PHY 554

Fundamentals of Accelerator Physics

Lecture 20: Collective effects part 2
Wake fields and Impedance

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Most (all) of materials come from USPAS/CERN school courses

https://uspas.fnal.gov/index.shtml

https://cas.web.cern.ch/





Today schedule

- Questions from the last lecture (on space charge effects)
- Last time Collective effects (part1)
  - Direct Space charge
  - Indirect space charge
  - From indirect space charge to wall wakes
- Collective effects (part II)
  - Concept of wake fields
  - Longitudinal and transverse wake fields and impedances
  - Energy balance
  - Impact of wake fields and impedance on the accelerator environment

Collective effects

- Particle accelerators use external electromagnetic fields to guide and accelerate charged particles.

In a real machine, however, there is another source of electromagnetic fields, the beam itself, which, interacting with the accelerators' devices, produces additional **self-induced fields** which perturb the particle's motion leading to the so-called **collective effects.**

- The self-induced fields are commonly divided into **space charge fields**, generated directly by the charge distribution and including the image currents circulating on the walls of a smooth, perfectly conducting pipe, and the **wakefields**, produced by the finite conductivity of the walls, resonant devices, or any geometrical variation of the beam pipe.

- Collective effects in particle accelerators are one of the key constituents for determining the ultimate particle accelerator performance. Their role is becoming increasingly important as particle accelerators are being pushed to ever higher intensity and beam brightness.

Introduction

In the previous lecture we discussed **space charge effects** and showed how these can limit the machine performance in that they can lead **to incoherent and coherent tune shifts**. We then moved on the a more general treatment of electromagnetic fields in simple structures where we were able to identify **yet another type of induced fields** originating from the **electromagnetic properties** of the surrounding material – **the wall wake**.

We then saw some animations of more general examples of induced fields in complex structures.

We now want to see how to treat such structures more formally to help us model these types of collective effects better. For this, we introduce the **concept of wake fields and impedances** and will talk about their impact on the machine and on the beam.

- Wake Field = the track left by a moving body (as a ship) in a fluid (as water); broadly : a track or path left
- Impedance = Fourier Transform (Wake Field)



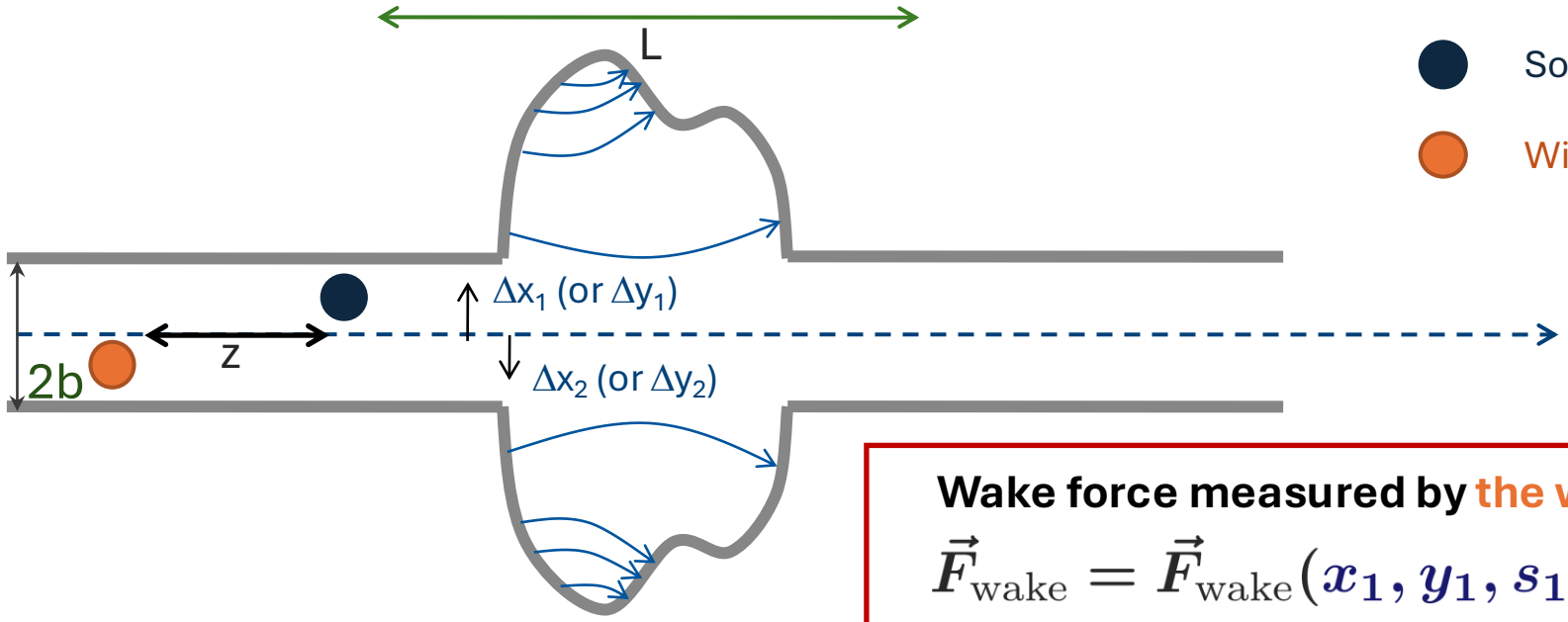
Collective effects part II

- In addition to space charge, self-induced electromagnetic forces can also be generated by the beam interacting with the different devices distributed along the accelerator, or due to the finite conductivity of the vacuum chamber.
- These devices may have a complex geometry: kickers, bellows, RF cavities, diagnostics components, collimators, and other special devices. The study of these fields requires solving the Maxwell equations in a given structure taking the beam current as source.
- This is a quite complicated task for which it has been necessary to develop dedicated computer codes, which solve the electromagnetic problem in the frequency or in the time domain.
- There are several useful codes developed for the electromagnetic design of accelerator devices such as CST Studio Suite, GDFIDL, ABCI, ECHO2D/3D, and others.

Definition of the wake fields

sometimes in short, we call them just wakes ☺

- We will use a little trick: we consider **two point particles** – one source, one probe
- The forces will be a function of the locations of both the source and the probe particles:

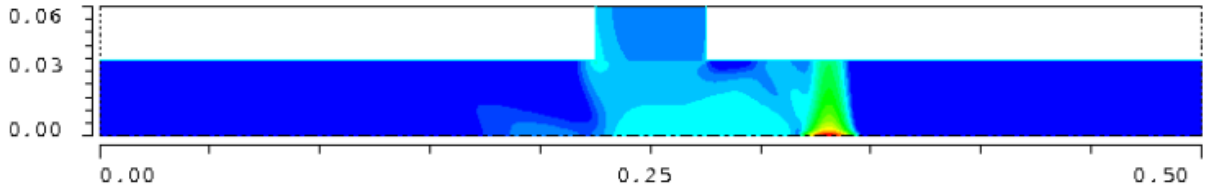


Wake force measured by the witness particle

$$\vec{F}_{\text{wake}} = \vec{F}_{\text{wake}}(x_1, y_1, s_1, x_2, y_2, s_2, t)$$

Note that: $\vec{F}_{\text{wake}} = q_2 \left(\vec{E}(x_1, y_1, s_1, x_2, y_2, s_2, t) + v_z \vec{e}_z \times \vec{B}(x_1, y_1, s_1, x_2, y_2, s_2, t) \right)$

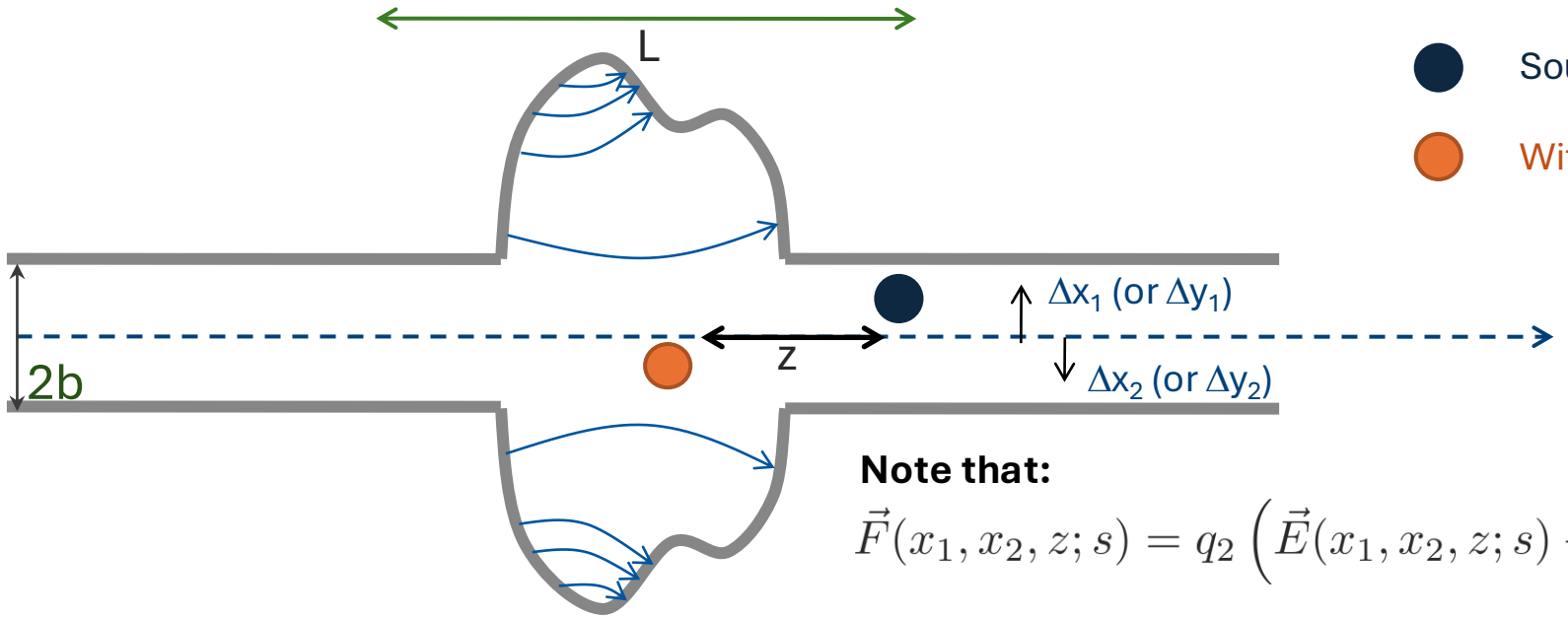
How can we treat these phenomena effectively in our models?



- **The rigid beam approximation**
- **The impulse approximation**
  - The wake field does not affect the motion of the beam
  - The wake field does not affect the motion of the test charge (only the energy or momentum change is calculated)
  - The interaction of the beam with the environment is not self consistent.

Nevertheless, one can use results from wake field calculations for a turn by turn tracking code - sometimes cutting a beam into slices.

