Comparison of Modulator Simulation with Theory

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Numerical setting

- Domain
  - Transversal : 2.4e-3m ($8 \times \textit{Debyelength}$) periodic boundary
  - Longitudinal : 1e-5m ($10 \times \textit{Debyelength}$) periodic boundary
- Grid number : 20 per Debye length
- Electron with uniform distribution
- Ion at center

Formula

$$\frac{\delta E(z_l)}{E_0} = \frac{< v_z >}{c}$$
$$= -\frac{1}{e n_0 \pi a^2 c} I_d(\gamma_0 z_l, \frac{L_{mod}}{\beta_0 \gamma_0 c})$$

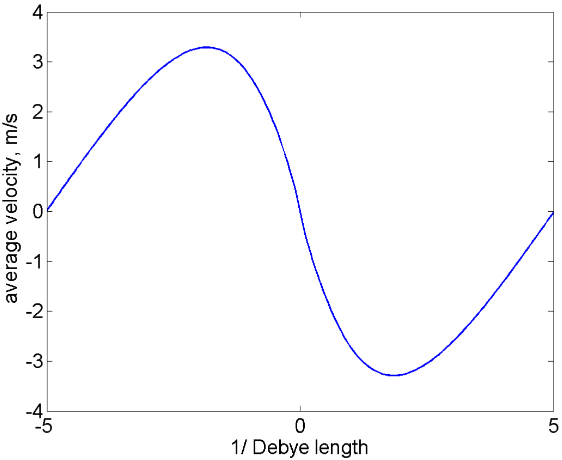
Formula

$$I_d(z,t) = -\frac{Z_i e \omega_p^2}{\pi}\int_0^t d\tau \left(z+v_{0,z}\tau\right)\left\{\frac{a_z \sin\left(\omega_p\tau\right)}{\left[\bar{\beta}^2\tau^2+\left(z+v_{0,z}\tau\right)^2\right]\left[1+\bar{\beta}^2\tau^2+\left(z+v_{0,z}\tau\right)^2/a^2\right]} - \cos\left(\omega_p\tau\right)\left[\frac{\arctan\left(\left|z+v_{0,z}\tau\right|/\left(\bar{\beta}\tau\right)\right)}{\left|z+v_{0,z}\tau\right|} - \frac{\arctan\left(\sqrt{\left(z+v_{0,z}\tau\right)^2+a^2}/\left(\bar{\beta}\tau\right)\right)}{\sqrt{\left(z+v_{0,z}\tau\right)^2+a^2}}\right]\right\}$$

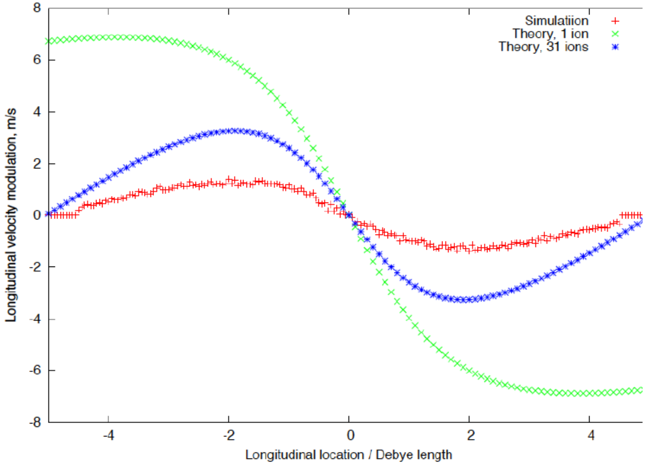
Formula

- Formula is for a single ion in open space
- We use superpositions of a series of ions along longitudinal direction to model the periodic boundary

