Appendix F: Inhomogeneous solution

Even though calculations are tedious, they are also transparent and straightforward. General expression for the inhomogeneous equation of 2n ordinary linear differential equations is found by a standard trick of variable constants (method developed by Lagrange), i.e. assuming that \( R = M(s)A(s) \):

\[
\frac{dR}{ds} = R' = D \cdot R + C; \quad M' = DM;
\]

\[
R = M(s)A(s) \Rightarrow M'A + MA' = DMA + C
\]

\[
R(0) = 0 \Rightarrow A_o = 0 \quad \text{(F-1)}
\]

\[
A' = M^{-1}(s)C \Rightarrow A = \int_0^s M^{-1}(z)Cdz = \left( \int_0^s e^{-Dz}dz \right) \cdot C
\]

with well known result of:

\[
R = e^{Ds} \left( \int_0^s e^{-Dz}dz \right) \cdot C. \quad \text{(F-2)}
\]

or

\[
R = M_{4 \times 4}(s) \left( \int_0^s M^{-1}_{4 \times 4}(z)dz \right) \cdot C. \quad \text{(F-3)}
\]

If you use computer, eq. (F-3) is one to use. For analytical folks, you should go through a tedious job is combining all terms together into final form:

\[
R(s) = \sum_{k=1}^n \left[ \prod_{l \neq k} \left( \frac{D - \lambda_l I}{\lambda_k - \lambda_l} \right) \sum_{j=0}^{n_k-1} \left( \frac{D - \lambda_j I}{\lambda_k - \lambda_j} \right) \right] \left[ \sum_{n=0}^{n_k-1} \left( \frac{D - \lambda_k I}{\lambda_l - \lambda_k} \right) \frac{s^n}{n!} \sum_{p=0}^{n_k-1} (-1)^{p+1} (D - \lambda_k I)^p \cdot C \cdot \left[ \sum_{q=0}^{p+1} \frac{s^{p-q}}{(p-q)!} \lambda_k^{q+1} = \frac{e^{\lambda_k}}{\lambda_k^{p+1}} \right] \right]
\]

\[
(s^{p-q}) \frac{\lambda_k^{q+1}}{(p-q)!} = \frac{e^{\lambda_k}}{\lambda_k^{p+1}} \quad \text{(F-4)}
\]