Homework PHY 554 #4.

HW 1 (3 points): Cavities filled with ferrite material are used for RF system requiring large frequency tuning range. The frequency is controlled by applying external magnetic field, B_{ext}, to the ferrite material and by doing so to change it magenta permeability $\mu(B_{ext})$. A 300 m in circumference AGS synchrotron accelerates polarized protons from total energy of 2.5 GeV to 25 GeV.

(a) Calculate the range of the beam revolution frequency in AGS;
(b) Assuming 100% filling by ferrite, what should be ratio of μ_{max} to μ_{min}. Where μ should have maximum value?

Note: RF systems operate on a fixed integer harmonic of the revolution frequency.

Solution:

(a) Rest energy of a proton is 0.9837 GeV. It means that Lorentz factor changes from 2.66 to 26.6 and v/c changes from 0.9269 to 0.9993, e.g. revolution frequency increases 1.0781 fold during acceleration from 927.5 kHz to 999.987 kHz (e.g.1 MHz!).

(b) Since the frequency of an RF cavity scales the same as speed of light in the media:

$$\omega_{res} = \frac{\omega_o}{\sqrt{\varepsilon\mu}} \rightarrow \mu \propto \omega_{res}^{-2}$$

one should reduce μ 1.162-fold to accommodate necessary change in resonant frequency.

HW 2 (2 points): In RF cavity operating at 500 MHz, amplitude of the magnetic field at the part surface is 500 Gs or 500 Oe. Find power losses per square meter of the surface for:

(a) Cu cavity*
(b) SRF cavity with surface resistance, $R_s = 5\ 10^{-9}$ Ohm.

How much water you can heat from 20 C° to 40 C° in one hour (3,600 second) by cooling such Cu cavity?

*Hint: you may use the conductivity of Cu or scale R_s from results shown in Lecture 11. Thermal capacitance of water is 4,179 J/kg/ C°.

Solution:

We should use formula for surface losses for a good conductor (it is in SI units):

$$\frac{P_{loss}}{A} = \frac{1}{2} R_s \left|\vec{\mathbf{H}}_{//}\right|^2$$

The most confusing is to transfer H from CGS (Gs = Oe) units to SI (A/m) with coefficient $1000/4\pi$: $H=3.98\ 10^4$ A/m: With $A=1\ m^2$ power lost is simply

$$P_{loss} = \frac{1}{2} R_s \left|\vec{\mathbf{H}}_{//}\right|^2 A = \frac{1}{2} R_s \left|\vec{\mathbf{H}}_{//}\right|^2$$

and for SRF cavity we would have 3.96 W losses per one square meter of the surface. For Cu surface impedance scales with the frequency

$$R_s = \sqrt{\frac{\omega\mu}{\sigma}} [\Omega]$$

In slide 8, lecture 11 we shown that for Cu R_s=10 mOhm at frequency of 1.5 GHz, which is 3 time higher than in our case. Thus

$$R_s(Cu, 500\,Mhz) = \frac{10 m\Omega}{\sqrt{3}} \approx 5.8 m\Omega$$

and power loss density is 4.57 MW per m^2. In one hour the EM field generates 1.65 10^{10} J in 1 m^2 of the Cu surface. Heating one kg (e.g. one liter) of water by 20K requires 83.6 kJ: hence this power will heat from 20 C^o to 40 C^o 197 tons of water! e.g. a cube approximately 6m x 6 m x 6m.

HW 3 (4 points): Superconducting RF pillbox cavity operating at 2K temperature would quench when the surface magnetic field reaches above 0.1 T (e.g. 1,000 Gs or 1,000 Oe).

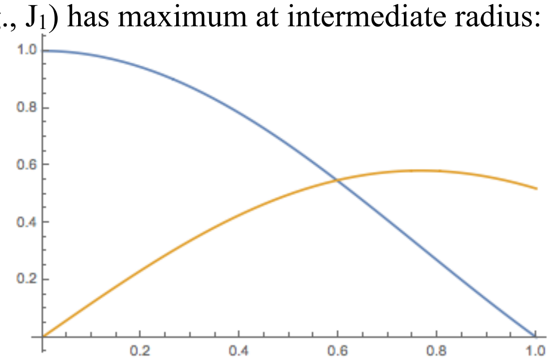
(a) For such pillbox cavity operating in fundamental TM_{010} mode find maximum attainable accelerating electric field on axis of the cavity.
(b) For Rs = 5 nanoOhm, calculate thermal losses in such cavity operating at 20 MV/m (Hint do not forget side walls!)

