PHY 554
Fundamentals of Accelerator Physics

Lecture 19: Collective effects part 1
Space Charge Effects

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Most (all) of materials come from USPAS/CERN school courses

https://uspas.fnal.gov/index.shtml

https://cas.web.cern.ch/





Today schedule

- Questions/recap from the last lecture (on synchrotron radiation)
- Collective effects (part1)
  - Direct Space charge
  - Indirect space charge
  - From indirect space charge to wall wakes
- Next time collective effects (part II)
  - Wake fields/ impedance

SR limits high energy lepton colliders

- For high-energy accelerators such as colliders SR was considered a waste product and a serious limitation

- LEP (CERN, 1990s) eventually reached 70 GeV, but at a circumference of 27 km ($\rho \approx 3$ km ➔ Bpeak = 0.078 T) and yet, every last free drift space was occupied by RF cavities in an attempt to replenish energy lost to SR

(for the **homework** estimate how much energy is lost per turn at LEP)

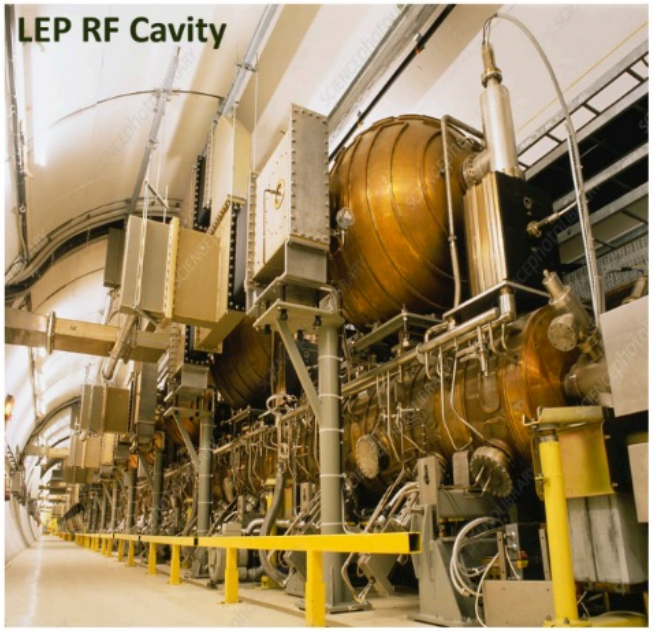
- LEP eventually replaced by LHC (hadron machine, SR suppressed by 10^{13})

Will use formula from the lecture for electrons:

$$\Delta E\,[\mathrm{keV}] \quad \approx \quad 88.5\,\frac{E\,[\mathrm{GeV}]^4}{\rho\,[\mathrm{m}]}$$

ΔE_{70GeV} = 88.5 *70^4/3000 =708295 keV = **708.3 MeV**

In year 2000 LEP at CERN* achieved energy of 100 GeV
ΔE_{100GeV} = 88.5 *100^4/3000 =2950 MeV = 2.95 GeV
The energy lost per turn was almost 3%. In order to, compensate these loss CERN used distributed RF in every location when it was possible



*) https://cds.cern.ch/record/2305213/files/fulltext.pdf

Collective effects

- Particle accelerators use external electromagnetic fields to guide and accelerate charged particles.

In a real machine, however, there is another source of electromagnetic fields, the beam itself, which, interacting with the accelerators' devices, produces additional **self-induced fields** which perturb the particle's motion leading to the so-called **collective effects.**

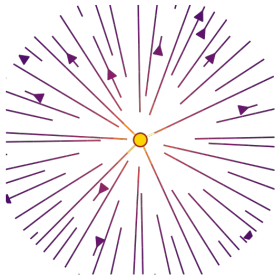
- The self-induced fields are commonly divided into **space charge fields**, generated directly by the charge distribution and including the image currents circulating on the walls of a smooth, perfectly conducting pipe, and the **wakefields**, produced by the finite conductivity of the walls, resonant devices, or any geometrical variation of the beam pipe.

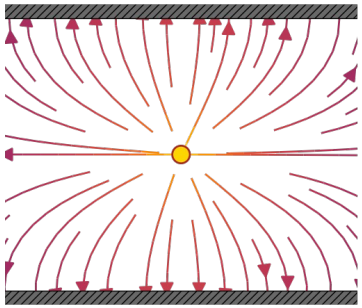
- Collective effects in particle accelerators are one of the key constituents for determining the ultimate particle accelerator performance. Their role is becoming increasingly important as particle accelerators are being pushed to ever higher intensity and beam brightness.

Space charge effect (direct vs indirect)

- A beam of charged particles induces electromagnetic fields when circulating inside the vacuum chamber.
- These self induced fields depend on:
  - beam current (intensity) and particle distribution (← **collective forces!)**
  - geometry and material of the surrounding vacuum chamber and machine elements

- Let's consider two aspects of the space charge effect separately





Direct space charge:
Interaction of charged particles in free space

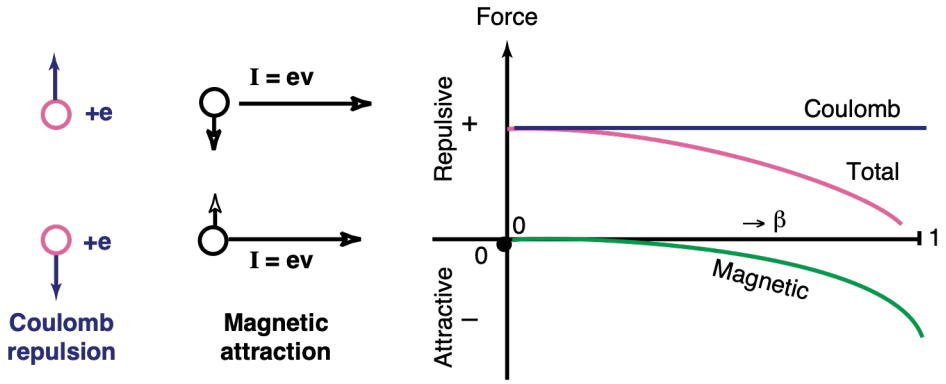
Indirect space charge:
Interaction with image charges and currents induced in perfect conducting walls and ferromagnetic materials close to the beam pipe

Space charge effects:

- cause **tune shifts** (transverse and longitudinal both incoherent (direct) and coherent (direct and indirect))
- can result in **longitudinal instability** (negative mass instability)

Two point charges with the same velocity

- Consider two point charges with the same charge q and with same velocity $v_1=v_2=v$ on parallel trajectories
  - In the rest frame, we know already the electric and magnetic fields generated by a “source” particle.
  - The force on the “test” particle is given by **the Lorentz force**.
- The attractive magnetic force tends to compensate the repulsive electric force
  - At rest the two particles experience only the repulsive Coulomb force
  - When travelling with velocity v the particles represent two parallel currents which attract each other by the induced magnetic field
  - The forces **become equal at the speed of light and thus cancel**



Using the correct Lorentz transforms one can compute the fields in the lab frame generated by a source particle with velocity v

$$E_r = \frac{e}{4\pi\epsilon_0}\frac{\gamma}{r^2} \qquad B_\phi = \frac{\beta E_r}{c}$$

→ Lorentz force acting on the test particle

$$F_r = e\left(E_r - v\,B_\phi\right) = e\left(E_r - \beta^2 E_r\right)$$

$$= \frac{eE_r}{\gamma^2} = \frac{e^2}{4\pi\epsilon_0\,\gamma r^2}$$

Coasting beam with uniform charge density

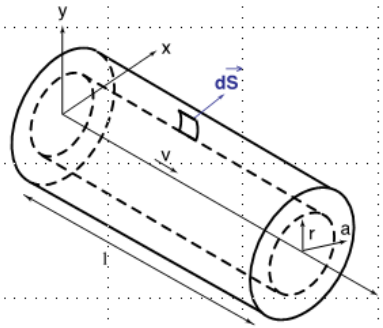
- Assume a coasting beam of circular cross section with radius a and uniform charge density $\rho = \lambda/(\pi\, a^2)\ [\mathrm{C/m^3}]$ moving at constant velocity $v = \beta c$

Maxwell's equation: $\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}$

Gauss' law

$$\iiint \vec{\nabla}\cdot\vec{E}\, dV = \iint \vec{E}\, d\vec{S}$$

$$\Rightarrow\ \pi r^2 l \frac{\rho}{\epsilon_0} = 2\pi r l E_r$$

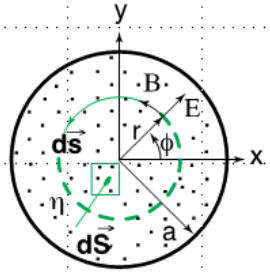


Electric field –

With $\lambda = \rho\,\pi a^2$ it follows:

$$E_r = \frac{\lambda}{2\pi\epsilon_0}\frac{r}{a^2}, \qquad r < a$$

Maxwell's equation: $\vec{\nabla}\times\vec{B} = \mu_0\vec{J}$



Stokes' law

$$\iint \vec{\nabla}\times\vec{B}\, d\vec{S} = \oint \vec{B}\, d\vec{s}$$

$$\Rightarrow\ \pi r^2 \mu_0 J = 2\pi r B_\phi$$

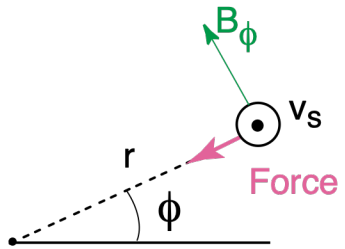
Magnetic field –

With $J = \beta c\rho = \beta c\frac{\lambda}{\pi a^2}$ and $\mu_0 = \frac{1}{\epsilon_0 c^2}$ it follows:

$$B_\phi = \frac{\lambda\beta}{2\pi\epsilon_0 c}\frac{r}{a^2}, \qquad r < a$$

Coasting beam with uniform charge density (cont.)

