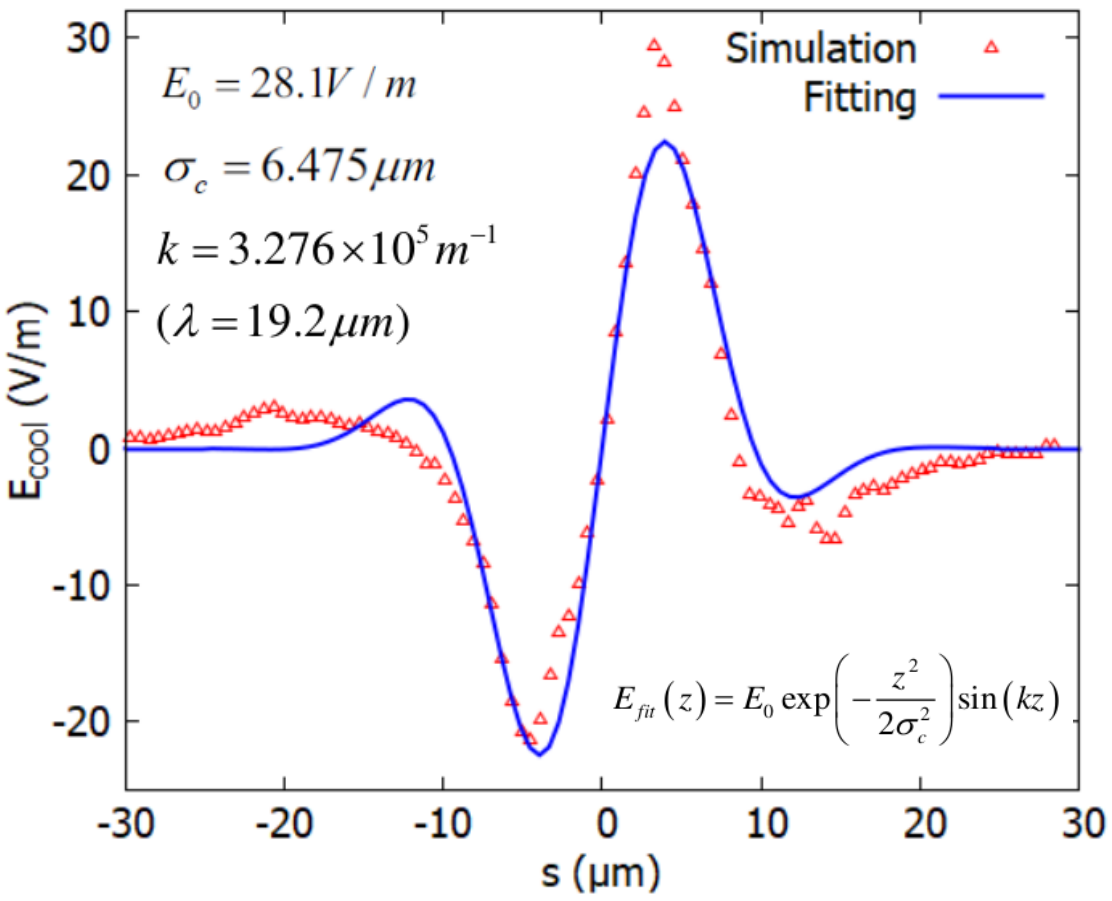
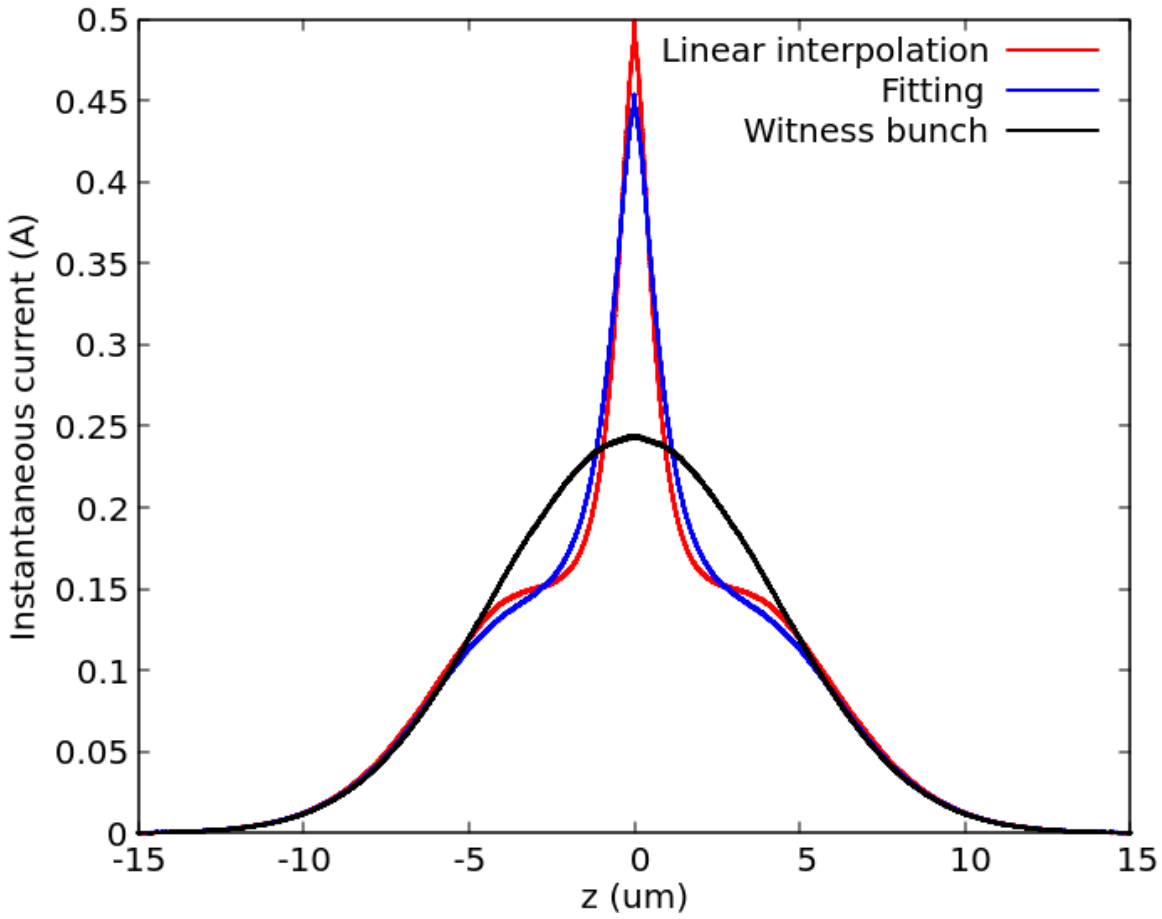
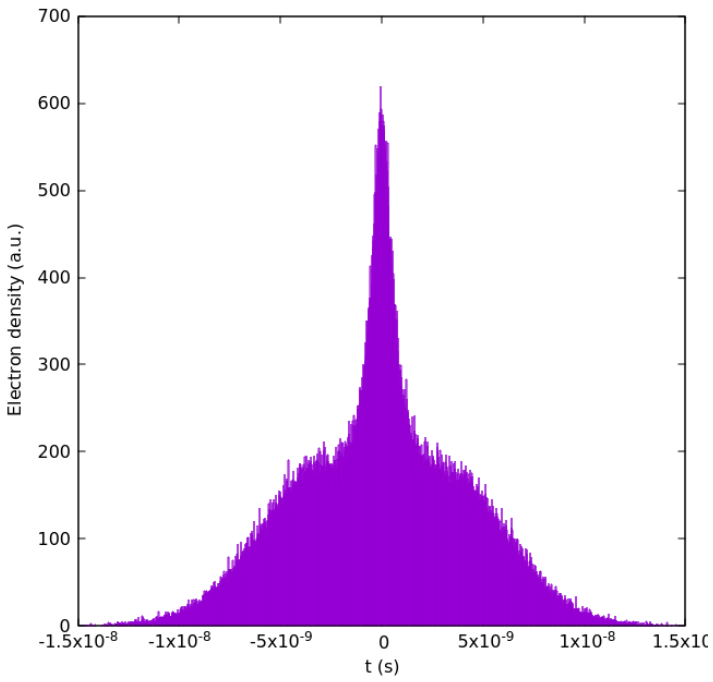
Tracking of the Ions for the CeC experiment with Cooling Force Depending on Longitudinal Location of the Ion

