Review of Beam Diagnostics

Diktys Stratakis
Brookhaven National Laboratory
Stony Brook University

PHY 542
April 13, 2015
Beam transport

• Each particle is defined by position and momentum:

\[ \vec{x} = (x, p_x, y, p_y, z, p_z) \]

• More convenient is to use position and divergence

\[ \vec{\xi} = (x, x', y, y') \]

\[ \vec{\xi} = (x, x' = \frac{p_x}{p_z}, y, y' = \frac{p_y}{p_z}, z, \frac{\Delta p}{p}) \]

\[ v = \sqrt{v_x^2 + v_y^2 + v_z^2} \approx v_z \]

• Assuming no coupling the transverse motion can be represented by two dimensional vectors \( u = (x, x') \) and \( v = (y, y') \)

Phase-space

Paraxial approximation
Beam transport

- A good approximation for the beam distribution in phase-space is an ellipse

- Four parameters describe it:
  - $\beta$ related to beam size
  - $\alpha$ related to tilt
  - $\gamma$ related to the previous two
  - $\epsilon$ is related to the area of the ellipse and is used to gauge the beam quality

- Equation for ellipse:
  \[
  \gamma x^2 + 2\alpha xx' + \beta x'^2 = \varepsilon_x
  \]
Beam phase-space in a drift

\[ \mathbf{M} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \]

\[ \begin{pmatrix} \hat{\beta}_1 \\ \hat{\alpha}_1 \\ \hat{\gamma}_1 \end{pmatrix} \rightarrow \text{Lens} \rightarrow \begin{pmatrix} \hat{\beta}_2 \\ \hat{\alpha}_2 \\ \hat{\gamma}_2 \end{pmatrix} \]

- Beam size in 2:

\[ x_{\text{rms}} = \sqrt{\hat{\beta}^2 \epsilon_x} = \sqrt{\left(m_{11}^2 \hat{\beta} \hat{\alpha} - 2m_{11}m_{12} \hat{\alpha} \hat{\gamma} + m_{12}^2 \hat{\alpha} \hat{\gamma}\right) \epsilon_x} \]
Transport matrix

\[
\begin{pmatrix}
\hat{\beta}_2 \\
\hat{\alpha}_2 \\
\hat{\gamma}_2
\end{pmatrix} =
\begin{pmatrix}
m_{11}^2 & -2m_{11}m_{12} & m_{12}^2 \\
-m_{11}m_{21} & 1 + 2m_{12}m_{21} & -m_{12}m_{22} \\
m_{21}^2 & -2m_{21}m_{22} & m_{22}^2
\end{pmatrix}
\begin{pmatrix}
\hat{\beta}_1 \\
\hat{\alpha}_1 \\
\hat{\gamma}_1
\end{pmatrix}
\]

- Can be proven, but not during lecture…
Beam phase-space in a drift

- For a thin lens quadrupole and drift, the transfer matrix $M$ is:

$$ M = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix} = \begin{pmatrix} 1 - L/f & L \\ -1/f & 1 \end{pmatrix} $$

$$ x_{\text{rms}} = \sqrt{\beta_2 \epsilon_x} = \sqrt{(m_{11}^2 \beta_1 - 2m_{11}m_{12} \alpha_1 + m_{12}^2 \gamma_1) \epsilon_x} $$

$$ x_{\text{rms}} = \sqrt{\beta_2 \epsilon_x} = \sqrt{\left(1 - L/f\right)^2 \beta_1 - 2 \left(1 - L/f\right)L \alpha_1 + L^2 \gamma_1} \epsilon_x $$
Example: ATF

Quadrupole
In your simulation…

- Vary quad strength and record images on screen

Emittance from fit: 0.84 um
Emittance from fit: 0.87 um
Quad scan technique is limited...

- Make *a priori* assumption that the phase-space distribution is an ellipse
- Ignores space-charge effects
- Alternative options?
- Pepper-pot technique...
- Phase-Space Tomography (My PhD dissertation)…
Pepper pot technique (1)
Pepper pot technique (2)

- Resolution sometimes can be poor
- Problematic for very small beams
An object in n-dimensional space can be recovered from a sufficient number of projections onto (n-1)-dimensional space.
Introduction to tomography

Question: How we can rotate the phase-space distribution?
Beam profiles in phase-space

\[ c_\theta(x) = \iiint f(x, x', y, y') \, dx' \, dy \, dy' = \mu_\theta(x) \]
Beam profile example
Example
Example of complex beams

Heavy ion fusion
Reconstruction of complex beams

Experimental and numerical study of phase mixing of an intense beam

D. Stratakis, R. A. Kishek, I. Haber, S. Bernal, M. Reiser, and P. G. O’Shea
Institute for Research in Electronics and Applied Physics, University of Maryland, College Park, Maryland 20742, USA
(Received 28 July 2008; published 1 June 2009)

We study, experimentally and numerically, the relaxation of an initially nonuniform intense beam in an alternating-gradient transport line. A nonlinear distribution consisting of five interacting beamlets is created and tracked for longer than seven plasma periods with the help of tomographic phase-space mapping. Emittance growth is initially rapid, but slows down as the nonuniform distribution homogenizes in a few plasma periods. Both growth rates are found to depend on the beam current.
Another example of complex beams