Wake function definition



Note that:

$$\vec{F}(x_1, x_2, z; s) = q_2 \left(\vec{E}(x_1, x_2, z; s) + v_z \vec{e}_z \times \vec{B}(x_1, x_2, z; s) \right)$$

We define the **wake function as the integrated force on the witness particle** (associated to a change in energy):

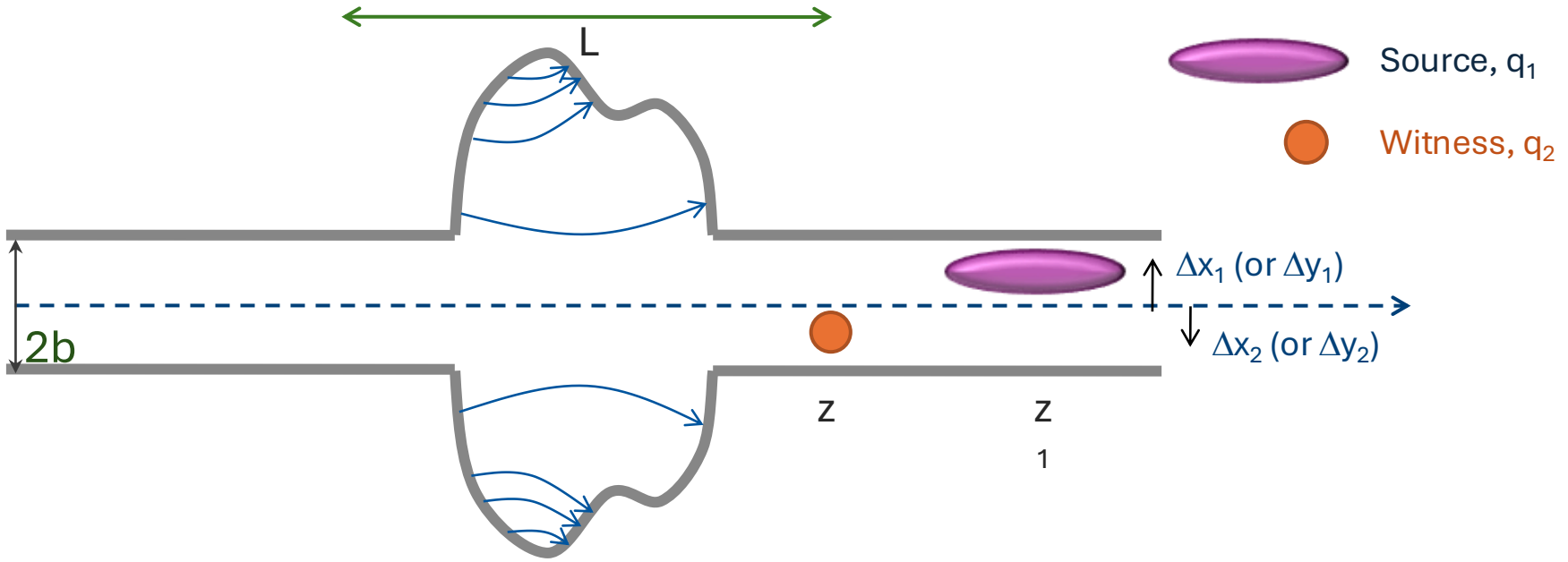
- Let's focus on the horizontal plane. In general, for two point-like particles, we have

$$\Delta E_2 = \int F(x_1, x_2, z; s)\, ds = -q_1 q_2\, \boldsymbol{w(x_1, x_2, z)} \qquad z \equiv s_2 - s_1\,, \qquad s \equiv s_1$$

w is proportional to **the integrated forces felt by the witness particle**:

- It is an **intrinsic property of any given device**, representing its **electromagnetic impulse response**.
- It is a function of **the transverse offsets and the longitudinal separation** of the source and the witness particles.

Wake potential for a distribution of particles



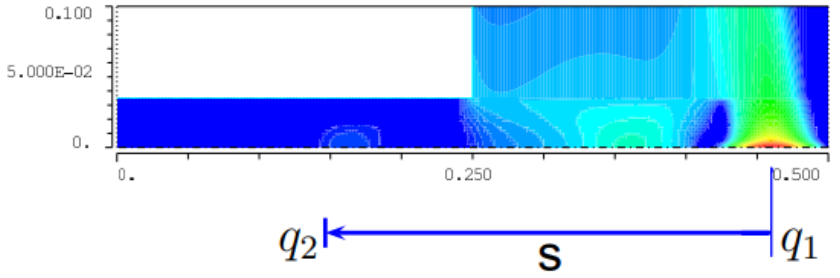
We define the **wake function as the integrated force** on the witness particle (associated to a change in energy):

- For an extended particle distribution this becomes (superposition of all source terms)

$$\Delta E_2(z) = -\sum_i q_i\, q_2\, \boldsymbol{w(x_i, x_2, z - z_i)} \longrightarrow \int \boxed{\lambda_1(x_1, z_1)}\, \boldsymbol{w(x_1, x_2, z - z_1)}\, dx_1 dz_1$$

Forces become dependent on the **particle distribution function**

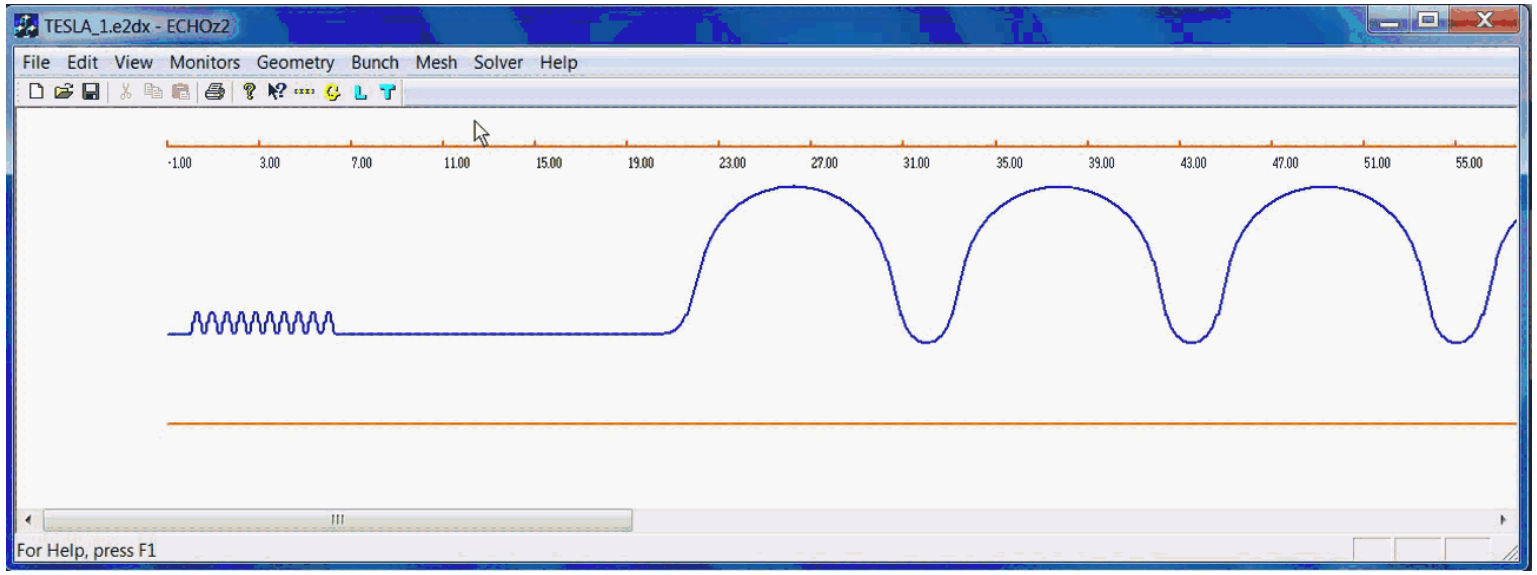
Wake function and potential general definition



We have introduced the concept of **the wake function** and have seen how these can simplify our handling of induced electromagnetic fields within complex structures. The wake function is the **electromagnetic response** of a structure and is in fact an **intrinsic property** of any such structure.

In practice, we will never compute the full wake function, we will separate between **longitudinal and transverse wake fields**.
We then treat these in an expansion which significantly simplifies our treatment. Complementary to the wake fields one can also move to frequency domain and use the impedance.







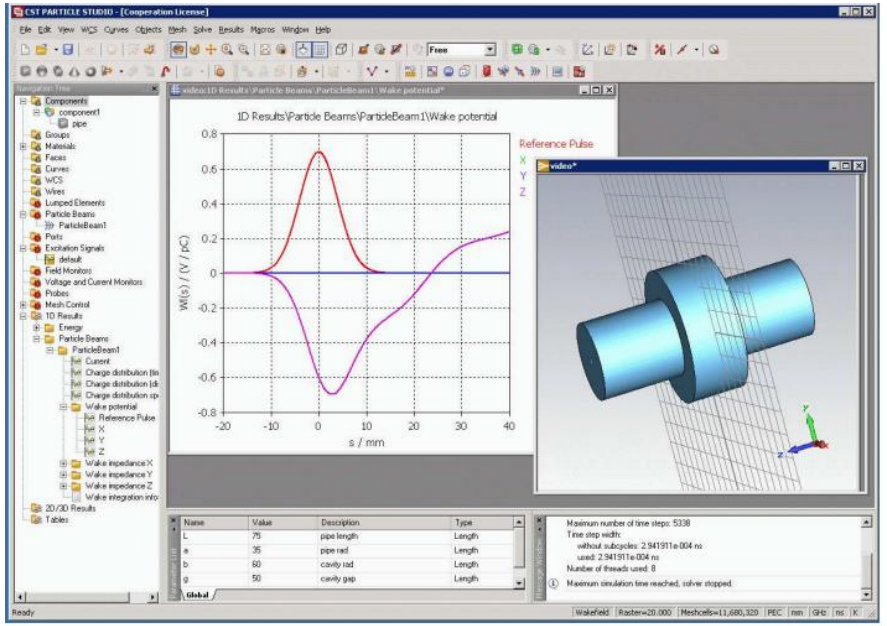
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Numerical calculations