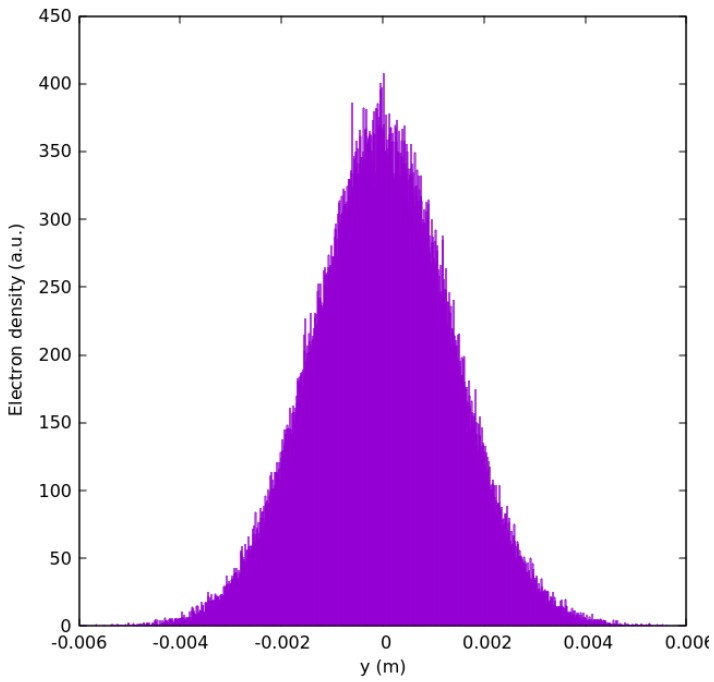
G. Wang

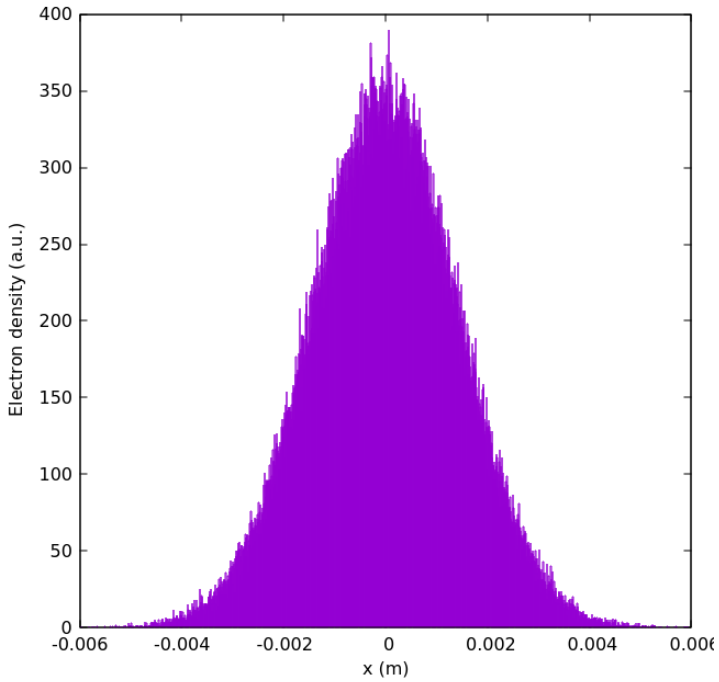
What has been done differently…
Fitting-> linear interpolation





