Comparison of PCA and MBEC for EIC

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Introduction

PCA was proposed in 2018 in: V.N. Litvinenko, G. Wang, D. Kayran, Y. Jing, J. Ma and I. Pinayev, "Plasma-Cascade micro-bunching Amplifier and Coherent electron Cooling of a Hadron Beams", 2018 (arXiv:1802.08677) [*PCA-paper*].

I analyze PCA using the model of slices (hadron and electron point charges are replaced by thin slices with a Gaussian surface charge distribution) developed for MBEC[1]. I assume the nominal parameters of the MBEC cooler from EIC CDR and consider the case of the highest proton energy of 275 GeV. No optimization has been done to maximize the PCA cooling rate.

[1] G. Stupakov, "Cooling rate for microbunched electron cooling without amplification", PRAB **21**, 114402 (2018); G. Stupakov and P. Baxevanis, "Microbunched electron cooling with amplification cascades", PRAB **22**, 034401 (2019); P. Baxevanis and G. Stupakov, "Transverse dynamics considerations for microbunched electron cooling", PRAB **22**, 081003 (2019).

MBEC cooler from EIC CDR

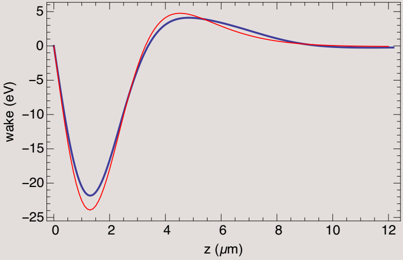
Table 3.69: Summary of Coherent Electron Cooling parameters

Parameter	Value
Electron beam energy [MeV]	149.8
Electron rms normalized emittance [µm]	2.8
Electron beam average current [mA]	98.5
Average electron β_x at M/K [m]	40
Average electron β_y at M/K [m]	40
Electron rms beam size at M/K [mm]	0.61
Average proton β_x at M/K [m]	40
Average proton β_y at M/K [m]	40
Proton horizontal beam size [mm]	1
Proton vertical beam size [mm]	0.2
Required rms electron energy spread	10^{-4}
Electron beam bunch charge [nC]	1
Electron beam bunch length [cm]	0.4
First electron Chicane R_{56} [cm]	0.71
2nd electron Chicane R_{56} [cm]	0.71
3rd electron Chicane R_{56} [cm]	0.71
Electron rms beam size at the amplification sections [mm]	0.15
Average electron β function at the amplification sections [m]	2.5
Quarter plasma oscillation wavelength at the amplification sections $\lambda_p/4$ [m]	29
Drift space between chicanes L_d [m]	43
Length of the modulator section [m]	40
Length of the kicker section [m]	40
Overall length of the cooler lattice [m]	166
Proton beam R_{56} between modulator and kicker [cm]	0.195
Dispersion D_x of protons in modulator and kicker sections [m]	1.17
Horizontal phase advance of protons between M/K sections [rad]	$0.036 + 2n\pi$
Cooling time (longitudinal/transverse) [min]	≈ 100

The wake function in MBEC

I will try to evaluate the wake function in PCA. The wake function is the energy kick that one proton exerts on itself after the signal from the modulator is amplified and propagated to the kicker. The knowledge of this wake function is enough to calculate the cooling time and diffusion (see S. Nagaitsev et al. arXiv:2102.10239). I will try to calculate the PCA wake and compare it with that of MBEC for the peak current $I_e = 30$ A and the nominal EIC CDR parameters.

MBEC wake function at $I_e = 30$ A

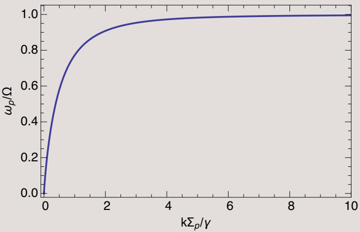


The wake is antisymmetric function, $w(-z) = -w(z)$.

