ M_{21} & M_{22} & 0 & M_{26} \\ M_{51} & M_{52} & 1 & M_{56} \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x \\ \theta_x \\ s \\ \Delta p/p \end{bmatrix}$$

with

$$M_{11} = \sqrt{\frac{\beta_2}{\beta_1}}(\cos\mu + \alpha_1 \sin\mu), \qquad M_{12} = \sqrt{\beta_1\beta_2}\sin\mu$$

$$M_{21} = \frac{\alpha_1 - \alpha_2}{\sqrt{\beta_1\beta_2}}\cos\mu - \frac{1+\alpha_1\alpha_2}{\sqrt{\beta_1\beta_2}}\sin\mu, \quad M_{22} = \sqrt{\frac{\beta_1}{\beta_2}}(\cos\mu - \alpha_2\sin\mu)$$

Matrix elements describing *x-s* coupling are bound up by motion symplecticity (remember the first day of lectures?):

$$\mathbf{M}^T\mathbf{U}\mathbf{M} = \mathbf{U}$$

Then one obtains:

$$M_{16} = D_2 - M_{11}D_1 - M_{12}D_1'$$
$$M_{26} = D_2' - M_{21}D_1 - M_{22}D_1'$$
$$M_{51} = M_{21}M_{16} - M_{11}M_{26}$$
$$M_{52} = M_{22}M_{16} - M_{12}M_{26}$$

Similar to a ring slip-factor, we can introduce P-to-K slip factor:

ring slip-factor $\eta = 1/\gamma^2 - \alpha$

P-to-K slip-factor $\eta_{12} = \dfrac{M_{51}D_1 + M_{52}D_1' + M_{56}}{2\pi R}$

describes the longitudinal displacement for a particle with momentum deviation $\Delta p/p$ in the absence of betatron oscillations

Brookhaven National Laboratory

OSC basics: Damping Rates

The linearized longitudinal kick to a particle due to its interaction with its own amplified radiation in the kicker:

$$\delta p/p = \kappa k \left(M_{51}^{(1)} x^{(1)} + M_{52}^{(1)} \theta_x^{(1)} + M_{56}^{(1)} \Delta p/p \right) \qquad k = 2\pi/\lambda$$

$\mathbf{x}^{(1)}$ vector of particle coordinates in the pick-up

$\mathbf{M}^{(1)}$ pick-up-to-kicker transfer matrix

Using the **perturbation theory and symplecticity of undamped motion** one obtains the cooling rates**:**

$$\boxed{\begin{aligned} \lambda_x &= -\frac{\kappa}{2}\left(2\pi R \eta_1 + M_{56}^{(1)}\right) \\ \lambda_s &= \pi \kappa R \eta_1 \end{aligned}}$$

Then the sum of the decrements**:**

$$\lambda_x + \lambda_s = -\frac{\kappa}{2} M_{56}^{(1)}$$

If $\alpha_1 = \alpha_2 = D_1' = D_2' = 0$**:**

$$\lambda_x = -\frac{\kappa}{2} \frac{D_1 D_2}{\sqrt{\beta_1 \beta_2}} \sin\mu_1$$

$$\lambda_s = -\frac{\kappa}{2}\left(M_{56}^{(1)} - \frac{D_1 D_2}{\sqrt{\beta_1 \beta_2}} \sin\mu_1 \right)$$

OSC basics: Cooling Range

The cooling force is linear only for small oscillations, in our case cooling force can be expressed as:

$$\frac{\delta p}{p} = \kappa \sin\left(a_x \sin\psi_x + a_p \sin\psi_p\right)$$

dimensionless amplitudes due to betatron and synchrotron motion expressed in units of the phase advance of the laser wave

$$a_p = k\left(M_{51}^{(1)} D_1 + M_{52}^{(1)} D_1' + M_{56}^{(1)}\right)\left(\frac{\Delta p}{p}\right)$$

$$a_x = k\sqrt{\tilde{\varepsilon}\left(\beta_1 M_{51}^{(1)^2} - 2\alpha_1 M_{51}^{(1)} M_{52}^{(1)} + \left(1+\alpha_1^2\right) M_{52}^{(1)^2}/\beta_1\right)}$$

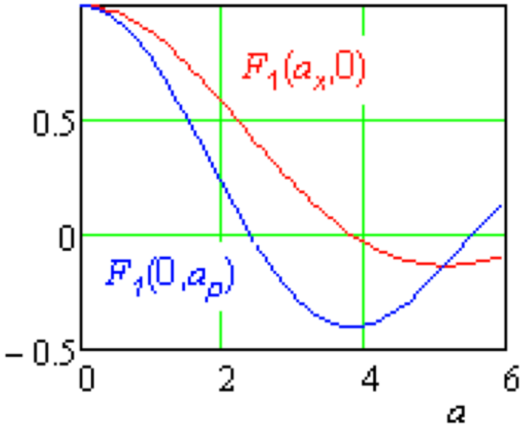
$$\tilde{\varepsilon} = x^2/\beta + 2\alpha x\theta + (1+\alpha^2)\theta^2/\beta$$

Averaging momentum kicks over betatron and synchrotron oscillations, one obtains the form factors for the transverse and longitudinal damping rates:

phase shift of the transverse cooling force

$$\begin{bmatrix} \lambda_1(a_x,a_p)/\lambda_1 \\ \lambda_2(a_x,a_p)/\lambda_2 \end{bmatrix} = \begin{bmatrix} F_1(a_x,a_p) \\ F_2(a_x,a_p) \end{bmatrix} = \begin{bmatrix} 2/\left(a_x \cos\psi_c\right) \\ 2/a_p \end{bmatrix} \int \sin\left(a_x \sin(\psi_x + \psi_c) + a_p \sin\psi_p\right) \begin{bmatrix} \sin\psi_x \\ \sin\psi_p \end{bmatrix} \frac{d\psi_x}{2\pi}\frac{d\psi_p}{2\pi}$$

$$\begin{bmatrix} F_1(a_x,a_p) \\ F_2(a_x,a_p) \end{bmatrix} = 2\begin{bmatrix} J_0(a_p)J_1(a_x)/a_x \\ J_0(a_x)J_1(a_p)/a_p \end{bmatrix}$$



- Damping rates oscillate with growth of amplitudes.
- For a given degree of freedom, damping rates change the sign at its own amplitude equal to $\mu_{11} \approx 3.832$ and at the amplitude of $\mu_{01} \approx 2.405$ for the other degree of freedom.
- Both cooling rates have to be positive for all amplitudes → stability condition:

$$a_{x,p} \le \mu_{01} \approx 2.405$$

Brookhaven National Laboratory

OSC basics: Cooling Range

Stability condition $a_{x,p} \le \mu_{01} \approx 2.405$ yields the stability boundaries for the emittance and the momentum spread:

$$\varepsilon_{\max} = \frac{\mu_{01}^2}{k^2\left(\beta_p M_{51}^{(1)^2} - 2\alpha_p M_{51}^{(1)} M_{52}^{(1)} + \gamma_p M_{52}^{(1)^2}\right)}$$

$$\left(\frac{\Delta p}{p}\right)_{\max} = \frac{\mu_{01}}{kS_{12}}$$

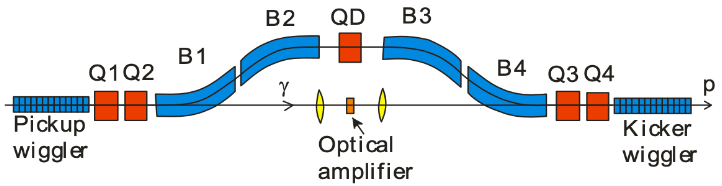
$$S_{12} \equiv 2\pi R\eta_{12} = M_{56} - D_1 D_2\left(\frac{1+\alpha_1\alpha_2}{\sqrt{\beta_1\beta_2}}\sin\mu_1 + \frac{\alpha_2-\alpha_1}{\sqrt{\beta_1\beta_2}}\cos\mu_1\right) - D_1 D_2'\sqrt{\frac{\beta_2}{\beta_1}}(\cos\mu_1 + \alpha_1\sin\mu_1)$$
$$+ D_1' D_2\sqrt{\frac{\beta_1}{\beta_2}}(\cos\mu_1 - \alpha_2\sin\mu_1) - D_1' D_2'\sqrt{\beta_1\beta_2}\sin\mu_1 \,,$$

