# PHY 554
# Fundamentals of Accelerator Physics
## Lecture 3: History of Accelerators

September 7, 2016

Vladimir N. Litvinenko

http://case.physics.stonybrook.edu/index.php/PHY554_fall_2016

We continue discussing development of accelerators and learning "accelerator slang"

**Era of Synchrotrons.** By 1940s all the above acceleration principles: DC, resonant and induction (betatron) had been successfully demonstrated. Having a solid-core magnets – with sizes reaching 10s of meters – becomes impossible fit and scientists started developing accelerators comprised of a separate lump magnets and RF structures.

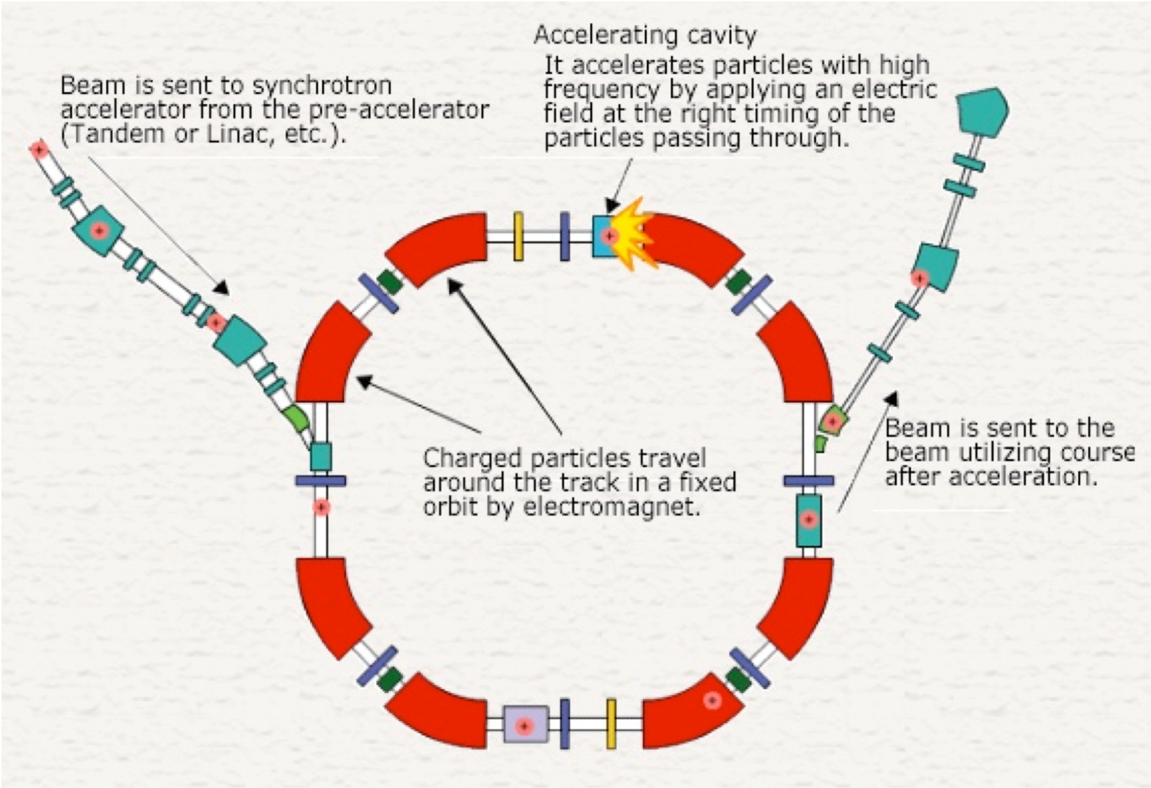



*Fig. 2.17. A simple synchrotron with injection, 8 dipole magnets and an accelerating RF cavity [5].*

Synchrotrons (in contrast with storage rings) were designed to accelerate particles from injection energy to ejection energy to send it either to the next accelerator (some complexes had chains of three-four synchrotrons with increasing energy reach) or to a target. RF cavity serves as turn after turn energy booster for the beam while the magnets have to follow (which is always slow process as you already know!) the increasing energy of the beam with the increase of their field – the process called ramping.

The operational principles of synchrotron, when you know them, are very straight forward:

(a) the particle motion (e.g. magnetic fields and time of flight) and the accelerating field in RF cavity have to be synchronized (hence the name synchrotron);
(b) the motion in all three directions has to be stable.

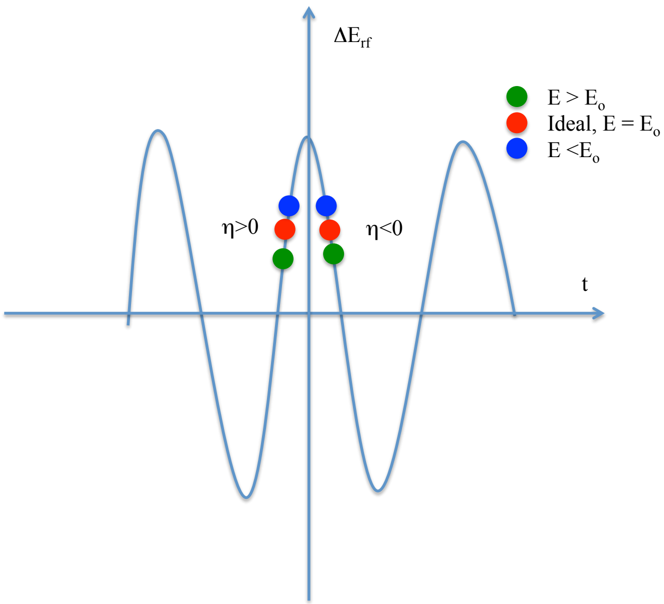
The fist problem was mostly engineering: one sets the ramping cycle of the magnets (frequently using the line AC frequency: 50 Hz in Europe, 60 Hz in US) and follow it up with necessary change of the RF frequency to beam synchronized with the accelerating cycle in the cavity:

$$T_o = \frac{C}{\mathrm{v}} = h_{RF} \cdot T_{RF} = \frac{h_{RF}}{f_{RF}}$$

Changing the RF frequency is mostly required in hadron synchrotrons, where particles do not reach relativistic velocities till very high energies. For example AGS (Alternating Gradient Synchrotron) accelerates protons from kinetic energy of 0.2 GeV to 28 GeV – this requires a nearly two-fold change of the RF frequency. You would learn later in the course that this is not a trivial but doable. Slightly different story is for electrons – it is relatively easy to accelerate electrons to tens of MeV before injecting them into a synchrotron. Usually then the available aperture of the vacuum chamber is sufficient to accommodate a slight variation of the electrons velocity. This answers the first requirements – what about second?