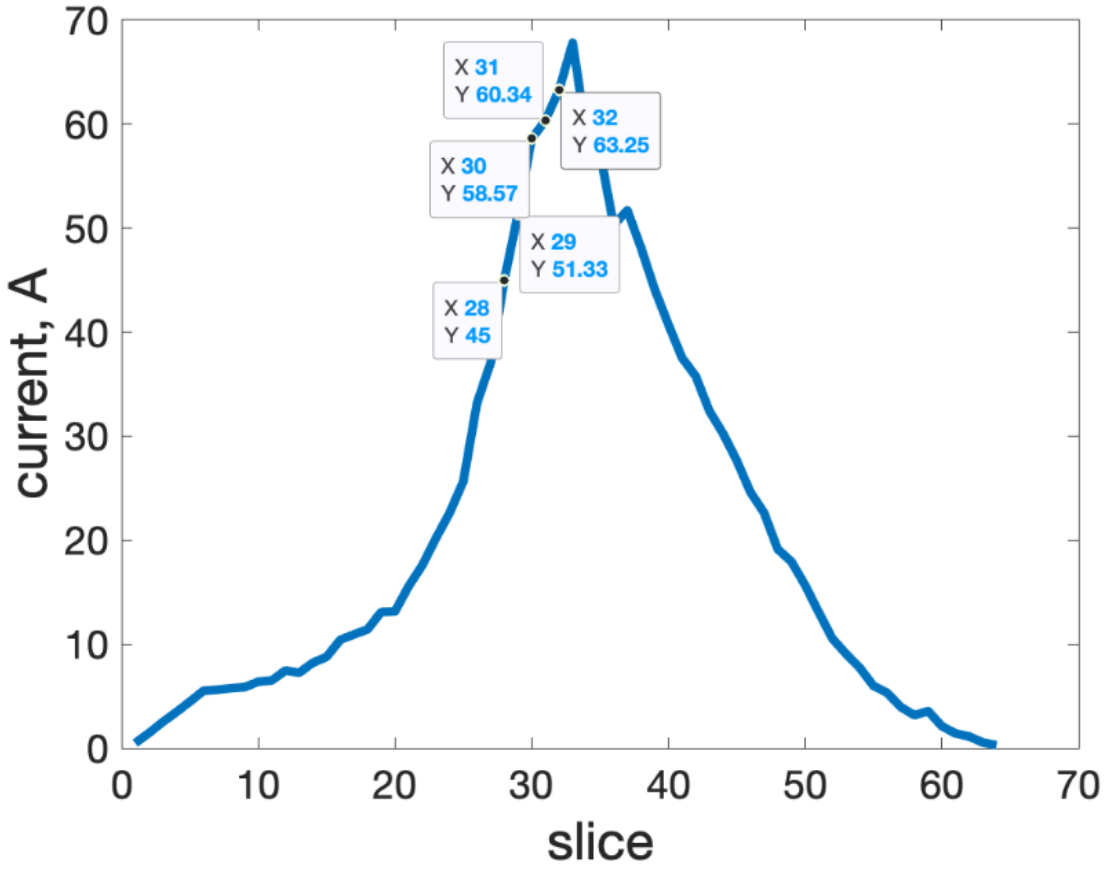


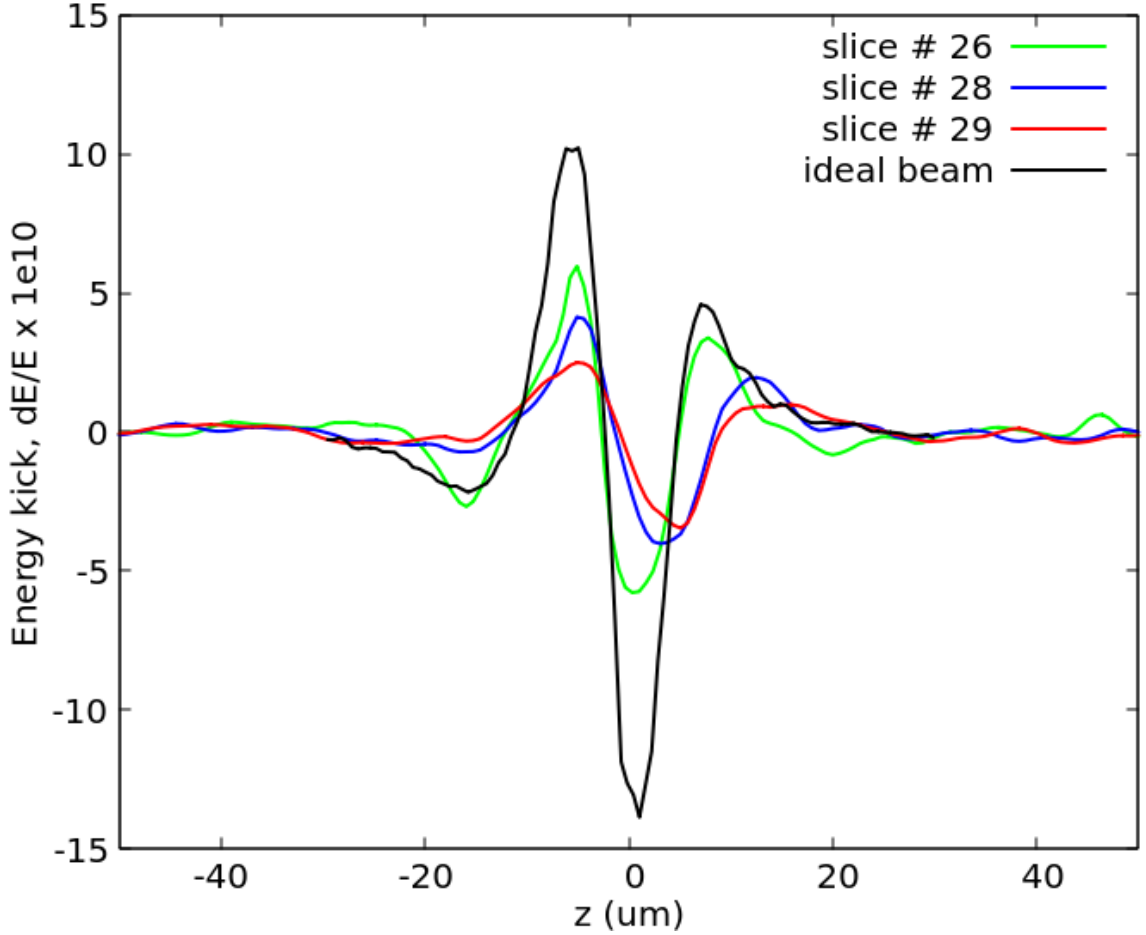




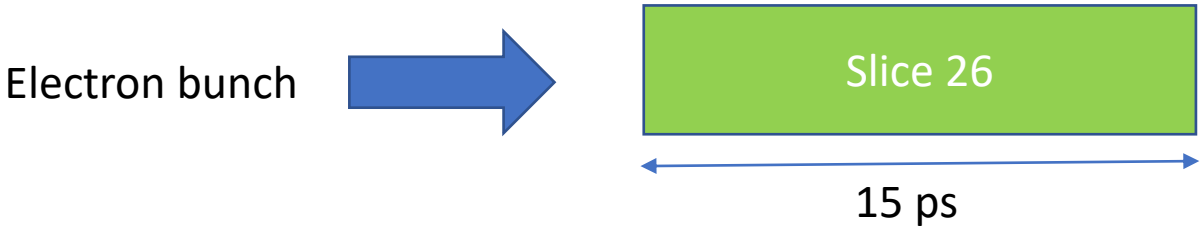
