

PHY 554

Fundamentals of Accelerator Physics

Lecture 11: Synchrotron Radiation

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Most (all) of materials come from
USPAS courses

<https://uspas.fnal.gov/index.shtml>



U.S. Particle Accelerator School

America's National School of Accelerator Science, Engineering and Technology



Lector/Instructor

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- Adjunct Professor at Stony Brook University , since 2015
- PhD in Accelerator Physics in 2001

Main area of scientific interest:

- Electron coolers
- Energy Recovery Linacs
- Free Electron Laser
- Beam dynamics of Charged particles

In Free time I Enjoy

Outdoor: camping, kayaking, rafting, hiking

At home: reading, watching movies, playing with my kids

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Why are we interested in photons?

- Photons are ideal probe of matter for physics, chemistry, biology, medicine, material science, environmental science, and many more

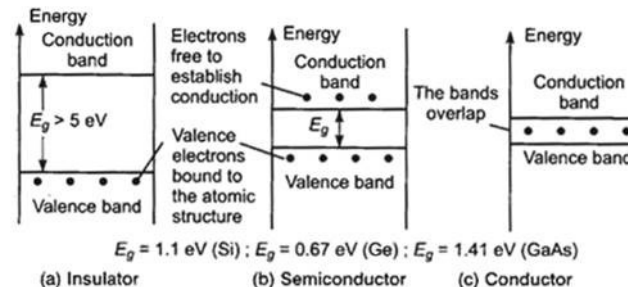
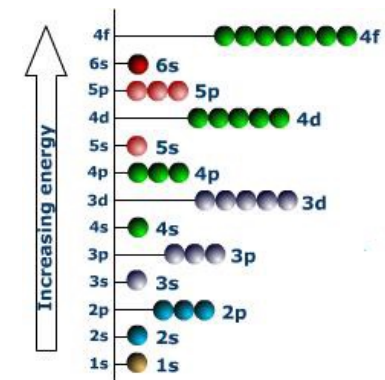
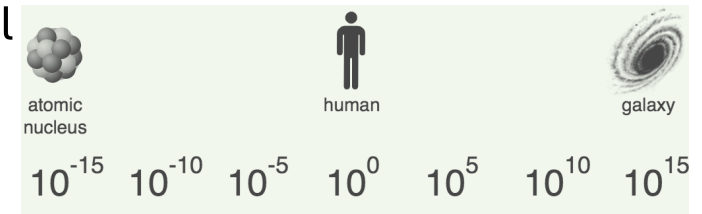
- Spectral resolution: photon wavelength λ sets roughly resolution $\rightarrow$ sub-atomic scales require 1 Å or 12 keV

- Energy resolution: electrons in atoms/molecules are located in orbitals with different energy levels $\rightarrow$ to excite electrons (photon absorption or scattering) or to ionize atoms, requires >few eV, ideally tunable (to scan, eg. spectroscopy)

- Temporal resolution: to probe fast phenomena need high time resolution $\rightarrow$ short exposure

time & high intensity (as in stroboscope)

- e-motion in atoms: ≈ 0.1 fs
- molecular vibrations: ≈ 1 fs
- magnetic domain flips: ≈ 1 ps
- protein folding: > 100 μ s



So what does the ideal photon source look like?

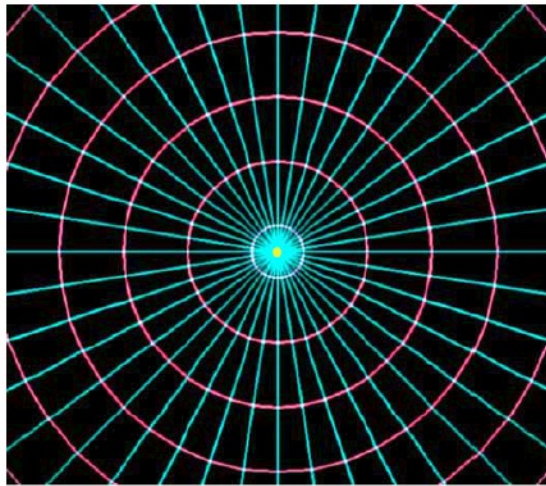
- Ideally, we'd want a short pulse of (quasi-) mono-energetic photons with appropriate (and tunable) wavelength
- In order to perform experiments in limited amount of time want as many photons per pulse as possible
- To some of you this might start to sound like a laser...
- but for acceptable levels of ph/pulse lasers are limited to few nm, and lasers are not freely tunable over a broader range
- Fortunately, an electron (or positron) synchrotron emitting synchrotron radiation (SR) generates huge numbers of ph/pulse at tunable wavelengths over a very broad energy range → mm to $<1 \text{ \AA}$

NSLS-II at Brookhaven National Lab

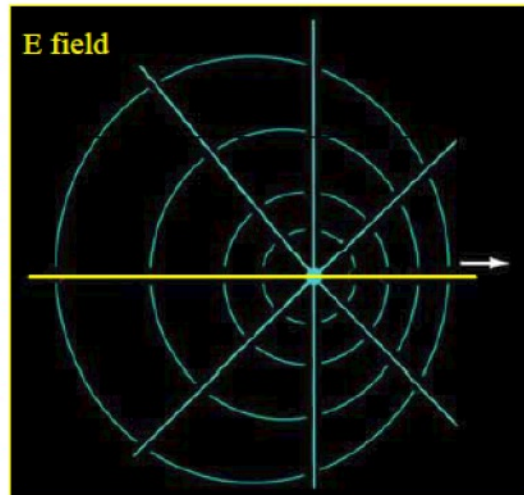


<https://www.bnl.gov/nsls2/10/>

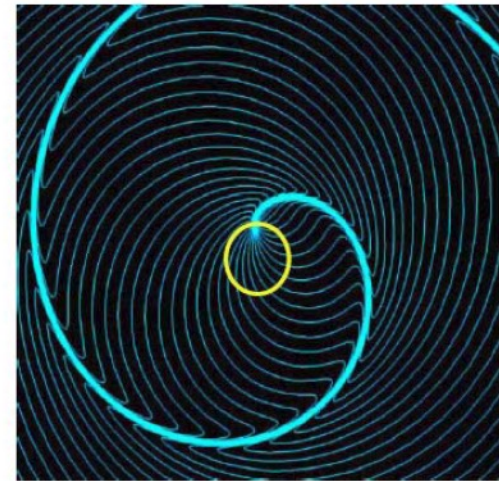
What is synchrotron radiation



Static field for a charge at rest



When a particle moves with a constant velocity, field moves with particle.



When a particle gets accelerated, some part of the field moves away from the particle to infinity: **radiation.**

The electromagnetic radiation emitted when charged particles are accelerated radially, $\vec{a} \perp \vec{v}$, is called **synchrotron radiation**.