- Assume a coasting beam of circular cross section with radius a and uniform charge density $\rho = \lambda/(\pi\, a^2)\ [\mathrm{C/m^3}]$ moving at constant velocity $v = \beta c$



Electric and magnetic components have opposite signs and scale between each other with $\beta^2 \to$ there is perfect cancellation when $\beta = 1$

$$E_r = \frac{\lambda}{2\pi\epsilon_0}\frac{r}{a^2}, \quad r < a \qquad B_\phi = \frac{\lambda\beta}{2\pi\epsilon_0 c}\frac{r}{a^2}, \quad r < a$$

Lorenz force for the geometry studied

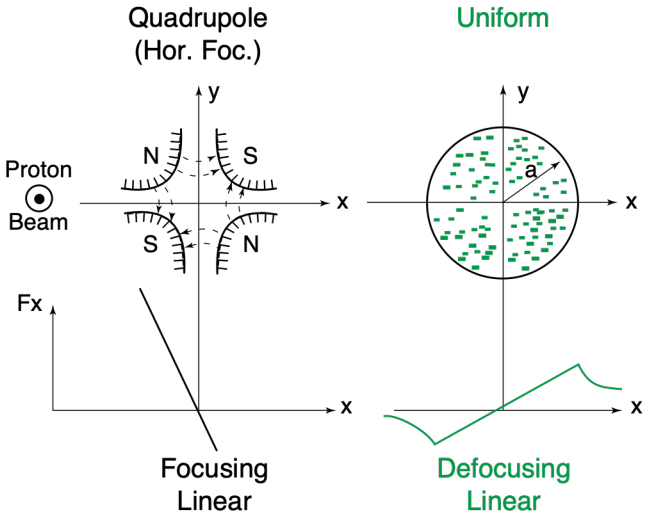
$$\begin{aligned} F_r &= e\,(E_r - v_s\, B_\phi) \\ &= \frac{e\lambda}{2\pi\epsilon_0}\left(1-\beta^2\right)\frac{r}{a^2} \\ &= \frac{e\lambda}{2\pi\epsilon_0}\frac{1}{\gamma^2}\frac{r}{a^2} \end{aligned}$$

The direct space charge force is linear in x and in y:

$$F_x = \frac{e\lambda}{2\pi\epsilon_0\gamma^2 a^2}\, x \qquad F_y = \frac{e\lambda}{2\pi\epsilon_0\gamma^2 a^2}\, y$$

Coasting beam with uniform charge density (cont.)

- Assume a coasting beam of circular cross section with radius a and uniform charge density $\rho = \lambda/(\pi\, a^2)\ [\mathrm{C/m^3}]$ moving at constant velocity $v = \beta c$
- In this case the direct space charge force is linear in x and y



$$F_x = \frac{e\lambda}{2\pi\epsilon_0\gamma^2 a^2}\, x \qquad F_y = \frac{e\lambda}{2\pi\epsilon_0\gamma^2 a^2}\, y$$

Direct space charge is **like a defocusing quadrupole**...

... however, direct space charge is always **defocusing in both planes**, while quadrupole is focusing in one and defocusing in the other plane

Homework #1:
Calculate space charge force for 2D gaussian transverse distribution

Space charge effects cause **tune shifts**

Direct space charge tune shift

- Since the uniformly charged coasting beam acts **like an additional quadrupole**, it will contribute to the normal transverse focusing with an **additional quadrupole focusing term**:

  - Hill's equation

$$y'' + \left(K_y(s) + K_y^{SC}(s)\right)\, y = 0$$

$$F_y = \frac{e\lambda}{2\pi\epsilon_0\gamma^2 a^2}\, y\,, \qquad r_0 = \frac{e^2}{4\pi\epsilon_0 m c^2}$$

  - Extra focusing term:

$$K_y^{SC}(s) = -\frac{1}{m\gamma\beta^2 c^2}\frac{F_y^{\mathrm{SC}}}{y} = -\frac{2r_0\lambda}{e\beta^2\gamma^3 a^2(s)}$$

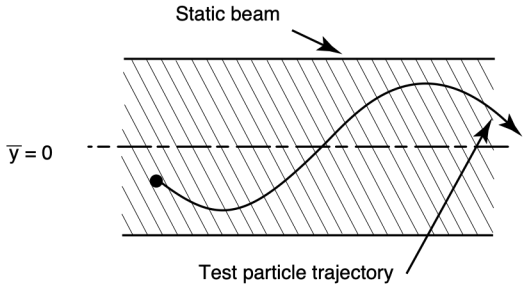
  - Tune shift:

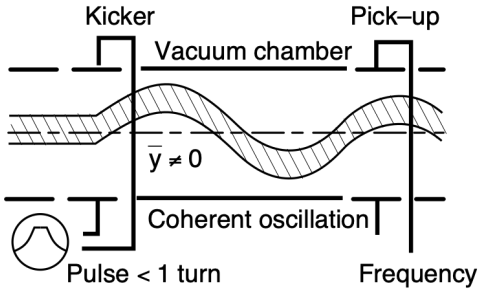
$$\Delta Q_y = \frac{1}{4\pi}\oint K_y^{SC}(s)\beta_y(s)\,ds = -\frac{1}{4\pi}\oint \frac{2r_0\lambda\beta_y(s)}{e\beta^2\gamma^3\, a^2(s)}\,ds = \underbrace{-\frac{r_0 R\lambda}{e\beta^2\gamma^3}\left\langle\frac{\beta_y(s)}{a^2(s)}\right\rangle}$$

Tune shift of a particle subject to the direct space charge fields of a uniform charge distribution

Space charge and incoherent vs. coherent motion

- Incoherent motion
  - The beam consists of many particles, each of which moves inside the beam with **its individual betatron amplitude, phase, and even tune** (under influence of direct space charge) – amplitude and phase **are distributed at random over all particles**
  - An outside observer (using a beam position monitor) does **not see any of this random betatron motion** – beam and source of the direct space-charge field do not move
- Coherent motion
  - A static beam given a transverse fast deflection starts to perform **betatron oscillations as a whole** – the source of direct space charge is now moving and individual particles continue their incoherent motion around **the common coherent trajectory**
  - In the accelerator environment, the **coherently oscillating beam** induces image charges/currents which are oscillating as well – this leads to a coherent tune shift

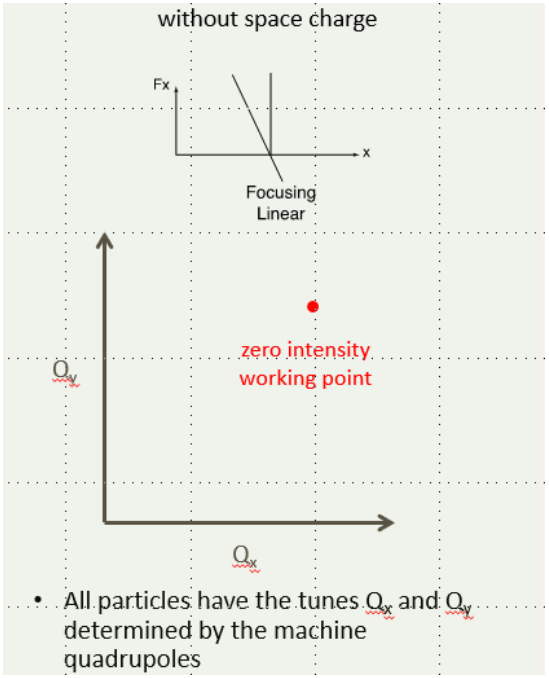


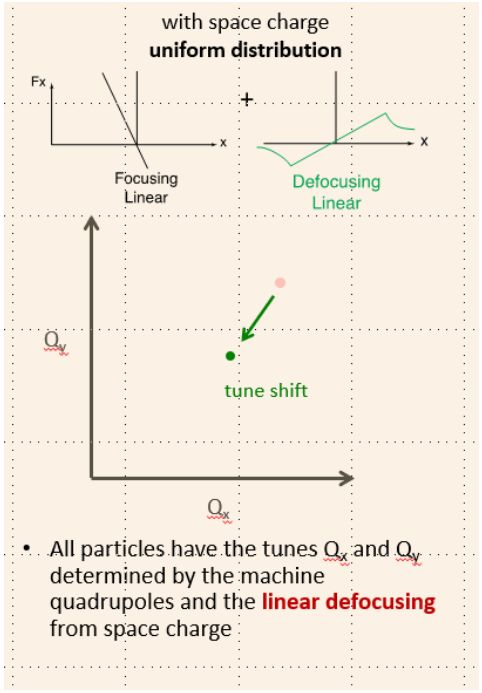


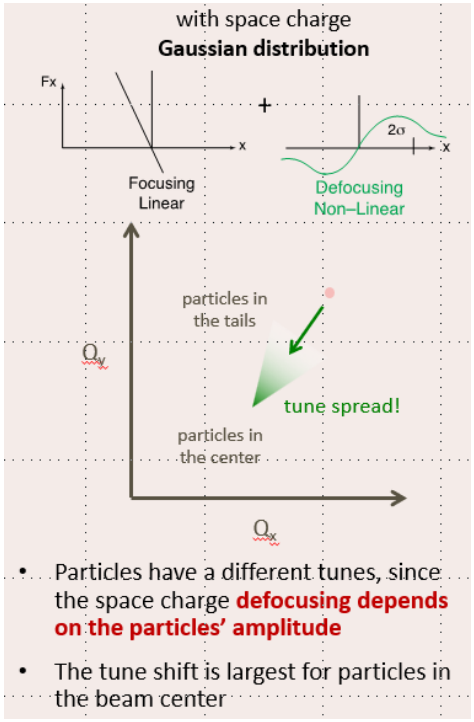
Direct space charge – impact on machine performance

We have seen the impact of direct space charge for the case of a uniform coasting beam. We also got a first idea of the scaling laws for direct space charge in general.

Let's have a look now at the more realistic case of non-uniformly charged, bunched beams and the **impact of direct space charge** on the beam and on the **machine performance**.





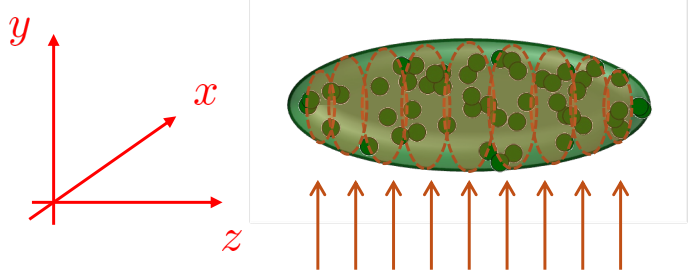


Coasting vs. bunched – Gaussian

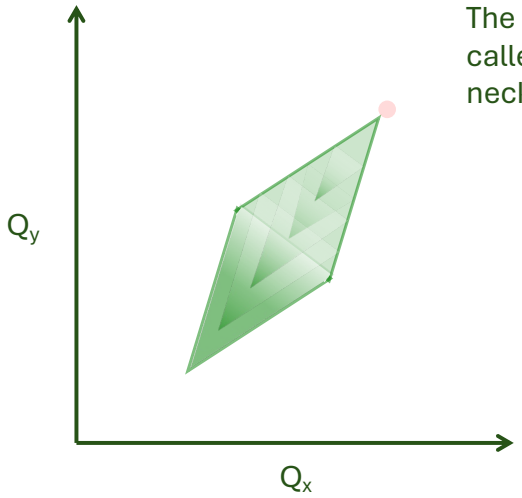
- In beams with **Gaussian transverse distribution** we observed already for coasting beams with constant line density a tune spread due to the nonlinear force and the resulting dependence on the transvers particle amplitude
- In case a Gaussian beam **is also bunched, an additional tune spread** is induced by the variation of the line density

- The longitudinal variation of the transverse space-charge force due to the line density fills the gap until the zero-intensity working point
- The tune of individual particles is modulated by twice the synchrotron period

The tune spread is reduced for particles further away from the peak line density



For bunched beams the tune spread is as big as the maximum tune shift!