**But what about longitudinal motion**, i.e. a particle slightly out of synchronism or slight off-energy? Would they survive or disappear? Veksler discovered the phase (auto-focusing) stability in circular accelerators by introducing the time of flight dependence of the particles energy (frequently called a slip-factor):

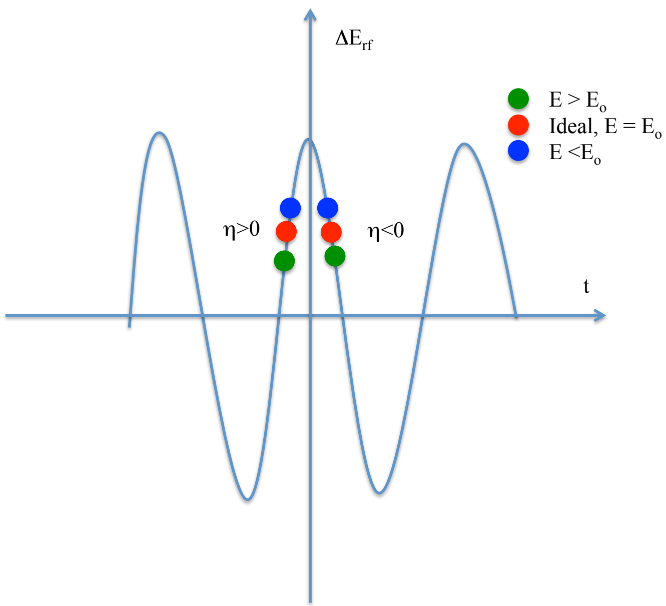
$$\eta = \frac{d\ln T_o}{d\ln E} = \frac{d\ln C}{d\ln E} - \frac{d\ln \mathrm{v}}{d\ln E} \tag{2.12}$$

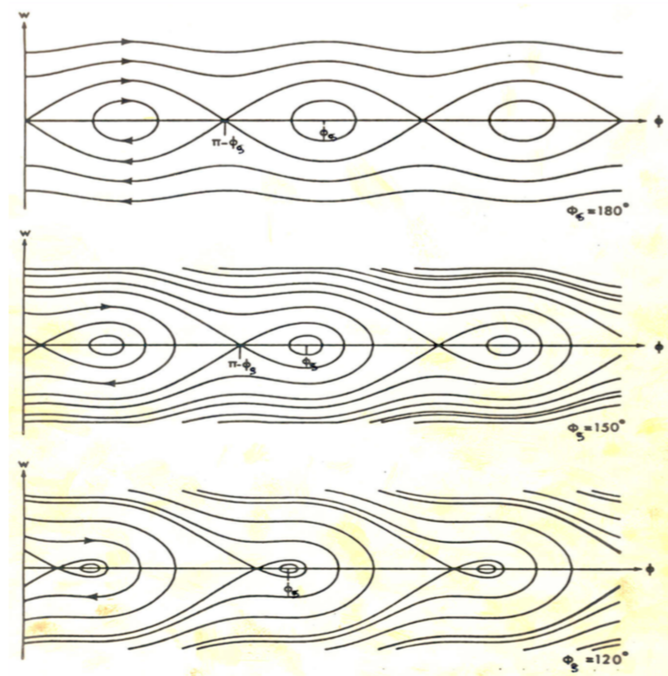



Veksler discovered that proper choice of accelerating (see fig. 2.18) provides for stability of longitudinal (phase – means RF phase) motion. It means that a particle with a phase or energy deviation will execute stable oscillations, which are called synchrotron oscillations.

Thus, longitudinal motion is stable (with an appropriate choice of phase and accelerating rate). What about transverse motion?

$$T_o = \frac{C}{v} = h_{RF} \cdot T_{RF} = \frac{h_{RF}}{f_{RF}}$$




By 1940s the principle of weak focusing for transverse motion was well know and this was working assumption that bending magnets have a gradient of the field splitting focusing between horizontal and vertical oscillations.

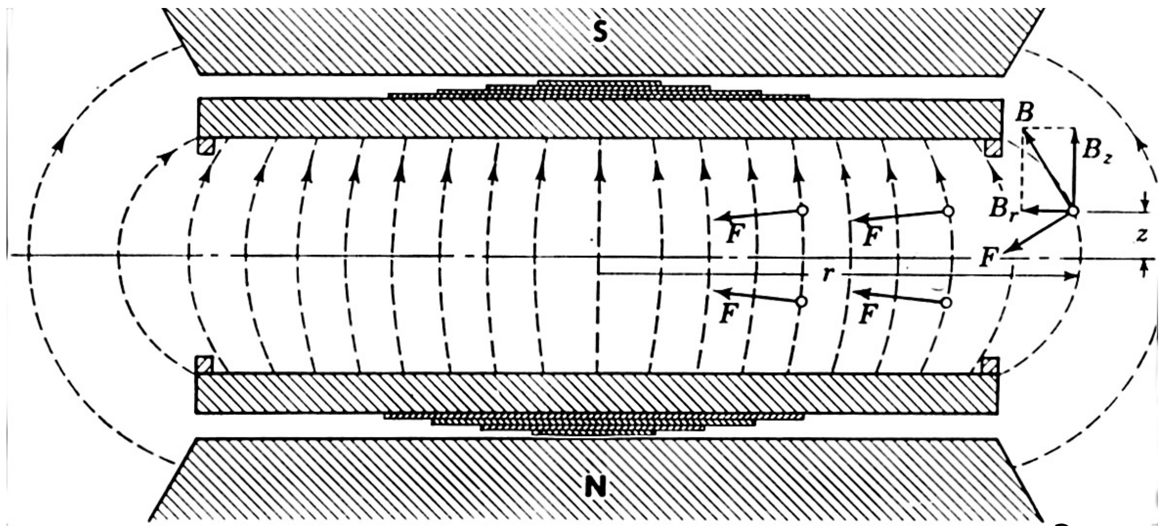
# Weak (transverse) focusing, plane orbit symmetry



To solve this problem let's expand the equations of motion near the ideal closed orbit:

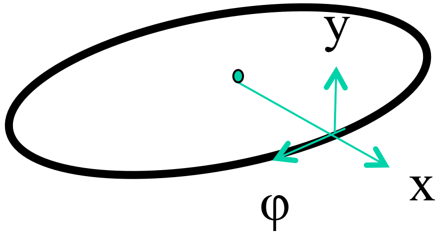
$$\vec{r} = \hat{r}\cdot(\rho + x) + \hat{y}\cdot y; \rho = \frac{pc}{eB_o};\ B_o = B_y(x=0, y=0) = const;$$

$$\vec{B}(\vec{r}) \cong \hat{y}B_o + \hat{r}\left(\frac{\partial B_x}{\partial x}x + \frac{\partial B_x}{\partial y}y\right) + \hat{y}\left(\frac{\partial B_y}{\partial x}x + \frac{\partial B_y}{\partial y}y\right);$$