How to generate synchrotron radiation

- **Quantum field theory:** particles moving in free space are surrounded by cloud of additional virtual particles (incl. photons)
- These particles are extremely short-lived and remain very close to free particle → cannot detect or use (hence "virtual")
 - However, if free particle undergoes strong transverse acceleration it can detach itself from "cloud" of **virtual photons** → photons become real → can be detected and used
 - Strong transverse acceleration is why we use e^- & e^+ in accelerators to generate SR photons → q/m ratio is maximized compared to hadrons → bending magnets create sharp curved trajectories → reduces emitted photon wavelength & more photons generated
(we'll see why shortly)

How to generate SR (Classical picture)

Joseph Larmor
1857-1942
Irish Physicist



- **Classical picture:** Joseph Larmor, 1897: instantaneous total power radiated by accelerated charged particle (as in eg. antenna)

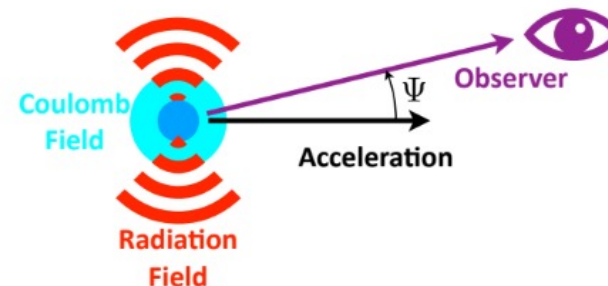
$$P = \frac{q^2}{6\pi\epsilon_0 c^3 m^2} \left(\frac{d\vec{p}}{dt} \right)^2$$

$\epsilon_0 = 8.85 \times 10^{-12} \text{ A s/V m}$
Permittivity of free space

→ without **acceleration** there is no radiation

- Power varies as $\sin^2 \Psi$ where Ψ is angle between acceleration $\frac{d\vec{p}}{dt}$ and point of observation → power emitted perpendicular to acceleration (as Hertz dipole)

$$\frac{dP}{d\Omega} = \frac{q^2}{16\pi^2\epsilon_0 c^3 m^2} \left(\frac{d\vec{p}}{dt} \right)^2 \sin^2 \Psi$$



How to generate SR (very high-speed ($\sim c$) particles)

Alfred Liénard, 1898: Extended Larmor's theory to very high-speed particles $\rightarrow$ use for relativistic particles undergoing centripetal acceleration along circular trajectory:

Alfred-Marie Liénard
1869-1958
French Physicist



$$P = \frac{q^2 c}{6\pi\epsilon_0 (mc^2)^2} \left[\left(\frac{d\vec{p}}{d\tau} \right)^2 - \frac{1}{c^2} \left(\frac{dE}{d\tau} \right)^2 \right]$$

where we have used

$$dt \rightarrow d\tau = \frac{1}{\gamma} dt$$

4-vector **P**

$$\left(\frac{d\mathbf{P}}{d\tau} \right)^2 \rightarrow \left(\frac{d\vec{p}}{d\tau} \right)^2 - \frac{1}{c^2} \left(\frac{dE}{d\tau} \right)^2$$

$$\begin{aligned} p &= \gamma m \beta c \\ E &= \gamma m c^2 \\ &= \frac{pc}{\beta} \end{aligned}$$

- This calls for an important distinction. But first, recall that

$$E^2 = (mc^2)^2 + p^2 c^2 \rightarrow E \frac{dE}{d\tau} = c^2 p \frac{dp}{d\tau} \rightarrow \frac{dE}{d\tau} = v \frac{dp}{d\tau}$$

So in general we can arrive at:

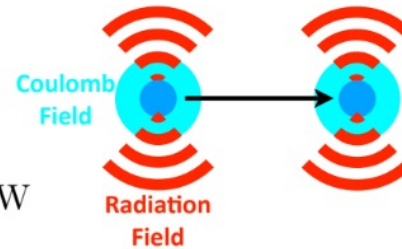
$$P = \frac{q^2 c}{6\pi\epsilon_0 (mc^2)^2} \underbrace{\left[1 - \frac{v^2}{c^2} \right]}_{1/\gamma^2} \left(\frac{d\vec{p}}{d\tau} \right)^2 = \frac{q^2 c}{6\pi\epsilon_0 (mc^2)^2} \left(\frac{d\vec{p}}{dt} \right)^2$$

As a result, we have two ways to generate SR

- **Case 1:** Linear acceleration: $\vec{p} = p_{\parallel} \rightarrow \frac{dp_{\parallel}}{dt} = \frac{dE}{dx}$

$$P_{\parallel} = \frac{q^2 c}{6\pi\epsilon_0 (mc^2)^2} \left(\frac{dp_{\parallel}}{dt} \right)^2 = \frac{q^2 c}{6\pi\epsilon_0 (mc^2)^2} \left(\frac{dE}{dx} \right)^2 \approx 10^{-16} \text{ W}$$

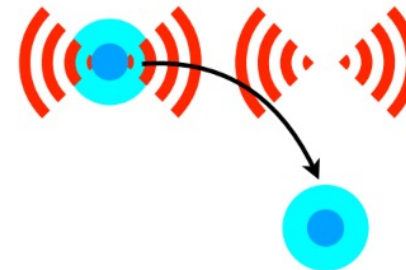
Even in best linacs <100 MV/m



- **Case 2:** Circular acceleration: $\vec{p} = \vec{p}_{\perp}$, $E = \text{const}$

$$\begin{aligned} P_{\perp} &= \frac{q^2 c}{6\pi\epsilon_0 (mc^2)^2} \left(\frac{dp_{\perp}}{d\tau} \right)^2 = \frac{q^2 c}{6\pi\epsilon_0 (mc^2)^2} \gamma^2 \left(\frac{dp_{\perp}}{dt} \right)^2 \\ &= \frac{q^2 c}{6\pi\epsilon_0 (mc^2)^2} \gamma^2 \frac{E^2}{\rho^2} \\ &= \frac{q^2 c}{6\pi\epsilon_0 \rho^2} \gamma^4 \end{aligned}$$

$$\frac{dp}{dt} = \frac{dp}{d\varphi} \frac{d\varphi}{dt} = p\omega = \frac{p v}{\rho} \approx \frac{pc}{\rho} = \frac{E}{\rho}$$



Radiated power increases dramatically with energy $\rightarrow$ sets practical limit for maximum energy in a circular accelerator $\rightarrow$ but also makes them highly suitable as sources of synchrotron radiation

What else to remember ?

- Note however, we want not just high E , but rather high $\gamma \rightarrow$ this is why light sources choose to accelerate leptons (e^- , e^+) instead of hadrons (p^+ , p^-)

$$\frac{P_e}{P_p} = \left(\frac{m_p c^2}{m_e c^2} \right)^4 \approx 10^{13}$$

- Let us now calculate the radiated energy per turn:

$$\begin{aligned} U_{\text{loss}} = \Delta E &= \oint P_{\text{SR}} dt \approx P_{\text{SR}} T_{\text{rev}} = P_{\text{SR}} \frac{2\pi\rho}{c} \\ &= \frac{q^2 \gamma^4}{3\epsilon_0 \rho} \\ \Delta E [\text{keV}] &\approx 88.5 \frac{E [\text{GeV}]^4}{\rho [\text{m}]} \end{aligned}$$

Advanced Light Source (ALS) at LBNL



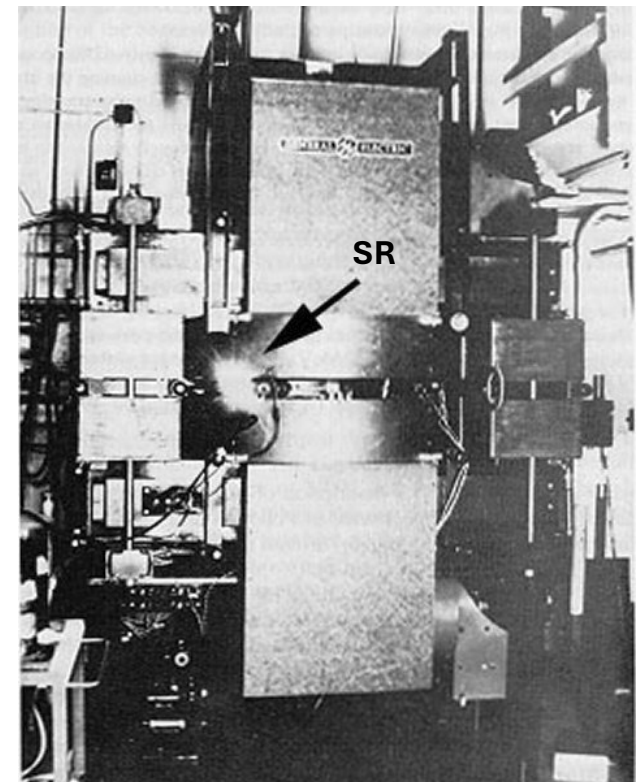
Example: ALS @ 1.9 GeV, $\langle\rho\rangle = 4.2$ m $\rightarrow \Delta E = 275$ keV

<https://als.lbl.gov/>

SR Light sources need high gamma

- Discovery of SR took so long because we first needed accelerators capable of achieving required energies
 - Synchrotron radiation was named after its discovery in Schenectady, New York from a **General Electric** synchrotron accelerator **built in 1946 and announced in May 1947** by Frank Elder, Anatole Gurewitsch, Robert Langmuir .
 - Synchrotron radiation is the **main constraint to accelerate electrons to very high energy** and hence is bad for high energy physics application, such as colliders.
 - However, it was then realized that the radiation can be so **helpful for other branches of science** such as biology, material science and medical applications.
- As a result, dedicated storage rings have been built to generate synchrotron radiation, which are called light sources.