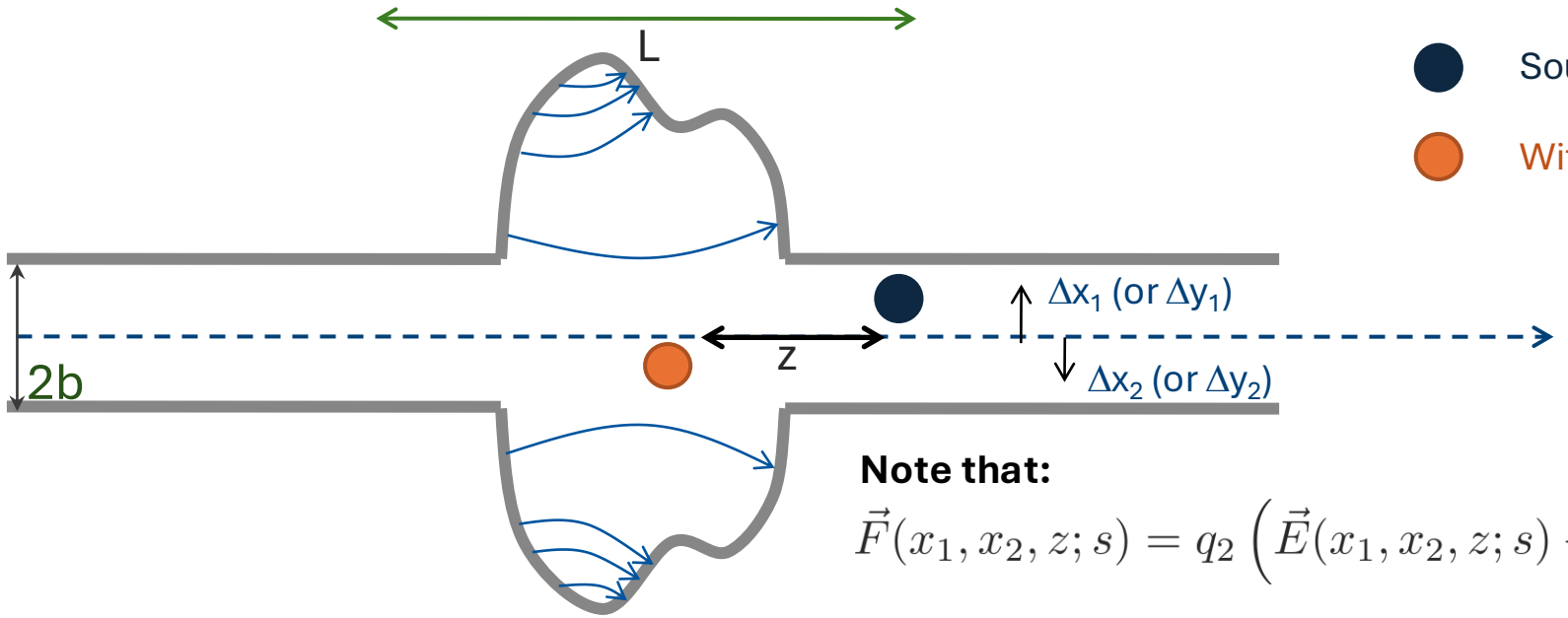
- There exist several numerical codes to calculate wakefields. Examples are:
- Non commercial codes:
  - ABCI (2D, r-z geometry) , Yong Ho Chin, KEK
    http://abci.kek.jp/abci.htm
  - Echo 2D/3D, Igor Zagorodnov, DESY
    https://echo4d.de/category/echo2d/
- Commercial codes (3D)
  - GdfidL
  - CST (Particle Studio, Microwave Studio)

The Maxwell equations are solved on a grid

Numerical calculation of the wakepotential with CST Particle Studio:



Wake function definition (cont.)



Note that:

$$\vec{F}(x_1, x_2, z; s) = q_2 \left(\vec{E}(x_1, x_2, z; s) + v_z \vec{e}_z \times \vec{B}(x_1, x_2, z; s) \right)$$

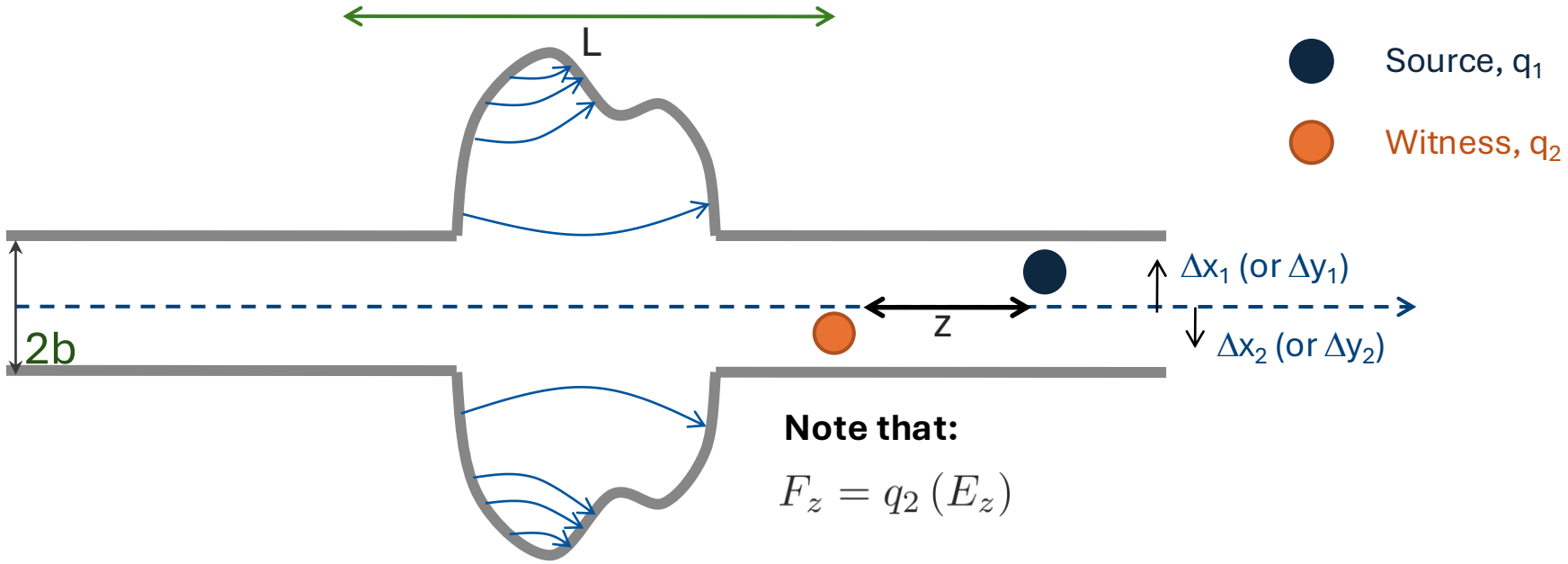
We define the **wake function as the integrated force on the witness particle** (associated to a change in energy):

- Let's focus on the horizontal plane. In general, for two point-like particles, we have

$$\Delta E_2 = \int F(x_1, x_2, z; s)\, ds = -q_1 q_2\, \boldsymbol{w(x_1, x_2, z)}$$

$$z \equiv s_2 - s_1\,, \qquad s \equiv s_1$$

Longitudinal wake function



Note that:

$$F_z = q_2 \left(E_z\right)$$

- Longitudinal wake fields

Can be simulated or measured

$$\int F_z(\Delta x_1, \Delta x_2, z; s)\, ds = -q_1 q_2 \left(W_{\parallel}(z) + O(\Delta x_1) + O(\Delta x_2)\right)$$

Zeroth order with source and test centered
usually dominant

Higher order terms
Usually negligible for small offsets

Longitudinal wake function / impedance

$$W_{\parallel}(z) = -\frac{\Delta E_2}{q_1 q_2} \quad \xrightarrow[q_2 \to q_1]{z \to 0} \quad W_{\parallel}(0) = -\frac{\Delta E_1}{q_1^2}$$

- The value of the wake function in z=0 is related to **the energy lost by the source particle** in the creation of the wake
- The **wake function** of an accelerator component is basically its **Green function in time domain** (i.e., its response to a pulse excitation)

$$\Delta E_2(z) \propto \int \lambda_1(z_1)\, W_{\parallel}(z - z_1)\, dz_1$$

→ Very useful for macroparticle models and simulations, because it can be used to describe the driving terms in the single particle equations of motion!

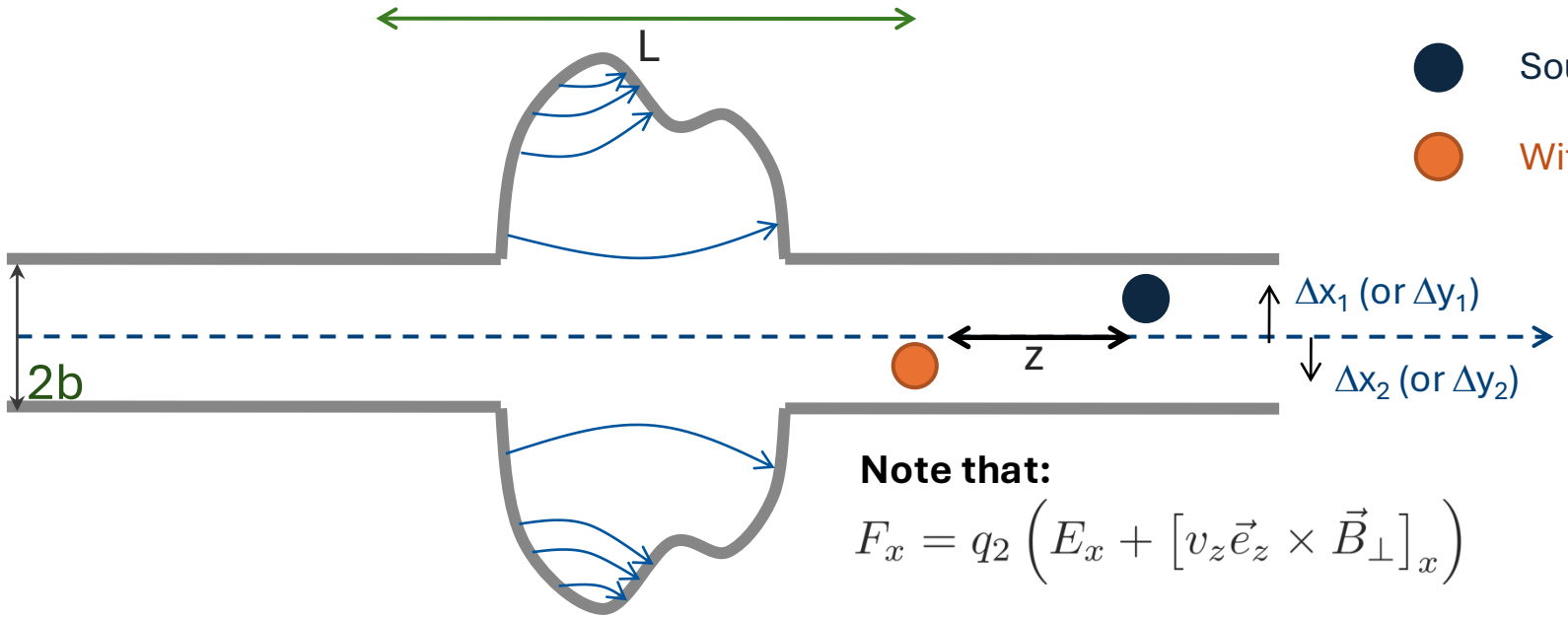
- We can also describe it as a **transfer function in frequency domain** (Fourier Transformation)

→ This is the definition of **transverse beam coupling impedance** of the element under study

$$Z_{\parallel}(\omega) = \int_{-\infty}^{\infty} W_{\parallel}(z) \exp\left(-\frac{i\omega z}{c}\right) \frac{dz}{c}$$

$Z_{\parallel}(\omega)$: $[\Omega]$ — $W_{\parallel}(z)$: $[\Omega/s]$

Transverse wake functions



Note that:

$$F_x = q_2 \left(E_x + \left[v_z \vec{e}_z \times \vec{B}_\perp \right]_x \right)$$