For further analysis we introduce the relative cooling ranges as ratios of cooling area boundaries $(\Delta p/p)_{\max}$ and $\varepsilon_{\max}$ to the rms values of momentum spread, σ_p , and horizontal emittance, ε .

$$n_{\sigma s} = (\Delta p/p)_{\max}/\sigma_p, \;\; n_{\sigma x} = \sqrt{\varepsilon_{\max}/\varepsilon}$$

- Transverse cooling range does not depend on the dispersion in the pickup undulator but depends on the beta-function in it.
- Beam cooling in a collider requires $n_{\sigma s},\; n_{\sigma x} \ge 4$

OSC basics: Beam Optics and Its Limitations



$$\lambda_x = -\frac{\kappa}{2}\frac{D_1 D_2}{\sqrt{\beta_1\beta_2}}\sin\mu_1$$

$$\lambda_s = -\frac{\kappa}{2}\left(M_{56}^{(1)} - \frac{D_1 D_2}{\sqrt{\beta_1\beta_2}}\sin\mu_1\right)$$

need these terms in the brackets to be different – place a defocusing quad in the chicane!

Leaving leading terms in the thin lens approximation, and assuming that the bends have zero length and don't produce horizontal focusing:

path lengthening in chicane

trajectory offset in chicane

$$M_{56}^{(1)} \approx 2\Delta s,\ \ S_{12} \approx 2\Delta s - \Phi D^* h,$$

$$\lambda_x/\lambda_s \approx \Phi D^* h/(2\Delta s - \Phi D^* h)$$

$$\Phi = 1/F$$

Assuming equal damping rates, the cooling ranges are:

$$n_{\sigma s} \approx \frac{\mu_{01}}{k\sigma_p \Delta s},$$

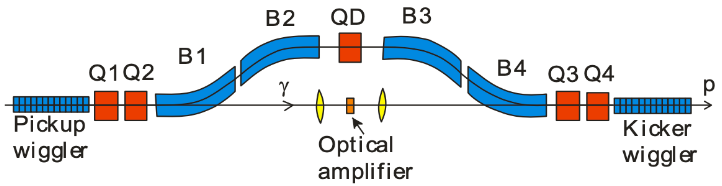
$$n_{\sigma x} \approx \frac{\mu_{01}}{2k\Delta s}\sqrt{\frac{A^*}{\varepsilon}}$$

The cooling dynamics is determined by the following parameters:

- Initial r.m.s. momentum spread and emittance
- Wave number of optical amplifier
- Dispersion invariant $A^* = {D^*}^2/\beta^*$
- Path length delay

Note: one can significantly affect the optics parameters by changing the distribution of cooling rates. An increase of horizontal damping can allow a reduction of the optical amplifier wavelength, but at the same time it makes more difficult to handle an increase of beta-function in the cooling area required to keep sufficiently large value for the horizontal cooling range.

OSC basics: Beam Optics and Its Limitations



Another important limitation on the beam optics is associated with the higher order contributions to the sample lengthening coming from the betatron and synchrotron motions.

Particle angle $\theta(s)$ introduces the relative delay $\Delta s/s = \theta^2(s)/2$

Orbit lengthening:

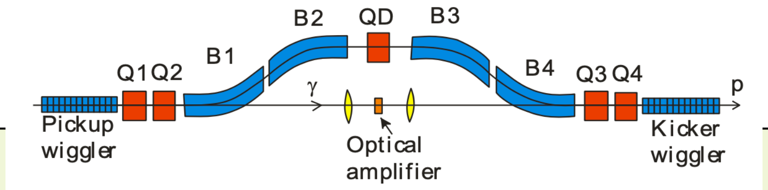
$$\Delta s_2 = \frac{1}{2}\int_{-L_c/2}^{L_c/2}\left(\sqrt{\frac{\varepsilon}{\beta(s)}}\left(\sin(\mu(s)-\mu_0)+\alpha(s)\cos(\mu(s)-\mu_0)\right)\right)^2 \mathrm{d}s$$

$$= \frac{\varepsilon}{2}\int_{-\mu_c/2}^{\mu_c/2}\left(\sin(\mu-\mu_0)+\alpha(\mu)\cos(\mu-\mu_0)\right)^2 \mathrm{d}\mu$$

- Averaging of cooling force with the second order lengthening shows that **to avoid shrinking of cooling boundary the second order contribution** at the cooling boundary **has to be less or about half of the first order contribution**.
- That yields the requirement on an acceptable value of the second order contribution computed at the boundary of cooling range $k\Delta s_2 \le 1.5$
- In our example, the compensation is achieved by placing a sextupole in between dipoles of each chicane leg.
- It decreases the sample lengthening by more than an order of magnitude so that the contribution for the horizontal plane is smaller than for the vertical one.

Brookhaven National Laboratory

OSC basics: Radiation from an Undulator



Goal: Compute the OSC damping rates.
What do we need for that: longitudinal kick which a particle receives in the kicker undulator from its own radiation.
How are we going to do that:

1. Find electric field of the radiation on the focusing lens surface
2. Compute the electric field in the kicker by integrating the field distribution on the lens
3. Find the longitudinal kick in the kicker

Assumptions:

1. The distances from the pickup center to the lens and from the lens to the kicker center are equal and are much larger than the pickup and kicker lengths; so large that the depth of field would not result in a deterioration of the interaction.
2. The pickup and kicker undulators are flat, have the same length and the same number of periods.

Step 1: Radiation from a pick-up undulator is determined by the Liénard-Wiechert equation

vector from the point of radiation to the point of observation

$$\mathbf{E}(\mathbf{r},t)=\frac{e}{c^2}\frac{(\mathbf{R}-\boldsymbol{\beta}R)(\mathbf{a}\cdot\mathbf{R})-\mathbf{a}(R-(\boldsymbol{\beta}\cdot\mathbf{R}))}{(R-(\boldsymbol{\beta}\cdot\mathbf{R}))^3},\quad \mathbf{a}=\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t},\quad \mathbf{R}=\mathbf{r}-\mathbf{r}'$$

Particle moving through an undulator:

amplitude of particle angle oscillations

$$v_x=c\theta_e\sin(\omega_u t'+\psi),\quad t'=t-R/c,$$
$$v_y=0,$$
$$v_z=c\left(1-\frac{1}{2\gamma^2}-\frac{\theta_e^2}{2}\sin^2(\omega_u t'+\psi)\right)$$

- By substituting velocities in LW equation one obtains the horizontal component of electric field in the far zone.
- The vertical and longitudinal components of the electric field are averaged out at the focus and therefore can be omitted.