Plot



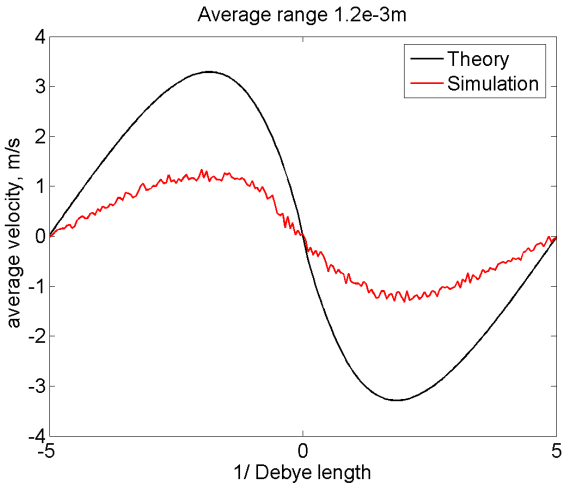
Previous comparison by Gang



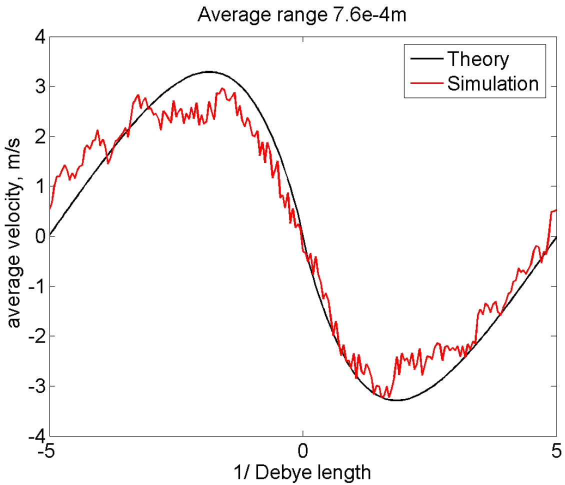
Comparison

- Velocity modulation changes when we measure it using different averaging range in transversal direction
- Theory uses $\sigma = 6.83e - 4m$ as the averaging range

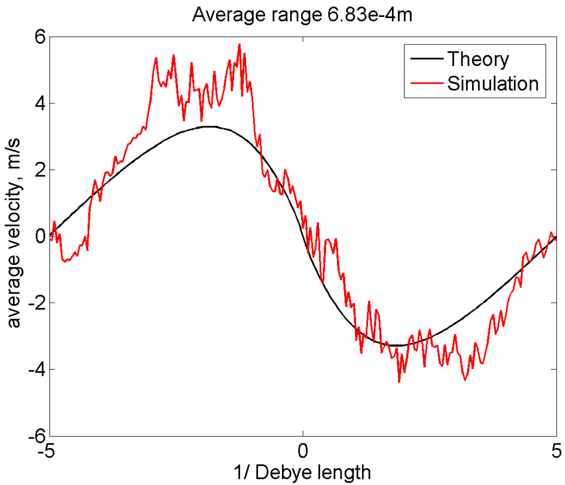
Comparison


Average range 1.2e-3m
Theory
Simulation
average velocity, m/s
1/ Debye length
4
3
2
1
0
-1
-2
-3
-4
-5
0
5

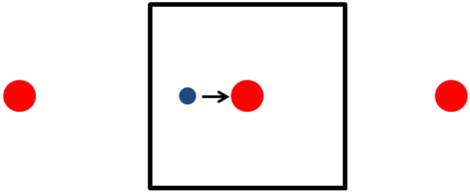
Comparison


Average range 7.6e-4m
Theory
Simulation
average velocity, m/s
1/ Debye length
4
3
2
1
0
-1
-2
-3
-4
-5
0
5

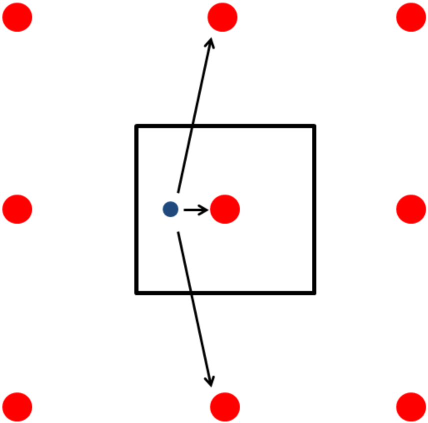
Comparison


Average range 6.83e-4m
Theory
Simulation
average velocity, m/s
1/ Debye length
6
4
2
0
-2
-4
-6
-5
0
5