Transverse core (linear interpolation)?

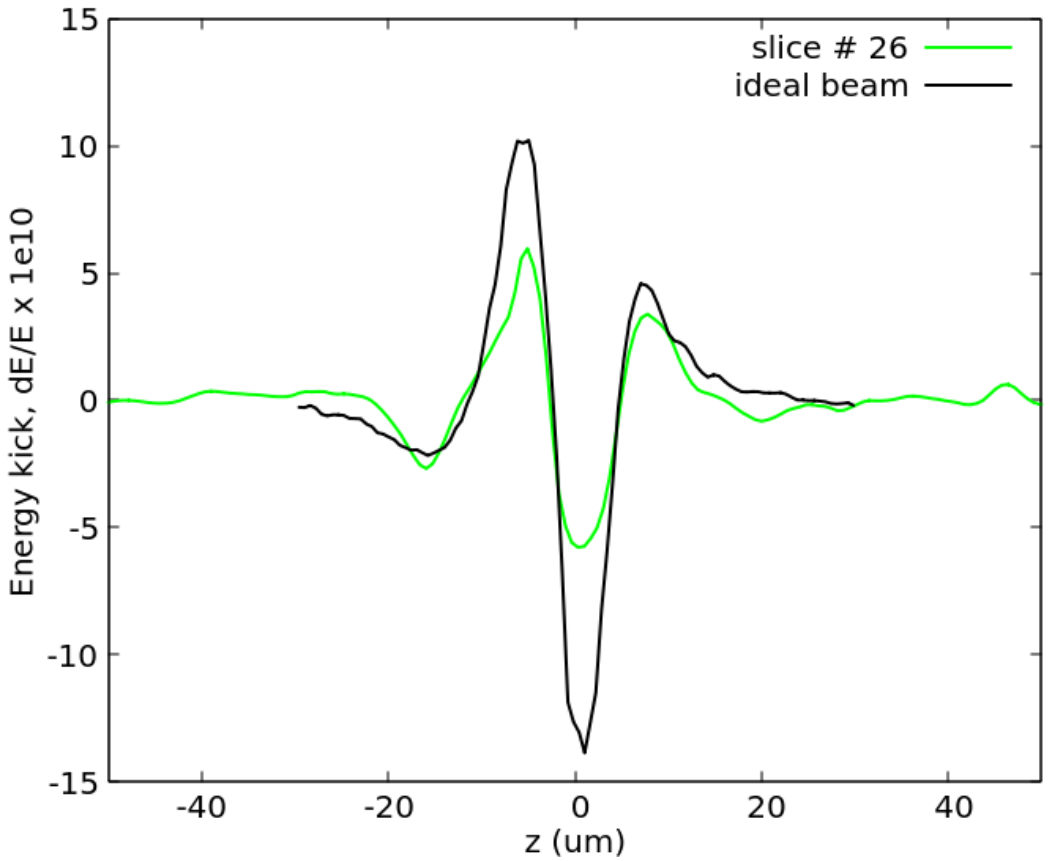
Energy kick from Ideal beam vs Simulated beam