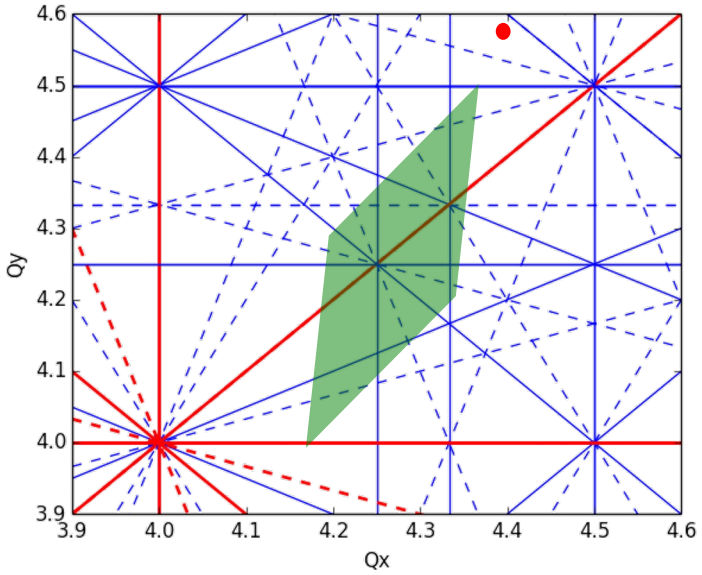


The tune footprint is also called the space charge necktie due to its shape

Brightness/luminosity limitation due to space charge

- A space charge tune spread beyond 0.5 can barely be tolerated without excessive **emittance blow-up and/or particle loss due to resonance crossing**
  - Dipole errors in the machine excite the integer resonances (Q=n)
  - Quadrupole errors excite the half integer resonances (Q=n+1/2)
  - Higher order resonances can be excited due to sextupoles and multipole errors

- Imagine that a beam with a tune spread of beyond 0.5 is injected
- Particles in the beam core will cross the integer resonance resulting in **emittance blow-up** and a reduction of the tune spread
- Particles in the beam tails can be pushed onto the half integer resonance resulting in **losses due to aperture restrictions**



Direct space charge tune shift

- After some reshuffling we notice some of the **fundamental properties of the direct space charge tune shift**:

$$\left.\begin{aligned} \Delta Q_{x,y} &= -\frac{r_0 R\lambda}{e\beta^2\gamma^3}\left\langle \frac{\beta_{x,y}(s)}{a^2(s)} \right\rangle \\ a(s) &= \sqrt{\frac{\beta_{x,y}(s)\hat{\varepsilon}^n_{x,y}}{\beta\gamma}} \end{aligned}\right\} \Longrightarrow \Delta Q_{x,y} = -\frac{r_0 R\lambda}{e\beta\gamma^2\hat{\varepsilon}^n_{x,y}}$$

$$r_0 = \frac{e^2}{4\pi\epsilon_0 mc^2} = \begin{cases} 1.54\cdot 10^{-18}\ \text{m (proton)} \\ 2.82\cdot 10^{-15}\ \text{m (electron)} \end{cases}$$

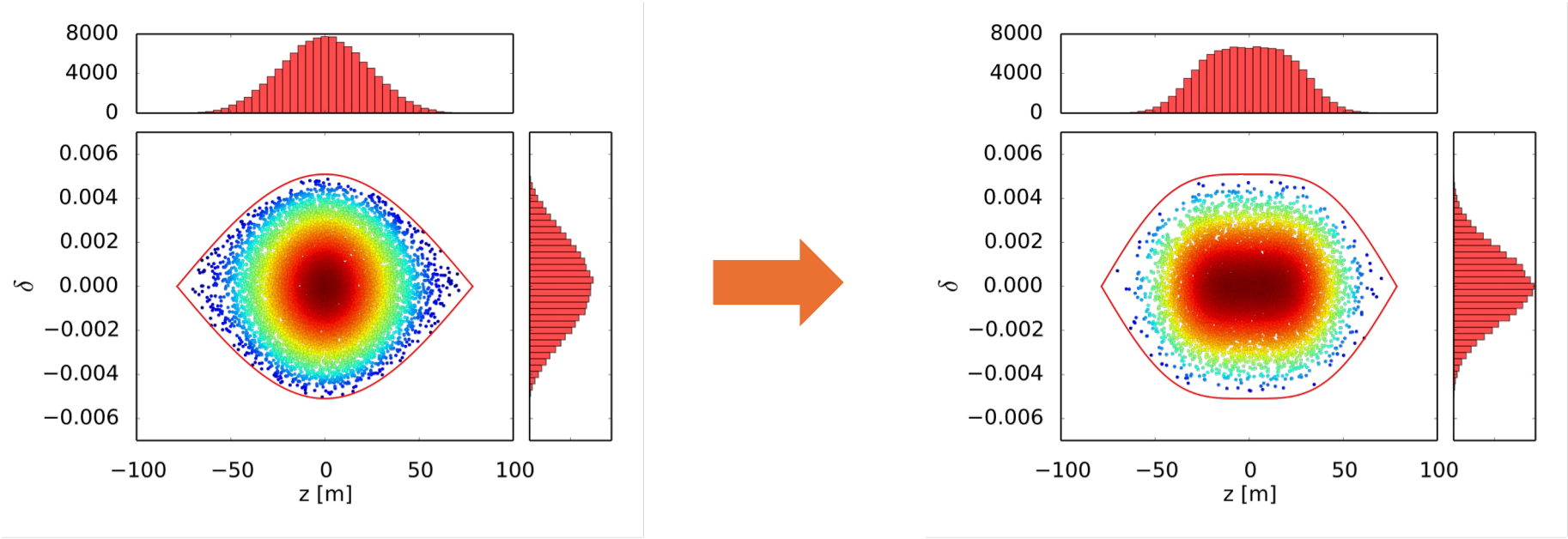
- is negative, because space charge transversely always defocuses
- is proportional **to the line density** and thus to the number of particles in the beam
- decreases **with energy like $1/(\boldsymbol{\beta\gamma^2})$** (when expressed in terms of normalized emittance) and therefore vanishes in the ultra-relativistic limit
- does not depend on the local beta functions or beam sizes but is inversely proportional to the normalized emittance (here the emittance includes all particles!)

Mitigation techniques

Maximum tune shift (circular Gaussian)

$$\Delta\hat{Q}_{x,y} = -\frac{r_0 C \hat{\lambda}}{2\pi e \beta \gamma^2} \frac{1}{2\,\varepsilon^n_{x,y}}$$

- Decrease the peak line density by
  - maximizing the bunch length
  - flattening the bunch profile with a specially configured (double harmonic) RF system



Mitigation techniques

Maximum tune shift (circular Gaussian)

$$\Delta\hat{Q}_{x,y} = -\frac{r_0 C\hat{\lambda}}{2\pi e\beta\gamma^2}\frac{1}{2\,\varepsilon^n_{x,y}}$$

- Decrease the peak line density by
  - **maximizing the bunch length**
  - flattening the bunch profile with a specially configured (double harmonic) RF system
  - using bunch distributions with small peak density (e.g. parabolic instead of Gaussian)
  - reducing the central density of the particle distribution (e.g. “hollow bunches”)

$$\Delta\hat{Q}_{x,y} = -\frac{r_0 C\hat{\lambda}}{2\pi e\beta\gamma^2}\frac{1}{2\,\varepsilon^n_{x,y}}$$

- Increase the beam energy by
  - accelerating the beam as quickly as possible
  - **increasing the injection energy** (usually requires an upgrade of the pre-injector)

$$\Delta\hat{Q}_{x,y} = -\frac{r_0 C\hat{\lambda}}{2\pi e\beta\gamma^2}\frac{1}{2\,\varepsilon^n_{x,y}}$$

- Minimize the machine circumference
  - when designing/building a new accelerator since the **space charge detuning is an integrated effect**

Low Energy Cooler for Electron Ion Collider: cooling beam at injection limited by space charge tune shift

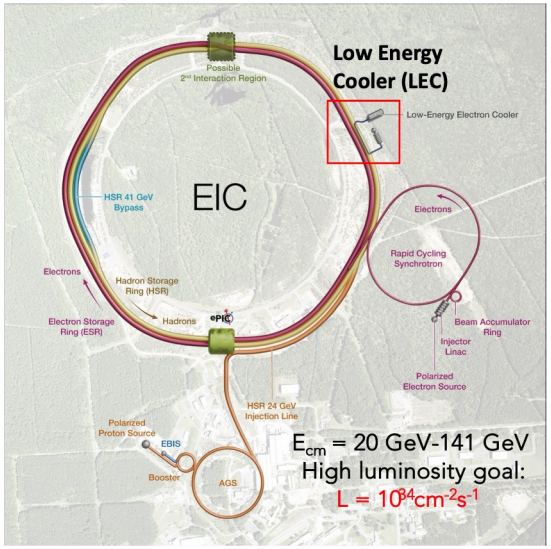
- Electron Ion collider at BNL is aiming to achieve luminosity

$L = 10^{34}\,cm^{-2}s^{-1}$

- High luminosity requires high intensity and low emittances beams

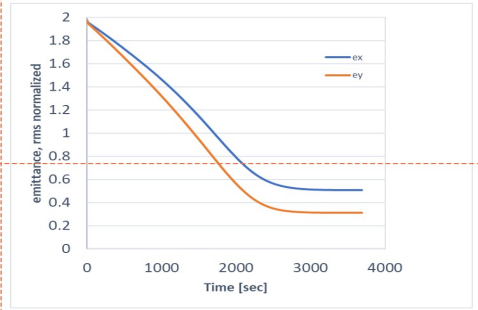
$$L = f_c \frac{N_1 N_2}{A} \cong f_c \frac{N_1 N_2}{2\pi\sqrt{\beta_{x1}\varepsilon_{x1} + \beta_{x2}\varepsilon_{x2}}\sqrt{\beta_{y1}\varepsilon_{y1} + \beta_{y2}\varepsilon_{y2}}}$$





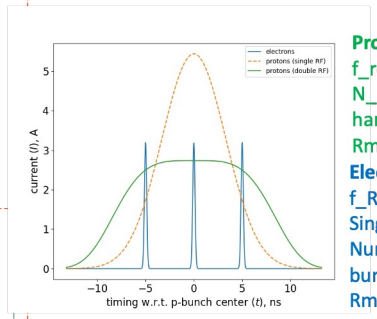
Cooling simulation

Average current of electrons: I_{av}=74mA
Electrons rms angles in the cooling section: **25 urad**
Effective cooling section length: **168 meters**



Cooling simulations of protons at γ=25, with decoupled transverse motion (**IBS+Cooling only,** using single harmonic RF).
Longitudinal emittance is kept constant during cooling process.
After cooling, normalized rms emittances of protons **ex, ey=0.5,0.3 um** (horizontal emittance can be increased further as needed.)