- Transverse wake fields

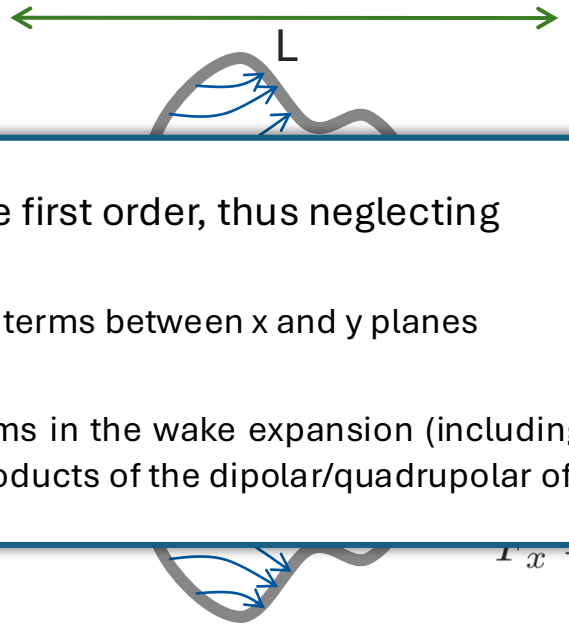
First order expansion in transverse coordinates of source and witness particles

$$\beta c\, \Delta p_{x_2} = \int F_x(\Delta x_1, \Delta x_2, z; s)\, ds = -q_1 q_2 \left(W_{C_x}(z) + W_{Dx}(z)\, \Delta x_1 + W_{Q_x}(z)\, \Delta x_2 \right)$$

$$\longrightarrow \frac{\Delta p_{x_2}}{p_0} = \Delta x_2'$$

Transverse deflecting kick of the witness particle from transverse wakes

Transverse wake functions



Source, q_1

Witness, q_2

Δx_1 (or Δy_1)

Δx_2 (or Δy_2)

We have truncated to the first order, thus neglecting

⇒ First order coupling terms between x and y planes

⇒ All higher order terms in the wake expansion (including mixed higher order terms with products of the dipolar/quadrupolar offsets)

$$F_x = q_2 \left(E_x + \left[v_z \vec{e}_z \times \vec{B}_\perp \right]_x \right)$$

- Transverse wake fields

Can be simulated or measured

$$\beta c\, \Delta p_{x_2} = \int F_x(\Delta x_1, \Delta x_2, z; s)\, ds = -q_1 q_2 \left(W_{C_x}(z) + W_{Dx}(z)\, \Delta x_1 + W_{Q_x}(z)\, \Delta x_2 \right)$$

Zeroth order for asymmetric structures → **Orbit offset**

Dipole wakes – depends on **source particle** → **Orbit offset & detuning**

Quadrupole wakes – depends on **witness particle** → **Detuning**

Transverse wake functions/ impedance

$$W_{D_x}(z) = -\frac{\beta^2 E_0}{q_1 q_2}\frac{\Delta x_2'}{\Delta x_1} \qquad W_{Q_x}(z) = -\frac{\beta^2 E_0}{q_1 q_2}\frac{\Delta x_2'}{\Delta x_2}$$

- The **wake function** of an accelerator component is basically its **Green function in time domain** (i.e., its response to a pulse excitation):

$$\Delta x_2'(z) \propto \int \left[\langle x_1 \rangle \cdot \lambda_1\right](z_1)\, W_{D_x}(z - z_1)\, dz_1$$

  → Very useful for macroparticle models and simulations, because it can be used to describe the driving terms in the single particle equations of motion!

- We can also describe it as a **transfer function in frequency domain** (Fourier Transformation)
  → This is the definition of **transverse beam coupling impedance** of the element under study

Dipolar

$$Z_{D_x}(\omega) = i\int_{-\infty}^{\infty} W_{D_x}(z)\exp\left(-\frac{i\omega z}{c}\right)\frac{dz}{c}$$

Quadrupolar

$$Z_{Q_x}(\omega) = i\int_{-\infty}^{\infty} W_{Q_x}(z)\exp\left(-\frac{i\omega z}{c}\right)\frac{dz}{c}$$

$[\Omega/\mathrm{m}]$

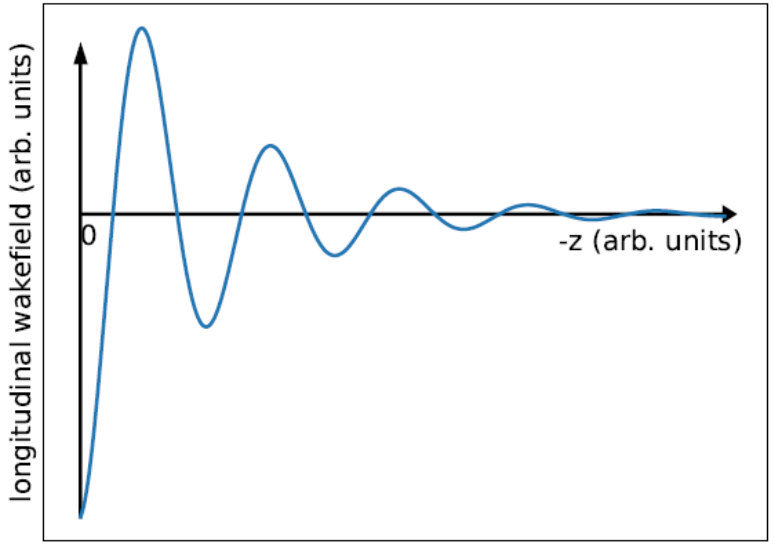
Loss factor and beam-loading theorem

- The loss factor represents the energy lost by a charge in passing through a structure due to the electromagnetic fields of the charge itself and normalized by q^2.

$$k_{\parallel} = \frac{U(z=0)}{q_0^2}. \qquad w_{\parallel}(z) = \frac{U(z)}{qq_0},$$

- Although in general the loss factor is given by the longitudinal wakefield at z = 0, for charges travelling with the speed of light, $w_{\parallel}(z = 0)$ is discontinuous at z = 0 due to the causality principle

- The exact relationship between the loss factor and the wakefield is given by the beam-loading theorem



Example of wakefield with β = 1.

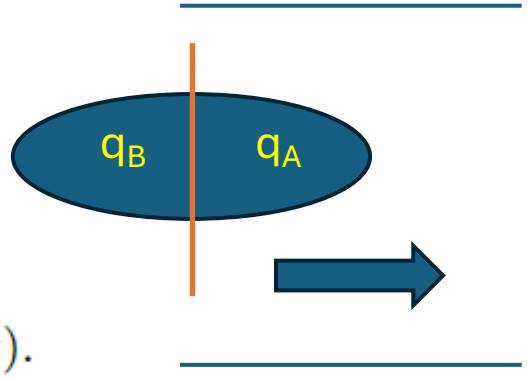
$$k_{\parallel} = \frac{w_{\parallel}(z \to 0^-)}{2}.$$

Demonstration of the beam loading theorem

For a simple demonstration of the beam-loading theorem, let us consider a charge q moving at the speed of light that is split into two charges of value q/2 at a distance z one from the other. Let us call A the head charge and B the tail one. The energy lost by A can be only due to the loss factor because of the causality principle.

$$U_A = q_A^2 k_{\parallel} = \frac{q^2}{4} k_{\parallel},$$

$$U_B(z) = q_B^2 k_{\parallel} + q_B q_A w_{\parallel}(z) = \frac{q^2}{4} k_{\parallel} + \frac{q^2}{4} w_{\parallel}(z).$$



$q=q_A+ q_B$
$q_A= q_B=q/2$

$k_{\parallel}$ the loss factor of the vacuum pipe

When $z \to 0$ the sum $U_A + U_B$ must be equal to the loss factor of the original charge q,

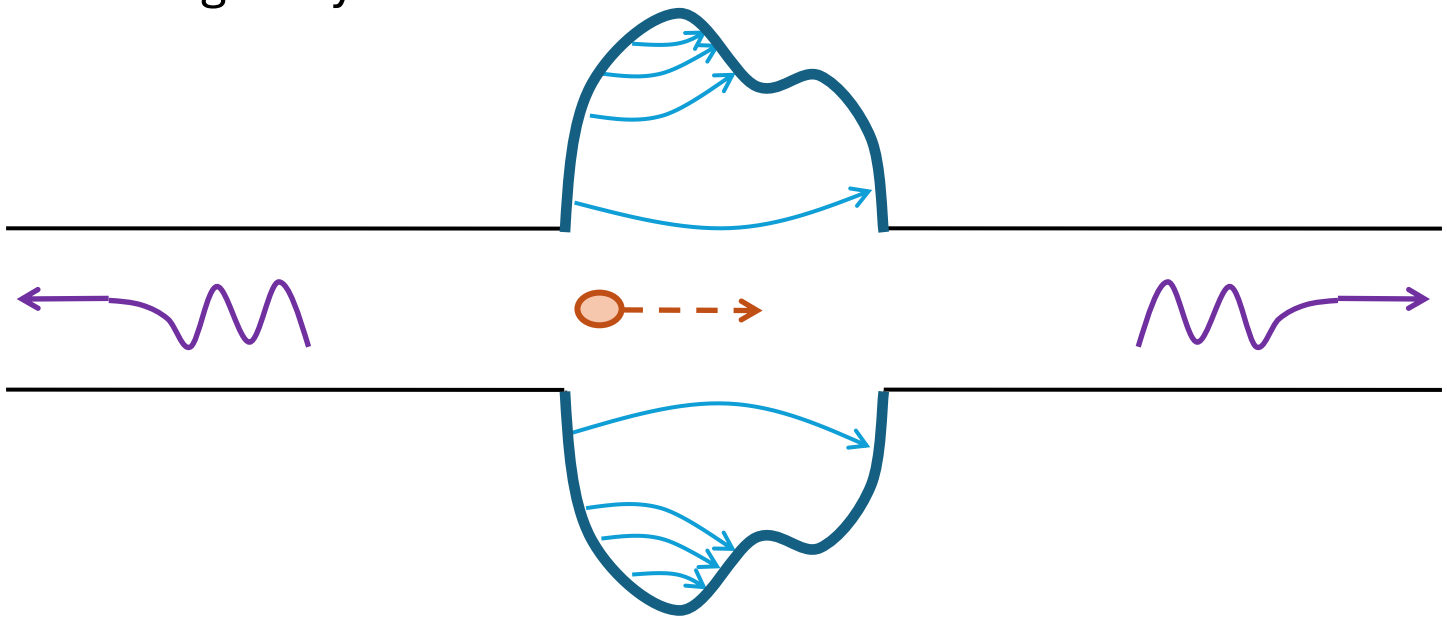
$$q^2 k_{\parallel} = U_A + U_B(z \to 0^-) = \frac{q^2}{2} k_{\parallel} + \frac{q^2}{4} w_{\parallel}(z \to 0^-) \quad \Rightarrow \quad k_{\parallel} = \frac{w_{\parallel}(z \to 0^-)}{2}.$$

The energy balance

$$W_{\parallel}(0) = \frac{1}{\pi}\int_0^{\infty} \mathrm{Re}\left(Z_{\parallel}(\omega)\right)\, d\omega = -\frac{\Delta E_1}{q_1^2}$$

What happens to the energy lost by the source?