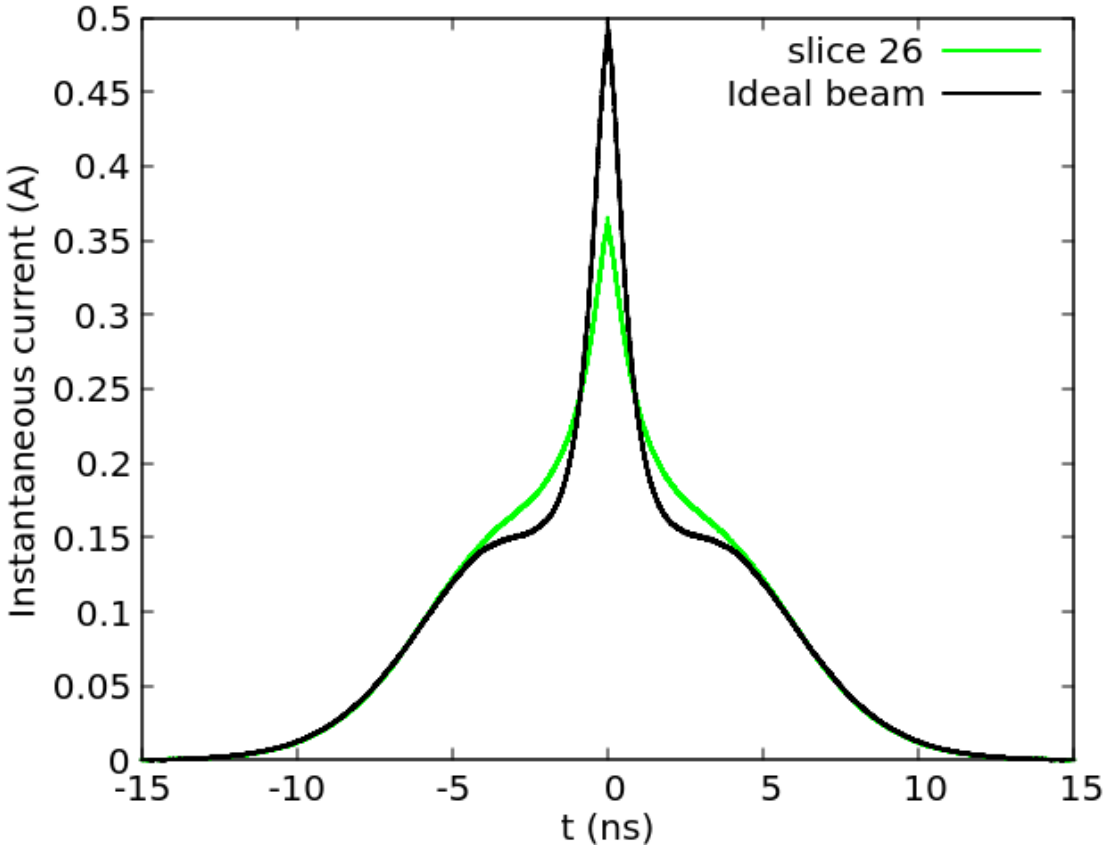




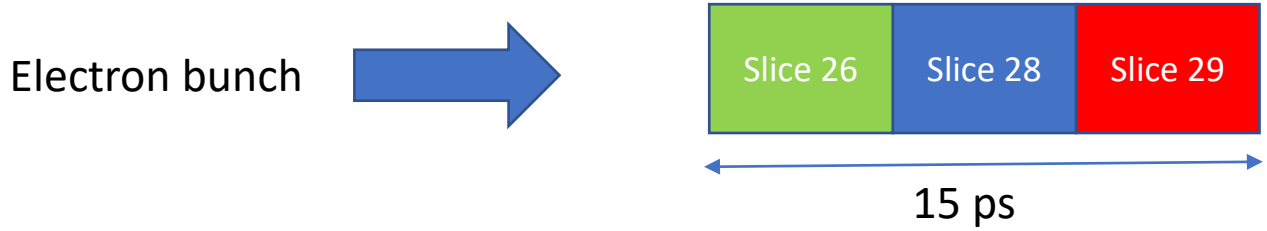
Using energy kick from the best slice, i.e. slice # 26 and assume the kick applies for 15 ps of electrons