Beam Structure in the Cooling Section



Protons bunch structure:
f_rep=24.6 MHz
N_p=2.8e11, I_peak=2.7 A (with 2nd harmonic RF)
Rms length=1m
Electrons bunch structure:
f_RF=197 MHz
Single bunch: Q_e=1 nC
Number of bunches in macro-bunch: 3
Rms length=0.04m

2nd harmonic RF alleviates space-charge effects reducing peak current of protons to about 3A (space charge tune shifts after cooling $\Delta Q_{sc,x,y}$=0.06, 0.1), allowing vertical emittance to be cooled without introducing additional heating of horizontal emittance

Horizontal Tune Shift (ΔQ_x)

$$\Delta Q_x = -\frac{r_0 N R}{2\pi\beta\gamma^2\sqrt{\beta_x\varepsilon_{n,x}}\left(\sqrt{\beta_x\varepsilon_{n,x}} + \sqrt{\beta_y\varepsilon_{n,y}}\right)}$$

Vertical Tune Shift (ΔQ_y)

$$\Delta Q_y = -\frac{r_0 N R}{2\pi\beta\gamma^2\sqrt{\beta_y\varepsilon_{n,y}}\left(\sqrt{\beta_x\varepsilon_{n,x}} + \sqrt{\beta_y\varepsilon_{n,y}}\right)}$$

Part of the HW#3

Indirect space charge effect

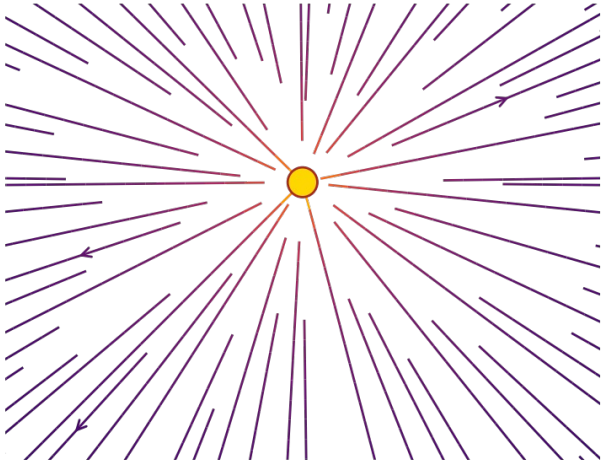
Direct space charge leads to a **purely incoherent tune shift** and a tune spread leading to the characteristic tune footprint with its **typical necktie shape** for bunched beams. Direct space charge does not lead to any coherent tune shifts (on the centroid motion).

We will now look at the effect of **indirect space charge**. We will briefly look at the **different sources** of indirect space charge, that it can lead to both **incoherent as well as coherent tuneshifts** and how these tune shifts are usually parameterized using the **Laslett coefficients**.

Direct vs. indirect space charge

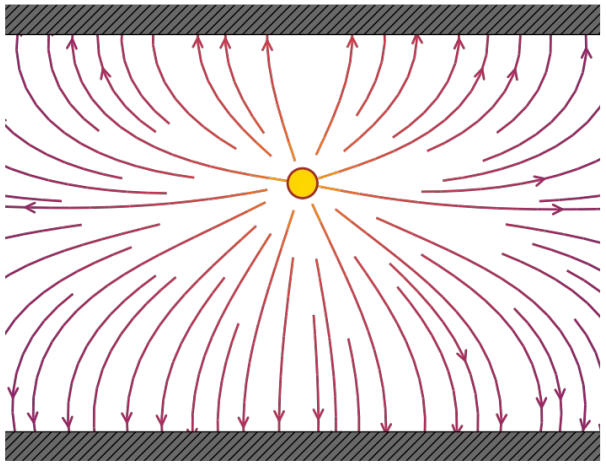
- Direct space charge – central force
- The space symmetry endowed by free space **eliminates any impact on the centroid** motion

→ **Exclusively incoherent tune shifts**

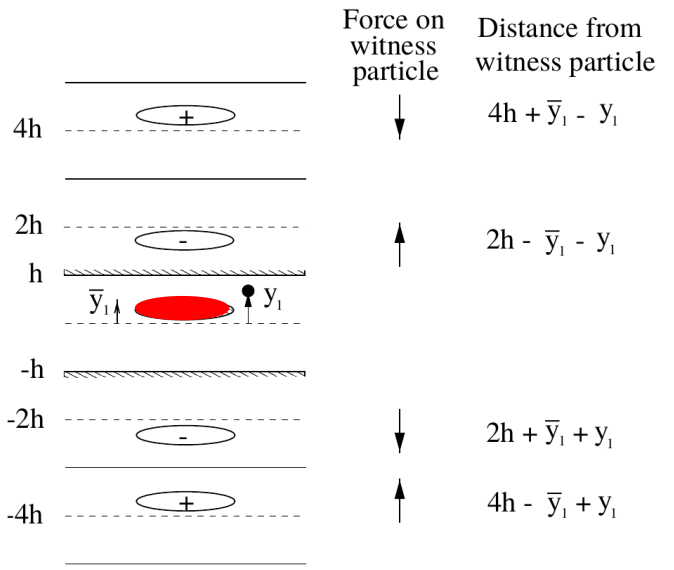


- Indirect space charge – image charge forces
- The free space symmetry is broken by the geometrical arrangement of the conducting parallel plates; this gives a **net impact on the centroid** motion

→ **Both incoherent and coherent tune shifts**



Direct vs. indirect space charge

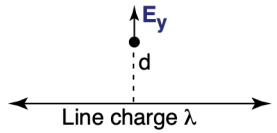


Boundary conditions $\bar{y}_1, y_1 \ll h\,, \quad E_{\|} = 0$

- A flat vacuum chamber can be approximated by two perfect conducting parallel plates placed at vertical positions +h and –h
- Let the **beam centroid be displaced vertically by $\bar{y}$**
- The **witness particle is at y**
- The **boundary condition** at the two perfect conducting plates is satisfied by superposing **an infinite number of image line charges** with alternating signs as shown in the sketch
- The resulting electric field as a **function of beam and witness particle offsets** can be computed as

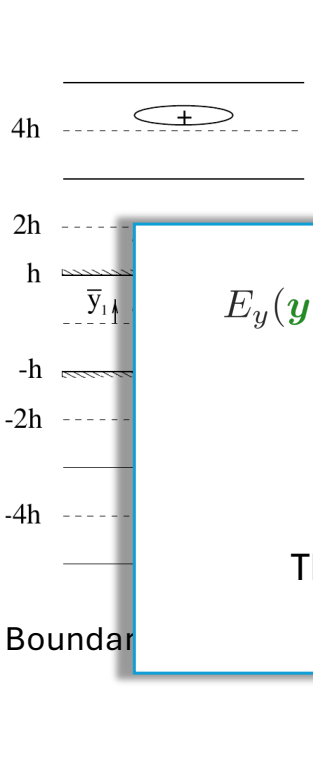
Electro- and magneto-static forces can be computed with the **method of image charges:**

- **We consider the beam as line charge density λ with infinite length**
- **We iteratively add charges to satisfy the boundary conditions. The resulting sum of fields converges to the final net field.**
- **The electric field at distance d is given by Gauss' law**



$$E_y = \frac{\lambda}{2\pi\epsilon_0}\frac{1}{d}$$

Direct vs. indirect space charge



- A flat vacuum chamber can be approximated by two perfect conducting parallel plates placed at vertical positions +h and –h
- Let the **beam centroid be displaced vertically by ȳ**

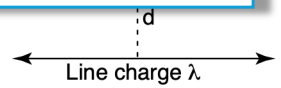
Electro- and magneto-static forces can be computed with the **method of image charges:**

- **We consider the beam as line** … **infinite**

… **arges to** … **oundary** … **esulting** … **erges to** … **distance** … **w**

$$E_y(\boldsymbol{y},\bar{\boldsymbol{y}}) = \frac{\lambda}{2\pi\epsilon_0}\Big[+\frac{1}{2h-\bar{\boldsymbol{y}}-\boldsymbol{y}}-\frac{1}{2h+\bar{\boldsymbol{y}}+\boldsymbol{y}}+\frac{1}{6h-\bar{\boldsymbol{y}}-\boldsymbol{y}}-\frac{1}{6h+\bar{\boldsymbol{y}}+\boldsymbol{y}}+\cdots$$
$$-\frac{1}{4h+\bar{\boldsymbol{y}}-\boldsymbol{y}}+\frac{1}{4h-\bar{\boldsymbol{y}}+\boldsymbol{y}}-\frac{1}{8h-\bar{\boldsymbol{y}}-\boldsymbol{y}}+\frac{1}{8h+\bar{\boldsymbol{y}}+\boldsymbol{y}}+\cdots\Big]$$

This is an infinite sum which, however, actually converges…



$$E_y(\boldsymbol{y},\bar{\boldsymbol{y}}) = \frac{\lambda}{\pi\epsilon_0 h^2}\left[(\bar{\boldsymbol{y}}+\boldsymbol{y})\,\frac{\pi^2}{32}+(\bar{\boldsymbol{y}}-\boldsymbol{y})\,\frac{\pi^2}{96}\right] = \frac{F_y}{e}$$

$$E_y = \frac{\lambda}{2\pi\epsilon_0}\frac{1}{d}$$

Image charge forces

- Static **electric fields vanish** inside a conductor for any finite conductivity, while **static magnetic fields pass through** (unless in case of very high permeability)

- This is **no longer true for time varying fields** – electromagnetic fields will **penetrate the material according to the skin depth δ_w** which depends on the frequency and the material properties

- We distinguish between **magnetic dc fields** (low frequency where $\delta_w >> \Delta_w$) and **magnetic ac fields** (high frequency where $\delta_w << \Delta_w$)

- We have seen how **electric image charge force** are induced by electrostatic fields in the presence of (perfectly) conducting boundaries

