Homework 16. Due November 15

Problem 1. 15 points.

A point particle with charge Q moves with velocity $\vec{v}=(0,0,v_z)$ in the longitudinal direction. The particle has a horizontal offset x=a. Show that, in the cylindrical system, the charge distribution due to the particle can be written as:

$$\rho(r,\theta,z,t)=\sum_{m=0}^{\infty}\frac{Q_m\cos(m\theta)}{\pi a^{m+1}(1+\delta_{m,0})}\delta(z-v_z t)\delta(r-a)$$

with $Q_m=Qa^m$ and $\delta_{m,0}=\begin{cases}1 & \text{if } m=0\\ 0 & \text{otherwise}\end{cases}$

Hint 1. prove that the delta function in cylindrical coordinate is $\rho(x,y,z)=\frac{1}{r}\delta(r-a)\delta_p(\theta)\delta(z)$

with $\delta_p(\theta)=\sum_{k=-\infty}^{\infty}\delta(\theta-2\pi k)$ (Jackson page 120 footnote)

Hint 2. perform Fourier transformation to $\delta_p(\theta)$