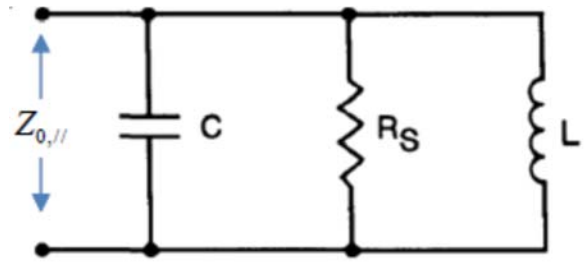
Home Work 17

1. (10 points)

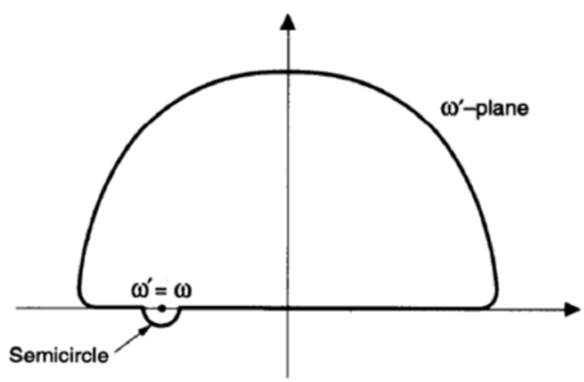


The impedance of a resonator model can be related to a circuit shown above. Show the impedance of above circuit can be expressed as

$$Z_{0,//} = \frac{R_s}{1 + iQ\left(\frac{\omega_R}{\omega} - \frac{\omega}{\omega_R}\right)} ,$$

and find the expression for Q and ω_R in terms of C, R_s, and L.

2. (10 points) Perform a contour integral of $\frac{Z_{//}(\omega')}{\omega' - \omega}$ in the complex ω' plane over the upper half plane along the contour shown in the figure.



Show that if $Z_{//}(\omega')$ converges sufficiently fast as $|\omega'| \to \infty$,

$$Z_{//}(\omega) = -\frac{i}{\pi} P.V. \int_{-\infty}^{\infty} \frac{Z_{//}(\omega')}{\omega' - \omega} d\omega' , \tag{1}$$

and eq. (1) leads to Kramers-Kronig relations.

$$\mathrm{Re}\left[Z_{//}(\omega)\right] = \frac{1}{\pi} P.V. \int_{-\infty}^{\infty} \frac{\mathrm{Im}\left[Z_{//}(\omega')\right]}{\omega' - \omega} d\omega'$$

$$\mathrm{Im}\left[Z_{//}(\omega)\right] = -\frac{1}{\pi} P.V. \int_{-\infty}^{\infty} \frac{\mathrm{Re}\left[Z_{//}(\omega')\right]}{\omega' - \omega} d\omega'$$

.