Because of the symmetry $\frac{\partial B_y}{\partial y} = \frac{\partial B_x}{\partial x} = 0;$ and $curl\vec{B} = 0 \Rightarrow G = \frac{\partial B_y}{\partial x} = \frac{\partial B_x}{\partial y}$

$$\vec{B}(\vec{r}) \cong \hat{y}B_o + G(\hat{r}y + \hat{y}x) + O\left(x^2, y^2\right); |x|, |y| << \rho$$




$$\vec{\mathrm{v}} = \frac{d}{dt}\left(\hat{r}\cdot\left(\rho_o + x\right) + \hat{y}\cdot y\right) = \frac{d\hat{r}}{dt}\cdot\left(\rho + x\right) + \hat{r}\frac{dx}{dt} + \hat{y}\cdot\frac{dy}{dt};$$

$$\frac{d\hat{r}}{dt} = \omega\hat{\varphi}; \frac{d\hat{\varphi}}{dt} = -\omega\hat{r}; \qquad \frac{d^2\hat{r}}{dt^2} = -\omega^2\hat{r}$$

$$\vec{\mathrm{v}}_o = \hat{\varphi}\cdot\omega\rho; \ \vec{\mathrm{v}} = \hat{\varphi}\cdot\omega\left(\rho + x\right) + \hat{r}\dot{x} + \hat{y}\cdot\dot{y}$$

$$\frac{d\vec{\mathrm{v}}}{dt} = \frac{d^2\hat{r}}{d^2t}\cdot\left(\rho_o + x\right) + \frac{d\hat{r}}{dt}\frac{dx}{dt} + \hat{r}\frac{d^2x}{dt^2} + \hat{y}\cdot\frac{d^2y}{dt^2}$$

$$\frac{d\vec{\mathrm{v}}}{dt} = \hat{r}\left\{\frac{d^2x}{dt^2} - \omega^2\cdot\left(\rho_o + x\right)\right\} + \hat{y}\cdot\frac{d^2y}{dt^2} + \omega\hat{\varphi}\dot{x}$$

Since energy is constant in magnetic field

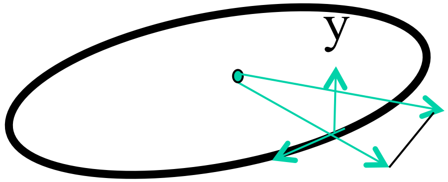
$$\frac{d\vec{p}}{dt} = \gamma m\frac{d\vec{\mathrm{v}}}{dt} = \frac{e}{c}\left[\vec{\mathrm{v}}\times\vec{B}\right] \cong \frac{e}{c}\left[\left\{\hat{\varphi}\cdot\omega\left(\rho_o + x\right) + \hat{r}\dot{x} + \hat{y}\cdot\dot{y}\right\}\times\left\{\hat{y}B_o + G\left(\hat{r}y + \hat{y}x\right)\right\}\right]$$

$$= \frac{e}{c}\left[-\hat{r}B_o\omega\left(\rho_o + x\right) + \omega\rho_o G\left(\hat{y}\cdot y - \hat{r}x\right) + \hat{\varphi}B_o\dot{x} + \mathrm{O}\left(\varepsilon^2\right)\right]$$

$$\frac{d\vec{\mathrm{v}}}{dt} = \hat{r}\left\{\frac{d^2x}{dt^2} - \omega^2\cdot\left(\rho_o + x\right)\right\} + \hat{y}\cdot\frac{d^2y}{dt^2} + \omega\hat{\varphi}\dot{x} = \frac{e}{\gamma mc}\left[-\hat{r}B_o\omega\left(\rho_o + x\right) + \omega\rho_o G\left(\hat{y}\cdot y - \hat{r}x\right) + \hat{\varphi}B_o\dot{x}\right]$$

$$\frac{d^2x}{dt^2} - \omega^2\cdot\left(\rho_o + x\right) = -\omega\left[\frac{eB_o}{\gamma mc}\left(\rho_o + x\right) + \frac{eG}{\gamma mc}\rho_o x\right] + \mathrm{O}\left(\varepsilon^2\right)$$

$$\frac{d^2y}{dt^2} = -\omega\rho_o\frac{eG}{\gamma mc}y + \mathrm{O}\left(\varepsilon^2\right)$$




$$\frac{d^2x}{dt^2} - \omega^2 \cdot (\rho_o + x) = -\omega\left[\frac{eB_o}{\gamma mc}(\rho_o + x) + \frac{eG}{\gamma mc}\rho_o x\right] + \mathrm{O}(\varepsilon^2)$$

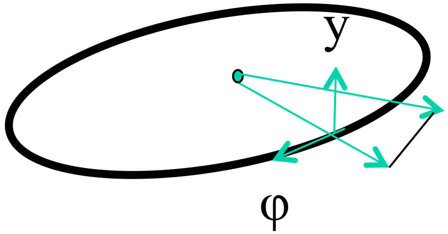
$$\frac{d^2y}{dt^2} = \omega\rho_o \frac{eG}{\gamma mc} y + \mathrm{O}(\varepsilon^2)$$

$$\boxed{dl = \mathrm{v}_o dt = \omega dt(\rho_o + x) \Rightarrow \omega = \frac{\mathrm{v}_o}{(\rho_o + x)} \cong \frac{\mathrm{v}_o}{\rho_o}\left(1 - \frac{x}{\rho_o}\right)}$$

$$\omega_o = \frac{\mathrm{v}_o}{\rho_o} = \frac{eB_o}{\gamma mc} \Rightarrow \rho_o = \frac{p_o c}{eB_o}$$

$$\frac{d^2x}{dt^2} + \omega_o^2(1-n)\cdot x \cong 0; \qquad \frac{d^2y}{dt^2} + n\omega_o^2 = 0; \quad n = -\frac{G\rho_o}{B_o}.$$

$$\boxed{\begin{gathered} Stability: \quad 0 < n < 1; \\ x = a_x \cos(\nu_x\omega_o t + \varphi_x); \quad y = a_y \cos(\nu_y\omega_o t + \varphi_y); \quad \nu_x = \sqrt{1-n}; \nu_y = \sqrt{n}; \\ \dot{x} = \nu_x\omega_o a_x \sin(\nu_x\omega_o t + \varphi_x); \quad \dot{y} = -\nu_y\omega_o a_y \sin(\nu_y\omega_o t + \varphi_y); \end{gathered}}$$




$$Stability: \quad 0 < n < 1;$$

$$x = a_x \cos\left(\nu_x \omega_o t + \varphi_x\right); \quad y = a_y \cos\left(\nu_y \omega_o t + \varphi_y\right); \quad \nu_x = \sqrt{1-n}; \nu_y = \sqrt{n};$$

$$\dot{x} = \nu_x \omega_o a_x \sin\left(\nu_x \omega_o t + \varphi_x\right); \quad \dot{y} = -\nu_y \omega_o a_y \sin\left(\nu_y \omega_o t + \varphi_y\right);$$

$$\text{Invariants}: \varepsilon_x = \frac{1}{\gamma mc} \oint dx\, dp_x; \varepsilon_x = \frac{1}{\gamma mc} \oint dy\, dp_y; \text{ length along trajectory } s = \omega_o t;$$

$$x = \sqrt{\beta_x \varepsilon_x} \cos\left(\frac{s}{\beta_x} + \varphi_x\right); \quad y = \sqrt{\beta_y \varepsilon_y} \cos\left(\frac{s}{\beta_y} + \varphi_y\right);$$

$$\frac{dx}{ds} = x' = -\sqrt{\frac{\varepsilon_x}{\beta_x}} \sin\left(\frac{s}{\beta_x} + \varphi_x\right); \quad \frac{dy}{ds} = y' = -\sqrt{\frac{\varepsilon_y}{\beta_y}} \sin\left(\frac{s}{\beta_y} + \varphi_y\right);$$

$$\beta_x = \frac{\rho_o}{\sqrt{1-n}} > \rho_o; \beta_y = \frac{\rho_o}{\sqrt{n}} > \rho_o;$$