70 MeV electron synchrotron, GE
Research Labs, Schenectady, NY, 1947



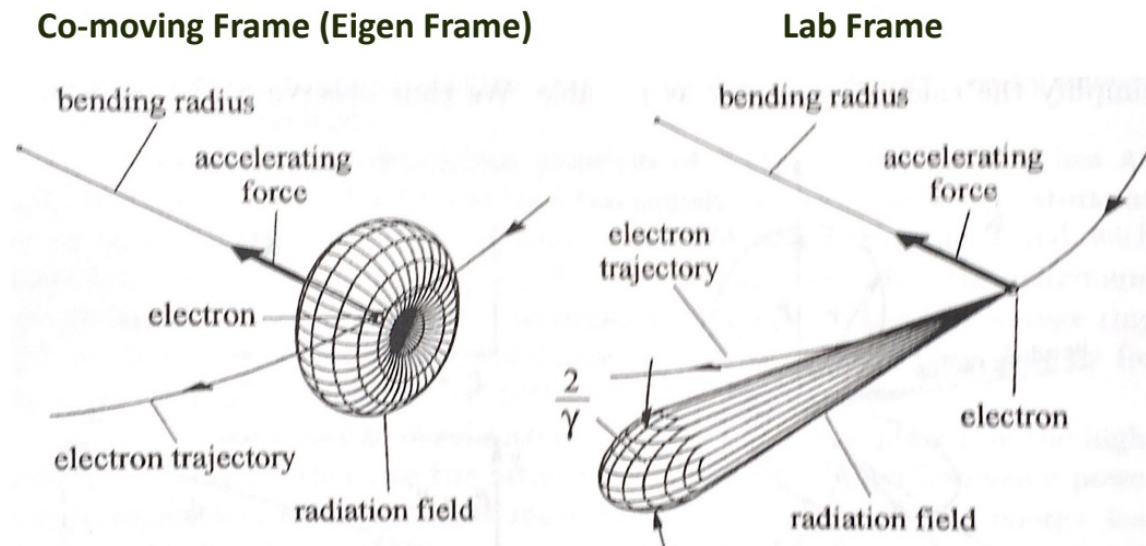
SR limits high energy lepton colliders

- For high-energy accelerators such as colliders SR was considered a waste product and a serious limitation
- LEP (CERN, 1990s) eventually reached 70 GeV, but at a circumference of 27 km ($\rho \approx 3 \text{ km} \rightarrow B_{\text{peak}} = 0.078 \text{ T}$) and yet, every last free drift space was occupied by RF cavities in an attempt to replenish energy lost to SR
(for the **homework** estimate how much energy is lost per turn at LEP)
- LEP eventually replaced by LHC (hadron machine, SR suppressed by 10^{13})



Angular Distribution of Synchrotron Radiation

- Due to acceleration of highly relativistic particles, emission is strongly focused and forward directed
- One can show that emission occurs into a cone with **half opening angle $1/\gamma$** (“**Lorentz boost**” → direct consequence of Lorentz transformation)



Spectral Distribution of Synchrotron Radiation

- Since we already know $\Theta \approx 1/\gamma$ we can calculate the part of the trajectory an observer can see:

$$\Delta l_{A \rightarrow B} = 2\Theta\rho \approx \frac{2\rho}{\gamma}$$

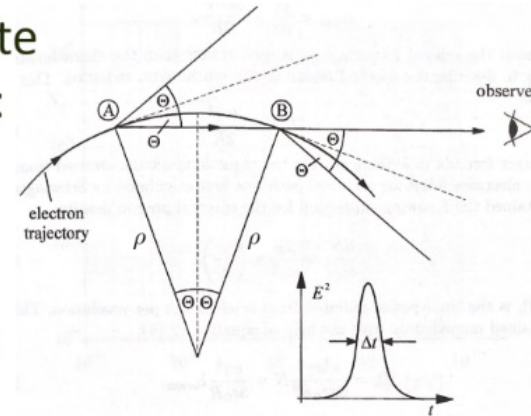
- Pulse length given by difference in time it takes the electron and photon to cover $\Delta l_{A \rightarrow B}$:

$$\begin{aligned} \Delta t &= t_e - t_\gamma = \frac{\Delta l}{\beta c} - \frac{\Delta l}{c} = \frac{\Delta l}{\beta c} (1 - \beta) \\ &= \frac{\Delta l}{\beta c} \frac{1 - \beta^2}{1 + \beta} \approx \frac{\Delta l}{\beta c} \frac{1}{2\gamma^2} \approx \frac{\rho}{c\gamma^3} \end{aligned}$$

Compression by $1/\gamma^2$ is Doppler effect
(radiation emitted by moving source)

- Literature sometimes quotes a better approximation: $\Delta t \approx \frac{4\rho}{3c\gamma^3}$
- Recall, a short EM pulse leads to a broad spectrum with $\Delta\omega \propto \frac{1}{\Delta t}$
- We define the "**characteristic frequency**" as:

$$\omega_{\text{char}} = \frac{2\pi}{\Delta t} = \frac{3\pi c\gamma^3}{2\rho}$$



Spectral Distribution of SR (cont.)

- Most common, however, is use of the "**critical frequency**"

$$\omega_c = \frac{\omega_{\text{char}}}{\pi} = \frac{3c\gamma^3}{2\rho}$$

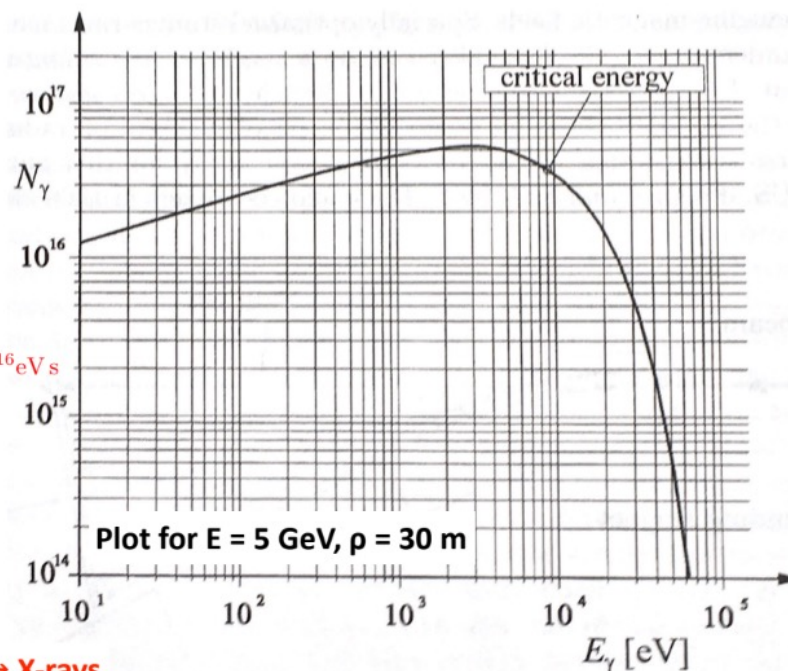
- It can be shown that ω_c divides SR into two regions of equal radiated power P
- In this context we commonly use the "**critical energy**"

$$E_c = \hbar\omega_c = \frac{3\hbar c\gamma^3}{2\rho} \quad \hbar = 6.582 \times 10^{-16} \text{ eV s}$$