OSC basics: Radiation from an Undulator

Only the first harmonic of the radiation interacts resonantly with the particle in the kicker undulator. Therefore, we keep only the first harmonic of radiation in further calculations:

$$E_\omega(r) = \frac{\omega(\theta)}{\pi} \int_0^{2\pi/\omega(\theta)} E_x(r,t) e^{-i\omega t} \mathrm{d}t, \;\; \omega(\theta) = 2\gamma^2 \omega_u / \left(1 + \gamma^2 \left(\theta^2 + \theta_e^2/2\right)\right)$$

To find the **electric field in the kicker undulator**, where the radiation is focused, we use the Kirchhoff formula:

$$E(r'') = \frac{1}{2\pi i c} \int_S \frac{\omega(\theta) E_\omega(r)}{|r'' - r|} e^{i\omega |r'' - r|/c} \mathrm{d}s$$

Considering a small undulator parameter $K = \gamma\theta_e \ll 1$:

$$E_x = \frac{4e\gamma^2\omega_u^2\theta_e}{3c^2}\left(1 - \frac{1}{(1+(\gamma\theta_m)^2)^3}\right)$$

angle subtending the lens from the pickup undulator

Averaging the energy transfer over oscillations in the kicker undulator, one obtains the amplitude of the energy change in the kicker undulator in the absence of optical amplification:

peak magnetic field of the undulator

$$\Delta E = \frac{2e^4 B_0^2 \gamma^2}{3m^2c^4} L_u \left(1 - \frac{1}{(1+(\gamma\theta_m)^2)^3}\right)$$

total undulator length

OSC basics: Damping Rates

In the absence of optical amplification and $\gamma\theta_{\max} \ll 1$ the amplitude of energy loss is equal to the total energy loss in both undulators:

$$\Delta E_{\mathrm{tot}} = \frac{2e^4 B_0^2 \gamma^2 L_u}{3m^2 c^4}$$

The interference of radiation of two undulators results in the energy loss being modulated with the path length difference on the travel from pickup to kicker:

$$\Delta E(\Delta s) = -\Delta E_{\mathrm{tot}} \left(1 + \cos(k\Delta s)\right)$$

For longitudinal motion:

$$\Delta E(\Delta p) = -\Delta E_{\mathrm{tot}} \left(1 + \cos\left(kS_{12}\frac{\Delta p}{p}\right)\right)$$

Finally, using the previously found expressions for the cooling rates:

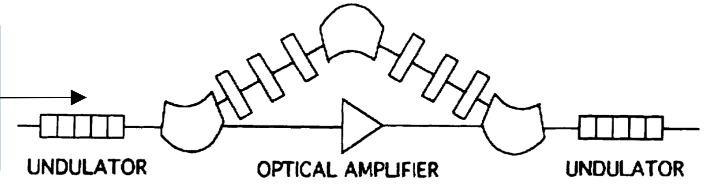
$$\lambda_x = -\frac{\kappa}{2}\left(2\pi R\eta_1 + M_{56}^{(1)}\right) \qquad k = \frac{2\gamma^2\omega_u}{c}$$

$$\lambda_s = \pi\kappa R\eta_1 \qquad \kappa = \frac{2e^4 B_0^2 \gamma}{3m^3 c^5} L_u K_a \left(1 - \frac{1}{(1 + (\gamma\theta_m)^2)^3}\right)$$

Transit-time method of OSC:

momentum deviation: $\delta_i = \Delta P_i / P$

betatron coordinate and angle: x_i, x_i'



Bypass trajectory:

trajectory of a reference particle

dispersion and its derivative at the 1st undulator

cosinelike and sinelike solutions of a homogeneous equation of motion

contribution of the bypass magnets to the original dispersion

$$l_i = l_0 + x_i I_\mathrm{U} + x_i' I_\mathrm{V} + \delta_i \left(\eta_0 I_\mathrm{U} + \eta_0' I_\mathrm{V} - I_\mathrm{D}\right)$$

$$I_\mathrm{U} = \int_L \frac{U(s)\mathrm{d}s}{\rho(s)}, \quad I_\mathrm{V} = \int_L \frac{V(s)\mathrm{d}s}{\rho(s)}, \quad I_\mathrm{D} = \int_L \frac{D(s)\mathrm{d}s}{\rho(s)}$$

bending radius of the magnets

Test particle radiates: $E_i = E_0 \sin(kz - \omega t + \phi_i)$ with $\lambda = \left[\lambda_\mathrm{u}\left(1 + K^2/2\right)\right]/2\gamma^2$

The particle arrives at the second undulator with

time delay: $\delta(\Delta t) = \Delta t_i - \Delta t_0;\ \Delta t_i = l_i/c,\ l_0 - c\Delta t_0 = \lambda/4$

phase shift: $\Delta\phi_i = k(l_i - l_0) = k\left[x_i I_\mathrm{U} + x_i' I_\mathrm{V} + \delta_i \left(\eta_0 I_\mathrm{U} + \eta_0' I_\mathrm{V} - I_\mathrm{D}\right)\right]$

Momentum change due to the coherent longitudinal kick:

number of undulator periods

In the second undulator, the particle interacts with its own radiation and changes momentum: $\delta P_i = g \dfrac{qE_0 M \lambda_\mathrm{u} K}{2c\gamma} \sin(\Delta\phi_i)$

Transverse kick from the energy kick:

For the case of a mirror symmetry, dispersion and its derivative for the second undulator are $\eta_0, -\eta_0'$

$\Delta x_i = -\eta_0 \left(\delta P_i / P\right);\ \Delta x_i' = \eta_0' \left(\delta P_i / P\right)$

Including incoherent kicks:

Change of the particle momentum including **incoherent kicks**:

$$\delta_{ic} = \delta_i + G\sin(\Delta\phi_i) + G\underbrace{\sum_{k\neq i}^{N_s}\sin(\Delta\phi_i + \psi_{ik})} \qquad \text{with} \quad \psi_{ik} = \phi_i - \phi_k \quad \text{and} \quad G = g\frac{qE_0M\lambda_u K}{2c\gamma P}$$

For transverse motion and a mirror symmetrical lattice with $I_U = 0, I_V = 2D_0, \eta_0 = D_0$ ($2D_0$ contribution to the dispersion function in the 2nd undulator from the elements of the bypass):

$$x_{ic} = x_i + D_0 G\sin(\Delta\phi_i) + D_0 G\sum_{k\neq i}^{N_s}\sin(\Delta\phi_i - \psi_{ik}),$$

$$x'_{ic} = x'_i - \eta'_0 G\sin(\Delta\phi_i) + \eta'_0 G\sum_{k\neq i}^{N_s}\sin(\Delta\phi_i + \psi_{ik})$$