In 1944 Veksler and McMillan (independently) proposed synchrotron as a next step towards high energy accelerators. **First synchrotrons were built using weak focusing.** Naturally they we using room temperature magnets and their radius was growing. One important feature of weak focusing is that particles executes less that one oscillation per turn. It means that for a fixed transverse angle particle deviation from ideal orbit will be proportional to the machine radius – hence the aperture of the accelerators went up with their energy. Technicians climbed inside vacuum chambers, physicist had meetings inside magnet aperture… it short, a new type of monsters appeared.

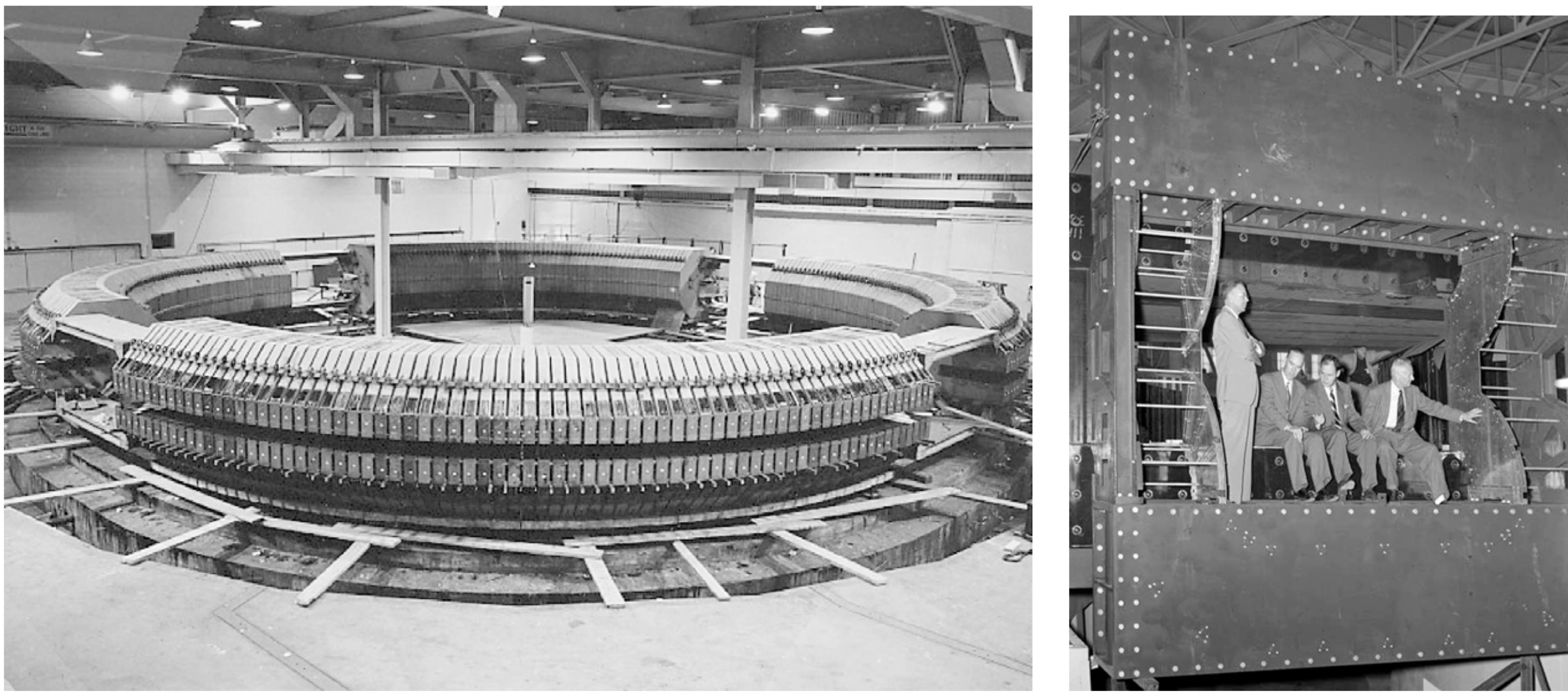


*Fig. 2.16. Left – BNL's Cosmotron and magnet aperture of 6 GeV weak-focusing Bevatron.*

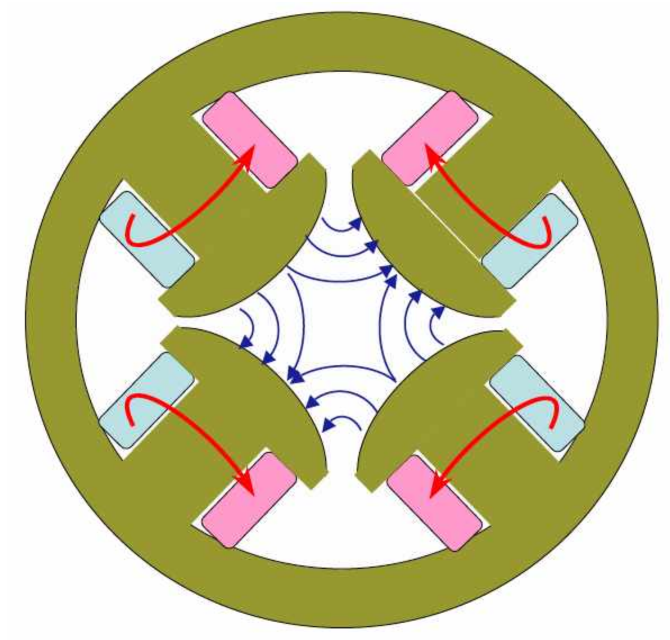


*Fig. 2.17. Typical quadrupole magnet.*

Physicist new about quadrupoles – magnets, which because of the Maxwell equations focused in one direction and defocus in the other:

$$curl\vec{B} = \hat{z}\left(\frac{\partial B_x}{\partial y} - \frac{\partial B_y}{\partial x}\right) = 0 \Rightarrow B_x = G \cdot x; B_y = G \cdot x$$

$$\vec{F} = \frac{e}{c}\left[\vec{\mathrm{v}} \times \vec{B}\right];\ \vec{\mathrm{v}}=\hat{z}\mathrm{v} \Rightarrow \vec{F} = \frac{eG}{c}\left(\hat{x} \cdot x - \hat{y} \cdot y\right) \tag{2.13}$$

depending on the sigh of the gradient, *G*.