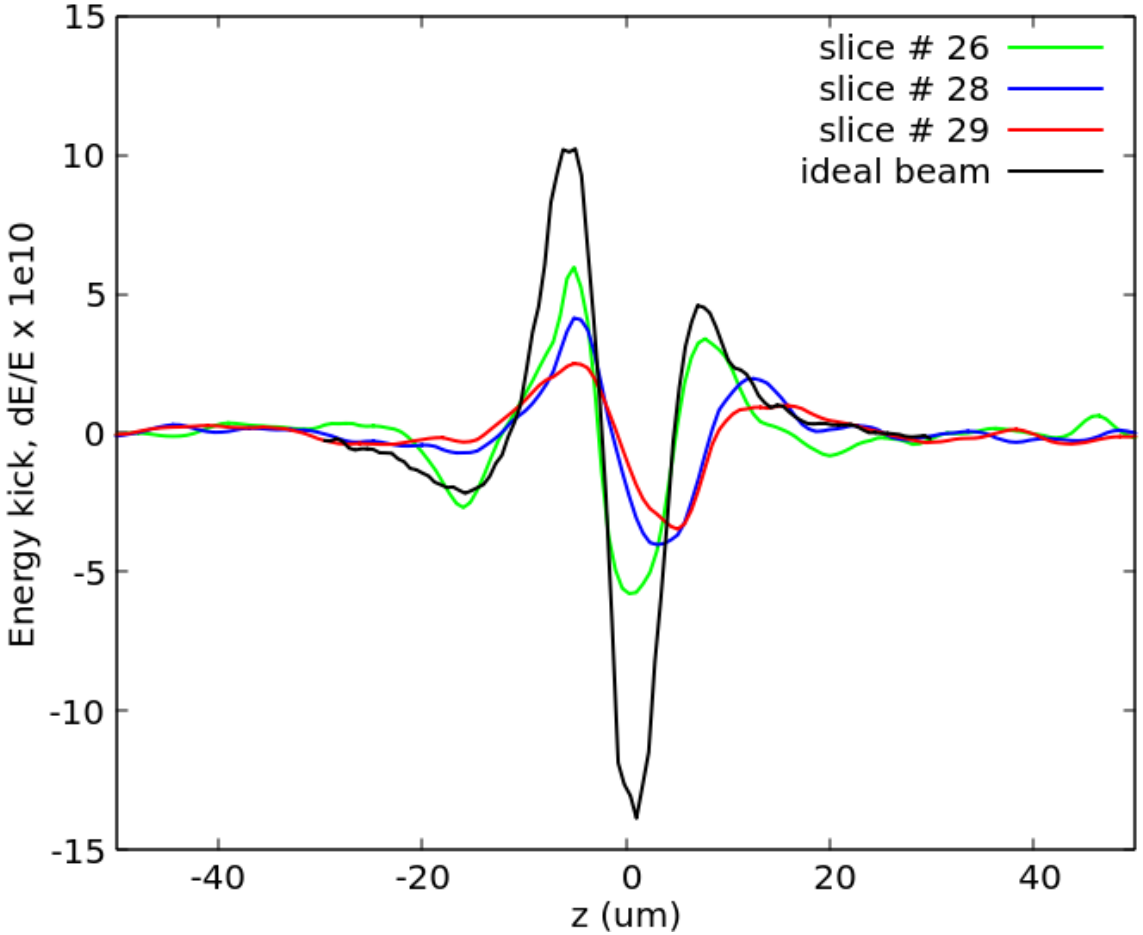


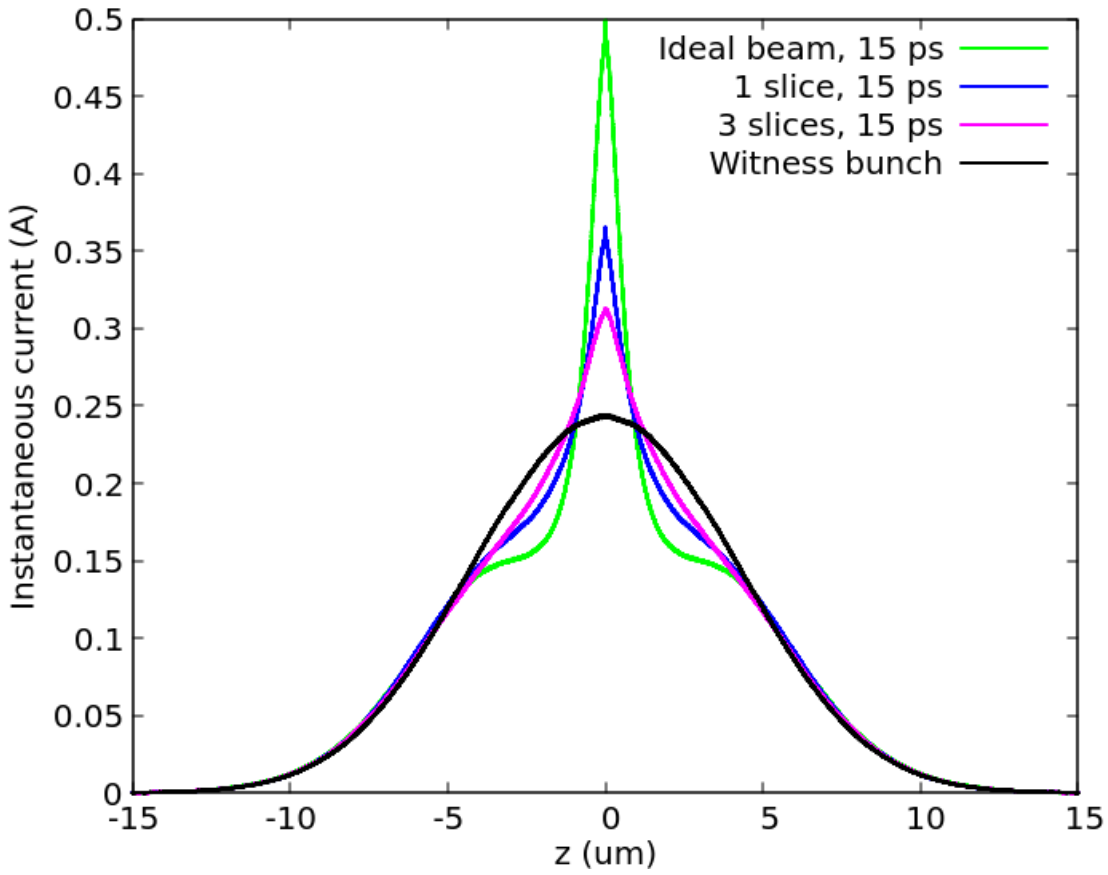




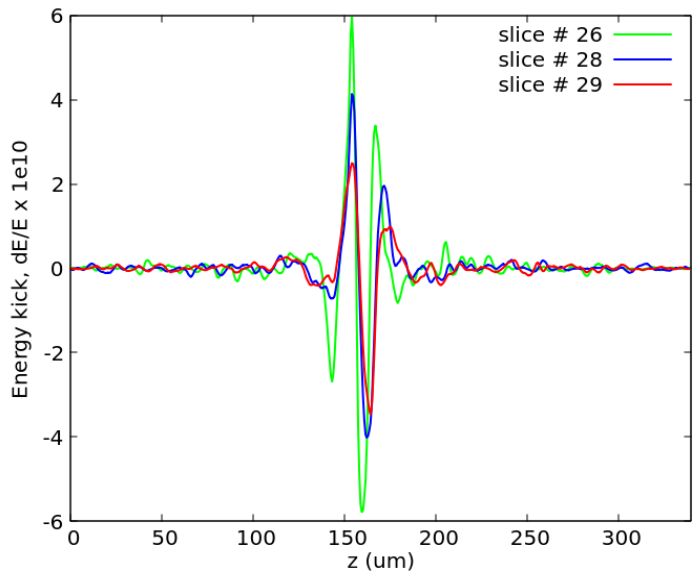
Using kicks from slice 26, 28 and 29 with each slice has 5 ps of duration