Evaluating the averages of the quantities, we obtain damping decrements:

$$\alpha_\delta = \frac{\overline{\Delta(\delta^2)}}{\overline{\delta^2}} = 2G(I_D - 2D_0\eta'_0)\,k\exp\left\{-\frac{\overline{\Delta\phi_i^2}}{2}\right\} - \frac{G^2 N_s}{2}\frac{1}{\sigma_\delta^2},$$

$$\alpha_x = \frac{1}{2}\left(\frac{\overline{\Delta(x^2)}}{\overline{x^2}} + \frac{\overline{\Delta(x'^2)}}{\overline{x'^2}}\right) = \frac{1}{2}\left[4G D_0\eta'_0\,k\exp\left\{-\frac{\overline{\Delta\phi_i^2}}{2}\right\} - \frac{G^2 N_s}{2}\left(\eta_0'^2 + \frac{D_0^2}{\beta^2}\right)\frac{\beta}{\epsilon_x}\right],$$

where $\overline{\Delta\phi_i^2} = k^2[(2D_0)^2\overline{x'^2} + (2D_0\eta'_0 - I_D)^2\overline{\delta^2}]$ and substitutions $\overline{\delta^2} = \sigma_{p'}^2$, $\overline{x^2} = \epsilon_x\beta$, and $\overline{x'^2} = \epsilon_x/\beta$ are used, and where ϵ_x is the beam emittance and β is the beta function in the undulator. The exponential term appearance in the first terms of Eqs. (6) is due to the sinelike dependence of the coherent kick from the particle's phase shift. We can now define the optimal G by maximizing

Optimal gain and limitations:

We can define the optimal gain by maximizing the sum of the decrements $\alpha_x + \alpha_\delta$

$$G = \frac{2(I_D - D_0\,\eta_0')\sigma_\delta^2 \boxed{k\ \exp\left\{-\overline{\Delta\phi_i^2}/2\right\}}}{N_s\left[1 + \frac{\beta}{2\epsilon_x}\ \sigma_\delta^2\left(\eta_0'^2 + \frac{D_0^2}{\beta^2}\right)\right]}\ , \qquad (7)$$

and

$$\alpha_x + \alpha_\delta = \frac{2(I_D - D_0\,\eta_0')^2\ \sigma_\delta^2\left[k\exp\left\{-\overline{\Delta\psi_i^2}/2\right\}\right]^2}{N_s\left[1 + \frac{\beta}{2\epsilon_x}\sigma_\delta^2\left(\eta_0'^2 + \frac{D_0^2}{\beta^2}\right)\right]}\ . \qquad (8)$$

The term in green box reaches its maximum at $\overline{\Delta\phi_i^2} = k^2\left[(2D_0)^2\frac{\epsilon_x}{\beta} + (2D_0\,\eta_0' - I_D)^2\sigma_\delta^2\right] = 1\ .$

Reduction of ε_x and σ_δ during the damping leads to decrease of $\overline{\Delta\phi_i^2}$ and the phase shifts of the individual particles:

- coherent components of the kicks are reduced
- incoherent components remain the same

Slowdown of the cooling rate!

Solution: adjust the bypass lattice during damping such that $2D_0$ and $(2D_0\eta_0' - I_\mathrm{D})$ are increased to compensate reduction of ε_x and σ_δ and keep $\overline{\Delta\phi_i^2}$ at a constant level.

Brookhaven National Laboratory

Cooling rate:

One can assume $(\eta_0'\sigma_\delta)^2 \approx \epsilon_x/\beta$ and $D_0^2 \ll \beta\eta_0'^2$, then for a case of equal decrements, when I_D is adjusted such that $I_\mathrm{D} = 3D_0\eta_0'$:

$$G \approx \frac{\sigma_\delta}{\sqrt{e}\,N_s} \,,$$

$$\alpha_x = \alpha_\delta \approx \frac{1}{2e\,N_s} \,.$$

Number of passes required for a 1/e reduction of the beam emittance is 2eNs, where

$$N_\mathrm{s} \simeq N\frac{M\lambda}{2Fl_b}$$

$N =$ number of particles in the bunch

$F =$ ratio of the beam transverse area in the undulator to the transverse coherence area of the light

$l_b =$ bunch length

Damping time due to the OSC:

$$\frac{\tau_{x,\delta}}{T} = \frac{eN}{\Gamma}\frac{\lambda}{Fl_b} \qquad \Gamma = \Delta\omega/\omega \approx 1/M$$

Brookhaven National Laboratory

Amplification factor:

Earlier we determined that: $G = g\frac{qE_0 M\lambda_\mathrm{u} K}{2c\gamma P}$ and $G \approx \frac{\sigma_\delta}{\sqrt{e}N_\mathrm{s}}$ if $(\eta_0'\sigma_\delta)^2 \approx \epsilon_x/\beta,\ D_0^2 \ll \beta\eta_0'^2,\ I_\mathrm{D} = 3D_0\eta_0'$

the only unknown

we want to estimate

$$g\frac{qE_0 M\lambda_\mathrm{u} K}{2c\gamma P} = \frac{\sigma_\delta}{\sqrt{e}N_\mathrm{s}}$$

At the waist of the light beam, the cross-section of the coherent mode of radiation $A \approx 2\lambda M\lambda_\mathrm{u}$

During one pass of the undulator with $K \approx 1$, the particle emits into the coherent mode $\sim q^2/\hbar c$ photons of the energy $\sim \hbar\omega$.

$\frac{c}{8\pi}E_0^2 A\Delta t_\mathrm{R} = kq^2,\ \Delta t_\mathrm{R} = M\lambda/c$ is the durations of the radiation pulse.

$$g \approx \frac{1}{\sqrt{e}}\frac{\varepsilon_{||}\Gamma F}{r_0 N} \qquad \varepsilon_{||} = \gamma l_b \sigma_\delta/\sqrt{2\pi}$$

- Reduction of the amplification factor to follow the emittance reduction is required during the damping process.
- The equilibrium emittance is reached when all sources of damping are balanced by all sources of emittance excitation.
- If the OSC is the only source of damping, absolute minimum emittance can be defined for the case where the only source of emittance excitation is the radiation fluctuation in the undulators:

$$\varepsilon_{||0} = \sqrt{e}\frac{r_0 N}{\Gamma F}$$

Brookhaven National Laboratory

Power limit:

So far, we assumed that we are not limited by the amplifier power.

If, in order to reach the optimal damping, the required output power of the amplifier exceeds the available power, then the available power of the amplifier will determine the damping time:

$$\frac{\tau_{x,\delta}}{T} \approx \left[\frac{N_{\mathrm{tot}}\lambda}{\overline{W}cTK^2}\frac{\sigma_\delta^2\left(E_b/q\right)^2}{Z_0}\right]^{1/2}$$

$\overline{W} =$ available average output power of the optical amplifier

$E_b =$ equilibrium beam energy

$Z_0 =$ free-space impedance

Brookhaven National Laboratory

Noise and Mixing:

NOISE

- The noise of the optical amplifier is due to ***spontaneous emission from the active media***, roughly equivalent to one noise photon in the value of the coherence at the amplifier front end.
- Compared to the noise due to the photons radiated in the first undulator $N_{\mathrm{s}} q^2/\hbar c$, the noise of the amplifier is negligible small.

MIXING

K-to-P:

- Complete rerandomization occur if particles change positions inside the bunch on $\sim M\lambda$
- Another possibility when the beam emittance is larger than $1/2k$ .Mixing in the transverse phase space will also occur.

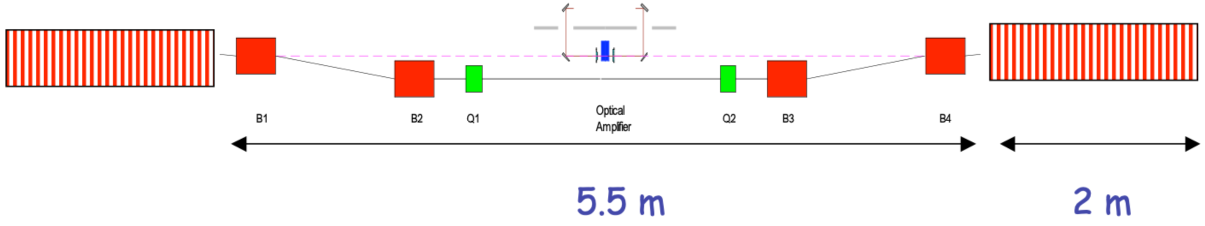
P-to-K:

- In the linear approximation, there is no mixing.
- If the beam emittance is larger than $1/2k$, the second-order geometric and chromatic aberrations can affect the synchronism between the particle and radiation.
- The bypass lattice and the optical system must have identical focusing properties, including the second-order geometric and chromatic aberrations.