It was (again) Christofilos who found a way out of this puzzle in 1949 by inventing a strong focusing. The idea is rather straight forward, again, after you know it: a combination of focusing and defocusing lens results is focusing:

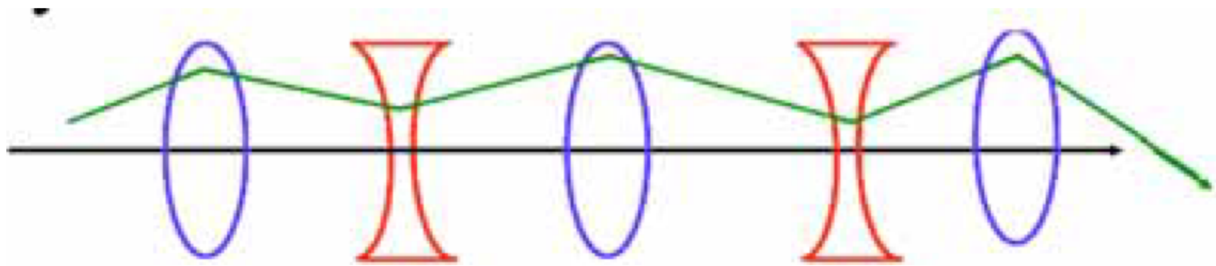


*Fig. 2.18. Principle of strong focusing (courtesy of W. Barletta)*

One can calculate a focusing lens and a defocusing lens with focal length of F separate distance L to find that the remaining focusing force in both directions to be (consider it as an exercise):

$$F_{eff} = \frac{F^2}{L} \tag{2.14}$$

This seemingly simple step, later combined with an exquisite theory developed at BNL by Courant and Snyder (the theory you would learn in this course [7]), made a real revolution. Modern accelerators based on the strong focusing have apertures from few cm to few millimeters (where and when needed).

**Era of storage rings and colliders**. The era of storage rings and colliders arrived on the shoulders of existing physics and technology already developed for synchrotrons. The new additions were superconducting magnets and superconducting RF systems. The main factor was also developing of ultra-high vacuum technology so beams can leave for hours and days in a properly designed storage rings.

It was natural to think about colliding beam in either the same storage ring where particles and antiparticles (electrons and positrons or protons and antiprotons) circulate in opposite directions and collide in a detector(s). A TEVATRON in FERMI-lab was based on this principle and well as LEP – both are closed now. Using two intersecting storage rings would allow colliding particles of any type with each other: this method is used in RHIC, LHC, B-factory.

As we discussed, the energy available for creating new particles in a collision is determined the the c.m. energy, which can be expressed as a scalar product of the total 4-momentum:

$$M = \frac{\sqrt{P_i P^i}}{c} = \frac{1}{c}\sqrt{= \frac{E^2}{c^2} - \vec{p}^{\,2}};_i P^i = p_1^i + p_2^i \tag{2.15}$$

As we discussed in our first class that colliding a relativistic particles $p_1^i = \gamma(m_1 c, m_1 \vec{\mathrm{v}})$ with a stationary particle $p_2^i = (m_2 c, 0)$ (a target) provides for a square root dependence of the available energy on the energy of the accelerator:

$$M = \sqrt{m_1^2 + m_2^2 + 2\gamma m_1 m_2}; \tag{2.16}$$

At the same time, two particles (with the same mass) colliding head-on $p_{1,2}^i = \gamma(mc, \pm m\vec{\mathrm{v}})$ can generate mass upto the total energy of two particles:

$$M = 2\gamma m \tag{2.17}$$

Thus, in late 1950 the ideas of colliding relativistic particles circulating in a storage ring was born. The skeptics who were using synchrotrons predicted this to be complete failure. The reasoning beyond this skepticism was so called luminosity of the collider. Processes in high energy and nuclear physics are described by a cross-section, $\sigma$. Then the number of the processes generated during collision of a particle with a target of transverse density $N_t / A$, where $N_t$ is the number of particles of interest in the target and A is its transverse area. If one will send $\dot{N}_b$ particles per second onto the target from an accelerator, the rete of the generated processes (events) will be given by:

$$R = \frac{dN_{events}}{dt} = \sigma \cdot \dot{N}_b \cdot \frac{N_t}{A} = \sigma \cdot L$$

$$L = \dot{N}_b \cdot \frac{N_t}{A} \tag{2.19}$$

where we introduced luminosity of the experiment, $L$. With $\frac{N_t}{A} \sim 10^{23} cm^{-2}$ of the solid target it is very hard to compete having from $10^9$ to $10^{11}$ particles per bunch.

Previous HW: Luminosity of CEBAF facility delivering 10 microamperes of electron beam on a 1 cm deep liquid $H_2O$ target at room temperature.

Density of the water is 1 gram per $cm^3$, e.g. electrons penetrate though the 1 cm of water see matter of 1 gram/ $cm^2$. $H_2O$ mole number is 18, it means that 18 grams of water contains Avogadro number of $H_2O$ molecules: $N_A = 6.02\times10^{23}$. Let remember that H contains one proton and one electron and $^8O_{16}$ contains 8 electrons and protons and 8 neutrons. It means that the target contains

$$n_{H_2O} = \frac{N_A}{18} \approx 3.34\cdot 10^{22}\,cm^{-2}$$

$$n_e = n_p = 10 n_{H_2O} = 3.34\cdot 10^{23}\,cm^{-2};$$

$$n_n = 8 n_{H_2O} = 2.68\cdot 10^{23}\,cm^{-2};$$

molecules, electron, protons and neutrons per unit transverse area. If one interested in transverse density of nucleons (e.g. neutrons and protons together) – it is simply the Avogadro number in this case. Now we need to calculate the flux of electrons per second coming from CEBAF accelerator, which is

$$\dot{N}_e = \frac{I}{e} = \frac{10^{-5}\,C/\sec}{1.6\cdot 10^{-19}} = 6.25\cdot 10^{13}\,\sec^{-1}$$

The luminosity for various collisions will be:

$$L_{e+H_2O} = \dot{N}_e n_{H_2O} \approx 2\cdot 10^{36}\,cm^{-2}\sec^{-1}$$

$$L_{e+e} = L_{e+p} = \dot{N}_e n_{e,p} \approx 2\cdot 10^{37}\,cm^{-2}\sec^{-1}$$

$$L_{e+n} = \dot{N}_e n_n \approx 1.7\cdot 10^{37}\,cm^{-2}\sec^{-1}$$

$$L_{e+nucleon} = L_{e+p} + L_{e+n} \approx 3.7\cdot 10^{37}\,cm^{-2}\sec^{-1}$$

By the way, CEBAF operates with 100 microamperes of electron beam and with typical luminosities ~ $10^{38}$ $cm^{-2}sec^{-1}$. Best modern colliders operating with amms of circulating beams currents are reaching into fewx$10^{35}$ $cm^{-2}sec^{-1}$ range of luminosities – still it is too hard to bit the Avagadro number! http://www.slac.stanford.edu/cgi-wrap/getdoc/slac-pub-11726.pdf

## Colliders

Let's consider two colliding beams consisting for individual bunches. Let's bunches collide with the collision rate $f_c$.