- Similarly, there are also **magnetics image current forces** – here we need to **differentiate between AC and DC forces**

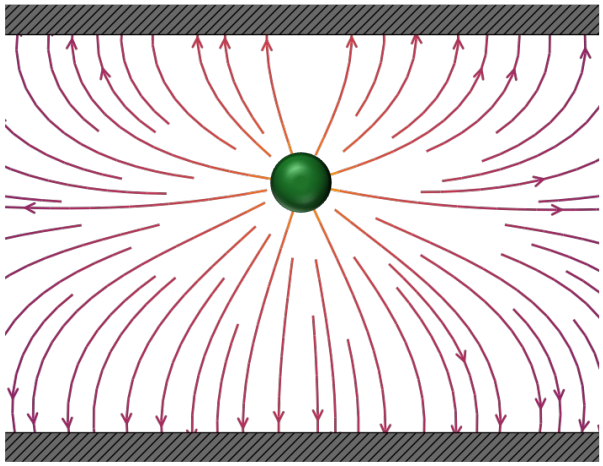
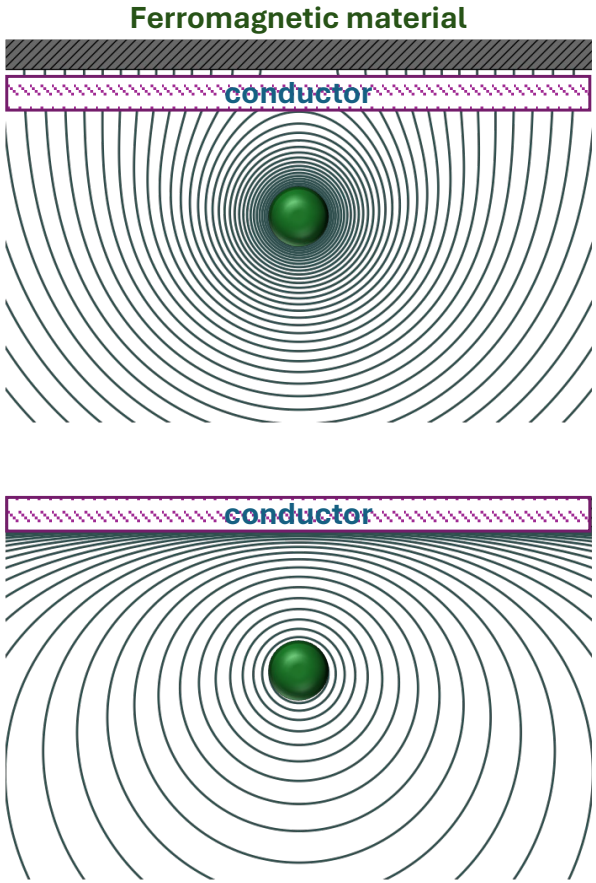


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At **low frequencies magnetic fields penetrate** and pass through the vacuum chamber, they can interact with bodies behind the chamber

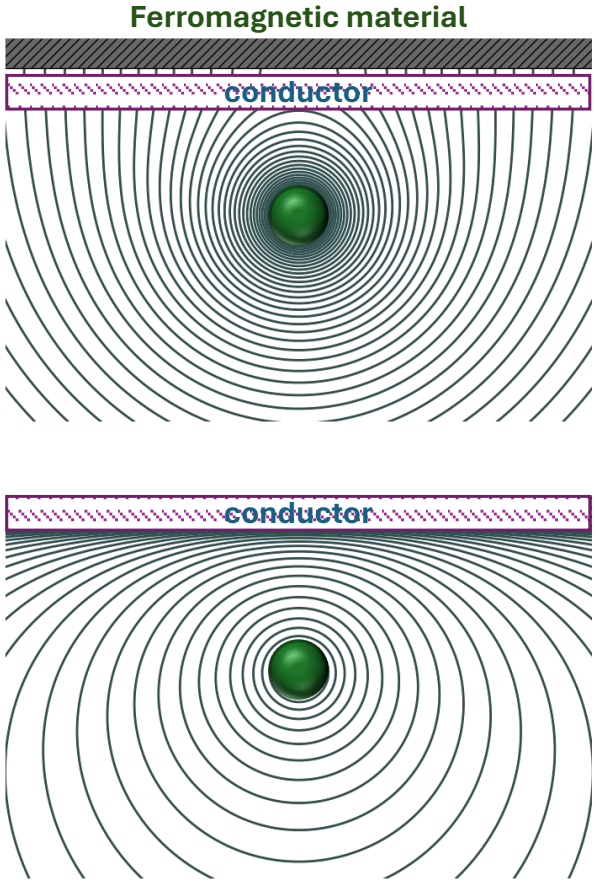
If the skin depth is very small (**rapidly varying fields**), **magnetic fields do not penetrate** and the field lines are tangent to the surface.

Image current forces

- In a very similar fashion as done for the electric case we can also here deploy the **method of image currents** to solve for the magnetic fields and forces
- We obtain forces of the form

$$\frac{F_y}{e} = -\frac{\lambda\beta^2}{\pi\epsilon_0 h^2}\left[(\bar{y}+y)\,\frac{\pi^2}{32} + (\bar{y}-y)\,\frac{\pi^2}{96}\right]$$

$$\frac{F_y}{e} = +\frac{\lambda\beta^2}{\pi\epsilon_0 h^2}\left[(\bar{y}+y)\,\frac{\pi^2}{32} - (\bar{y}-y)\,\frac{\pi^2}{96}\right]$$



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Tune shifts and Laslett coefficients

$$F_x \propto \frac{e\lambda}{\pi\epsilon_0 h^2} f(\bar{x}, x) \qquad F_y \propto \frac{e\lambda}{\pi\epsilon_0 h^2} f(\bar{y}, y)$$

- These electric image charge and magnetic ac and dc image current forces have the general similar form

$$\Delta Q_{x,y}^{\mathrm{inc}} = -\frac{2\langle\beta_{x,y}\rangle r_0 R}{e\beta^2\gamma} \frac{\varepsilon_1^{x,y}}{h^2} \qquad \Delta Q_{x,y}^{\mathrm{coh}} = -\frac{2\langle\beta_{x,y}\rangle r_0 R}{e\beta^2\gamma} \frac{\xi_1^{x,y}}{h^2}$$

- These forces can lead to **both incoherent as well as coherent tune shifts** – from the forces, the tune shifts can be computed. In fact, it turns out that the **tune shifts can be parameterized** via the **Laslett coefficients**. For example, for electric image charge forces between two perfectly conducting parallel plates, the incoherent and coherent tune shifts can be expressed as:

The Laslett coefficients can be evaluated **for different geometries** and are classified in **incoherent and coherent tune shifts for electric, magnetic ac and magnetic dc** image charges and currents.

Tune shifts and Laslett coefficients

Laslett coefficients	Circular $(w = h)$	Elliptical (e.g. $w = 2h$)	Parallel plates $(h/w = 0)$
ε_1^x	0	-0.172	$-\pi^2/48$
ε_1^y	0	$+0.172$	$+\pi^2/48$
ξ_1^x	$+1/2$	0.083	0
ξ_1^y	$+1/2$	0.55	$+\pi^2/16$
ε_2^x	$-\pi^2/24$	$-\pi^2/24$	$-\pi^2/24$
ε_2^y	$+\pi^2/24$	$+\pi^2/24$	$+\pi^2/24$
ξ_2^x	0	0	0
ξ_2^y	$+\pi^2/16$	$+\pi^2/16$	$+\pi^2/16$

Assuming parallel plates for the ferro-magnetic boundary for all geometries ...

* L. J. Laslett, LBL Document PUB-616, 1987, vol III

Note: they are always defined with respect to the *vertical half gap h or g*

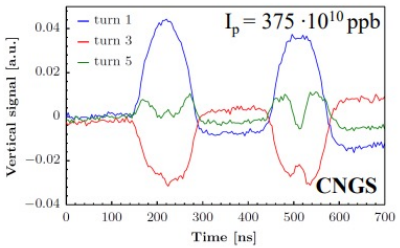
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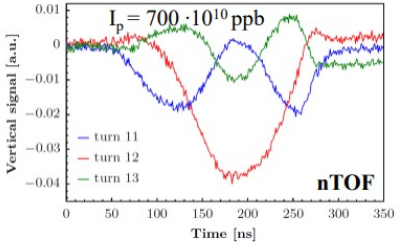
Example: PS injection oscillations

The observed intra-bunch motion was reproduced with an amazing precision with multi-particle simulations (HEADTAIL code) **including the indirect space charge effect** taking

Vertical intra-bunch oscillations

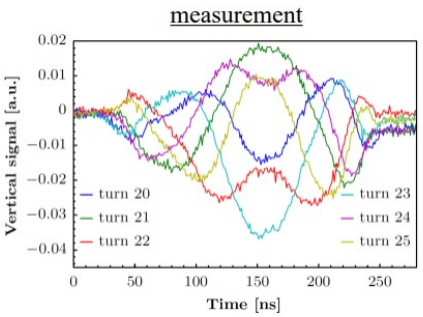
- wide-band pick-up used to observe the beam → signal proportional to displacement and line density
- in a linear machine, with a tune set to $q = 0.25$, one would expect the following signal
- however, for high-intensity beams a fast build up of **intra-bunch oscillations** (few turns after injection) is observed

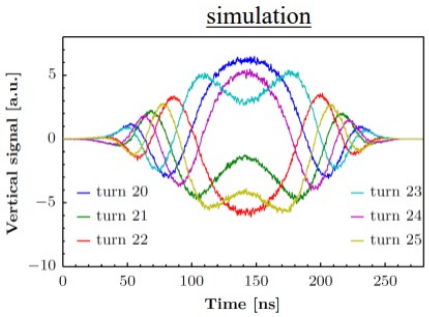




Simulation studies with HEADTAIL

- very good agreement between measurements and simulations obtained
- wake field based on **simple elliptic geometry** is sufficient to reproduce the measurements
- reduce this oscillations by reducing the injection error or using the transverse feedback