Brookhaven National Laboratory

Experimental Realization of OSC

- The OSC was proposed in 1994 but wasn't tested experimentally for a long time.
- There were suggestions of its experimental implementation in Tevatron and RHIC, but it was too risky to implement it on the operating collider and the work did not proceed beyond initial proposal.
- The first attempt to make a test of the OSC with small energy electrons was done in the BATES but it did not get enough support.



OSC estimates for RHIC

OSC study for RHIC {M. Babzien *et al*, Phys Rev STAB 7, 012801 (2004)}

Ions	Gold	Protons	Protons	Protons
Gamma	106	106	266	266
Number of ions/bunch	1.5E9	2.E11	2.E11	2.E11
Number of bunches	112	112	112	112
Laser wavelength [micron]	12	12	**1.9**	12
Wiggler period [cm]	27	27	27	**130**
Wiggler length [m]	3	3	3	**20**
Wiggler parameter	0.056	0.14	0.14	0.67
Cooling time [hrs]	1.1	2.2	2.2	1.1

V. Yakimenko, PRELIMINARY, OSC Workshop (MIT, 2006)

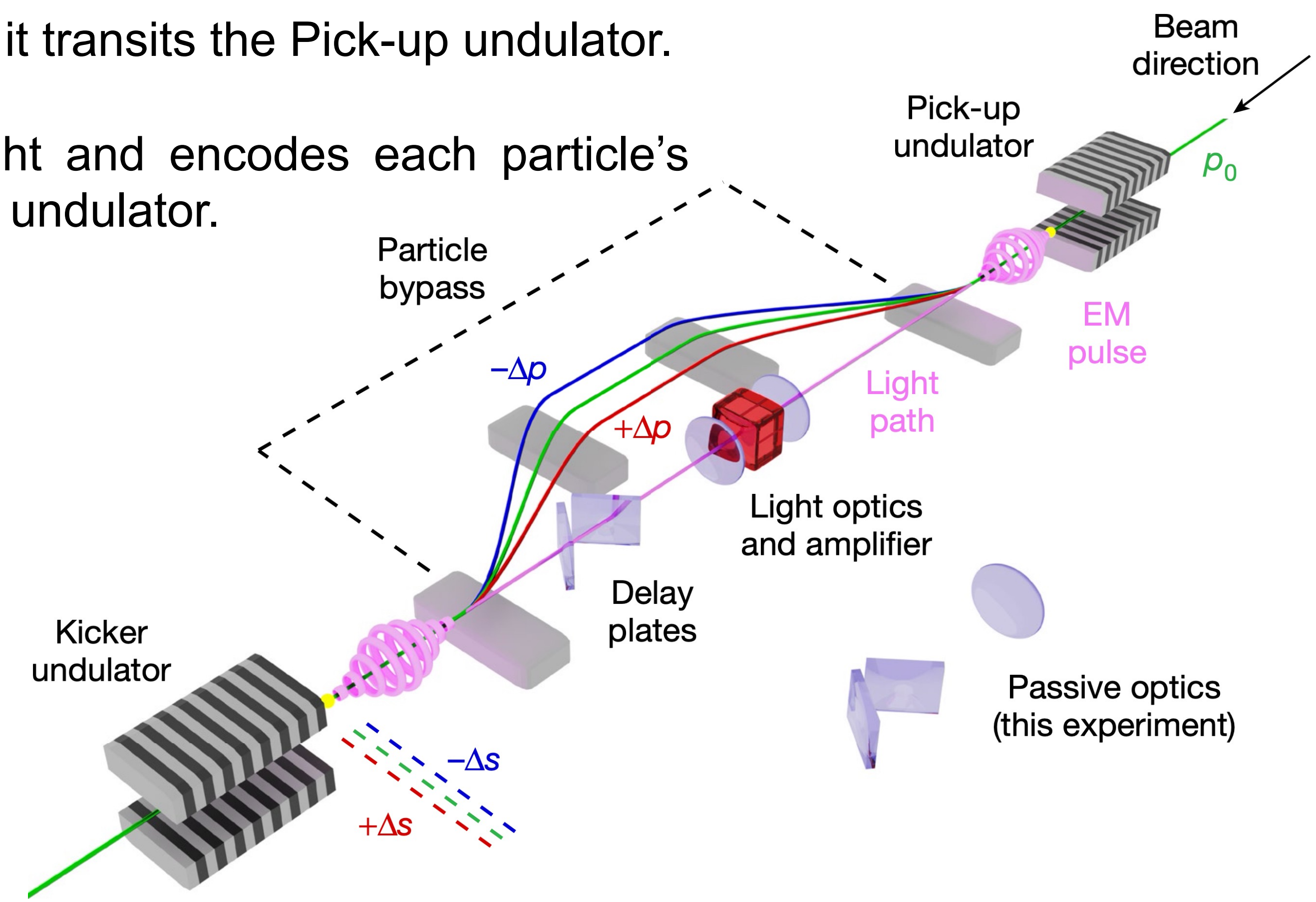
- Amp saturation at 20 W
- Wiggler peak field 10 T
- No SC, EC

Experimental Realization of OSC: schematic

- Each particle produces a pulse of EM radiation as it transits the Pick-up undulator.

- A magnetic bypass separates the beam and light and encodes each particle's phase-space error on its arrival delay at the kicker undulator.

- Trajectories for particles with positive (red) and negative (blue) momentum deviations are shown, along with their corresponding arrival delays, relative to the reference particle (green).