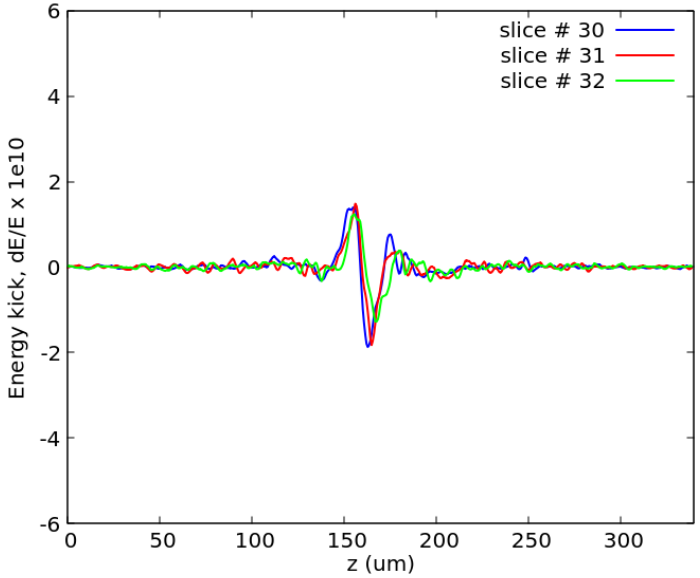




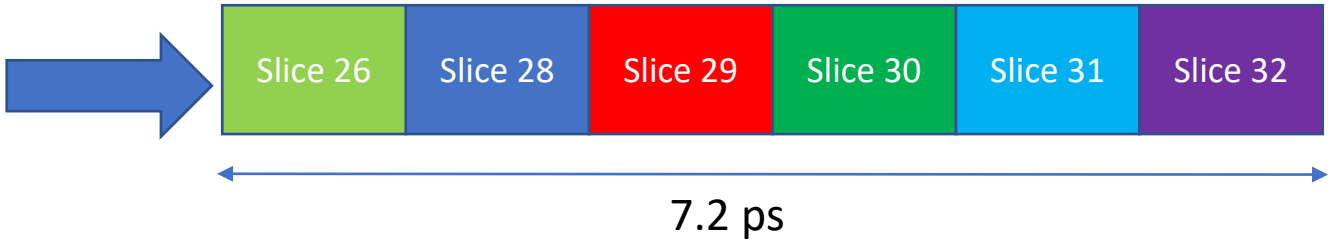


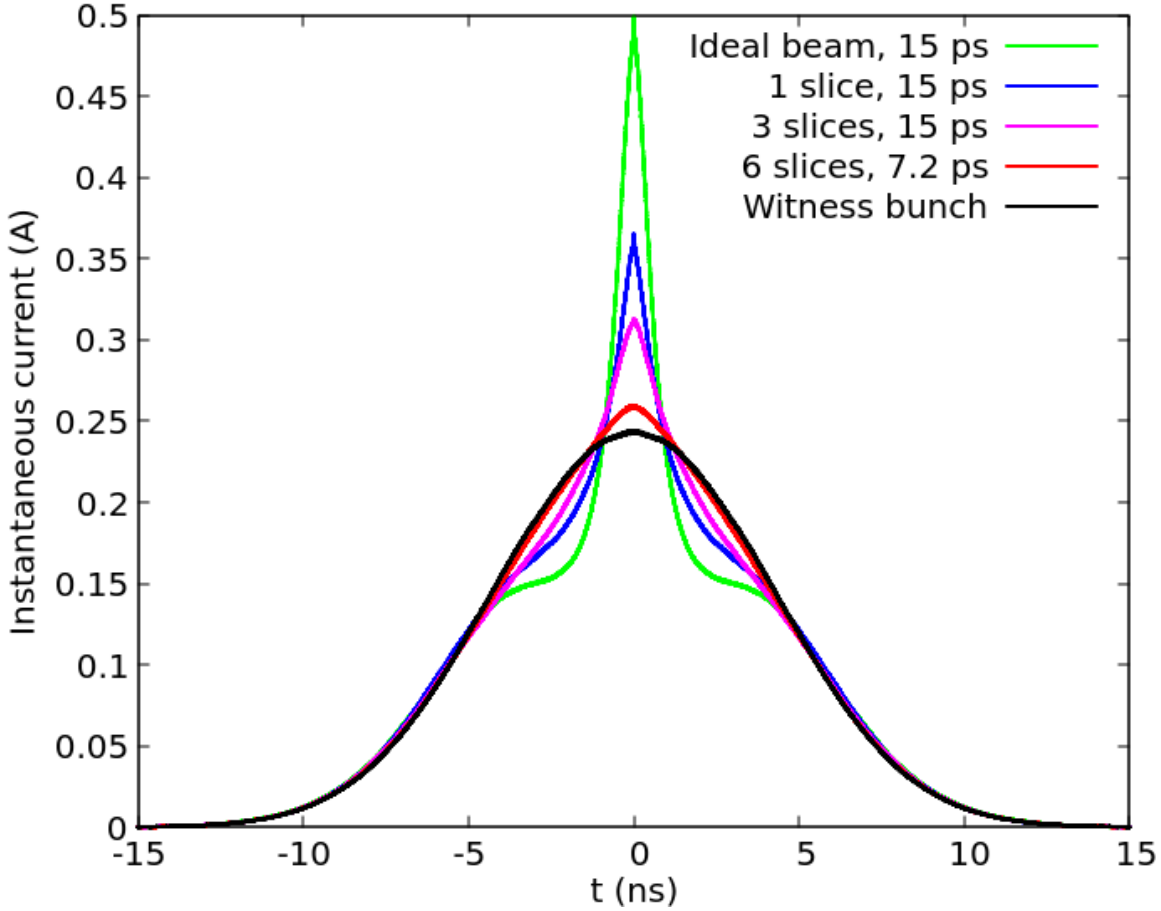