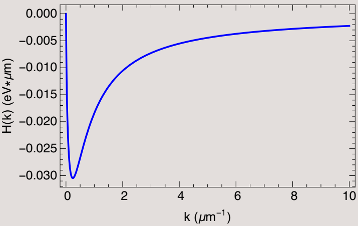
Two results from the slice model

In our model of MBEC, particles are represented by slices with a Gaussian transverse distribution of charge. We derived the dependence of the plasma frequency versus k, $\omega_p(k) = \Omega f(k\Sigma_p/\gamma)$ (Σ_p is the rms transverse size)

$$\Omega = \frac{c}{\Sigma_p}\sqrt{\frac{I_e}{I_A\gamma^3}}$$



Spectrum of the energy perturbation in the electron beam induced by one proton in the modulator. The wavenumber $k = 1$ µm corresponds to the frequency of ≈ 50 Thz.



Plasma frequency depends on wavenumber and s

When the transverse size of the beam varies with s, $\Sigma_p(s) \propto \sqrt{\beta(s)}$, we have $\omega_p(k,s)$. The equation for the Fourier component $\delta\hat{n}_k(s)$ reads:

$$\frac{d^2\delta\hat{n}_k}{ds^2} + \frac{1}{c^2}\omega_p^2(s,k)\delta\hat{n}_k = 0$$

Here $\delta\hat{n}_k = \int \delta n(z) e^{-ikz} dz$.

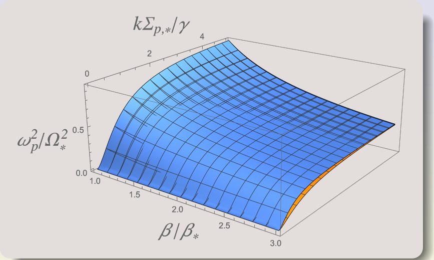
Because $\Omega^2 \propto \Sigma_p^{-2} \propto 1/\beta$ a good approximation for the function $\omega_p^2(s,k)$ is ($\beta_* = \beta(s_*)$ - minimal beta)

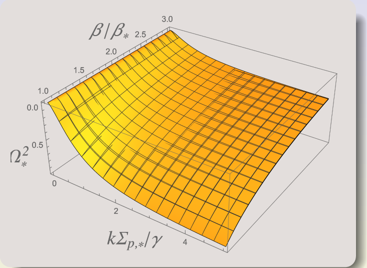
$$\omega_p^2(s,k) \to \frac{\omega_p^2(s_*,k)}{\beta(s)/\beta_*}$$

Approximation for $\omega_p(s, k)$

$$\omega_p^2(s,k) \to \frac{\omega_p^2(s_*,k)}{\beta(s)/\beta_*}$$

This approximation is also used in PCA-paper.

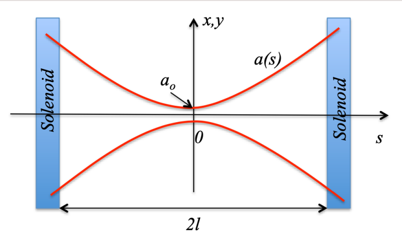




Blue surface is $\omega_p^2(s,k)$, yellow surface is $\dfrac{\omega_p^2(s_*,k)}{\beta(s)/\beta_*}$.

One cell

Our goal is to find the amplification in one cell (see the figure from PCA-paper). Between the solenids we have $\beta/\beta_* = 1 + s^2/\beta_*^2$.



$$\frac{d^2 \delta \hat{n}_k}{ds^2} + \frac{\omega_p^2(s_*, k)/c^2}{1 + s^2/\beta_*^2} \delta \hat{n}_k = 0$$

This equation is also studied in PCA-paper. It has a solution in terms of hypergeometric function.

$$\frac{d^2}{d\hat{s}^2}\tilde{q}_k + \frac{2k_{sc}^2}{1+\left(k_\beta^2 + 2k_{sc}^2\right)\hat{s}^2} \cdot \tilde{q}_k = 0.$$

$$\frac{d^2x}{d\hat{s}^2} + \frac{\omega^2}{1+\kappa^2\hat{s}^2} \cdot x = 0; \;\; d = \frac{\sqrt{\kappa^2 - 4\omega^2}}{4\kappa}$$

$$x = a_1 \cdot {}_2F_1\left(-\frac{1}{4} - d, -\frac{1}{4} + d, \frac{1}{2}, -\kappa^2\hat{s}^2\right) + a_2 \cdot \kappa\hat{s} \cdot {}_2F_1\left(-\frac{1}{4} - d, -\frac{1}{4} + d, \frac{3}{2}, -\kappa^2\hat{s}^2\right)$$

PCA-paper ignores the dependence of ω_p versus k.