- Illustrative example **ALS**:
1.9 GeV, average bending radius $\rho = 4.2$ m:

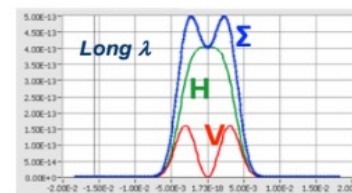
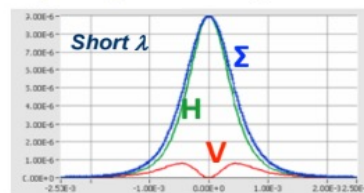
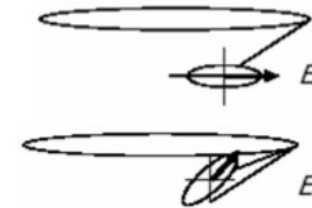
$$\lambda_c = \frac{c}{f_c} = \frac{2\pi c}{\omega_c} = \frac{4\pi\rho}{3\gamma^3} = 0.34 \text{ nm}$$

$E_c = 3.6 \text{ keV} \rightarrow \text{X-rays}$



Polarization of Synchrotron radiation

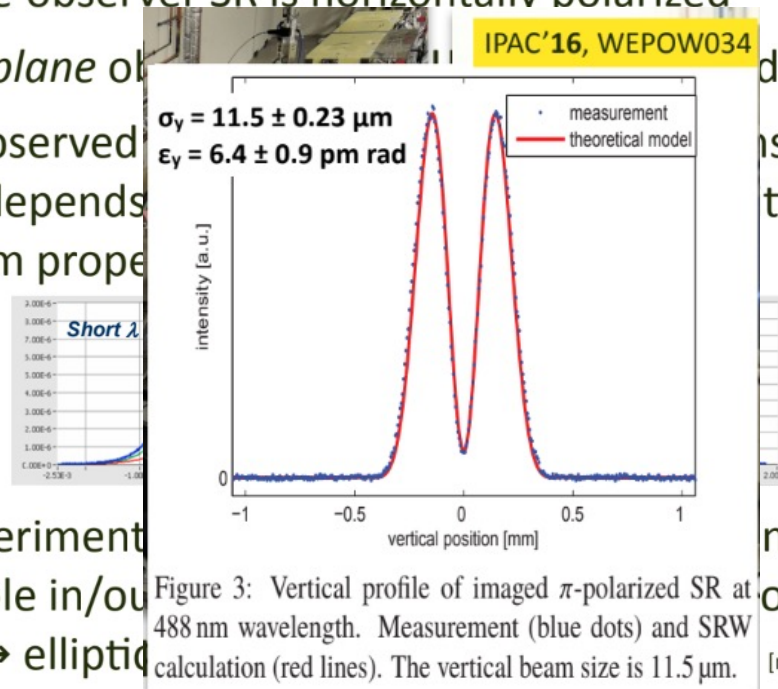
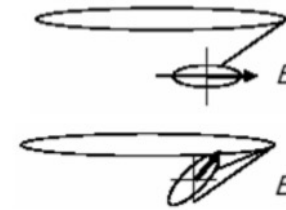
- To an *in-plane* observer SR is horizontally polarized
- To an *out-of-plane* observer SR is elliptically polarized
- In general, observed SR contains various polarizations; distribution depends on wavelength, observer's position/angle, electron beam properties, etc.



- Many SR experiment require specific polarization → instead of moving sample in/out of plane, use special devices to create specific polarization → elliptically polarizing undulator (EPU)
- Interesting diagnostics application: "***π-pol. method***" → measure very small vertical electron beam size with just visible SR (no x-rays required)

Polarization of Synchrotron radiation

- To an *in-plane* observer SR is horizontally polarized
- To an *out-of-plane* observer SR is vertically polarized
- In general, observed polarization distribution depends on electron beam properties and observation angle



- Many SR experiments use a moving sample in/out of the beam to create specific polarization → elliptical polarization
- Interesting diagnostics application: " **π -pol. method**" → measure very small vertical electron beam size with just visible SR (no x-rays required)

instead of
to create specific
[more on that in just a moment]

Flux & Brightness

- These are the key benchmarks of a SR source
- **Flux** = no. of photons in $\Delta\omega/\omega$ per unit time:

$$\mathcal{F} = \frac{\dot{N}_{\text{ph}}}{(\Delta\omega/\omega)} \quad \leftarrow \text{Bandwidth}$$

- **Brightness** = flux per solid angle and unit area:

$$\mathcal{B} = \frac{\mathcal{F}}{4\pi^2 \Delta x \Delta y \Delta\theta \Delta\phi} = \frac{\dot{N}_{\text{ph}}}{4\pi^2 \Delta x \Delta y \Delta\theta \Delta\phi (\Delta\omega/\omega)}$$

- When dealing with SR emitted by e^-/e^+ beams it's convenient to use:

$$\mathcal{B} = \frac{\mathcal{F}}{4\pi^2 \sigma_x \sigma_{x'} \sigma_y \sigma_{y'}} = \frac{\dot{N}_{\text{ph}}}{4\pi^2 \sigma_x \sigma_{x'} \sigma_y \sigma_{y'} (\Delta\omega/\omega)}$$

- If the beam is a point source, it can be shown the emitted SR obeys:

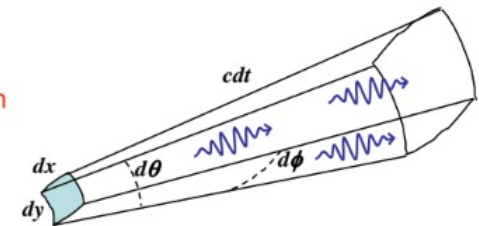
$$\epsilon_{\text{ph}} = \sigma_r \sigma_{r'} = \frac{\lambda}{4\pi} \rightarrow \mathcal{B}_{\text{max}} = \dot{N}_{\text{ph}} \frac{4}{\lambda^2} (\Delta\omega/\omega)^{-1}$$

Assuming photon beam Gaussian

Apparent rms source size

Apparent rms source divergence

Phase space volume of SR photons (depends only on wavelength)



Flux & Brightness (cont.)

- However, in reality photons emitted from e-/e+ beams with finite emittance → we need to account for that in brightness by using

$$\sigma_{x,\text{tot}} = \sigma_r \oplus \sigma_x = \sqrt{\sigma_r^2 + \sigma_x^2}, \quad \sigma_{x',\text{tot}} = \sigma_{r'} \oplus \sigma_{x'} = \sqrt{\sigma_{r'}^2 + \sigma_{x'}^2}$$

$\uparrow \epsilon_x \beta_x$
 $\uparrow \epsilon_x / \beta_x$

and likewise for y . If e-/e+ beam is a point source or $\epsilon_{x,y} \ll \epsilon_{\text{ph}} \rightarrow \mathcal{B}$ will approach $\mathcal{B}_{\text{max}}$ and we achieve high degree of transverse coherence

- It can be shown that for photons emitted from an insertion device of length L (applying the rather coarse approximation of Gaussian beams), the intrinsic photon beam size & divergence are given by