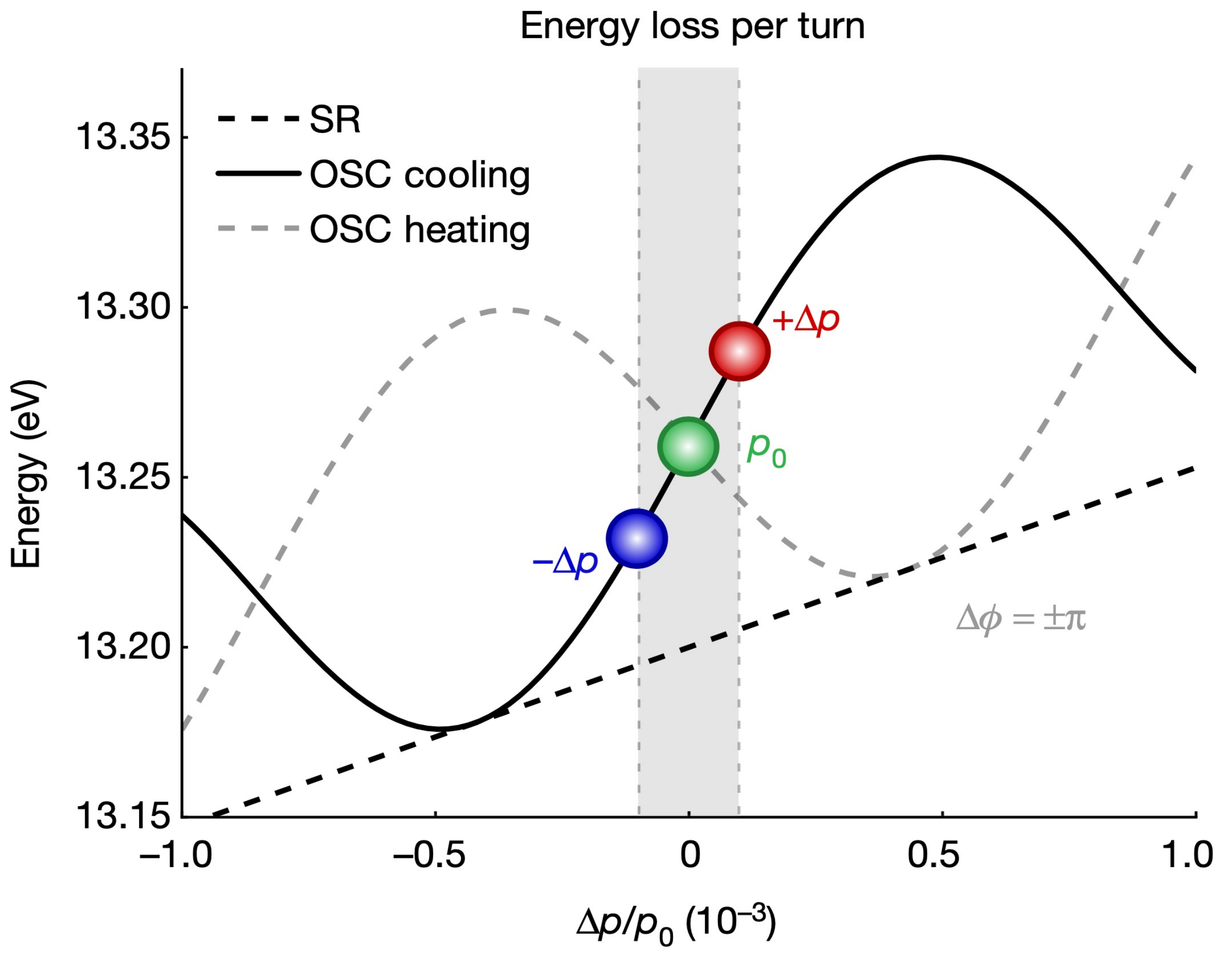


- The pick-up radiation is amplified and focused into the kicker undulator.

- The particles' interaction with the pick-up radiation inside the kicker produces corrective energy kicks when the system is tuned for the cooling mode

Brookhaven National Laboratory

Experimental Realization of OSC: energy loss per turn



- Example of energy loss per turn with (black solid line) and without (black dashed line) OSC as a function of a particle's relative momentum deviation $\Delta p/p_0$.
- Detuning the delay system by half of the fundamental wavelength (a shift of $\Delta\phi = \pm\pi$ in the radiation phase) places the OSC system in a heating mode (grey dashed line).
- The grey shaded region corresponds to the relative r.m.s. energy spread for the reported configuration (about 10^{-4}).

Brookhaven National Laboratory

OSC experiment at IOTA

The Integrable Optics Test Accelerator (IOTA) is a 40 m circumference, electron and proton storage ring at FNAL.

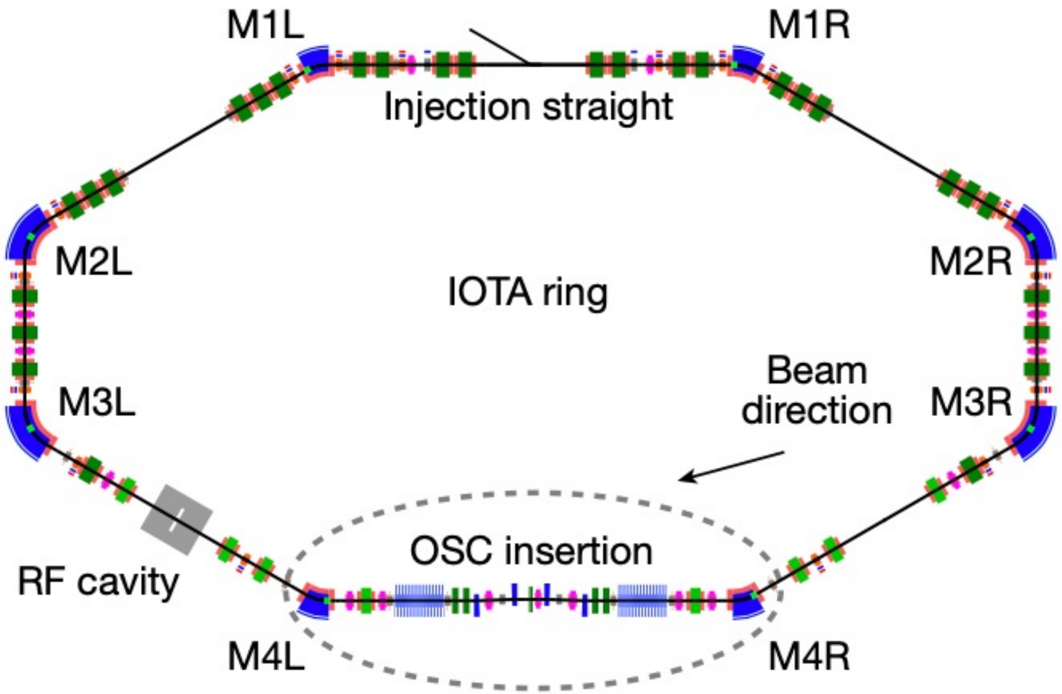


Table 1 | Design-performance parameters for IOTA OSC[9]

Design momentum, p_0 (MeV c^{-1})	100
Revolution frequency (MHz)	7.50
Radio frequency (MHz)	30.00
Momentum compaction	4.91×10^{-3}
Relative r.m.s. momentum spread, σ_p/p_0[a]	0.986×10^{-4}
Horizontal emittance: x–y uncoupled, ε_0 (nm)[a]	0.857
Total bypass delay (mm)	0.648
Nominal radiation wavelength, λ_r (nm)	950
Maximum OSC kick per turn (meV)	60
Horizontal cooling acceptance, ε_{max} (nm)	72
Longitudinal cooling acceptance, $(\Delta p/p)_{max}$	5.7×10^{-4}
Bandwidth of the OSC system (THz)	19
Sum of emittance OSC rates (s^{-1})	38
SR emittance damping rates, $[z,x,y]$ (s^{-1})	2.06, 0.94, 0.99

[a]Equilibrium values do not include the effects of OSC.

