PHY 554. Homework 4.

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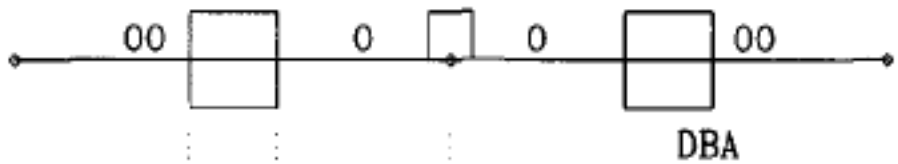
1 (4 point): Double Bend Achromat (DBA)



A system with D = D' = 0 at both start and end is called an achromat system. A DBA section contains two dipoles for orbit bending and one quadrupole in between for optics matching. It has been widely used in designing low emittance storage rings.

To match the dispersion after the DBA, we need to impose a symmetric condition. For half of the DBA section, we have

$$\begin{pmatrix} D_c \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -1/(2f) & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & L_1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & L & L\theta/2 \\ 0 & 1 & \theta \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Thus the D' at the center of the DBA section is 0. Please express focal length of the quadrupole required for this condition in terms of L(dipole length), L1(drift between dipole and quadrupole) and θ (dipole angle). And write down the resulting dispersion Dc at the middle of the quadrupole.

HW 2 (6 point): Combined function magnets

The dispersion function in a combined function magnet satisfies

$$D'' + K_x(s)D = \frac{1}{\rho}$$

a) show that the solution for constant Kx=K>0 is

$$D = a\cos\sqrt{K}s + b\sin\sqrt{K}s + 1/\rho K$$

let D_0 and D_0' be the initial dispersion function and its derivative at s = 0. Show that solution can be expressed as

$$\begin{pmatrix} D(s) \\ D'(s) \\ 1 \end{pmatrix} = M \begin{pmatrix} D_0 \\ D_0' \\ 1 \end{pmatrix}$$

and the transfer matrix

$$M = \begin{pmatrix} \cos\sqrt{K}s & \frac{1}{\sqrt{K}}\sin\sqrt{K}s & \frac{1}{\rho K}(1-\cos\sqrt{K}s) \\ -\sqrt{K}\sin\sqrt{K}s & \cos\sqrt{K}s & \frac{1}{\rho\sqrt{K}}\sin\sqrt{K}s \\ 0 & 0 & 1 \end{pmatrix}$$

b) show that the transfer matrix for constant Kx=K<0 is

$$M = \begin{pmatrix} \cosh\sqrt{|K|}s & \frac{1}{\sqrt{|K|}}\sinh\sqrt{|K|}s & \frac{1}{\rho|K|}(-1+\cosh\sqrt{|K|}s) \\ \sqrt{|K|}\sinh\sqrt{|K|}s & \cosh\sqrt{|K|}s & \frac{1}{\rho\sqrt{|K|}}\sinh\sqrt{|K|}s \\ 0 & 0 & 1 \end{pmatrix}$$

c) show that the transfer matrix for a pure sector dipole is

$$M = \begin{pmatrix} \cos\theta & \rho\sin\theta & \rho(1-\cos\theta) \\ -(1/\rho)\sin\theta & \cos\theta & \sin\theta \\ 0 & 0 & 1 \end{pmatrix}$$

d) in thin lens(small angle) approximation, show that the transfer matrices M for quadrupole and dipole become

$$M_{\mathrm{quad}} = \begin{pmatrix} 1 & 0 & 0 \\ -1/f & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad M_{\mathrm{dipole}} = \begin{pmatrix} 1 & \ell & \ell\theta/2 \\ 0 & 1 & \theta \\ 0 & 0 & 1 \end{pmatrix}$$