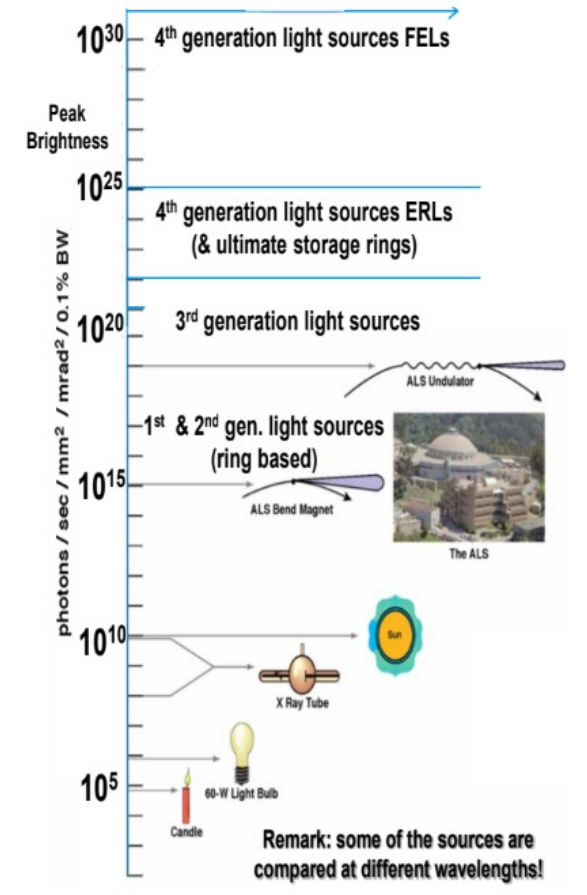
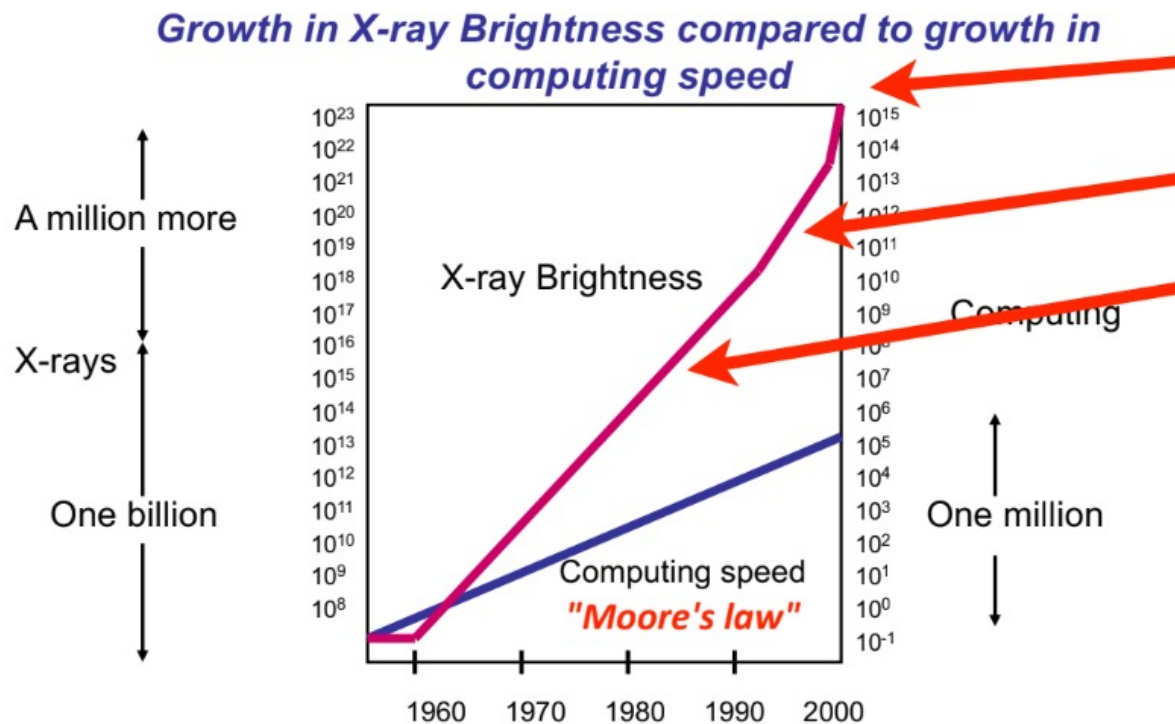
$$\sigma_r = \frac{\sqrt{2L\lambda}}{4\pi}, \quad \sigma_{r'} = \sqrt{\frac{\lambda}{2L}}$$

Depend only on wavelength and ID length

- We refer to e-/e+ beams being "**diffraction-limited**" when the contributions from e-/e+ beam and intrinsic photon beam become comparable → requires proper matching of optics: $\hat{\beta}_{x,y} = \sigma_r / \sigma_{r'} = L / 2\pi$

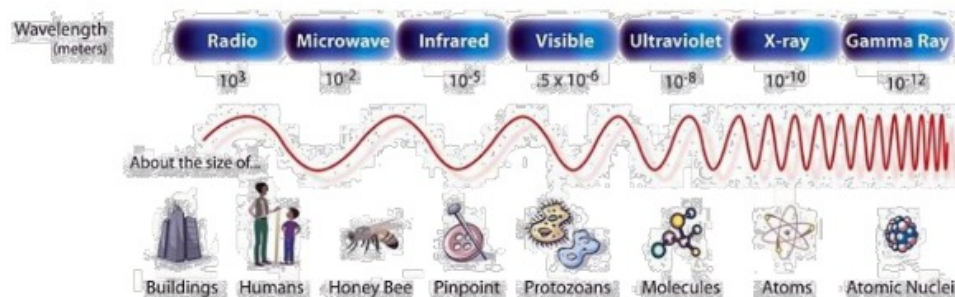
So how bright are synchrotron light sources?

- Extremely bright!



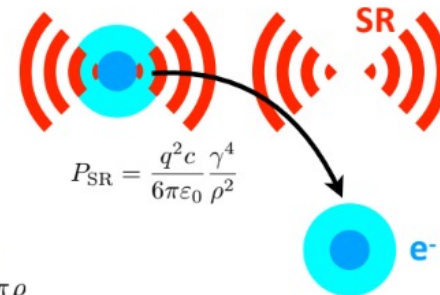
So how bright are synchrotron light sources?

- Extremely bright!
 - This great brightness is highly desirable because it allows for
 - faster experiments (time-resolved; also, consider overbooking of sources)
 - improved spacial/temporal/energy resolution
 - higher spatial coherence
 - Similar advantages to lasers in the visible range, but here we can offer wavelengths from IR all the way down to hard x-rays
 - And unlike table-top lasers, SR sources are tunable → yes, here you can have it all (almost)
- 



SR Damping & Effect on Emittance

- To better understand the physics of SR storage rings, let's examine how the emission of SR acts back on the electron (or positron) beam
- Recall, SR emitted only due to radial acceleration in storage ring (no meaningful contribution from longitudinal acceleration in RF cavities)
- Therefore, we calculate the energy lost per turn as:



$$P_{\text{SR}} = \frac{q^2 c}{6\pi\epsilon_0} \frac{\gamma^4}{\rho^2}$$

$$U_{\text{loss}} = \Delta E = \oint P_{\text{SR}} dt \approx P_{\text{SR}} T_{\text{rev}} = P_{\text{SR}} \frac{2\pi\rho}{c}$$

$$= \frac{q^2 \gamma^4}{3\epsilon_0 \rho}$$

- Now recall damping from Longitudinal Dynamics I:

$$a_s = \left. \frac{1}{2T_0} \frac{dU}{dE} \right|_{E_0} = \frac{1}{2T_0} \frac{d}{dE} \oint P_{\text{SR}}(E_0) dt = \frac{U_0}{2T_0 E_0} (2 + \mathcal{D})$$

$$\Delta \ddot{E} + 2a_s \Delta \dot{E} + \Omega_s^2 \Delta E = 0$$

$$\Delta E(t) = \Delta E_0 e^{-a_s t} e^{i\Omega_s t}$$

where $\mathcal{D}$ depends only on the magnetic lattice. In the simplest case

(iso-magnetic separate-function lattice):

$$\mathcal{D} = \alpha_c \frac{L_{\text{bend}}}{2\pi\rho}$$

SR Damping & Effect on Emittance (cont.)