Solution:
(a) In pillbox cavity

$$\mathbf{E}_z = E_o \cdot J_o\left(2.405\frac{r}{a}\right)\sin(\omega t);$$

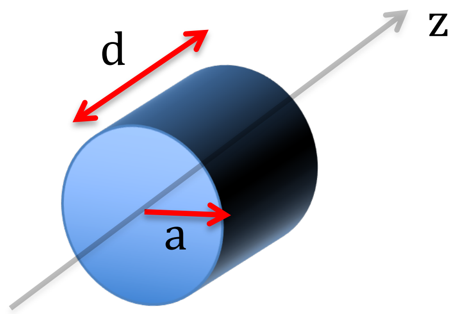
$$\mathbf{B}_\theta = E_o \cdot J_1\left(2.405\frac{r}{a}\right)\cos(\omega t)$$

and magnetic field (e.g., J_1) has maximum at intermediate radius:



$$J_1(x)_{\max} = 0.581865 \qquad at\, x = 1.84118$$

e.g., we have limit $\mathbf{B}_\theta = 0.581865 E_o \le 10^3 Gs$. Now we just need to remember that 1 Gs (CGS system) is equal to 299.79 V/c ~ 30kV/m. Hence we get limitation for accelerating field on axis ***Eo < 51.5 MV/m.***



(b) We need to integrate on 3 surfaces: front and back faces (circles with radius *a*) and a cylinder of radius *a* and length *d*. Reversing the relations, 20 MV/m is equivalent to B=667 Gs. Corresponding H_o=5.31 10^4 A/m.

$$\mathbf{H}_\theta = \frac{\mathbf{B}_\theta}{\mu_o} = H_o \cdot J_1\left(2.405\frac{r}{a}\right)\cos(\omega t);$$

$$P_{loss} = \frac{1}{2}R_s \oiint da\left|\vec{\mathbf{H}}_{//}\right|^2 = R_s\frac{H_o}{2}\left(2\oiint_{circle} J_1^2\left(2.405\frac{r}{a}\right)da + J_1^2(2.405)\oiint_{culinder} da\right)$$

$$P_{loss} = 2\pi R_s \frac{H_o^{\ 2}}{2}\left(2a^2\int_0^1 J_1^2(2.405x)x\,dx + adJ_1^2(2.405)\right)$$

$$\int_0^1 J_1^2(2.405x)x\,dx = 0.134757;\quad J_1^2(2.405) = 0.269475;$$

$$P_{loss} \simeq 2\pi R_s\frac{H_o^{\ 2}}{2}\cdot 0.27\left(a^2+ad\right) \approx 0.85\cdot H_o^{\ 2}\left(a^2+ad\right)$$

or, in other terms:

$$P_{loss}[W] \simeq 12*\left(a^2+ad\right)[m^2]$$

With ratio between the RF wavelength and the pillbox-cavity of

$$a = \frac{2.405}{2\pi}\lambda_{RF};\ \lambda_{RF} = \frac{c}{f_{rf}};$$

and assuming a maximum gain form the cavity with $d = \lambda_{RF}/2$, the losses become

$$P_{loss}[W] \simeq 12\cdot\frac{2.405}{2\pi}\left(\frac{2.405}{2\pi}+\frac{1}{2}\right)\lambda_{RF}^{\ 2}[m^2] \approx 4\lambda_{RF}^{\ 2}[m^2]$$

e.g. if *Rs* is a constant, than the losses are increasing with reducing the frequency! This is the case of rather low frequencies when the so-called residual resistivity is larger that regular SRF surface resistance.

As an example, for 500 MHz pillbox cavity we would have RF wavelength of 0.6 m, and losses will be 1.44 W.

HW 4 (6 points): For SRF Nb cavity the London penetration depth is equal to 40 nanometers.

(a) What is the density of superconducting electrons, n_s?
(b) For surface magnetic field of 500 Gs or 500 Oe, find the density of surface current.
(c) For frequency of 1 GHz, find value of electric field on the surface of the superconductor.
(d) Assuming conductivity on normal component (non-superconducting electron) of Nb is 3 x 10^8 S/m (e.g., conductivity of 6x 10^6 S/m at room temperature multiplied by RRR of 50), find what is the value of the normal component of the surface current.

Hint: assume that the superconducting conductivity is significantly higher than normal part.

Solution: Use lecture 12
(a) Use your favorite unit system, mine is clearly CGS:

$$\lambda = \sqrt{\frac{m}{\mu_o e^2 n_s}}(SI) = \sqrt{\frac{1}{4\pi r_e n_s}}(CGS); \quad r_e = \frac{e^2}{mc^2}$$

with classical radius of electron being 2.8 10^{-13} cm, λ= 4 10^{-6} cm

$$n_s = \frac{1}{4\pi r_e \lambda^2} = 1.76 \cdot 10^{22}\, cm^{-3}$$

(b) Again, in CGS units

$$\oint H\, dl = \frac{4\pi}{c} I \;\rightarrow\; \frac{I}{l} = \frac{c}{4\pi} H;$$

and in SI units

$$J_s = \frac{I}{l}\left[\frac{A}{m}\right] = H\left[\frac{A}{m}\right] = \frac{10^3 H[Gs]}{4\pi} \approx 40\left[\frac{kA}{m}\right]$$

(c) The current density at the surface

$$j_s = \frac{J_s}{\lambda} = \approx 10^{12}\left[\frac{A}{m^2}\right]; \; E = \mu_o j_s \cdot \omega \lambda^2 = 12.6\; V/m$$

(d) we need to remember expression for Ohms law (lecture 10, slide 8)

$$j_n = \sigma_n E = 3.8 \cdot 10^9\, A/m^2; \;\; J_n = j_n \lambda = 150\; A/m$$