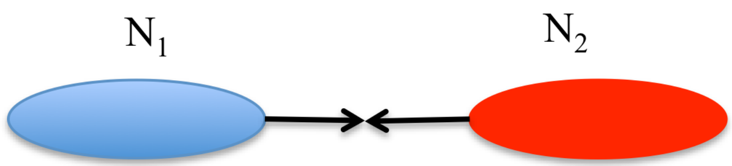



*Fig. 2.19. Two colliding beams*

Then during the collision the fist beam seas the density of particles in the second beam $\frac{N_t}{A} = \frac{N_2}{A}$. The first beam intensity is nothing that the collision (bunch) rate multiplied by the number of particles in the bunch 1: $\dot{N}_b = f_c N_1$. Plugging this in (2.19) we can write luminosity for colliding beams:

$$L = f_c \cdot \frac{N_1 N_2}{A} \tag{2.20}$$

Naturally, the success of the modern colliders was built upon colliding beams with very small transverse sizes, e.g. with a very high density and on high collision frequency. After this course you would know how the beam quality (emittance, $\varepsilon$ ) and the beam optics (beta-functions, $\beta$) affect the luminosity via $A = 4\pi\beta\varepsilon$ .

# HW 1 problem 3

$$p_p^\mu = \{E_p / c, p_p, 0, 0\} \qquad\qquad p_e^\mu = \{E_e / c, -p_e, 0, 0\}$$

First let's find the c.m. energy using 4-momenta of both particles

$$p_e^\mu = \{E_e / c, -p_e, 0, 0\}; p_p^\mu = \{E_p / c, p_p, 0, 0\};$$

$$p_c^\mu = \left\{\frac{E_p + E_e}{c}, p_p - p_e, 0, 0\right\}; \frac{E_{cm}{}^2}{c^2} = p_c^\mu p_c^\mu = \left(\frac{E_p + E_e}{c}\right)^2 - \left(p_p - p_e\right)^2;$$

$$E_{cm}{}^2 = \left(E_p + E_e\right)^2 - \left(p_p c - p_e c\right)^2 = E_p{}^2 - \left(p_p c\right)^2 + E_e{}^2 - \left(p_e c\right)^2 + 2\left(E_p E_e + p_p p_e c^2\right)$$

$$E_p{}^2 - \left(p_p c\right)^2 = \left(m_p c^2\right)^2; \; E_e{}^2 - \left(p_e c\right)^2 = \left(m_e c^2\right)^2;$$

$$E_{cm}{}^2 = m_p{}^2 c^4 + m_e{}^2 c^4 + 2 E_p E_e \left(1 + \beta_p \beta_e\right);$$

$$E_{cm} = \sqrt{m_p{}^2 c^4 + m_e{}^2 c^4 + 2 E_p E_e \left(1 + \beta_p \beta_e\right)}$$

# HW 1 problem 3 cont.

$$p_p^\mu = \{E_p / c, p_p, 0, 0\} \qquad p_e^\mu = \{E_e / c, -p_e, 0, 0\}$$

For ultra-relativistic case ($\gamma_p >> 1; \gamma_e >> 1$, $1 - \beta_p \beta_e << 1$) we can approximately write

$$E_{cm} \simeq 2\sqrt{E_p E_e}$$

(a) eRHIC with 20 GeV electrons and 250 GeV protons:
Exact $E_{cm}$ = 141.4242197 GeV, approximate 141.4213562 is accurate in 5 digits.

(b) LHeC with 60 GeV electrons with 7 TeV protons: $E_{cm}$ = 1.296 TeV GeV.
Difference between exact and proximate formulae 2.6E-7 is negligible.

**HW 1 (5 point):** Future Circular Collider (FCC, ) is under consideration by world physics community as a potentially next high energy collider.

(a) 1 point: The tunnel circumference would be 100 km https://en.wikipedia.org/wiki/Future_Circular_Collider. What average magnetic field is required to circulate 50 TeV proton beam?

The radius of curvature is defined by the particle momentum, charge and magnetic field (Lect 2, eq. (2.3)) and we defined an average "guiding" or "dipole" magnetic filed as defined by the ring circumference, C:

$$\rho = \frac{pc}{eB_y} = \frac{B\rho}{B_y} \to d\theta = \frac{ds}{\rho} = \frac{eB_y}{pc} ds \to$$

$$2\pi = \oint d\theta = \frac{e}{pc} \oint_C B_y \, ds = \frac{eC}{pc} \langle B_y \rangle$$

$$\langle B_y \rangle = \frac{2\pi \cdot pc}{eC} = \frac{2\pi \cdot B\rho}{C}$$

Now we can use eq. (2.4) and the fact that $pc = \sqrt{E^2 - m_p{}^2 c^4} \cong E$. Indeed, E=50 TeV and

$$E = 50{,}000\ GeV;\ m_p c^2 = 938.272046\ MeV \approx 0.938\ GeV$$

$$\gamma \cong 5.33 \cdot 10^4 \Rightarrow \frac{E - pc}{E} \equiv \frac{c - \mathrm{v}}{c} \equiv 1 - \beta \cong \frac{1}{2\gamma^2} = 1.76 \cdot 10^{-10}$$

Then

$$B\rho[T \cdot km] \cong \frac{pc[TeV]}{0.299792458} \cong 167\, T \cdot km$$

$$\langle B_y \rangle = \frac{2\pi \cdot B\rho}{C} = 10.48\, T$$

# HW 1 problem 1 cont.

(a) 1 point: It is also considered for electron-positron collider with beam energy up to 175 GeV. What average magnetic field is required to circulate 175 GeV electron or positron beam?

Similarly, electrons are ultra relativistic:

$$E = 175\ GeV;\ m_e c^2 = 0.511998910\ MeV \approx 0.511\ MeV$$

$$\gamma \cong 3.43 \cdot 10^5 \Rightarrow \frac{E - pc}{E} \equiv \frac{c - \mathrm{v}}{c} \equiv 1 - \beta \cong \frac{1}{2\gamma^2} = 4.26 \cdot 10^{-12}$$

$$B\rho\left[T \cdot km\right] \cong \frac{pc\left[TeV\right]}{0.299792458} \cong 0.584\,T \cdot km$$

$$\left\langle B_y \right\rangle = \frac{2\pi \cdot B\rho}{C} = 0.0367\,T = 367\,Gs$$

# HW 1 problem 1 cont.

(a) 2 points: Show that the same ring (set of magnets) can be used to circulate electrons and positrons with the same energy but moving in opposite (colliding) directions. Specifically, write equation of motion for an electron and a positron and show that they can travel by the same trajectory but in opposite directions

Consider equation of motion on the same trajectory $\vec{r}_o(t)$ set by magnetic field $\vec{B}(\vec{r})$. Than equation of motion is:

$$\frac{d\vec{p}}{dt} = \frac{e}{c}\left[\vec{\mathrm{v}} \times \vec{B}(\vec{r})\right]; \vec{\mathrm{v}}(t) \equiv \frac{d\vec{r}_o(t)}{dt}; \vec{p}(t) = \gamma m \vec{\mathrm{v}}(t)$$