Article

Experimental demonstration of optical stochastic cooling

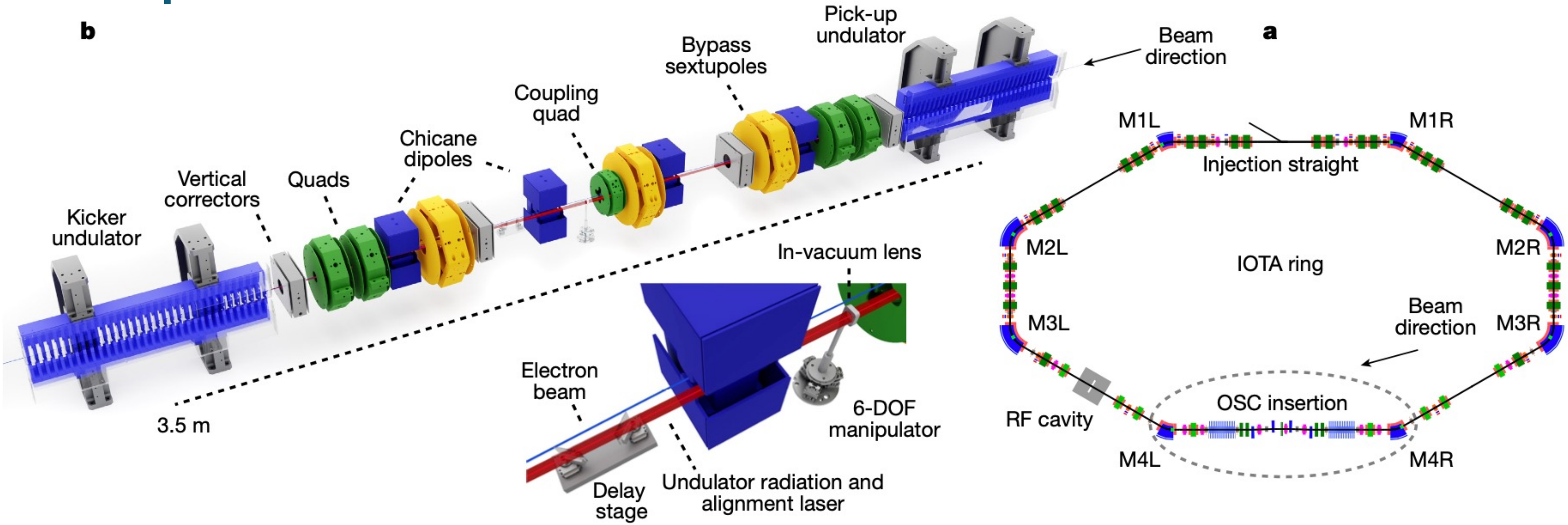
https://doi.org/10.1038/s41586-022-04969-7

Received: 16 March 2022

J. Jarvis[1✉], V. Lebedev[1✉], A. Romanov[1], D. Broemmelsiek[1], K. Carlson[1], S. Chattopadhyay[1,2,3], A. Dick[2], D. Edstrom[1], I. Lobach[4], S. Nagaitsev[1,4], H. Piekarz[1], P. Piot[2,5], J. Ruan[1], J. Santucci[1], G. Stancari[1] & A. Valishev[1]

Brookhaven National Laboratory

OSC experiment at IOTA



- The OSC insertion is ~6m-long straight section between IOTA's M4L and M4R dipoles.
- The PU and KU are identical undulators with N_u = 16 magnetic periods (4.84 cm each) and produce an on-axis fundamental radiation wavelength of λ_r = 950 nm for the design energy of 100 MeV.
- The radiation from the PU is relayed to the KU using a single in-vacuum lens with a focal length of 0.853 m at the fundamental wavelength.
- Although this configuration does not include optical amplification, it still produces strong cooling and enables detailed measurements of the underlying physics.

Brookhaven National Laboratory

Diagnostic

Undulator-radiation diagnostics:

- 2 cameras are used in combination with a filter wheel to image the fundamental, second or third harmonic from the KU and PU.
- These UR cameras are located at two separate image planes corresponding to different locations inside the KU.
- The PU light is mapped into the KU with an approximately negative-identity transformation.
- The imaging system then produces a single, relatively sharp image of the beam, for both the PU and KU, from the corresponding source plane.
- These images can be used to estimate the trajectory errors of the closed orbit in both undulators.

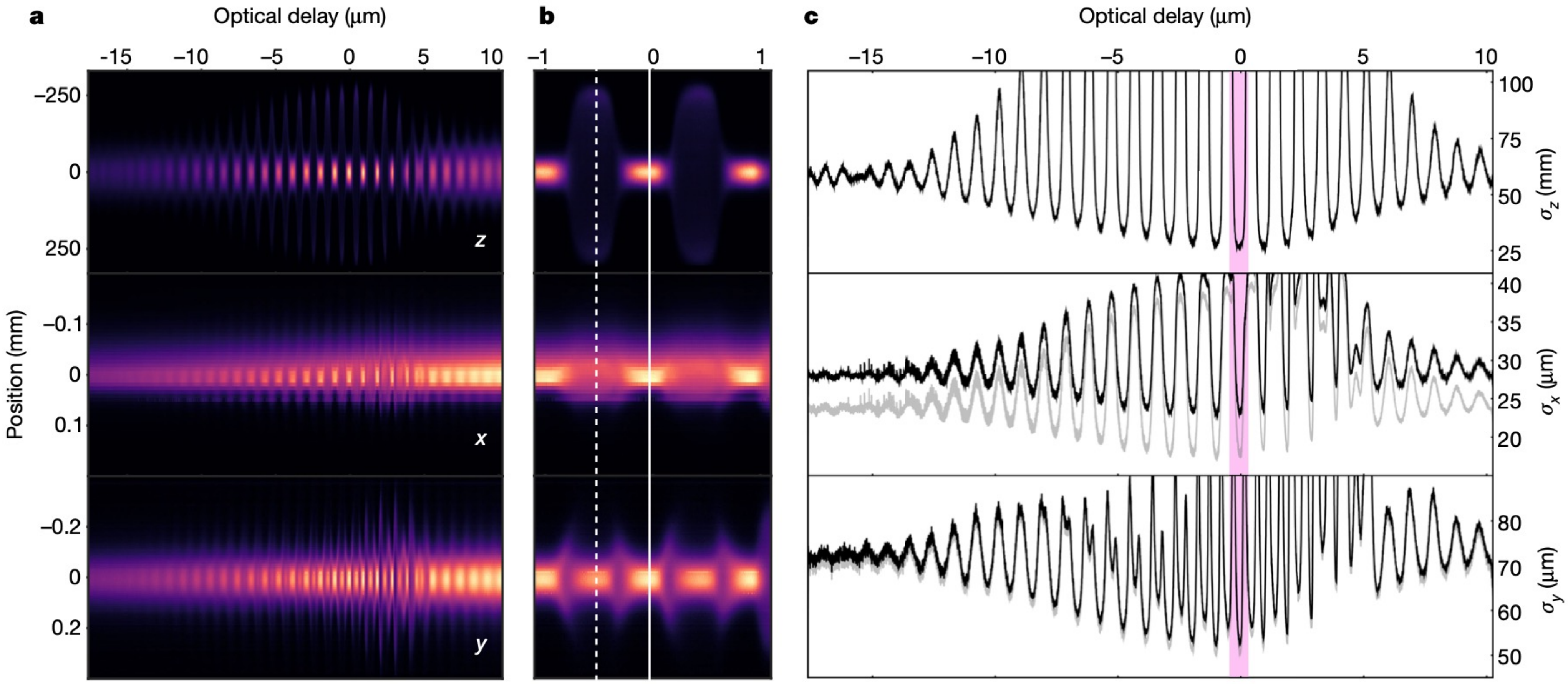
Longitudinal beam measurements:

- A dual-sweep streak camera was used to measure the beam's longitudinal distribution.
- A CMOS camera was used as the detector element, and the system was installed above the M3R dipole.
- A 50/50 non-polarizing beam splitter was used to direct half of the SR from the existing M3R SR beam-position monitor to the entrance slit of the streak camera.

The OSC interaction was characterized using a combination of slow-delay scans and fast on–off toggles through rapid changes of the delay setting.