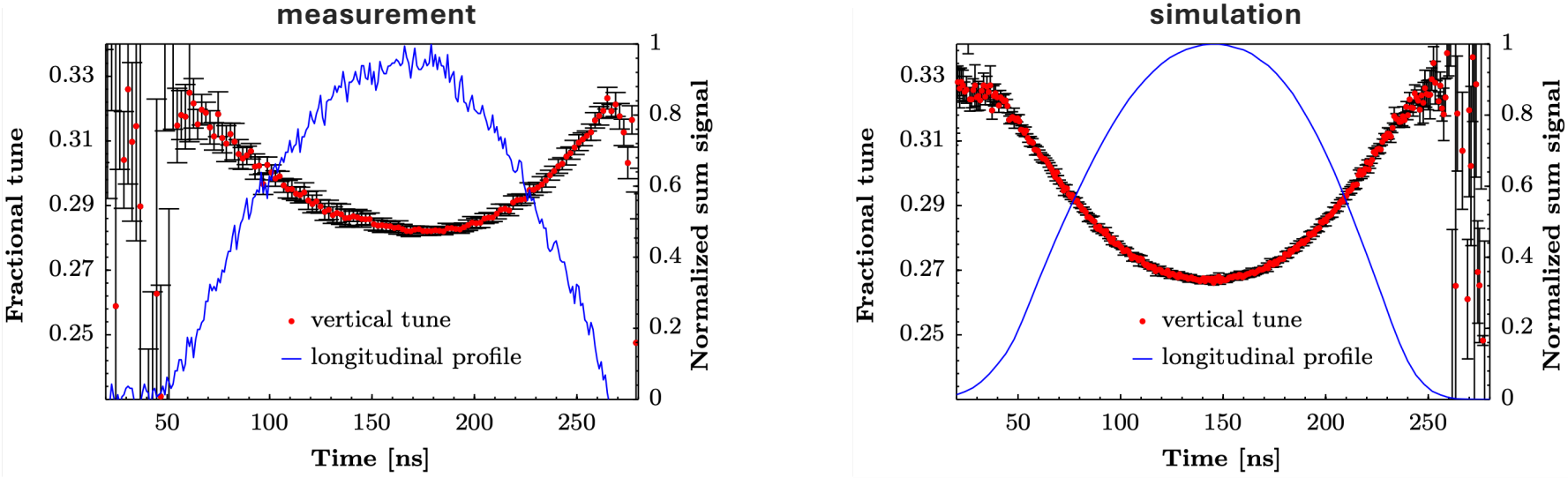


A. Huschauer et. al

https://indico.cern.ch/event/385807/contributions/919273/attachments/770052/1056207/LIU_PS_student_day.pdf

Example: PS injection oscillations

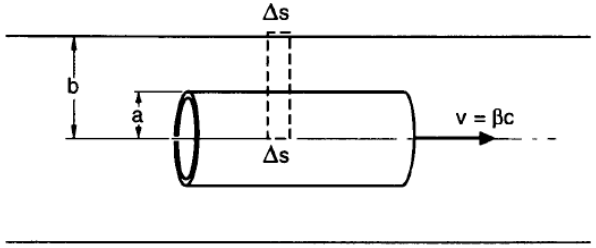
- The observed intra-bunch motion was reproduced with an amazing precision with multi-particle simulations (HEADTAIL code) **including the indirect space charge effect** taking
- It was understood from simulations that the observed intra-bunch motion was induced by the beam injected **off-center in combination with the indirect space charge effect**, which causes a tune shift along the bunch proportional to the local charge density



A. Huschauer et. al

Longitudinal space charge

- Longitudinal (direct) space charge can be also computed analytically (via Maxwell's equations) for cylindrically symmetric bunches of a given transverse density profile n(r):



$$E_s(r, s-\beta ct) = -\frac{1}{2\pi\epsilon_0\gamma^2}\,\lambda'(s-\beta ct)\left[\log\left(\frac{b}{r}\right) + \int_r^b 2\pi r' n(r')\log\left(\frac{r}{r'}\right)\,dr'\right]$$

- Note about this formula
  - For r=b the longitudinal electrical field vanishes as it should (perfect conducting pipe)
  - It diverges for b → ∞ because the analysis breaks down if the bunch length ≤ b/γ

Longitudinal space charge

It has the following interesting dependencies:

- It decreases with energy like 1/γ2 and **vanishes in the ultra-relativistic limit**
- It is proportional to the **opposite of the derivative of the line density –λ'**. This can be understood intuitively because it must be directed from a region with higher charge density to a region with lower charge density (i.e. it pushes with the opposite of the gradient of the line charge)

$$E_s(r, s-\beta ct) = -\frac{1}{2\pi\epsilon_0\gamma^2}\,\lambda'(s-\beta ct)\left[\log\left(\frac{b}{r}\right) + \int_r^b 2\pi r' n(r')\log\left(\frac{r}{r'}\right)\,dr'\right]$$

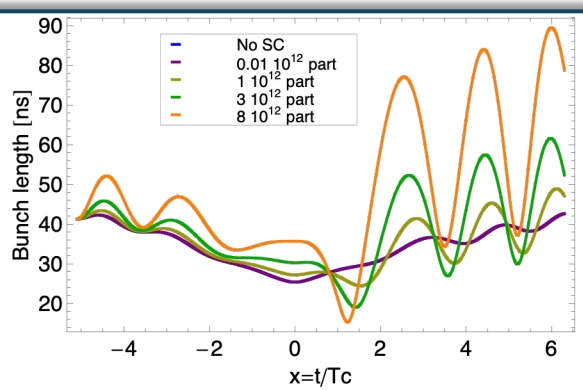
- Space charge would then spread out charge bumps. However, remember that only below transition energy, accelerated particles go faster and space charge has this smoothing action. Above transition, accelerated particles take a longer time to go around the accelerator and density peaks can be enhanced. This is the origin of the so-called **negative mass instability**. Momentum spread (unbunched beams) or synchrotron motion (bunched beams) can usually stabilize this effect.

Longitudinal space charge: synchrotron tune shift

$$\Delta Q_s \approx \frac{3\, e^2 g N_b \eta R^2}{8\pi\epsilon_0 \beta^2 \gamma^2 E_0 Q_{s_0} \hat{z}^3}$$

... example of synchrotron tune shift due to space charge in the radial center of a transverse parabolic bunch

- Similarly to the transverse plane, longitudinal space charge also leads to **a synchrotron tune shift** which is proportional to number of particles in the bunch N_b (linear)



The longitudinal space charge force **changes sign when crossing transition**:

- Quadrupolar oscillations are excited due to the sudden mismatch of the beam distribution at transition
- This effect gets stronger with increasing space charge force
- To avoid the mismatch, a jump of the RF voltage needs to be programmed similar to the phase jump

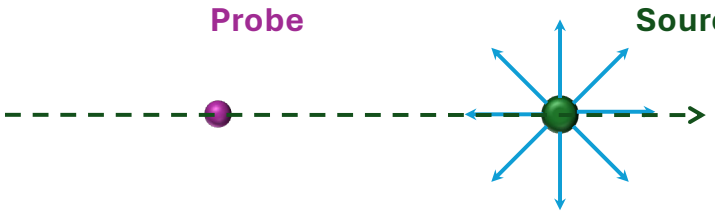
Remarks

- Direct space charge
  - interaction of the bunch particles with the self induced electro-magnetic fields in free space
  - results in an incoherent tune shift (or spread)
  - the space charge force along a bunch is modulated with the local line density along the bunch and this results in an additional tune spread
  - decreases with energy like $\beta^{-1}\gamma^{-2}$
  - is a typical performance limitation for low energy machines
- Indirect space charge
  - interaction with image charges and currents induced in perfect conducting walls and ferromagnetic materials close to the beam pipe
  - results in incoherent and coherent tune shifts (or spreads), some of which are proportional to the average line density and others to the local line density
  - the contributions to the coherent and incoherent tune shifts for different standard geometries are expressed in terms of Laslett coefficients
  - decreases with energy like $\beta^{-2}\gamma^{-1}$

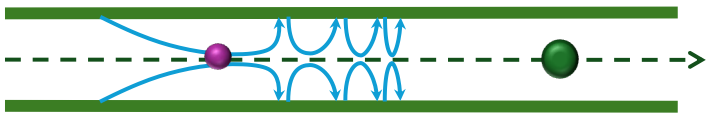
So far, we have introduced **direct and indirect space charge as collective effects**. The corresponding forces were not externally given but **dependent on the actual particle distribution** within the beam (remember, we looked at single particles as well as uniform and Gaussian distributions). The forces led to **incoherent and coherent tune shifts**.

We will now go a step further and investigate more complicated structures. We will try to find a smart way to deal with these structures. In the course of this, we will generalize and extend the direct and indirect space charge effects towards the **concept of wake fields and impedances**.

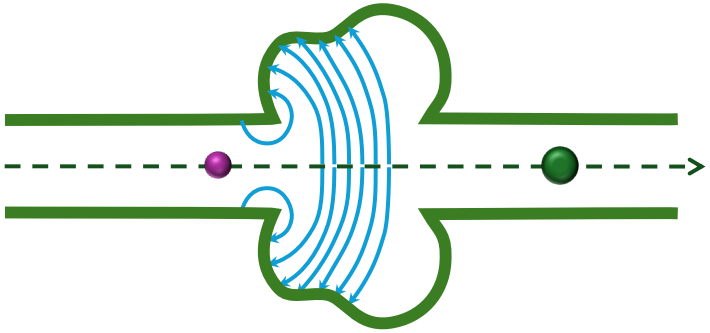
Electromagnetic fields of different sources



- Direct space charge
  - **Free space:** probe particles are effected directly by the source particles via the Lorentz force.



- Indirect space charge/**resistive wall wake**
  - **Smooth boundaries:** probe particles are effected by the source particles' induced image charges and currents.



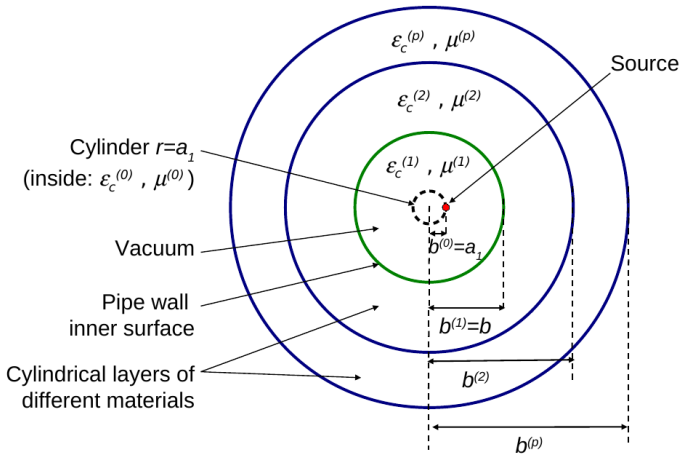
- Resonator wake fields
  - Discontinuities in boundaries: fields are excited by the source particles' distribution and can keep ringing, thus effecting trailing probe particles

From (in-)direct space charge to resistive walls…

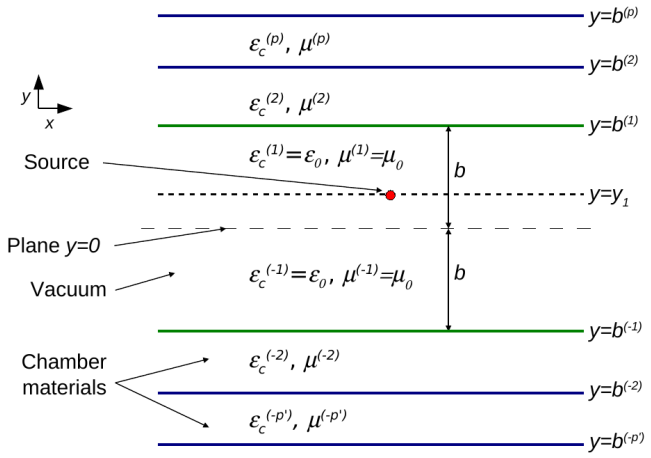
From (in-)direct space charge to resistive wall

- We consider a smooth multilayer structure, **longitudinally translation invariant and transversely bounded**.
- We consider a charged point particle traveling through the smooth multilayered structure.
- The **induced electromagnetic fields can be computed** for certain geometries by means of Maxwell's equations (longitudinal and transverse electromagnetic fields) with **field matching at the boundaries**. Two examples of such geometries are shown below.