Eigenmodes

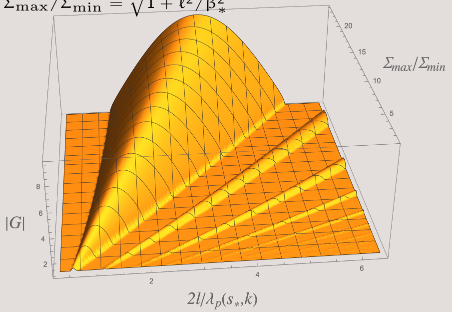
The equation for $\delta\hat{n}_k$ has special solutions

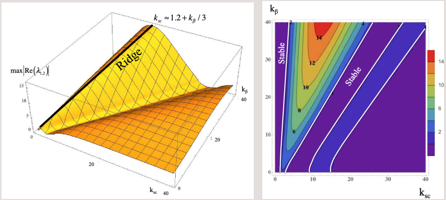
$$\begin{pmatrix} \delta\hat{n}_k \\ \delta\hat{n}'_k \end{pmatrix}_{s=2\ell} = G \begin{pmatrix} \delta\hat{n}_k \\ \delta\hat{n}'_k \end{pmatrix}_{s=0}$$

The derivative $\delta\hat{n}'_k = d(\delta\hat{n}_k)/ds$ is proportional to the energy perturbation in the beam, $\delta\hat{n}'_k \propto \delta\hat{\eta}_k$. Hence the gain is calculated for a specific eigenmode that has a well defined combination of the density perturbation $\delta\hat{n}_k$ and an energy perturbation $\hat{\eta}_k$. The other, orthogonal mode damps down as G^{-1}. An arbitrary initial perturbation in PCA should be decomposed into the two eigenmodes—one of them is amplified, the other is damped. (In MBEC we only amplify the density perturbations and use chicanes to convert an energy perturbation into a density one.)

Gain in one cell

$\Sigma_{\max}/\Sigma_{\min} = \sqrt{1 + \ell^2/\beta_*^2}$





The same result is obtained in the PCA-paper

Amplification $G \approx 10$ requires a large variation of the beta function $(\beta(\ell)/\beta_* \sim 500)$ and a long cell $(2\ell \approx 3\lambda_p)$.

PCA cooler

Let us choose $|G|_{\max} = 4$ which means $\Sigma_{\max}/\Sigma_{\min} = 7.5$. With 4 amplification cell we get $|G|_{\max} = 4^4 = 256$. There should be an even number of cells, because they flip the sign of the wake.

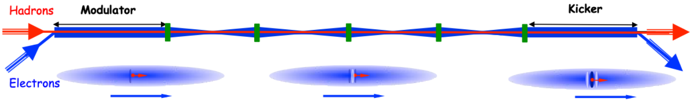
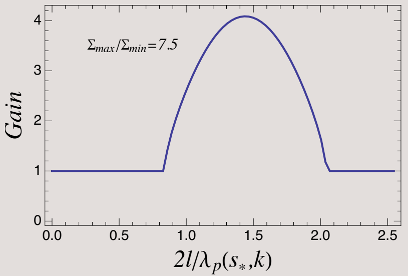


Figure 1.d. A layout of a CeC with a PCA. No beam's separation is necessary.

The perturbation from the modulator has only an energy perturbation, so only a fraction of it will be amplified. The output from the PCA has both the energy and the density perturbations, but the kick is produced by the density perturbation in e-beam, so only a fraction of output makes the energy kick.