Experimental Results: slow delay scan

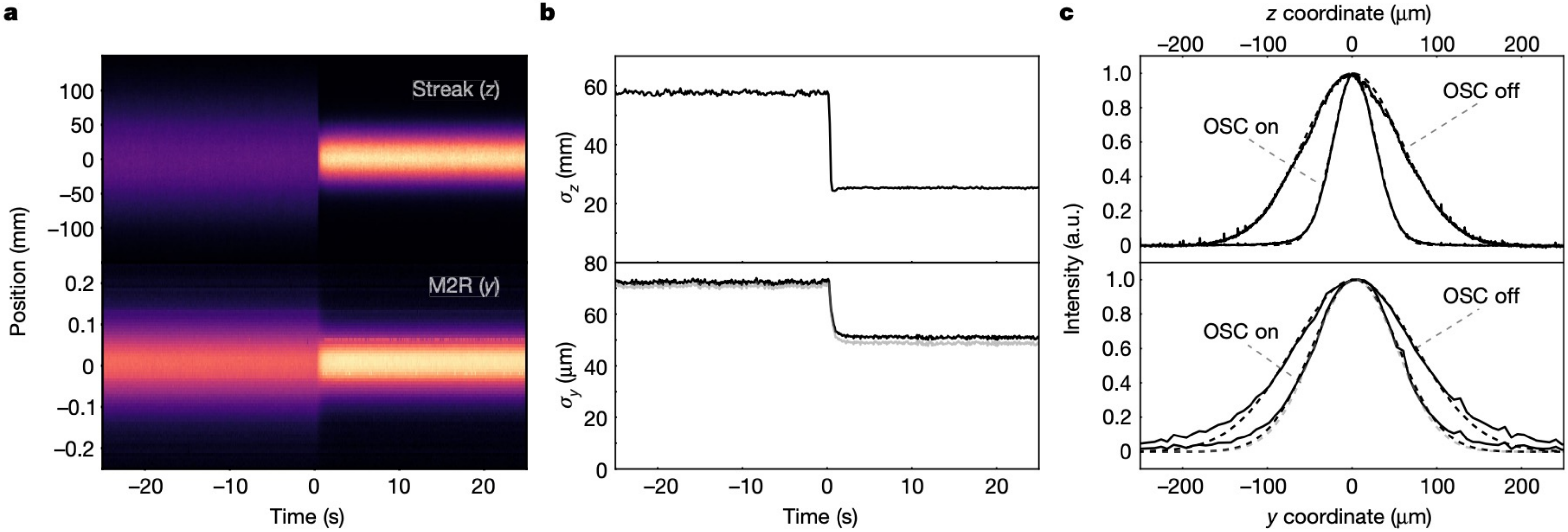
- At the beginning and the end of the scan, the particles and light are separated longitudinally = OSC is off, only SR damping.
- OSC alternates between cooling and heating modes with the total number of modulation periods $\approx 2N_u$.
- In the heating mode, large amplitudes in one plane can lead to an inversion of OSC in other planes (dashed white line).



Projected beam distributions for a delay scan in the 3D OSC configuration.

Experimental Results: fast toggle

- The delay system is initially misaligned by about $30\lambda_r$.
- At time $t = 0$, the system is moved to the strongest cooling zone.
- Single-dimensional beam distributions in z (streak camera) and y (M2R SR monitor) were recorded during an OSC toggle.
- Distributions averaged over time for the OSC-off and OSC-on states for the intervals of [−20, −10] s and [10, 20] s demonstrate the cooling effect.



Analysis of OSC rates

For very small beam current, where IBS is negligible, the r.m.s. emittance growth rate of small-amplitude motion is determined by the following equation:

$$\frac{\mathrm{d}\varepsilon_n}{\mathrm{d}t} = -2\lambda_n\varepsilon_n + B_n,\ \ n = 1, 2, s$$

$B_n =$ the diffusion driven by fluctuations from SR emission and scattering from residual gas molecules

In equilibrium: $\lambda_n = B_n/2\varepsilon_n$

Cooling rate can be computed from the ratio of r.m.s. beam sizes with σ_n and without σ_{n0} OSC:

$$\frac{\lambda_n}{\lambda_{n0}} = \left(\frac{\sigma_{n0}}{\sigma_n}\right)^2$$

Although all reported OSC measurements were done with a small beam current (about 50–150 nA), for a large fraction of the measurements IBS was not negligible:

$$\frac{\mathrm{d}\varepsilon_n}{\mathrm{d}t} = -2\lambda_n\varepsilon_n + B_n + A_n\frac{N}{\varepsilon_\perp^{3/2}\sqrt{\varepsilon_s}}$$

$\varepsilon_\perp = \varepsilon_1 = \varepsilon_2 =$ r.m.s. transverse emittance

$\varepsilon_s =$ r.m.s. longitudinal emittance

$A_\mathrm{n} =$ Constant determined from the measurement

Brookhaven National Laboratory

Analysis of OSC rates

Ratio of cooling rates with and without OSC:

ratio of vertical beam sizes without/with OSC

ratio of bunch lengths without/with OSC

$$\frac{\lambda_{\mathrm{vOSC}}}{\lambda_{\mathrm{vSR}}} = \left(\frac{\sigma_{\mathrm{vSR}}}{\sigma_{\mathrm{vOSC}}}\right)^2 \frac{1}{R_{\mathrm{IBS}}(1-R_{\mathrm{v}})+R_{\mathrm{v}}}, R_{\mathrm{IBS}} = \left(\left(\frac{\sigma_{\mathrm{vSR}}}{\sigma_{\mathrm{vOSC}}}\right)^3 \frac{\sigma_{\mathrm{sSR}}}{\sigma_{\mathrm{sOSC}}}\right)^{-1} \qquad R_{\mathrm{v}} \equiv \left(\frac{\sigma_{\mathrm{v0}}}{\sigma_{\mathrm{v2}}}\right)^2 = \left(\frac{\sigma_{\mathrm{v1}}}{\sigma_{\mathrm{v2}}}\right)^2 - \frac{(\sigma_{\mathrm{v1}}/\sigma_{\mathrm{v2}})^2-1}{1-(\sigma_{\mathrm{v1}}/\sigma_{\mathrm{v2}})^3(\sigma_{\mathrm{s1}}/\sigma_{\mathrm{s2}})/R_{\mathrm{N}}}.$$

ratio of initial and final vertical beam sizes

Obtained ratios for cooling rate:

transverse	$\lambda_{\mathrm{vOSC}}/\lambda_{\mathrm{vSR}} = 2.94$
longitudinal	$\lambda_{\mathrm{sOSC}}/\lambda_{\mathrm{sSR}} = 8.06$

When combined into a single plane, the total amplitude cooling rate of OSC is about 9.2 s^{-1}, which to a total emittance cooling rate of 18.4 s^{-1} and is about an order of magnitude larger than the longitudinal SR damping.

Brookhaven National Laboratory

Conclusion

- ***Optical Stochastic Cooling (OSC)*** uses free space EM waves as the signaling medium, undulators to couple the radiation to the circulating particle beam and optical amplifiers for signal amplification.

- The OSC can support the cooling rates orders of magnitude larger than have been achieved with the micro-wave stochastic cooling.

- ***OSC has been demonstrated experimentally and*** confirmed the realization of a stochastic beam-cooling technique in the terahertz-bandwidth regime with an increase in bandwidth of about 2,000 times over conventional SC systems.