Homework 9.

Problem 1. 4 x 4 points. FODO cell.

Consider a general FODO cell comprised of two quadrupoles F and D separated by two drift sections, e.g. the structure below:

$$F:\ K_F = \frac{e}{pc}\frac{\partial B_y}{\partial x}, l_F;$$

$$O1: l_1$$

$$D:\ K_D = \frac{e}{pc}\frac{\partial B_y}{\partial x}, l_D;$$

$$O2: l_2$$

(a) write matrix (both x and y or 4x4) of general FODO cell (not assuming any limitations on K F,D).

(b) write stability criteria (for x and y) for periodic lattice built of this FOD cell. Hint – do not try to solve it!

(c,d) make transition to short lens approximation and assume equal strength of

$$l_F K_F = -K_D l_D = \frac{1}{f} = const, l_{F,D} \to 0$$

$$l = l_1 = l_2$$

and

(c) show that both x and y motion can be stable (e.g. prove so called strong focusing: combination of focusing and defocusing length can provide focusing in both directions);

(d) define (e.g solve) the stability criteria for such cell.

Problem 2. 2x5 points. Find not-trivial solution for building an unit 2x2 transport matrix out of repeating cells:

$$M^4 = I; M \neq I$$

(a) show that one of the solutions $trace(M) = 0$; Hint: used $M^2 = -I$;

(b) for a "symmetric" FODO cell and finite length equally strong quadrupoles $K_F = -K_D = K; l_F = l_D = L;\ l_1 = l_2 = l$ write the condition that $M_x^4 = M_y^4 = I$, e.g. the 4x4 transport matrix is unit.