Cylindrical



Flat



From (in-)direct space charge to resistive wall

- We consider a smooth multilayer structure, **longitudinally translation invariant and transversely bounded**.
- We consider a charged point particle traveling through the smooth multilayered structure.
- The **induced electromagnetic fields can be computed** for certain geometries by means of Maxwell's equations (longitudinal and transverse electromagnetic fields) with **field matching at the boundaries**. Two examples of such geometries are shown below.

- It turns out, that the **resulting electromagnetic fields** can be decomposed into 3 components:

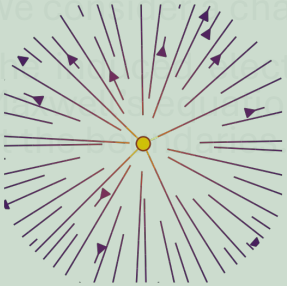
$$\vec{K}_{\text{Total}} = \vec{K}_{\text{direct}} + \vec{K}_{\text{boundaries}}$$

$$= \vec{K}_{\text{direct}} + \vec{K}_{\text{indirect}} + \vec{K}_{\text{resistive wall}}$$

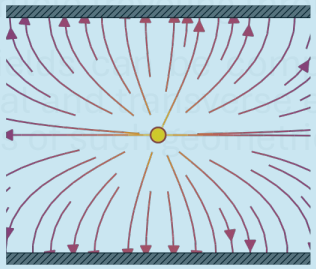
$\sigma \to \infty$ $\beta = 1$

From (in-)direct space charge to resistive wall

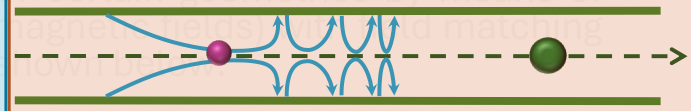
- A component which is **independent of the surrounding boundaries → direct space charge**



- A component which is independent of the surrounding material properties and purely **depended on the surrounding geometry → indirect space charge**



- A component which is **dependent on the surrounding material's electromagnetic properties → resistive wall**



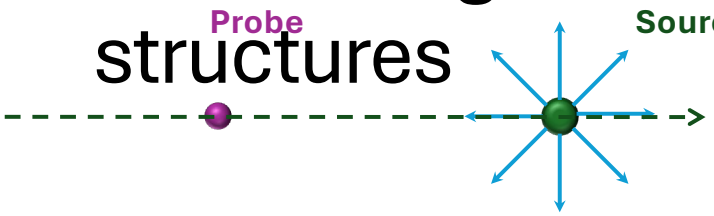
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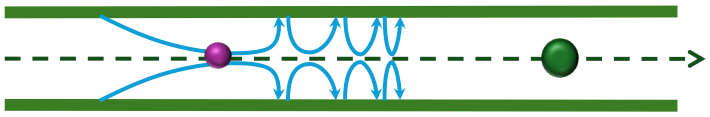
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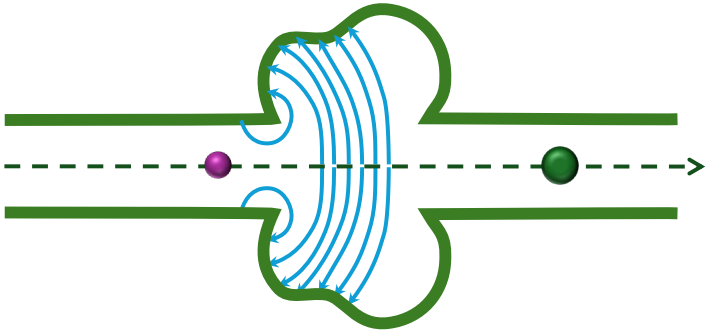
Electromagnetic fields in different types of structures



- Direct space charge
  - **Free space:** probe particles are effected directly by the source particles via the Lorentz force.

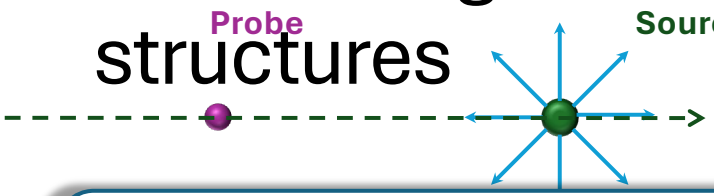


- Indirect space charge/**resistive wall wake**
  - **Smooth boundaries:** probe particles are effected by the source particles' induced image charges and currents.



- Resonator wake fields
  - **Discontinuities in boundaries:** fields are excited by the source particles' distribution and can keep ringing, thus effecting trailing probe particles

Electromagnetic fields in different types of structures

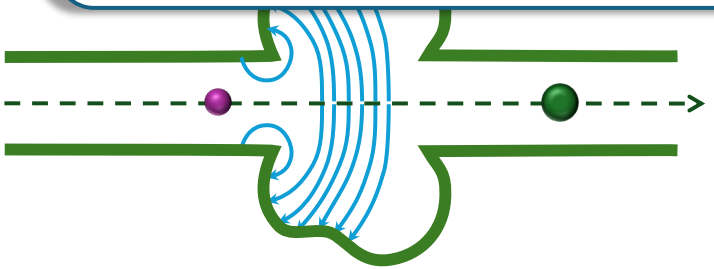


- Direct space charge
  - **Free space:** probe particles are effected directly by the source particles via the Lorentz force.

For more complicated geometries the (semi-) analytic methods no longer work. One must rely on **computational methods** to evaluate the induced electromagnetic fields (FDTD, FEM etc.).

Some examples:

…

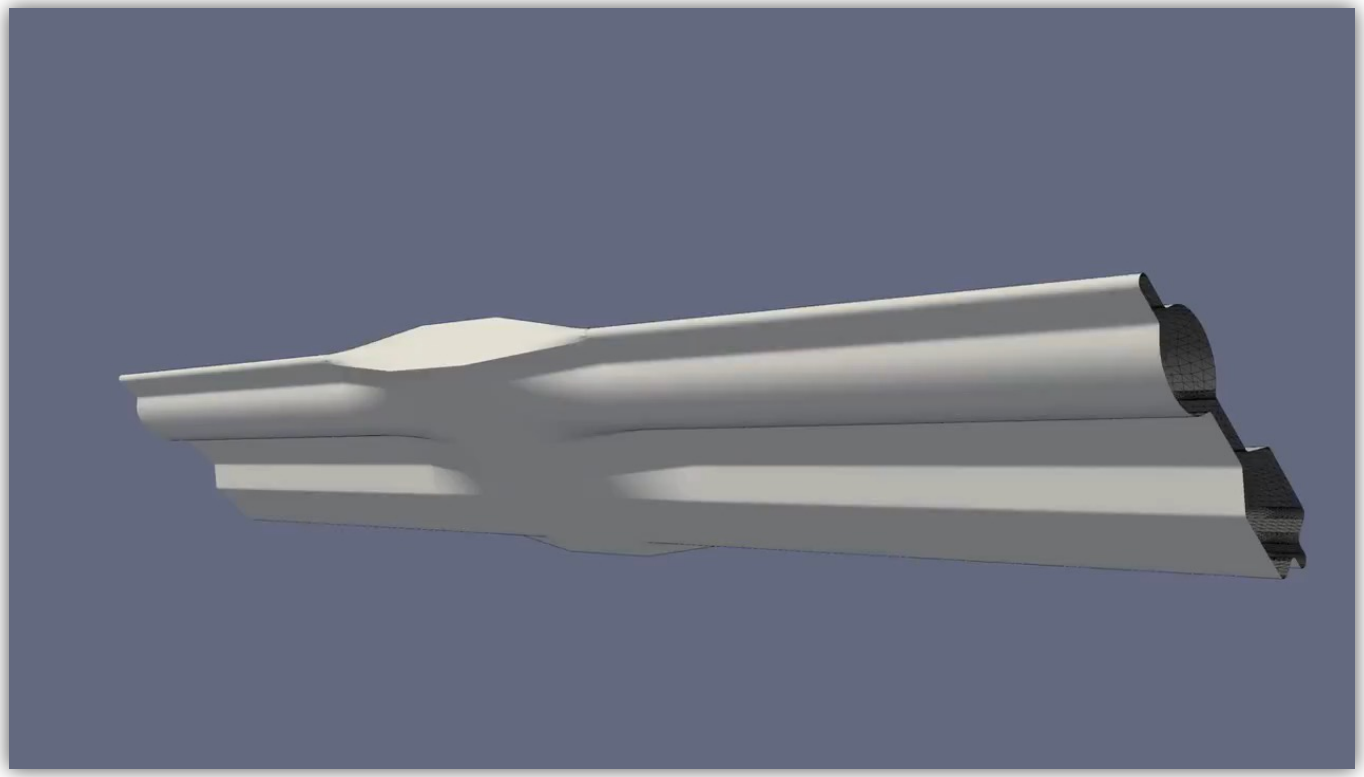


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Electromagnetic fields in complex structures