Since energy is preserved in magnetic field (neglecting radiation!) than $\gamma = const$ and velocity is a function of the trajectory

$$\frac{d^2\vec{r}_o(t)}{dt^2} = \frac{e}{\gamma mc}\left[\vec{\mathrm{v}} \times \vec{B}(\vec{r})\right] = \frac{e}{\gamma mc}\left[\frac{d\vec{r}_o(t)}{dt} \times \vec{B}\left(\vec{r}_o(t)\right)\right]; \ \vec{\mathrm{v}}(t) \equiv \frac{d\vec{r}_o(t)}{dt} = \vec{\mathrm{v}}\left(\vec{r}_o(t)\right) \qquad \text{(I)}$$

Now let change signs of the particle charge $e \to -e$ and velocity $\vec{\mathrm{v}} \to -\vec{\mathrm{v}}$ to derive equation of motion for an antiparticle in the same m

$$\frac{d^2\vec{r}_{ap}}{dt^2} = \frac{-e}{\gamma mc}\left[-\vec{\mathrm{v}} \times \vec{B}\left(\vec{r}_{ap}\right)\right] = \frac{-e}{\gamma mc}\left[\frac{d\vec{r}_{ap}}{dt} \times \vec{B}\left(\vec{r}_{ap}\right)\right];$$

# HW 1 problem 1 cont.

Now we can set antiparticle on the reverse trajectory:

$$\vec{r}_{ap}(t) = \vec{r}_o(t_o - t)$$

where $t_o$ is an arbitrary constant and check that its identical to equation of the particle (I):

$$\frac{d^2\vec{r}_o(t_o-t)}{dt^2} == \frac{-e}{\gamma mc}\left[\frac{d\vec{r}_o(t_o-t)}{dt} \times \vec{B}\left(\vec{r}_o(t_o-t)\right)\right]; \boldsymbol{\tau} = t_o - t; dt = -d\boldsymbol{\tau}$$

$$\frac{d^2\vec{r}_o(\boldsymbol{\tau})}{d\boldsymbol{\tau}^2} == \frac{e}{\gamma mc}\left[\frac{d\vec{r}_o(\boldsymbol{\tau})}{d\boldsymbol{\tau}} \times \vec{B}\left(\vec{r}_o(\boldsymbol{\tau})\right)\right] \qquad \text{(II)}$$

In short, the changing signs of the particle charge and velocity does not change the value and the direction of the force and trajectory bends the same way.

(a) 1 point: Can be the same trick used to circulate and collide two proton beams?

No, the sign of the force changes for proton propagating in the opposite direction. It means that its trajectory will bend in opposite direction: dipole separator. Surprisingly, electric field would do the trick, but it is not useful for high energies: 10 T filed corresponds to electric field of 3,000 MV/m!

## Light sources

You are well aware that electrons when accelerated (rotated in the bending magnets or "shacked" in wigglers and undulators. They radiate incoherent radiation with critical wavelength $\lambda \sim \rho / \gamma^3$ and $\lambda \sim \lambda_u / \gamma^2$ from undulators. Most of popular storage ring light sources operate in X-ray or soft-X-ray range of photon energies, which result in energies from 3 to 8 GeV.