Theory



Simulation



Improvement

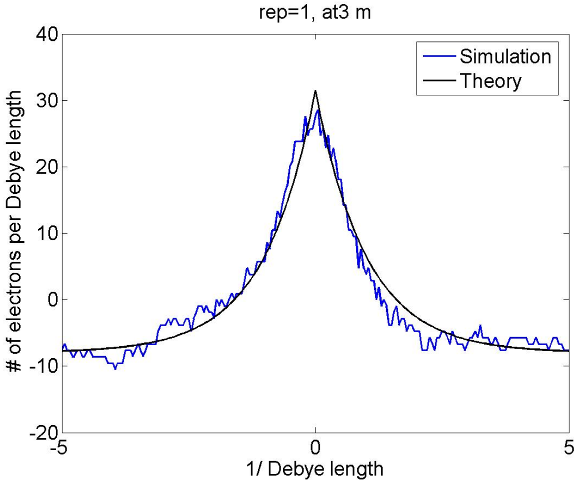
- Rerun simulations using larger domain to decrease the effect of periodic ions in transversal direction
- Measure the velocity modulation using reasonable averaging range

Formula

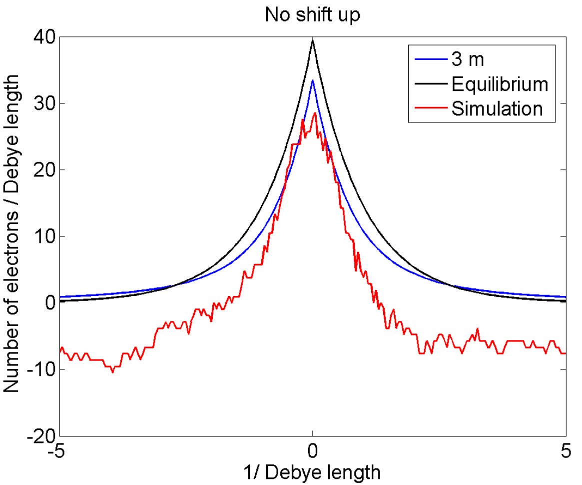
$$\lambda_1(z) = \int_0^{\omega_p t} d\tau \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} n(\vec{x}, \tau)\, dx\, dy = \frac{Z_i}{\pi a_z} \int_0^{\omega_p t} \frac{\tau \sin(\tau)}{(\bar{z} + \bar{v}_z \tau)^2 + \tau^2} d\tau$$

For $v_z = 0$, above formula reduces to $\lim_{t \to \infty} \lambda_1(z) = \frac{Z_i}{2a_z} \exp\left(-\frac{|z|}{a_z}\right)$