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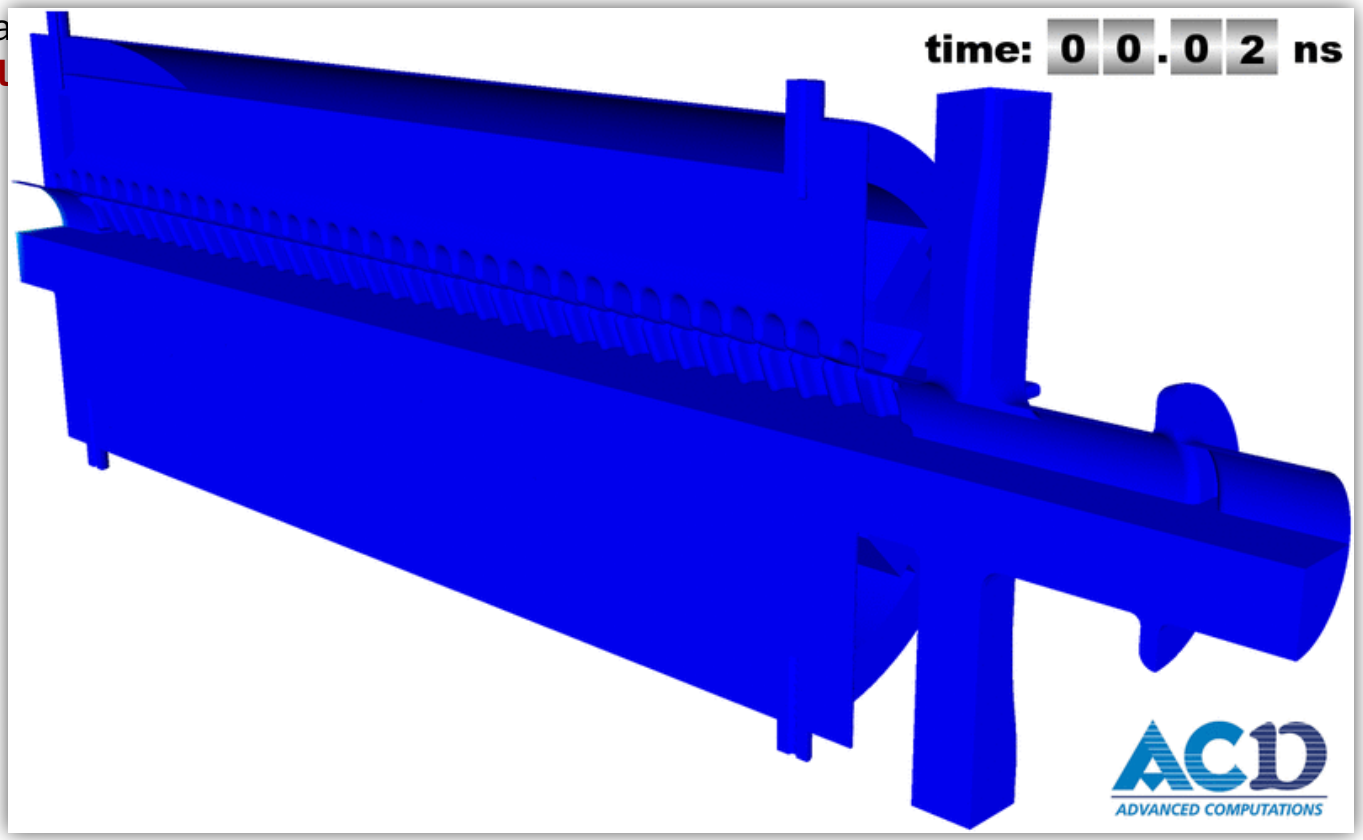
ERL vacuum chamber



Electromagnetic fields in complex structures

For more complica
computational

CLIC PETS



Electromagnetic fields in complex structures

For more complicated geometries the (semi-) analytic methods no longer work. One must rely on **computational methods** to evaluate the induced electromagnetic fields

In principle, we need to solve the **full set of Maxwell's equations at every time step** to obtain the electromagnetic fields at every location within the structure in order to evaluate to forces at the probe particle's locations. This becomes very tedious and **virtually impossible for a 27km ring such as the LHC**.

How can we treat these phenomena more effectively in our models?

We normally **use a set of assumptions** to simplify the problem:

- **Rigid beam approximation:** the beam traverses the discontinuity of the vacuum chamber rigidly
- **Impulse approximation:** what the beam really cares about is the integrated impulse as it completes the traversal of the discontinuity

Summary

In this lecture we discussed **direct and indirect space charge** and showed that this can lead **to both incoherent as well as coherent tune shifts**. We then moved on the a more general treatment of electromagnetic fields in simple structures where we were able to identify **yet another type of induced fields** originating from the **electromagnetic properties** of the surrounding material – **the wall wake**.

We have shown on the example of **multilayer structures** that one can in principle solve Maxwell's equations to obtain the induced electromagnetic fields by a given charge distribution. We saw that it is **possible to decompose these fields**. We were able to identify the part that is dependent on the electromagnetic properties of the surrounding material as the **wall wake** which already led to the idea of wake fields.

We have seen examples of induced electromagnetic fields within complex structures. We will now look at how we can deal with these types of fields more practically by introducing the **concept of wake fields**.

We also already looked at more general examples of induced fields in complex structures.

Next we will introduce the **concept of wake fields and impedances** and study the effect of these on the machine and on the beam.

References

- Advanced accelerator physics: https://indico.cern.ch/event/509762/timetable/

Homework (due to Nov. 11 50 points)

(20 points)

1. Calculate the transverse space charge force for coasting beam with axial symmetrical Gaussian transverse charge density:

$$f(r) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{r^2}{2\sigma^2}\right)$$

   a) Compare results with space charge effect for coasting beam with uniform charge density radius a=2*σ

2. (10 points) Based on the results of HW1 : what high order multipole (4 poles - quadrupole, 6 poles - sextupole, 8 poles- octupole, 10 poles –decapole or higher) is the best choice to reduce space charge tune spread due to gaussian transverse distribution? And why?

Homework (due to Nov. 11 50 points) (cont.)

3. (20 points) Calculate the transverse space charge force for coasting beam with axial symmetrical Gaussian transverse am of Electron Ion Collider. In order to operate EIC with both space charge tune shift below 0.1 and required emittances we plan to reduce peak current by introducing 2nd harmonic RF system. Based on the plot estimate:

- Tune shift at the injection energy (γ=25) without cooling and without second harmonic RF (gaussian current distribution)
- Tune shift after acceleration to the store energy (γ=293)

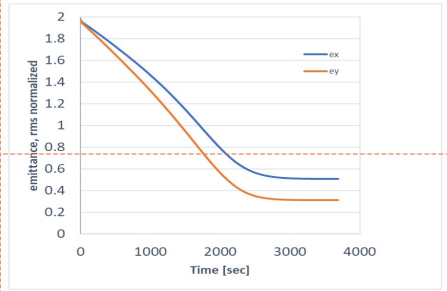
Horizontal Tune Shift (ΔQ_x)

$$\Delta Q_x = -\frac{r_0 N R}{2\pi\beta\gamma^2\sqrt{\beta_x\varepsilon_{n,x}}\left(\sqrt{\beta_x\varepsilon_{n,x}}+\sqrt{\beta_y\varepsilon_{n,y}}\right)}$$

Vertical Tune Shift (ΔQ_y)

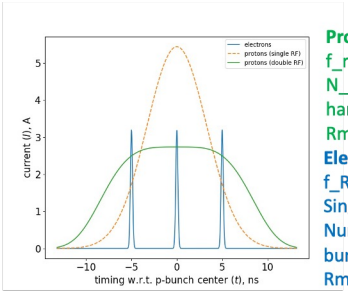
$$\Delta Q_y = -\frac{r_0 N R}{2\pi\beta\gamma^2\sqrt{\beta_y\varepsilon_{n,y}}\left(\sqrt{\beta_x\varepsilon_{n,x}}+\sqrt{\beta_y\varepsilon_{n,y}}\right)}$$

Average current of electrons: **I_{av}=74mA**
Electrons rms angles in the cooling section: **25 urad**
Effective cooling section length: **168 meters**



Cooling simulations of protons at γ=25, with decoupled transverse motion (**IBS+Cooling only,** using single harmonic RF).
Longitudinal emittance is kept constant during cooling process.
After cooling, normalized rms emittances of protons **ex, ey=0.5,0.3 um** (horizontal emittance can be increased further as needed.)

Beam Structure in the Cooling Section



Protons bunch structure:
f_rep=24.6 MHz
N_p=2.8e11, I_peak=2.7 A (with 2nd harmonic RF)
Rms length=1m
Electrons bunch structure:
f_RF=197 MHz
Single bunch: Q_e=1 nC
Number of bunches in macro-bunch: 3
Rms length=0.04m

2nd harmonic RF alleviates space-charge effects reducing peak current of protons to about 3A (space charge tune shifts after cooling $\Delta Q_{sc,x,y}$=0.06, 0.1), allowing vertical emittance to be cooled without introducing additional heating of horizontal emittance

Part 2 preview

The goal of the second part is to introduce the concepts of wake fields and impedances as a way to model the interaction an ensemble of particles with the accelerator environment – in particular the response of the accelerator environment to a given distribution of particles.

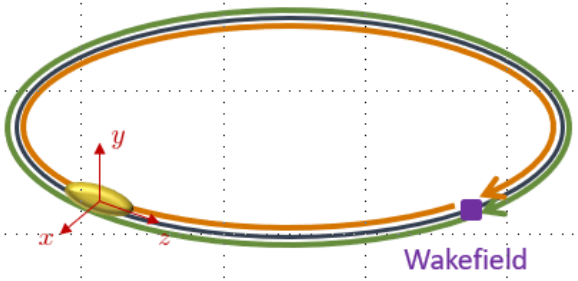
We will see how multi-particle systems can excite longitudinal and transverse wake fields in accelerator components and lead to undesired effects such as beam induced heating.

Part 2: Wake fields and impedances

- Longitudinal wake fields and impedances
- Beam induced heating
- Transverse wake fields and impedances
- The energy balance
- Numerical computation and measurement of impedances

Additional material

Tracking one full turn including the interaction with wake fields:



$$\left.\begin{pmatrix} x_i \\ x_i' \end{pmatrix}\right|_{k+1} = \mathcal{M}_i \left.\begin{pmatrix} x_i \\ x_i' \end{pmatrix}\right|_{k}$$

$$\left.\begin{pmatrix} z_i \\ \delta_i \end{pmatrix}\right|_{k+1} = \mathcal{I}\left[\left.\begin{pmatrix} z_i \\ \delta_i \end{pmatrix}\right|_{k}\right]$$

$$\left.(x_i')\right|_{k+1} = \left.(x_i')\right|_{k} + \mathcal{WK}$$

1. Initialise a macroparticle distribution with a given emittance
2. Update transverse coordinates and momenta according to the linear periodic transfer map – adjust the individual phase advance according to chromaticity and detuning with amplitude
3. Update the longitudinal coordinates and momenta according to the leap-frog integration scheme
4. Update momenta only (apply kicks) according to wake field generated kicks
5. Repeat turn-by-turn…