- In the global energy balance, the energy lost by the source splits into:
  - Electromagnetic energy of the **modes that remain trapped** in the object
    - → Partly dissipated on **lossy walls** or into purposely designed inserts or HOM absorbers
    - → Partly transferred to **following particles** (or the same particle over successive turns), possibly feeding into an instability!
  - Electromagnetic energy of **modes that propagate** down the beam chamber (above cut-off), eventually lost on surrounding lossy materials



The energy balance

$$W_{\parallel}(0) = \frac{1}{\pi}\int_0^{\infty} \mathrm{Re}\left(Z_{\parallel}(\omega)\right)\, d\omega = -\frac{\Delta E_1}{q_1^2}$$

What happens to the energy lost by the source?

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  - Electroma
    - → Partly di
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    eventually

), possibly feeding

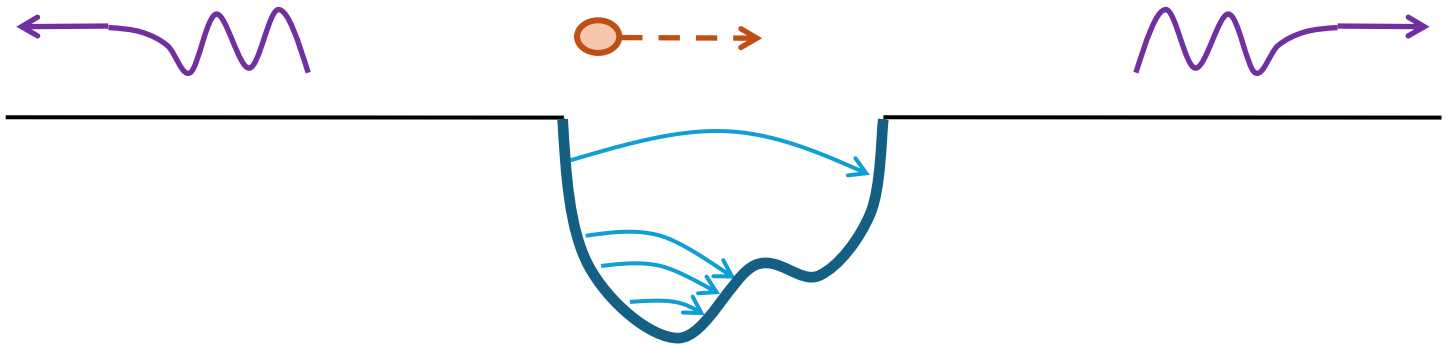
(above cut-off),

The energy loss of a particle bunch

⇒ causes **beam induced heating** of the machine elements (damage, outgassing)

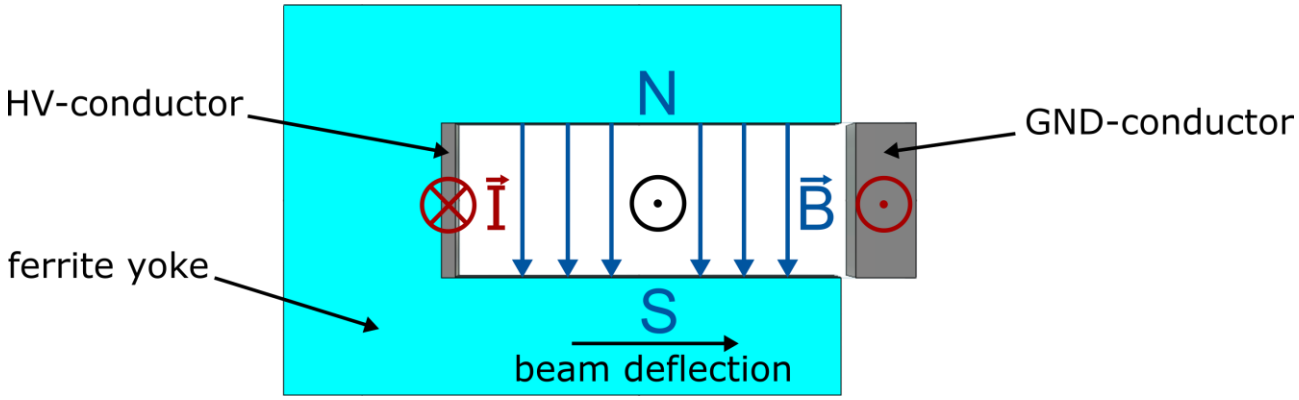
⇒ feeds into both **longitudinal and transverse instabilities** through the associated EM fields

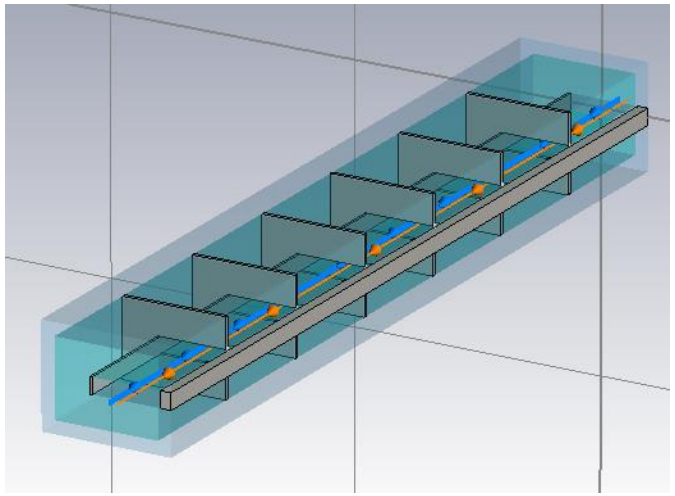
⇒ is compensated by the RF system determining a **stable phase shift**



Example: SPS extraction kickers

- Problem with SPS extraction kickers (MKE)
  - Extraction elements through which the beam passes every turn
    - Based on a fast pulsed magnet capable of deflecting the whole beam over one turn
    - Active only on turn in which beam has to be extracted, otherwise passive but with all its elements (ferrite, conductors) exposed to the beam
  - Use of beam for LHC filling (4x 200-ns spaced trains of 72x 25-ns spaced bunches) led to inacceptable heating of these elements)
    - Heating above Curie temperature leads to ferrite degradation → Beam cannot be extracted anymore from the SPS
    - Heating causes outgassing and strong pressure rise in the kicker sector, with consequent beam interlocking due to poor vacuum



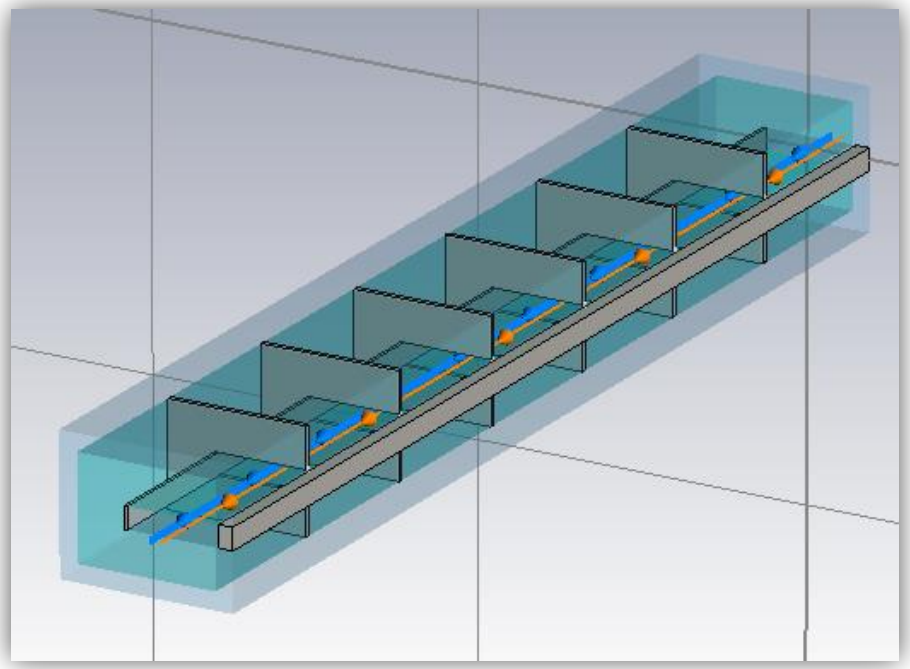


Example: SPS extraction kickers

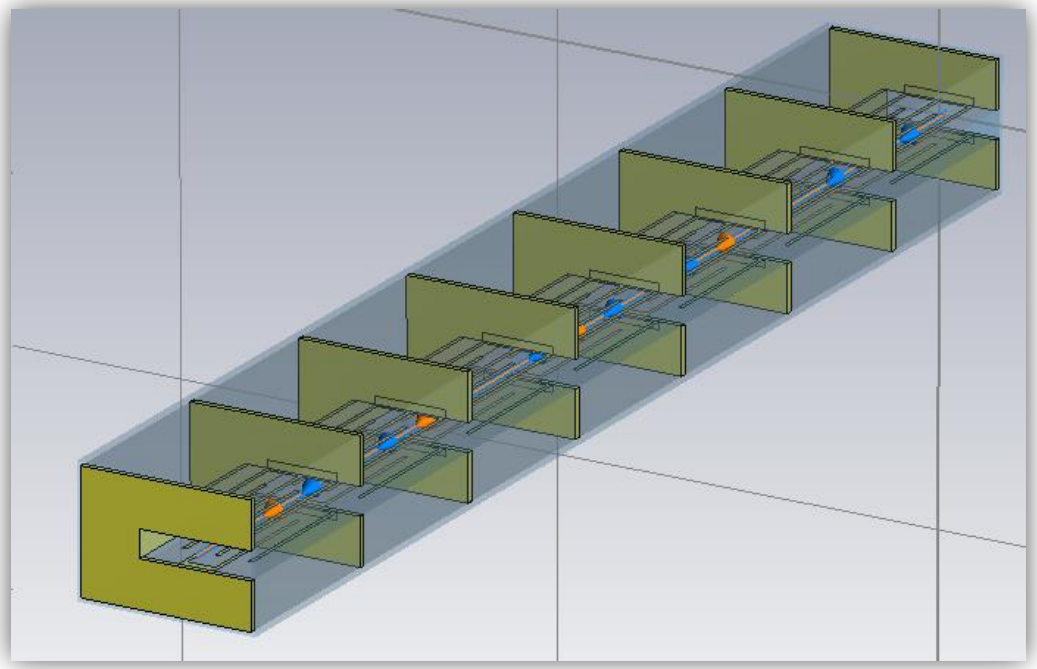
Two types of SPS extraction kickers (MKE):

1. Original design: several modules separated by conductor stripes (segmentation) with **bare ferrite blocks**, fed by an inner and an outer conductor
2. New design: like original, but modules have **'serigraphed' ferrite blocks** (i.e. with patterns of silver paste screen printed on the ferrite surface exposed to the beam)