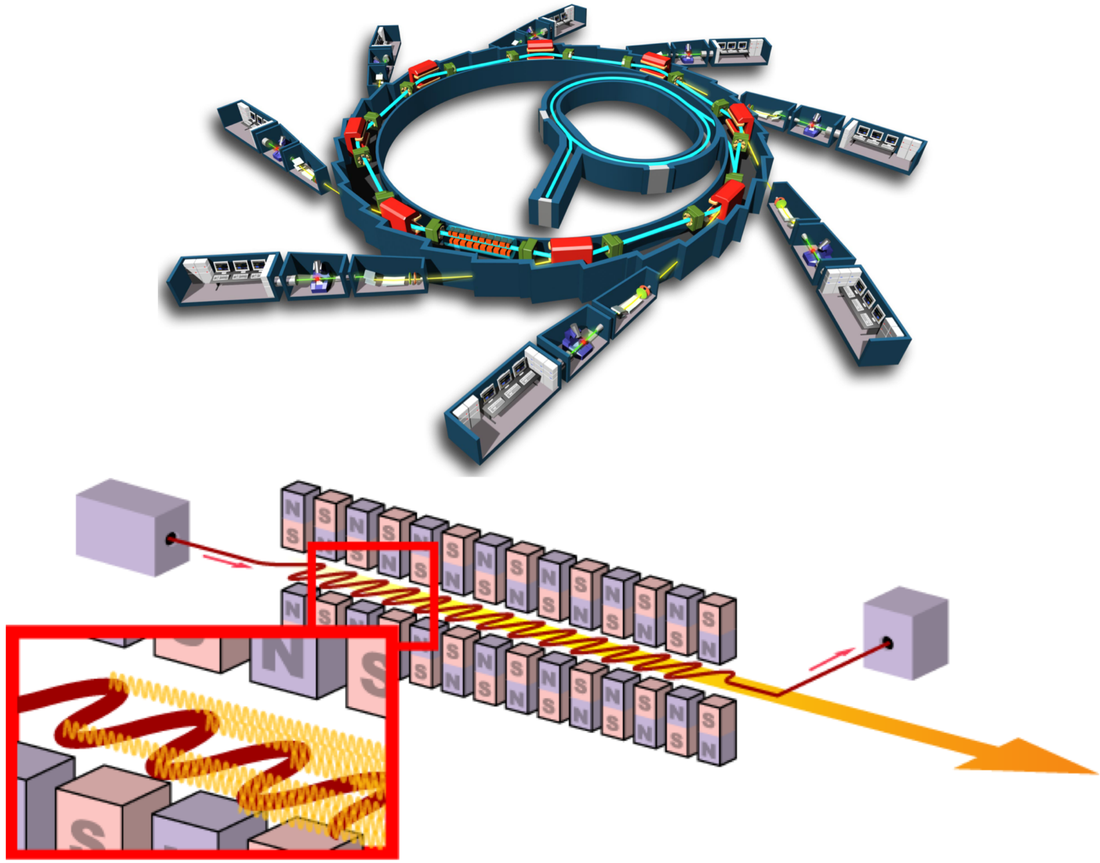


*Fig. 2.20. Typical layout of ring-based light source and an FEL*

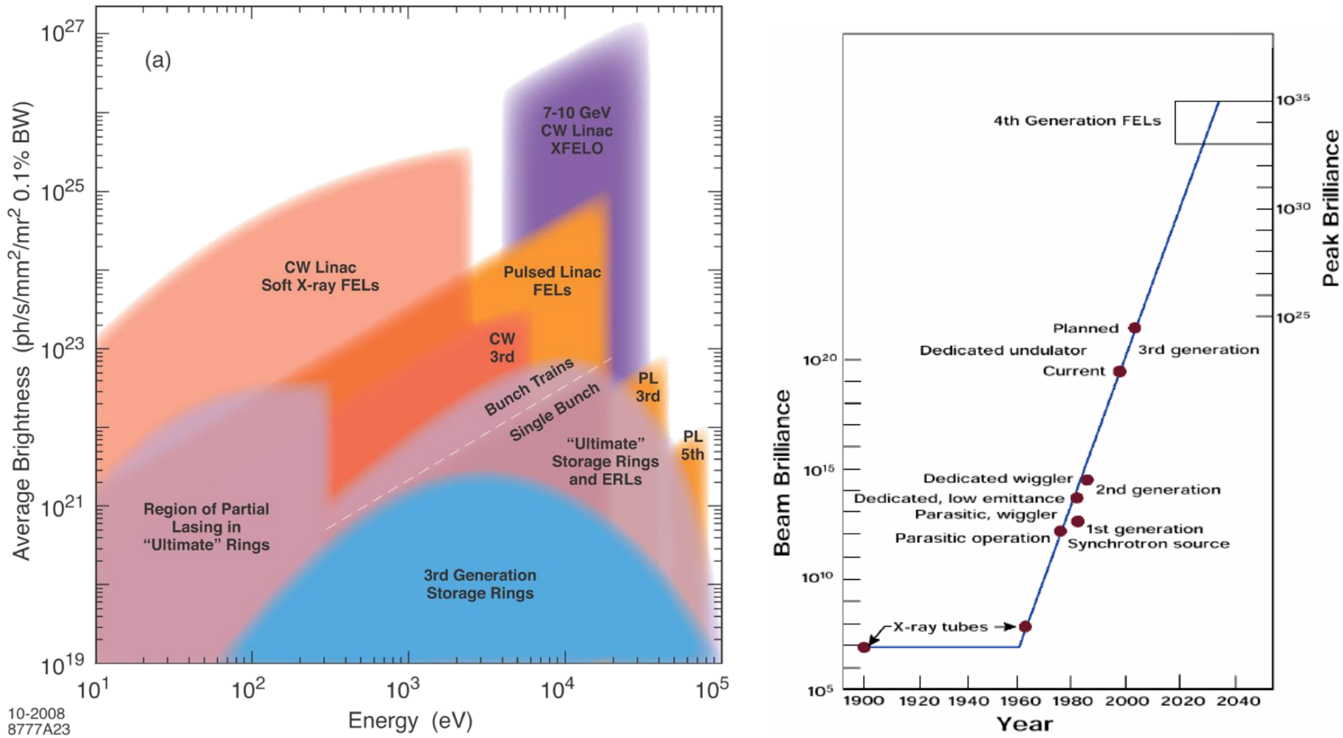



*Fig. 2.21. Average spectral brightness of light sources (courtesy of D.Robin) and equivalent of the Livingston plot for light sources*

$$B = \frac{\dot{N}_{ph}}{A \cdot \Omega} \cdot \frac{\delta E}{\sigma_E} \tag{2.21}$$

The quality of the generated photon beams is characterized by peak (or average) spectral brightness measured in number of photons per second radiated into a desirable energy spread from a unit areas into a unit spherical.

FELs are generating photon beams using instability of the system comprised of electron beam propagating in an undulator and TEM optical wave. Resulting X-ray beams have laser quality and X-ray FELs (currently operating only in pulsed mode) have peak spectral brightness exceeding that of othe light sources by about 10 orders of magnitude.

**References:**

[1] A BRIEF HISTORY AND REVIEW OF ACCELERATORS, P.J. Bryant, CERN, Geneva, Switzerland, http://cds.cern.ch/record/261062/files/p1.pdf

[2] J.D. Crockcroft and E.T.S. Walton, "Experiments with high velocity ions", Proc. Royal Soc., Series A 136 (1932), 619–30.

[3] R.J. Van de Graaff, "A 1,500,000 volt electrostatic generator", Phys. Rev., 387, (Nov.1931), 1919–20.

[4] http://cdn.preterhuman.net/texts/science_and_technology/physics/ , Principles of Charged Particle Accelerators/chap10.pdf

[5] www.werc.or.jp/english/reseadeve/activities/accelerator/accelerator/synchro/index.htm

[6] E.M. McMillan, "The synchrotron - a proposed high-energy particle accelerator", Phys. Rev., Letter to the editor, 68 (Sept. 1945), 1434.

[7] V. Veksler, J. of Phys, USSR, 9 (1945), 153–8.

[8] E.D. Courant and H.S. Snyder, Annals of Physics, 3, 1, 1958

[9] D.W. Kerst, "The acceleration of electrons by magnetic induction", Phys. Rev., 60 (July 1942), 47–53.

[10] M.S. Livingston, "Particle accelerators: A brief history", Harvard University Press,

Cambridge, Massachusetts (1969), 32.

[13] E.D. Courant, M.S. Livingston and H.S. Snyder, "The strong-focusing synchrotron – a new high-energy accelerator", Phys. Rev., 88 (Dec. 1952), 1190-6, and

[14] N.C. Christofilos, Unpublished report (1950), and
U.S. Patent no. 2.736,799, filed March 10, 1950, issued February 28, 1956.
Energy Accel., CERN, 1971 (CERN, Geneva, 1971), 501–4.

**HW 2 (2 points):** For a classical microtron having energy gain per pass of 1.022 MeV and operational RF frequency 3 GHz (3 x $10^9$ Hz) find required magnetic field (Hint: use k=1). What will be radius of first orbit in this microtron? 

Solution: Again, lets start from the radius of curvature

$$\rho = \frac{pc}{eB_y}$$

and calculate time of flight for a given energy

$$T = \frac{2\pi\rho}{\mathrm{v}} = \frac{2\pi}{eB_y}\cdot\frac{pc}{\mathrm{v}} = \frac{2\pi}{eB_y}\cdot\frac{E}{\mathrm{c}}$$

The energy at n-turn is equal to the rest energy electron energy 0.511 MeV plus n-fold energy gain:

$$E_n = mc^2 + n\cdot\Delta E; \quad \Delta E = 2mc^2;$$

$$E_n = (2n+1)mc^2 = (2n+1)\cdot 0.511\,MeV$$

Now we should use synchronization condition, e.g. that each turn should take an integer number of RF cycles

$$T_n = N(n)\cdot T_o;\ T_o = \frac{1}{f_{RF}};\ T_n = \frac{2\pi}{eB_y}\cdot\frac{E_n}{\mathrm{c}} = (2n+1)\frac{2\pi mc}{eB_y};$$

$$N(n) = k\cdot(2n+1);\ \frac{2\pi mc}{eB_y} = kT_o = \frac{k}{f_{RF}} \rightarrow B_y = \frac{1}{k}\frac{2\pi mc^2}{e(cT_o)}$$

where $k$ is a positive integer. Putting number together for $k=1$, we get

$$cT_o = \frac{c}{f_o} = 9.993\,cm \cong 10\,cm;$$

$$\frac{2\pi m_e c^2}{e} = 2\pi\frac{0.511...}{0.29979...} \cong 2\pi\cdot 1.705\ kGs\ cm = 10.71\ kGs\ cm$$

$$B_y \cong 1.071\ kGs$$