Cell parameters



We need to specify the length of the cell 2ℓ. I define it in units $\lambda_0 \equiv \lambda_p(s_*, k=\infty)$

$$\lambda_0 = 2\pi\sqrt{\frac{I_A \Sigma_{\rm min}^2 \gamma^3}{I_e}}$$

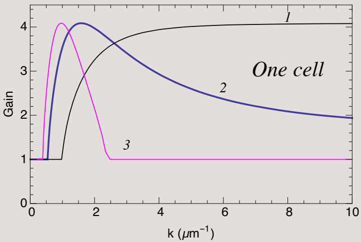
In CDR we have $I_e = 30$ A and $\Sigma_p = 0.15$ mm (beta function 2.5 m), so I assume $\Sigma_{\rm min} = 0.15$ mm.

This gives $\lambda_0 = 113$ m, hence the one-cell length is 100-200 m! In a VL slide for EIC PCA there is a number $I_e = 150$ A, this makes $\lambda_0 = 50$ m—still large.

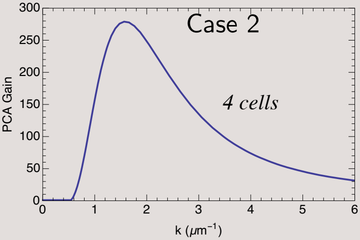
PCA amplification spectral characteristic

Consider three cases: 1) $2\ell = 1.4\lambda_0$, 2) $2\ell = 2\lambda_0$ and 3) $2\ell = 2.5\lambda_0$.

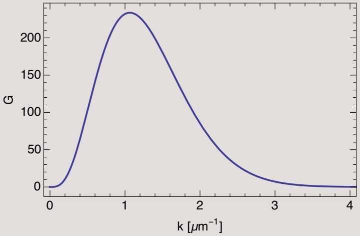
One cell amplification vs k



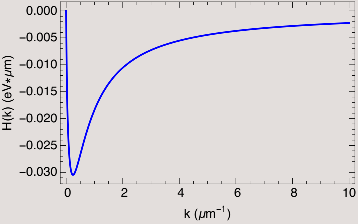
4 cells amplification vs k



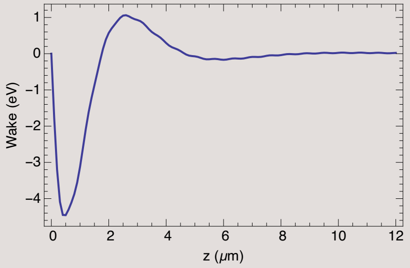
MBEC amplification vs k

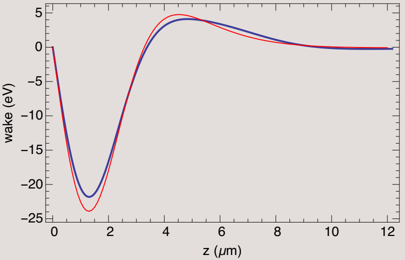


Spectrum from the modulator



PCA vs MBEC wake





The PCA wake is factor 4-5 smaller then MBEC. My explanation: it is likely due to the mismatch between the modulator energy perturbation and the eigenmode that is amplified (a factor of ~ 2) and that the kicker needs only δn in the amplified eigenmode (another factor of ~ 2).

Comments

- PCA cell is 2-3 λ_p in length (MBEC amplification section is $\frac{1}{4}\lambda_P +$ chicane). For the EIC CDR parameters with the peak current of 30 A, and $\gamma = 293$ PCA these cells are too long. Large gain also requires a large variation of the beta function in the cell. In the simplest configuration, M-PCA-K, there is a mismatch at M-PCA and PCA-K boundaries that lowers the wake.
- PCA may work for smaller energies, but for $\gamma = 293$ the system looks prohibitively long.
- The modulator and kicker response is peaked in the range ~ 50 THz; if the PCA amplifier bandwidth extends to ~ 1 PHz, there will be a mismatch between the amplifier and M/K.
- Going to high electron peak current ($I_e \sim 150$ A) would result in large electron noise affecting the cooling (even assuming the nominal Poisson-like shot noise in the e-beam).