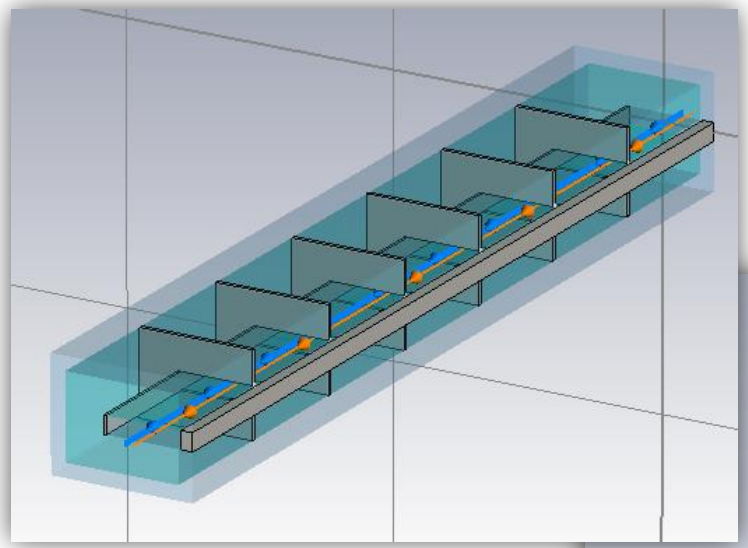
Original kicker



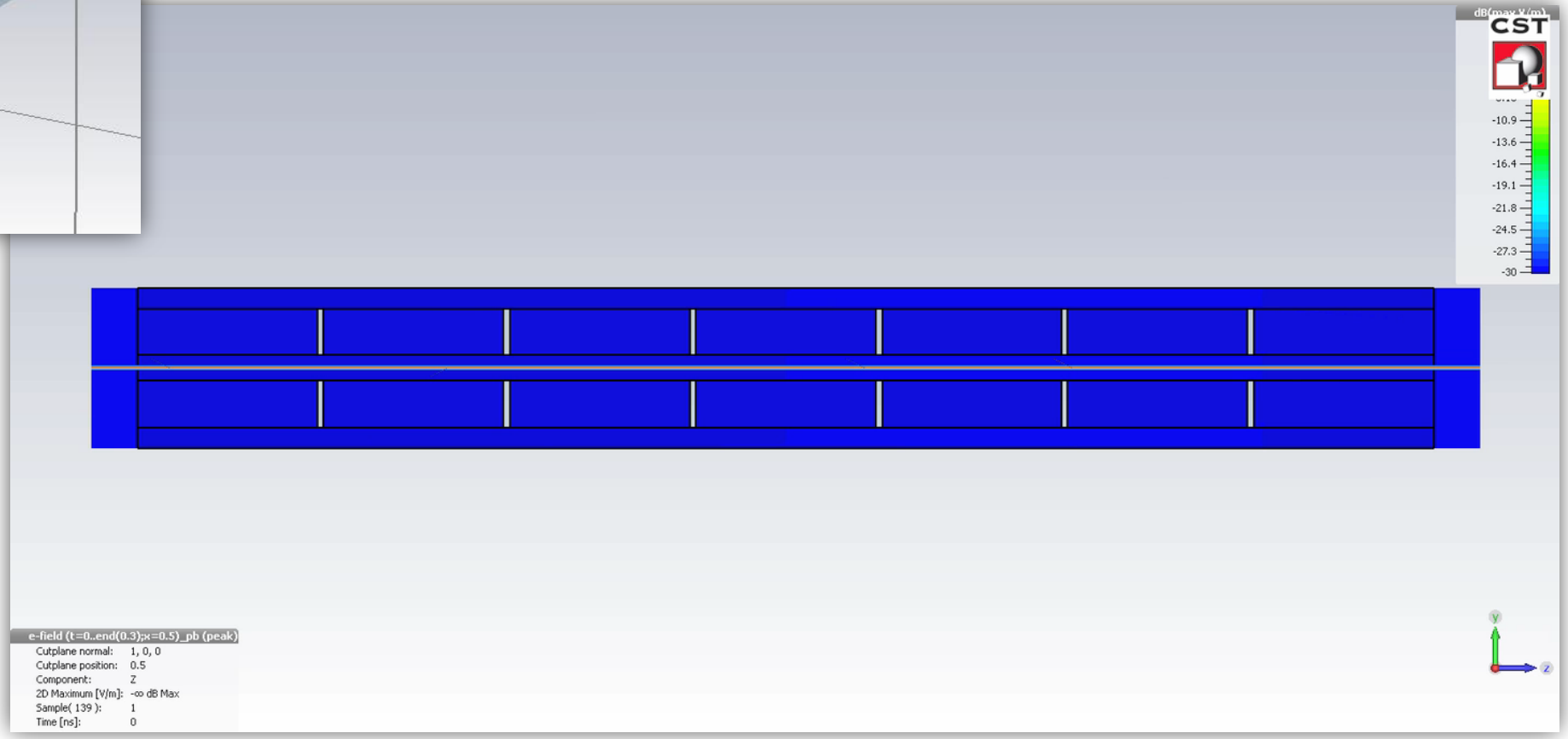
Serigraphed kicker



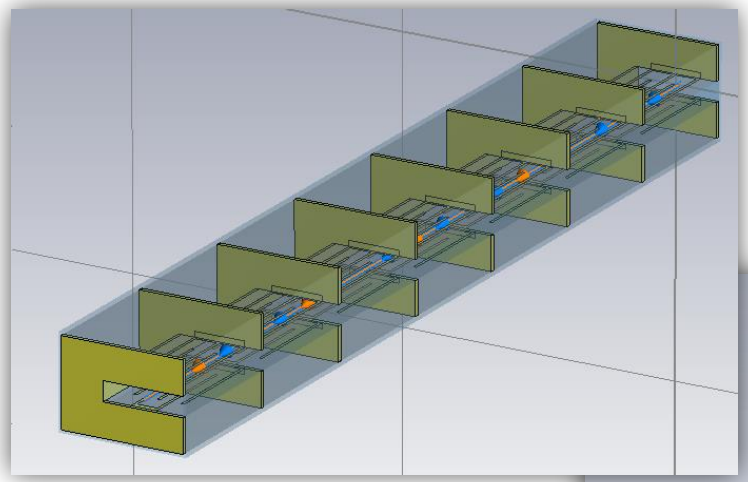
Examples: ferrite kicker – simple model



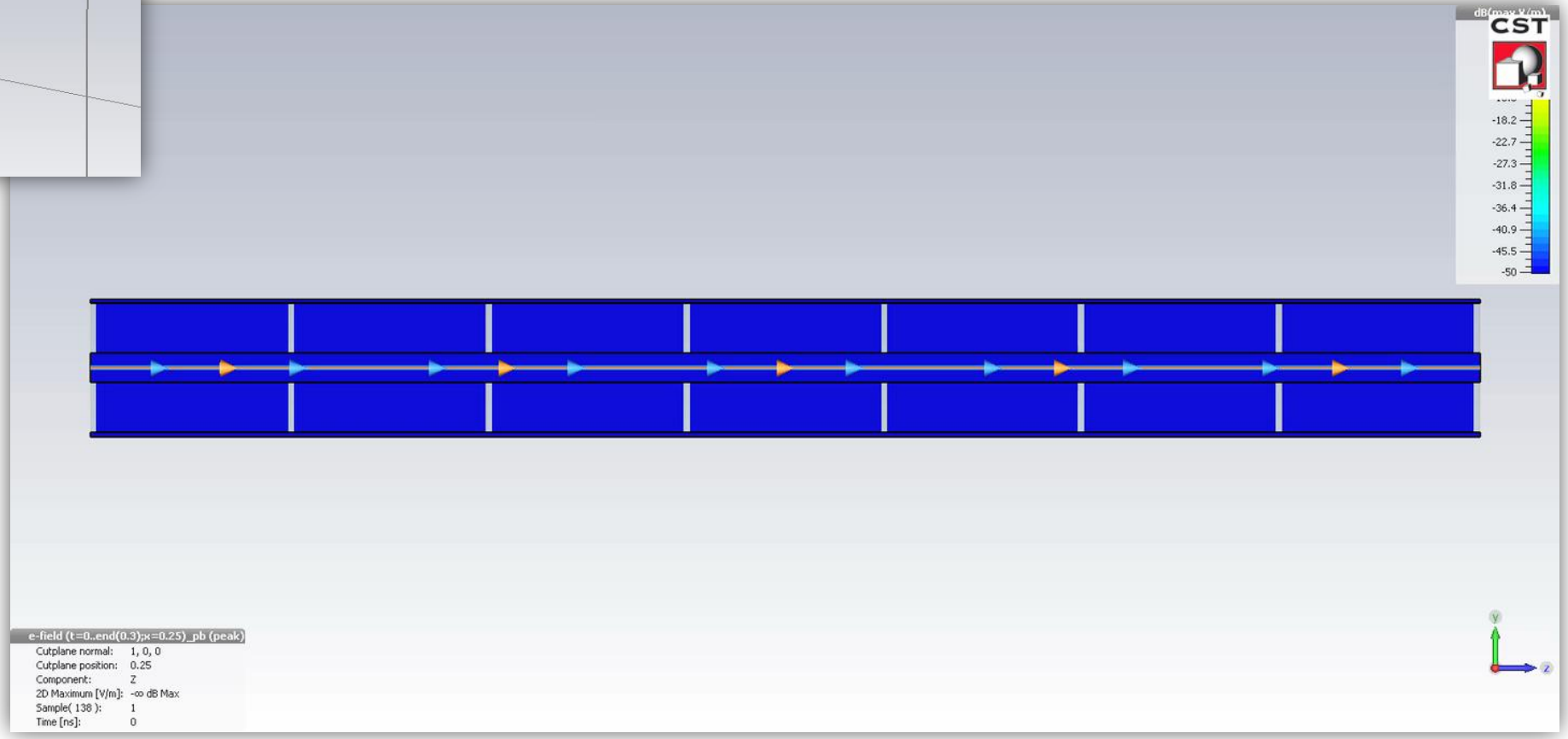
Original kicker **without serigraphy, typical broad-band behaviour**, here some ringing is due to the longitudinal segmentation



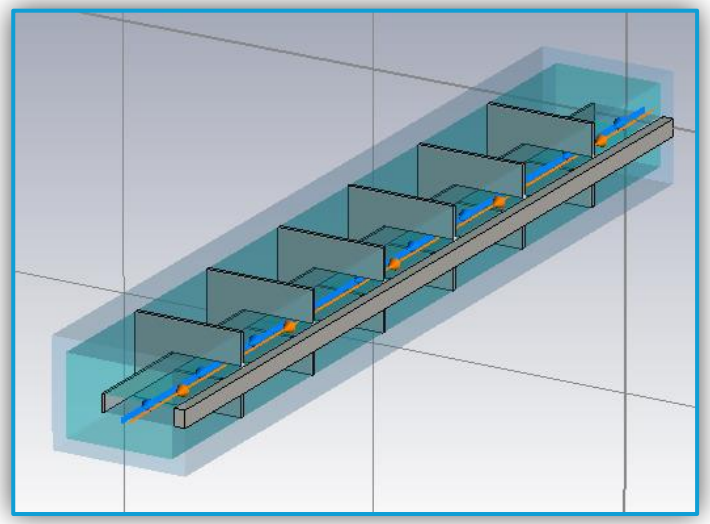
Examples: serigraphed kicker – simple model