- In the transverse plane it can be shown that a similar formalism can be used to describe damping of betatron motion due to SR:

$$\begin{aligned}a_x &= \frac{U_0}{2T_0 E_0} (1 - \mathcal{D}) \\a_y &= \frac{U_0}{2T_0 E_0}\end{aligned}$$

- Sometimes it's more convenient to use the "**partition numbers**" $J_{x,y,s}$:

$$a_{x,y,s} = \frac{U_0}{2T_0 E_0} J_{x,y,s} \longrightarrow \begin{aligned}J_x &= 1 - \mathcal{D} \\J_y &= 1 \\J_s &= 2 + \mathcal{D}\end{aligned}$$

- Partition numbers must obey the "**Robinson theorem**":

$$\sum_{i=x,y,s} J_i = 4$$

- This theorem tells us that in designing lattices (varying $\mathcal{D}$), although we can distribute damping among planes (i.e. trade off damping through synchrotron motion vs. betatron motion), the overall damping remains fixed

SR Damping & Effect on Emittance (cont.)

- For separate-function magnets generally $\mathcal{D} \ll 1 \rightarrow$ "natural" damping distribution:

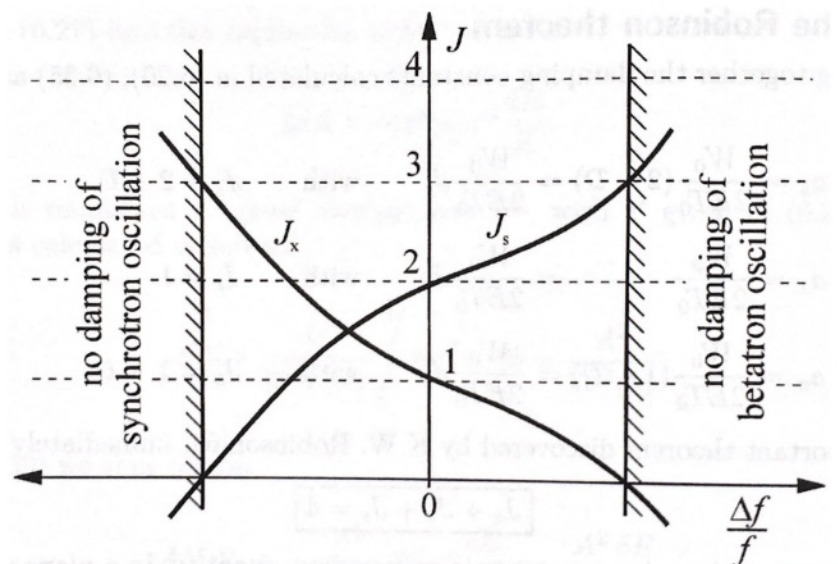
$$J_x \approx J_y \approx 1, \quad J_s \approx 2 \quad \leftarrow \text{Long. damping twice as strong as transverse}$$

- However, if we put strong focusing gradients into dipoles we observe

$$J_x \rightarrow 2, \quad J_y \approx J_s \approx 1 \quad \leftarrow \text{H damping twice as strong/fast}$$

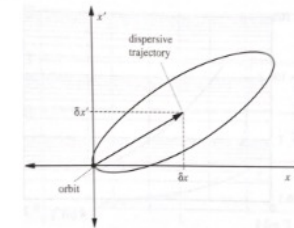
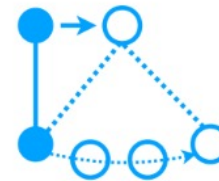
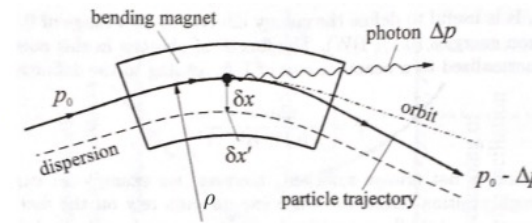
- For a given lattice, we can vary the accelerating RF to change the damping distribution, since

$$x_\delta(s) = D(s) \delta = D(s) \frac{\Delta f_{\text{rf}}}{f_{\text{rf}}} \frac{1}{\alpha_c}$$

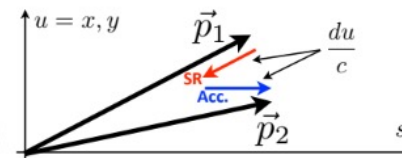


SR Damping & Effect on Emittance (cont.)

- So with all of this damping, why doesn't the e^- beam shrink to infinitely small dimensions?
- Recall that QM says emission of SR is a quantized, stochastic process



- Electron emits SR in dipole $\rightarrow$ suffers energy loss $\rightarrow$ electron is now on dispersive orbit with a betatron oscillation amplitude (even if its initial betatron amplitude was zero) around this dispersive orbit $\rightarrow$ will require sufficient SR damping (in the longitudinal and transverse) to return it to both E_0 and $x \approx x' \approx 0$
- However, since SR emission is a stochastic process $\rightarrow$ continuous excitation of betatron motion (stochastic energy loss excites oscillations, i.e. "**quantum excitation**") that competes with "**radiation damping**" process (photon emission $\rightarrow$ e^- recoil $\rightarrow$ betatron & synchrotron oscillations $\rightarrow$ eventually oscillations damp down via radiation)



SR Damping & Effect on Emittance (cont.)

- The equilibrium of these two competing processes is what defines the "**natural emittance**"

- Simple illustration:

- assume motion along dispersive orbit (without betatron oscillations)

$$x_\delta(s) = D(s) \delta, \quad x'_\delta(s) = D'(s) \delta$$

- we can now calculate the action (recall the C-S parameter) associated with this dispersive orbit:

Action, not ensemble emittance! $\rightarrow \varepsilon_x$

$$\begin{aligned} \varepsilon_x &= \gamma_x x_\delta^2 + 2\alpha_x x_\delta x'_\delta + \beta_x x'^2_\delta \\ &= \delta^2 \underbrace{(\gamma_x D^2 + 2\alpha_x D D' + \beta_x D'^2)}_{\mathcal{H}(s)} \end{aligned}$$

Simplified notation:
explicit s-dependence dropped

- We refer to $\mathcal{H}(s)$ as the "**dispersion invariant**". It depends only on the magnetic optics and the position s . We'll use this in a moment.

- Useful: maximum dispersion amplitude at s given by $\sqrt{\hat{\mathcal{H}}\beta_x(s)} \delta$
 $\leftarrow$ Peak $\mathcal{H}$ around ring

SR Damping & Effect on Emittance (cont.)

- It can be shown (M. Sands, 1971) that the ***natural emittance*** of an ideally flat storage ring is given by:

$$\varepsilon_x = \frac{55}{32\sqrt{3}} \frac{\hbar \gamma^2}{mc} \frac{I_5}{I_2 - I_4}$$

Matthew Linzee Sands
1919-2014
SLAC



where we have used the "***radiation integrals***" defined as:

$$I_1 = \oint \frac{D(s)}{\rho} ds \quad I_2 = \oint \frac{ds}{\rho^2} \quad I_3 = \oint \frac{ds}{|\rho^3|} \quad I_4 = \oint \frac{(1 - 2n)D(s)}{\rho^3} ds \quad I_5 = \oint \frac{\mathcal{H}(s)}{|\rho^3|} ds$$

Field index : $n = -k\rho^2$

- Note: s-dependence of ρ not explicitly shown
- Note: for drifts $1/\rho \rightarrow \infty$ so only need to evaluate integrals in dipoles
- Very important: this means magnetic lattice alone defines emittance. This is specific to rings (and results from their periodicity). In linacs or transfer lines emittance depends on initial ensemble properties $\rightarrow$ matching is important

SR Damping & Effect on Emittance (cont.)