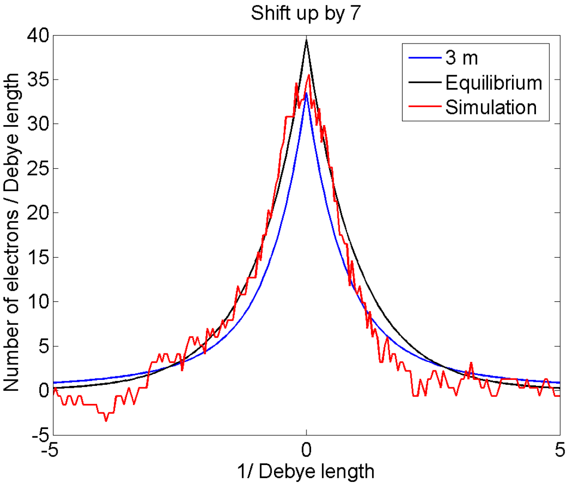
Previous Comparison



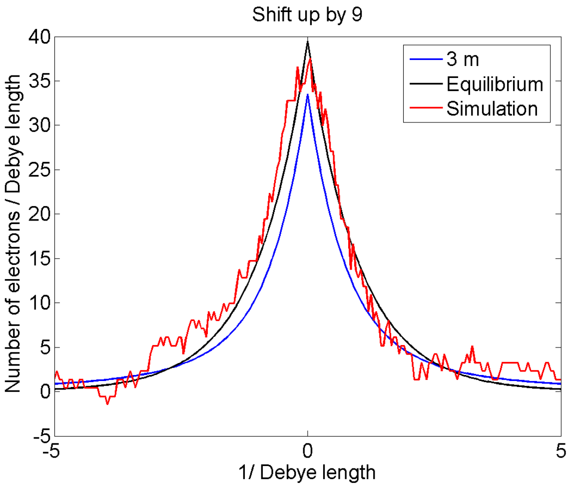
Comparison


No shift up
3 m
Equilibrium
Simulation
Number of electrons / Debye length
1/ Debye length
40
30
20
10
0
-10
-20
-5
0
5

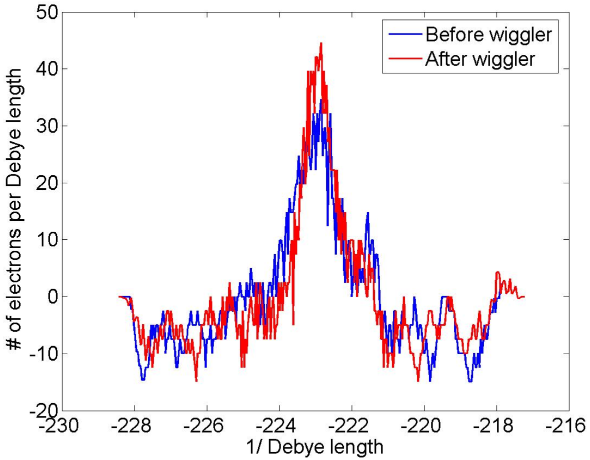
Comparison


Shift up by 7
Number of electrons / Debye length
1/ Debye length
3 m
Equilibrium
Simulation
40
35
30
25
20
15
10
5
0
-5
-5
0
5

Comparison

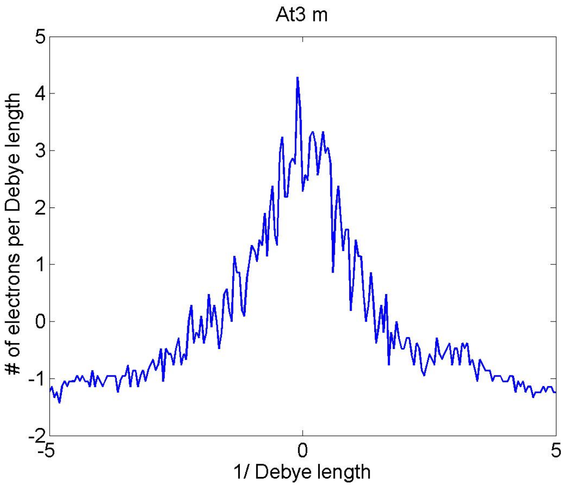

Shift up by 9
3 m
Equilibrium
Simulation
Number of electrons / Debye length
1/ Debye length
40
35
30
25
20
15
10
5
0
-5
-5
0
5

Previous wiggler simulation

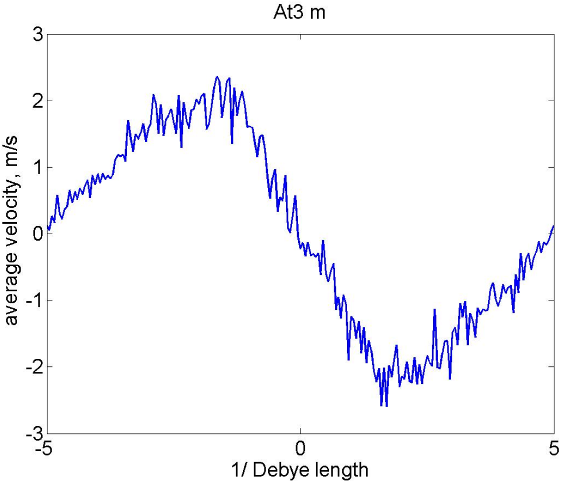


- Rerun modulator simulations using 1/10 of the previous electron number density
- Take the output of modulator as input of wiggler

Longitudinal number distribution



Longitudinal velocity distribution



Wiggler simulation