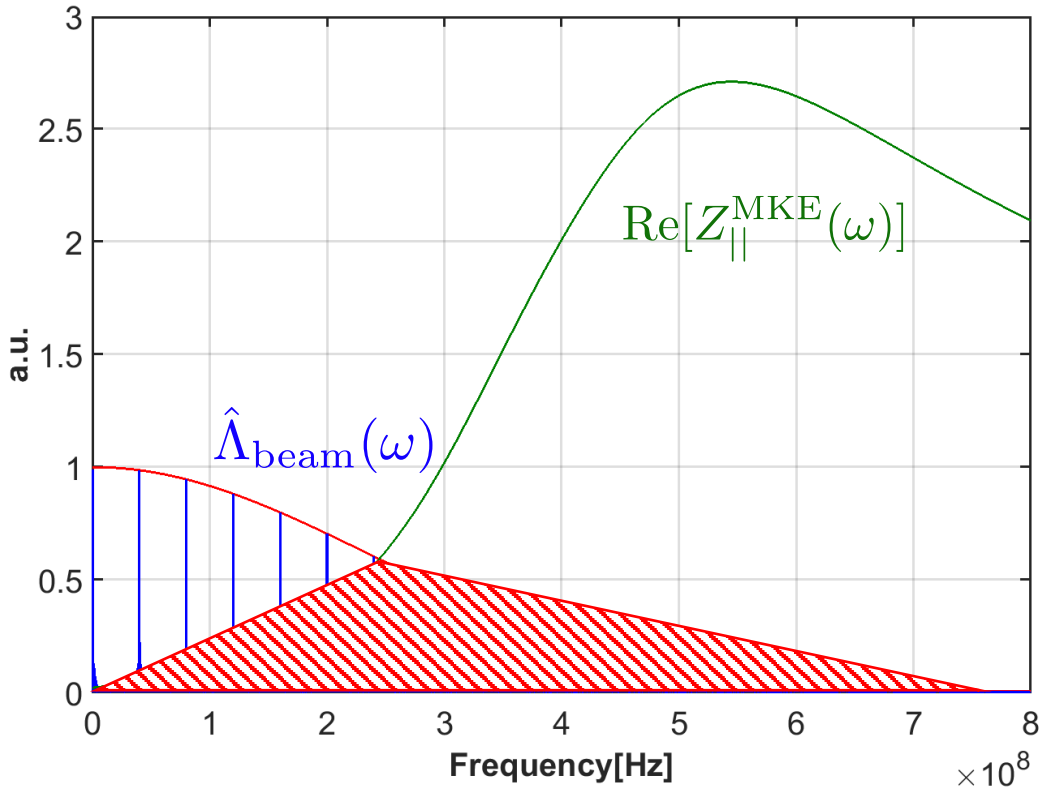


Serigraphed kicker **exhibits strong ringing** due to the EM trapping along the serigraphy fingers



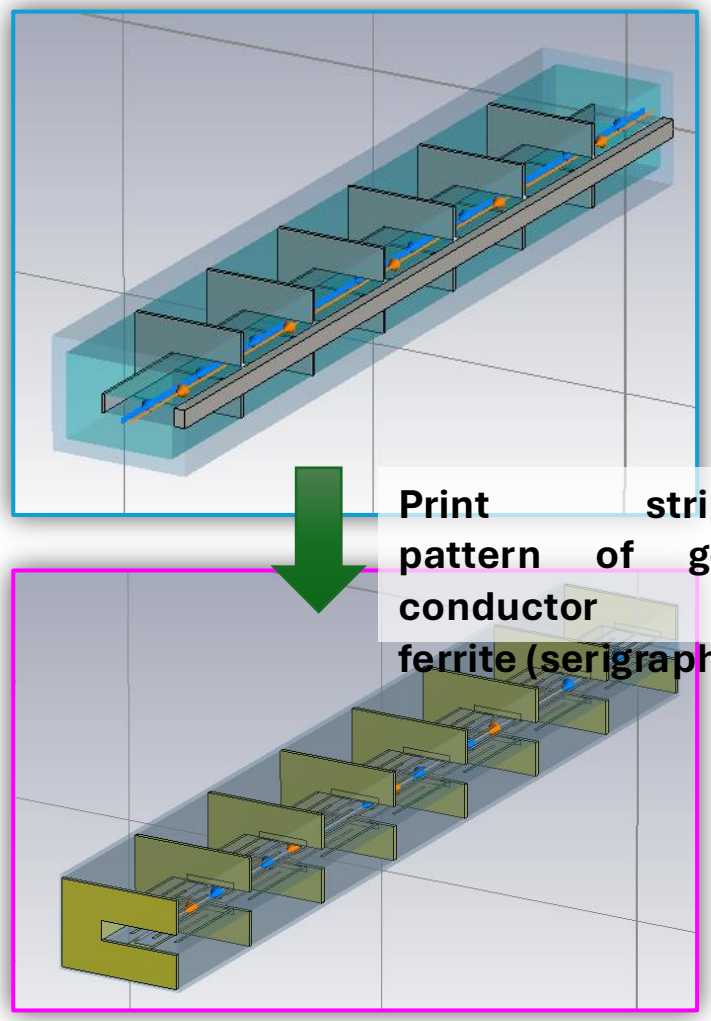
Examples: Serigraphy impact



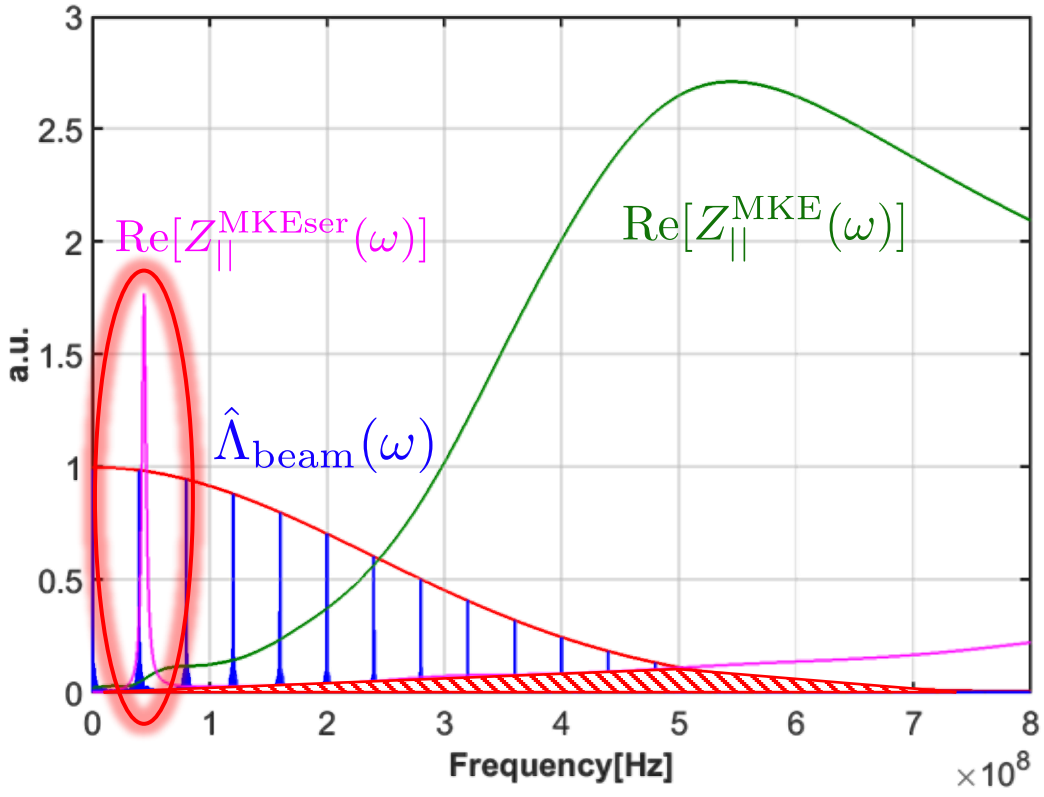


- The energy loss of the beam to the kicker structure is given by **the overlap of the impedance with the power beam spectrum**
- Kicker impedance already **becomes significant at frequencies for which the beam spectrum has not fully decayed**, causing the undesired heating
- We need to lower the kicker impedance → Impedance dominated by losses in ferrite → ferrite shielding

Examples: Serigraphy impact

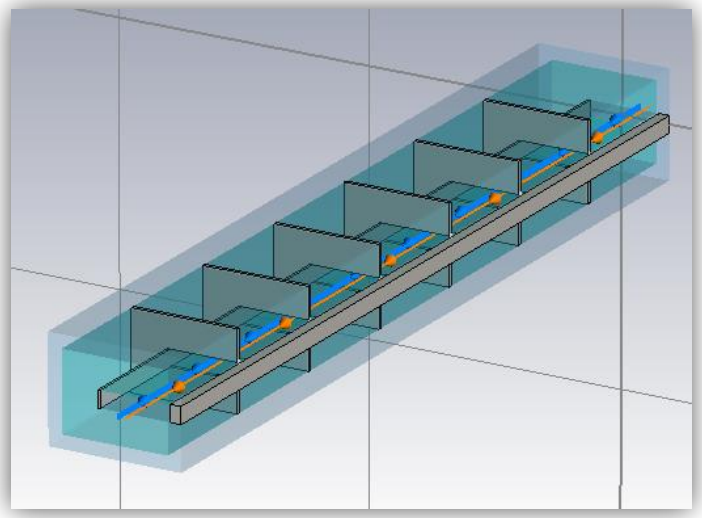


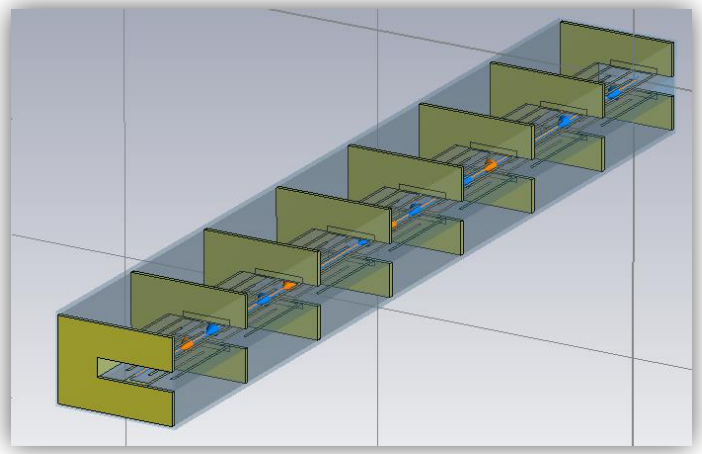
Print striped pattern of good conductor on ferrite (serigraphy)