- The radiation integrals are usually computed for us by optics codes, but for simple cases we can make good estimates

$$\varepsilon_x = \frac{55}{32\sqrt{3}} \frac{\hbar\gamma^2}{mc} \frac{I_5}{I_2 - I_4}$$

$$\mathcal{H} = \gamma_x D^2 + 2\alpha_x D D' + \beta_x D'^2$$

- There are many convenient applications of these integrals:

$$I_5 = \oint \frac{\mathcal{H}(s)}{|\rho^3|} ds$$

– Momentum compaction: $\alpha_c = \frac{I_1}{C}$

– Energy loss per turn: $U_0 = \frac{2r_e\gamma^3}{3} E_0 I_2$

$$\varepsilon_x \propto \gamma^2 \frac{I_5}{I_2 - I_4} \propto \gamma^2 \frac{I_5}{J_x U_0}$$

Low dispersion, large losses, and high $J_x \rightarrow$ low natural emittance

– Damping partitions: $\mathcal{D} = \frac{I_4}{I_2} \rightarrow J_x = 1 - \frac{I_4}{I_2}, J_s = 2 + \frac{I_4}{I_2}$

– Damping times: $\tau_{x,y,s} = a_{x,y,s}^{-1} = \frac{2T_0 E_0}{U_0} J_{x,y,s}^{-1} \approx \frac{3\mathcal{C}\langle\rho\rangle}{2\pi r_e c \gamma^3} J_{x,y,s}^{-1}$

$$\Delta\ddot{E} + 2a_s\Delta\dot{E} + \Omega_s^2\Delta E = 0$$

$$\Delta E(t) = \Delta E_0 e^{-a_s t} e^{i\Omega_s t}$$

– Natural energy spread: $\sigma_E = \sqrt{\frac{55}{32\sqrt{3}} \frac{\hbar\gamma^2}{mc} \frac{I_3}{2I_2 + I_4}}$

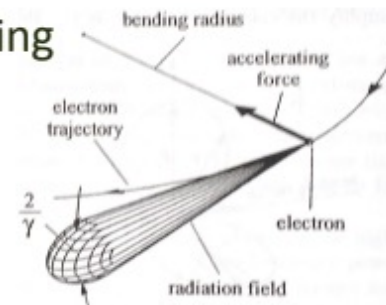
$I_2 J_s$

High-energy rings damp quickly

SR Damping & Effect on Emittance (cont.)

- But wait! In conventional (flat) storage rings, dipoles bend only in horizontal plane $\rightarrow D = D_x \rightarrow$ does $\varepsilon_y = 0$ thus follow from $D_y = 0$?

- Almost. Recall SR is emitted into cone with half opening angle $1/\gamma \rightarrow$ there is a small amount of emission momentum in vertical plane $\rightarrow$ this results in a small amount of recoil in vertical plane $\rightarrow$ excites small β_y oscillations $\rightarrow$ small "**natural vertical emittance**"



- In reality unavoidable errors in machine (alignment errors, magnet imperfections, orbit constraints) lead to a finite D_y which leads to ε_y usually several factors larger than natural limit, eg. Swiss Light Source measured $\varepsilon_y = 0.9 \text{ pm rad} \rightarrow$ less than 0.02% of ε_x and very close to natural limit (after years of correction and realignment efforts across entire storage ring)



SR Damping & Effect on Emittance (cont.)

- In practice, many light sources deliberately excite vertical dispersion (through skew quadrupoles) to increase ε_y in order to increase the bunch volume and thereby lower charge density $\rightarrow$ relaxes particle losses and intrabeam scattering effects
- It is therefore quite common to specify the **"emittance coupling" κ** using

$$\begin{aligned}\varepsilon_y &= \frac{\kappa}{\kappa+1} \varepsilon_0, & \varepsilon_0 : \text{natural emittance} \\ \varepsilon_x &= \frac{1}{\kappa+1} \varepsilon_0 \\ \kappa &= \varepsilon_y / \varepsilon_x\end{aligned}$$

- Brief take home message: lepton rings emit large amounts of SR leading to an equilibrium state characterized by $\varepsilon_{x,y,s}$ and Gaussian distributions in all 3 planes determined by energy, magnetic lattice, and RF acceleration parameters



Sources of SR: Dipole Magnets

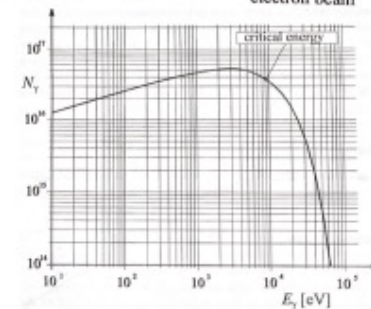
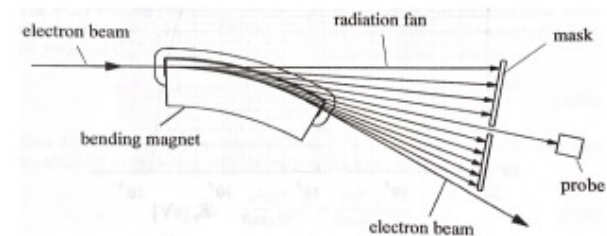
- Let us return to storage rings as sources of SR and study the actual emission. The most simple source is the bend magnet.
- We have already seen the energy distribution is split in halves (in terms of power) around the critical energy

$$E_c = \hbar\omega_c = \frac{3\hbar c\gamma^3}{2\rho} \quad \hbar = 6.582 \times 10^{-16} \text{ eV s}$$

- Practical units: $E_c [\text{eV}] = 665 E^2 [\text{GeV}] B [\text{T}]$

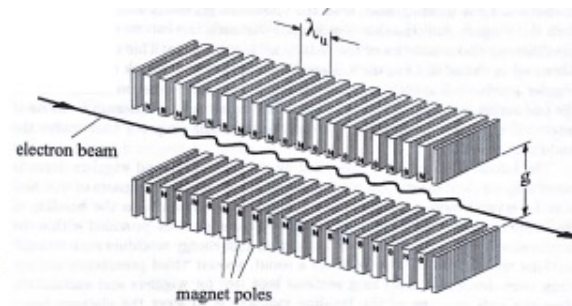
- Examples:

- ALS: 1.9 GeV, $B_{\text{peak}} = 1.5 \text{ T} \rightarrow E_c = 3.6 \text{ keV}$
- ALS superbend: $B_{\text{peak}} = 5 \text{ T} \rightarrow E_c = 12 \text{ keV}$
- SPring-8 (Hyogo, Japan): 8 GeV, $B_{\text{peak}} \approx 0.7 \text{ T} \rightarrow E_c = 30 \text{ keV}$



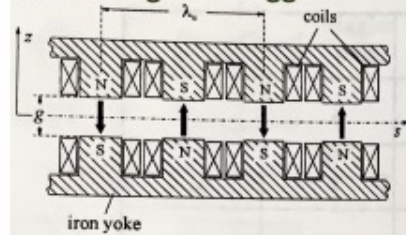
Sources of SR: W wigglers

- Array of alternating permanent magnets (invented by K. Halbach in Berkeley, 1981) → as if electron beam traversed many small bending magnets and emitted SR at each

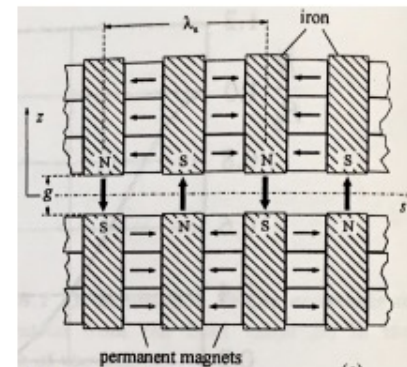
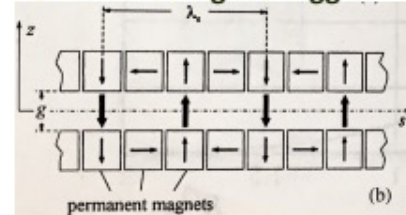


- Many different technical implementations of this idea...