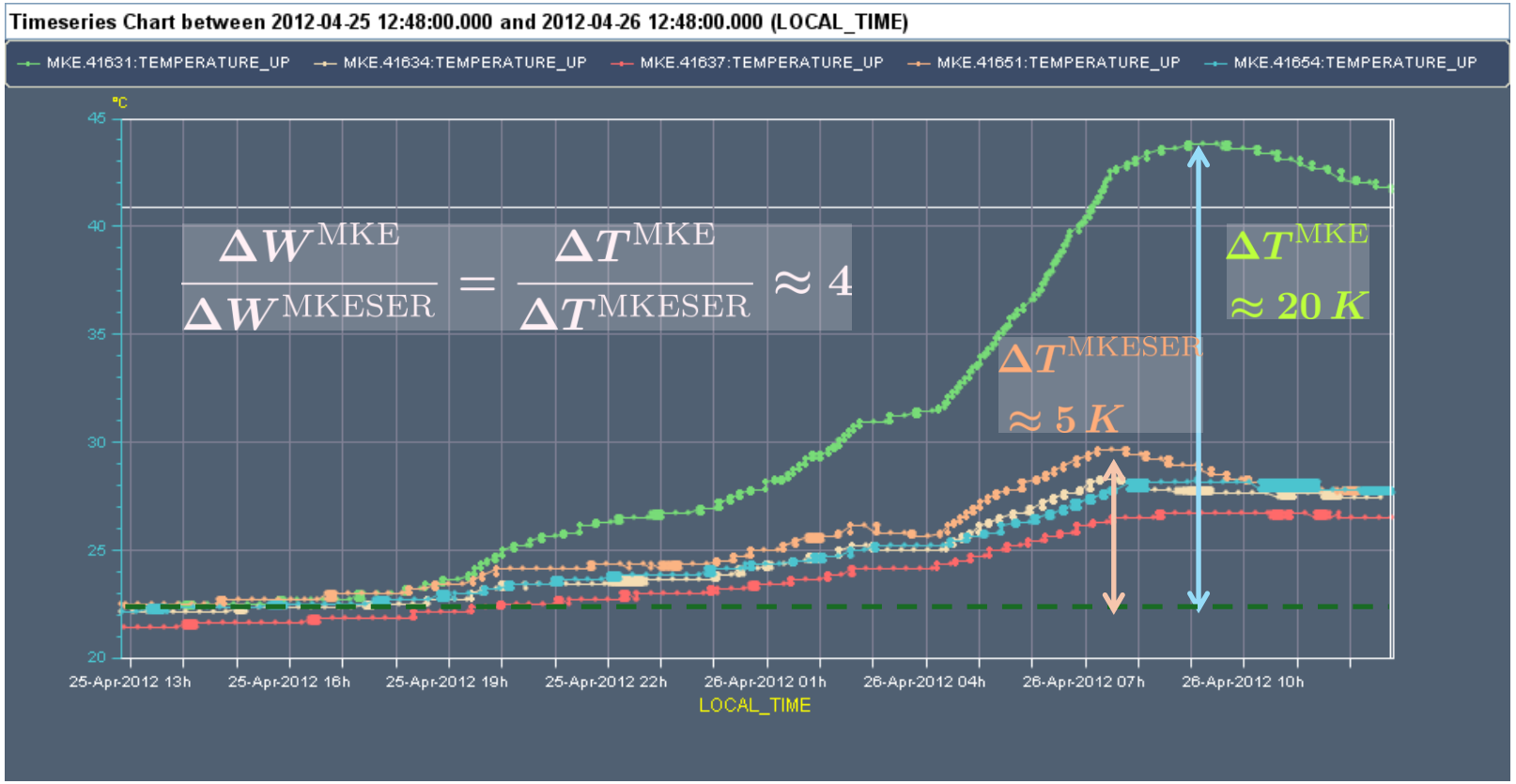
- The energy loss of the beam to the kicker structure is given by **the overlap of the impedance with the power beam spectrum**
- This almost suppresses the impedance over the bunch spectrum
- It however introduces **a low frequency peak**, which needs to be kept far from beam spectral lines → pay attention to do that for all needed bunch spacing

Examples: Serigraphy impact





~ 17h run with 25 ns beams at 26 GeV after technical stop



Summary

We have used the concept of wake fields in **the longitudinal and the transverse planes**, respectively. We have found that we usually do a decomposition of the wake function to obtain only the leading orders, namely, **constant, dipolar and quadrupolar wake fields**. We have also introduced the **impedance of the frequency domain representation** of the wake function.

We have learned that impedances can have a **detrimental impact** on both the **machine environment** as well as **the beam itself**.

A careful design of machine elements to **minimize the impedance** is therefore necessary.

Homework (due Nov 16) (50 points)

1. (10) Evaluate the space charge longitudinal impedance based on longitudinal space charge wake field

$$w_{\parallel}(z) = \frac{1}{qq_0}\int_0^{\ell} F_s(0,z)ds = -\frac{\ell}{4\pi\varepsilon_0\gamma^2}\left(1+2\ln\frac{b}{a}\right)\delta'(z),$$

2. (10 points) Evaluate the energy lost per unit length by a charge inside a given longitudinal distribution λ(s) (Gaussian) due to the longitudinal wakefield of the space charge and compare it with the longitudinal space charge force in r = 0.

$$F_s(r,s) = qE_s(r,s) = -\frac{q}{4\pi\varepsilon_0\gamma^2}\left(1-\frac{r^2}{a^2}+2\ln\frac{b}{a}\right)\frac{\partial\lambda(s)}{\partial s}.$$

3. (10 points) From the result of the previous exercise, evaluate the total energy lost per unit length by the whole bunch.

4. (20 points) Evaluate the energy spread (Umax − Umin) of a Gaussian bunch of RMS length σ due to the longitudinal wakefield of the space charge in a structure of length ℓ.

$$\lambda(s) = \frac{q_{\text{tot}}}{\sqrt{2\pi}\sigma_z}e^{-\frac{s^2}{2\sigma_s^2}},$$

Homework (due to Nov. 11 50 points)

(20 points)

1. Calculate the transverse space charge force for coasting beam with axial symmetrical Gaussian transverse charge density:

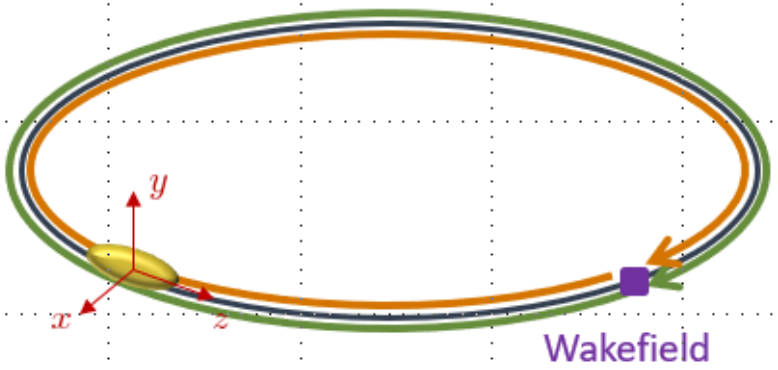
$$f(r) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{r^2}{2\sigma^2}\right)$$

   a) Compare results with space charge effect for coasting beam with uniform charge density radius $a=2*\sigma$

2. (10 points) Based on the results of HW1 : what high order multipole (4 poles - quadrupole, 6 poles - sextupole, 8 poles- octupole, 10 poles –decapole or higher) is the best choice to reduce space charge tune spread due to gaussian transverse distribution? And why?

Additional material

Tracking one full turn including the interaction with wake fields:



$$\left.\begin{pmatrix} x_i \\ x_i' \end{pmatrix}\right|_{k+1} = \mathcal{M}_i \left.\begin{pmatrix} x_i \\ x_i' \end{pmatrix}\right|_{k}$$

$$\left.\begin{pmatrix} z_i \\ \delta_i \end{pmatrix}\right|_{k+1} = \mathcal{I}\left[\left.\begin{pmatrix} z_i \\ \delta_i \end{pmatrix}\right|_{k}\right]$$

$$\left.(x_i')\right|_{k+1} = \left.(x_i')\right|_{k} + \mathcal{WK}$$

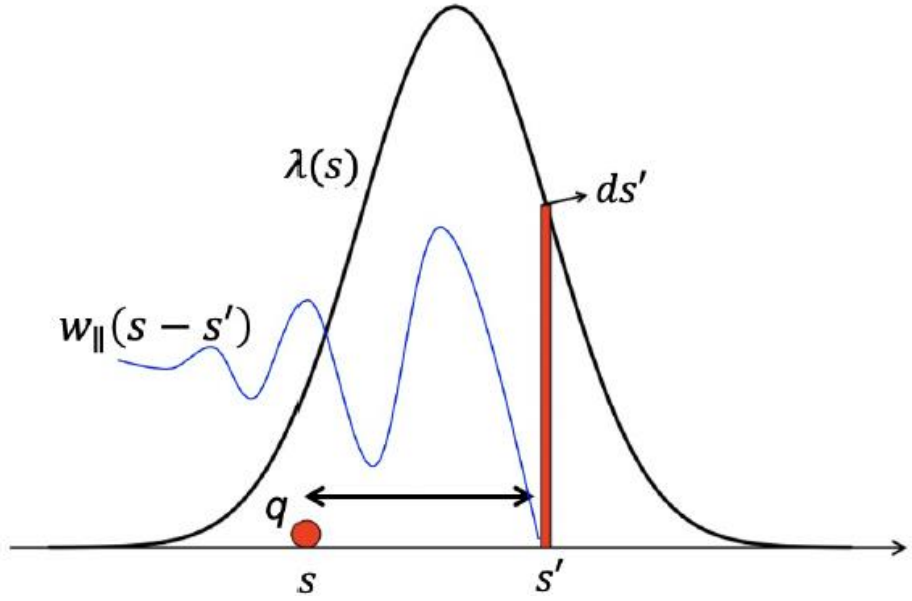
1. Initialise a macroparticle distribution with a given emittance
2. Update transverse coordinates and momenta according to the linear periodic transfer map – adjust the individual phase advance according to chromaticity and detuning with amplitude
3. Update the longitudinal coordinates and momenta according to the leap-frog integration scheme
4. Update momenta only (apply kicks) according to wake field generated kicks
5. Repeat turn-by-turn…

Wake potential and energy loss of a bunched distribution

The energy change of q due to all the charges dq(s′) that are in a position s′ inside the longitudinal distribution as

$$q_{\text{tot}} = \int_{-\infty}^{\infty} \lambda(s) ds$$

$$dU(s) = q dq(s') w_{\parallel}(s - s') = q w_{\parallel}(s - s') \lambda(s') ds'.$$



energy change over the whole bunch,

$$U(s) = q \int_{-\infty}^{\infty} w_{\parallel}(s - s') \lambda(s') ds'.$$

longitudinal wake potential

$$W_{\parallel}(s) = \frac{U(s)}{q q_{\text{tot}}} = \frac{1}{q_{\text{tot}}} \int_{-\infty}^{\infty} w_{\parallel}(s - s') \lambda(s') ds'.$$

The average energy change of the bunch due to the wake field

$$U_{\text{bunch}} = \frac{1}{q} \int_{-\infty}^{\infty} U(s) \lambda(s) ds = q_{\text{tot}} \int_{-\infty}^{\infty} W_{\parallel}(s) \lambda(s) ds.$$