Electromagnetic wiggler



Permanent magnet wiggler



Hybrid wiggler

...but beam physics remains the same for all

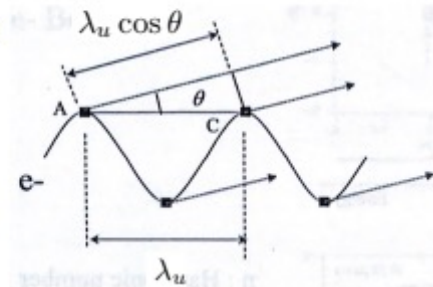
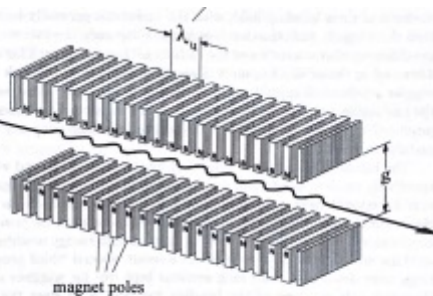


Sources of SR: Undulators

- Consider a "wiggler" but designed such that under certain angles of emission, constructive interference occurs, for example...
- If the wavelength period λ corresponds to the time difference between light traversing $\lambda_u \cos \theta$ vs. the time it takes the electron beam to go from A to C, emitted photons of wavelength λ are in phase and constructively interfere:

$$t_{AC} = \frac{\lambda_u}{v} = \frac{\lambda_u}{c \left[1 - \frac{1}{2\gamma^2} \left(1 + \frac{K^2}{2} \right) \right]}, \quad K = \frac{\lambda_u q \tilde{B}}{2\pi m c}$$

$$\lambda = \lambda_u \cos \theta - c t_{AC} \approx \frac{\lambda_u}{2\gamma^2} \left(1 + \frac{K^2}{2} + \gamma^2 \theta^2 \right)$$



$$\tilde{B} = B_0 \cosh^{-1} \left(\pi \frac{g}{\lambda_u} \right)$$

- Alternate explanation: constructive interference occurs when electrons, while traveling along one **undulator** period, slip by one radiation wavelength with respect to the (faster) photons

Lorentz boost

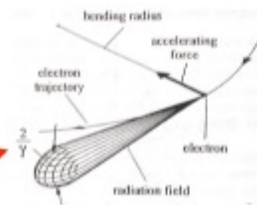
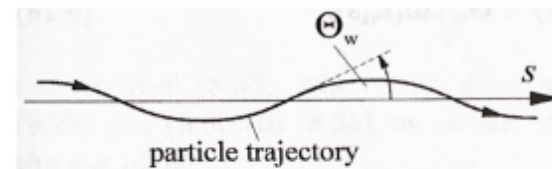
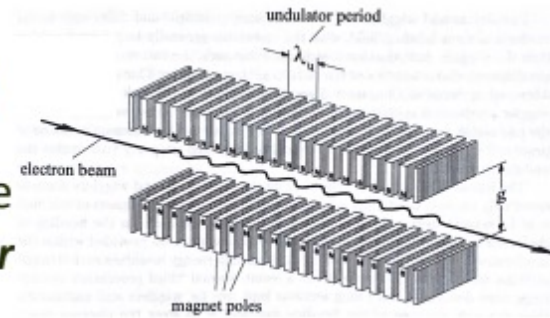
Sources of SR: Undulators (cont.)

- What allows for this constructive interference is small wiggling ($\lambda_u \geq 10$ mm) in an **undulator** compared to large deflections in a wiggler
- To better illustrate, let's consider the max. deflection an electron experiences as it crosses axis of the ideal wiggler/undulator:

$$x'_{\max} = \Theta_w = \frac{K}{\gamma} \quad K = \frac{\lambda_u q \tilde{B}}{2\pi mc}$$

$K \gg 1 \rightarrow \Theta_w \gg 1/\gamma \rightarrow$ wiggler (historical, spectrum $\approx$ bend magnet)

$K \approx 1 \rightarrow \Theta_w \approx 1/\gamma \rightarrow$ undulator (emission cone $\approx$ bend magnet)



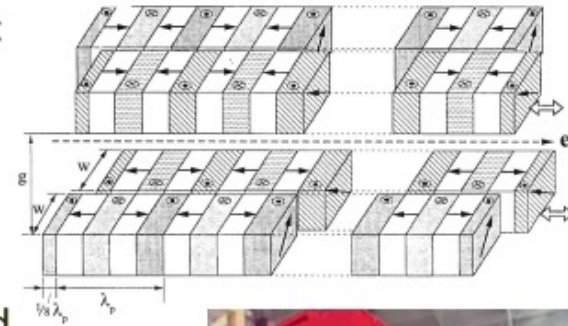
- Undulators are usually built to allow adjusting the magnetic gap $\rightarrow$ by adjusting gap, we can at any time change K parameter of the undulator $\rightarrow$ undulator becomes *wavelength tunable device* (unlike most lasers)

$$\tilde{B} = B_0 \cosh^{-1} \left(\pi \frac{g}{\lambda_u} \right)$$

$$\lambda \approx \frac{\lambda_u}{2\gamma^2} \left(1 + \frac{K^2}{2} + \gamma^2 \theta^2 \right)$$

Sources of SR: Elliptically Polarizing Undulators

- In a permanent magnet EPU the magnetic arrays (similar to planar undulator, but split in half along length of device) can be longitudinally shifted against each other → effectively modifying the "wiggling plane"
- This affects the polarization of the emitted photons → linear polarization (H, V, or inclined), circular polarization, elliptical polarization → renders complete polarization control of emitted SR
- Nowadays universal polarization control: measure various polarizations at detector as function of settings of EPU → invert this lookup table to maintain desired polarization at sample in spite of various optical aberrations/imperfections along entire beamline



Lattice Design for Light Source Storage Rings

- Guiding principle is brightness

$$\mathcal{B} = \frac{\dot{N}_{\text{ph}}}{4\pi^2 \sigma_x \sigma_{x'} \sigma_y \sigma_{y'} (\Delta\omega/\omega)}$$

and therefore we design for:

- Store large current, maintain it constant (top off injection)
- Small $\epsilon_{x,y}$ (ideally zero, but unrealistic) $\rightarrow$ design to minimize $\epsilon_{x,y}$ compared to ϵ_{ph} or at the very least so that electron beam optics are well matched to photon beam $\hat{\beta}_{x,y} = \sigma_r / \sigma_{r'} = L / 2\pi$
"Diffraction-limited Storage Rings"
"4th Generation Storage Rings"
- Leave enough space for IDs $\rightarrow$ long straights with low dispersion (minimize dispersive contribution to electron beam size)
- Keep cost manageable $\rightarrow$ design for lowest possible beam energy (that still allows covering required spectral range) and circumference (large circumference lowers emittance, but real estate & civil engineering are usually very expensive)

ACCELERATORS HISTORICAL TIMELINE

1947: First direct observation of synchrotron radiation at GE
1947: L. Alvarez builds first proton linear accelerator
1947: Ginzton builds first electron linear accelerator
1950-1952: Concept of strong-focusing is invented by E. Courant, H. Snyder, J. Blewett, N. Christofilos
1954: R.R. Wilson et. al. builds first strong-focusing synchrotron at Cornell
1956: Hartmann uses synchrotron radiation for first spectroscopy experiments
1960: First electron-positron collider: ADA at Frascati
1966: SLAC 2-mile electron linac
1972: First proton-proton collider: ISR at CERN
1981: First proton-antiproton collider: SPS at CERN (273 GeV x 273 GeV)
1983: First large-scale superconducting accelerator: Tevatron at Fermilab
1987: Tevatron proton-antiproton collider begins operation (900 GeV x 900 GeV)
198x: LEP at CERN
1990: First electron-proton collider: HERA at DESY (27.5 GeV e- x 902 GeV p)
2000: RHIC at Brookhaven National Laboratory, New York USA (gold on gold)
2008: First operation of LHC at CERN
2022: FRIB first operation

Credit: Sarah Cousineau et al

Useful materials to study

- USPAS lectures

<https://uspas.fnal.gov/materials/materials-table.shtml>

- CERN lectures

<https://e-publishing.cern.ch/index.php/CYRSP/article/view/1608/1321>

- J.D. Jackson, Classical electrodynamics, 3rd ed. (New York, Wiley, 1999), pp. 237–294.