Using kicks from 6 slices with 1.2 ps of duration for each slice





Electron bunch



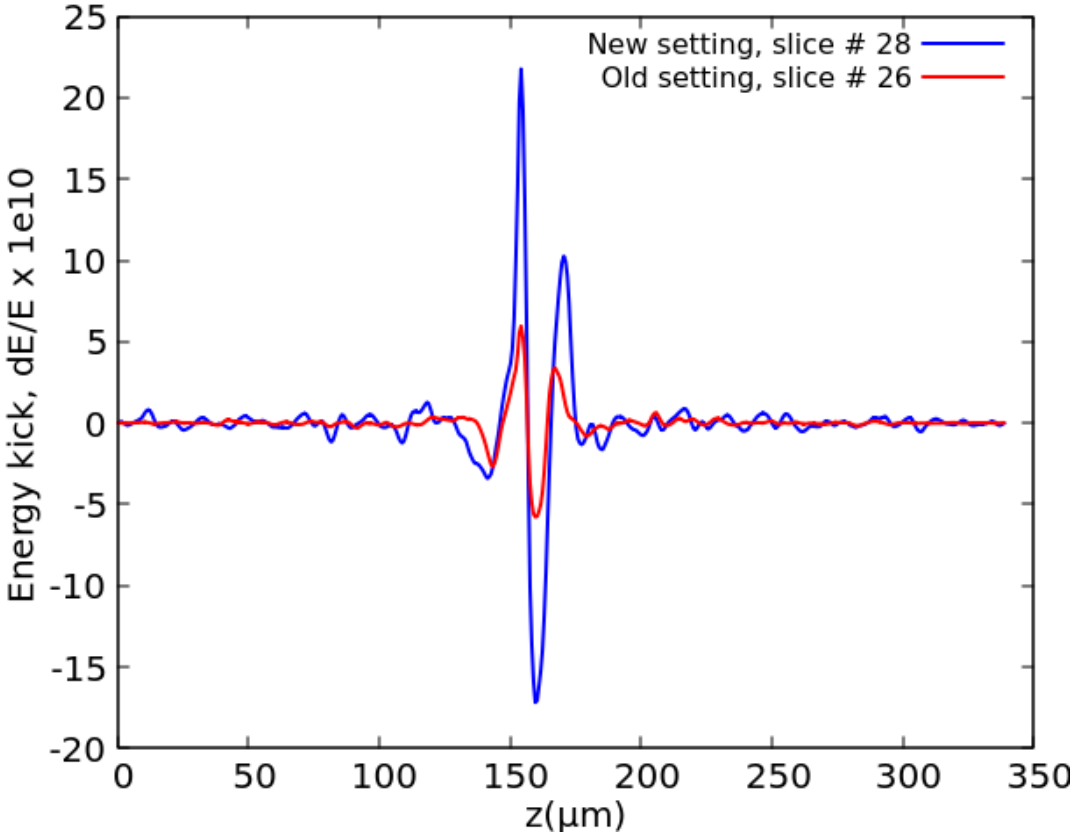


Summary

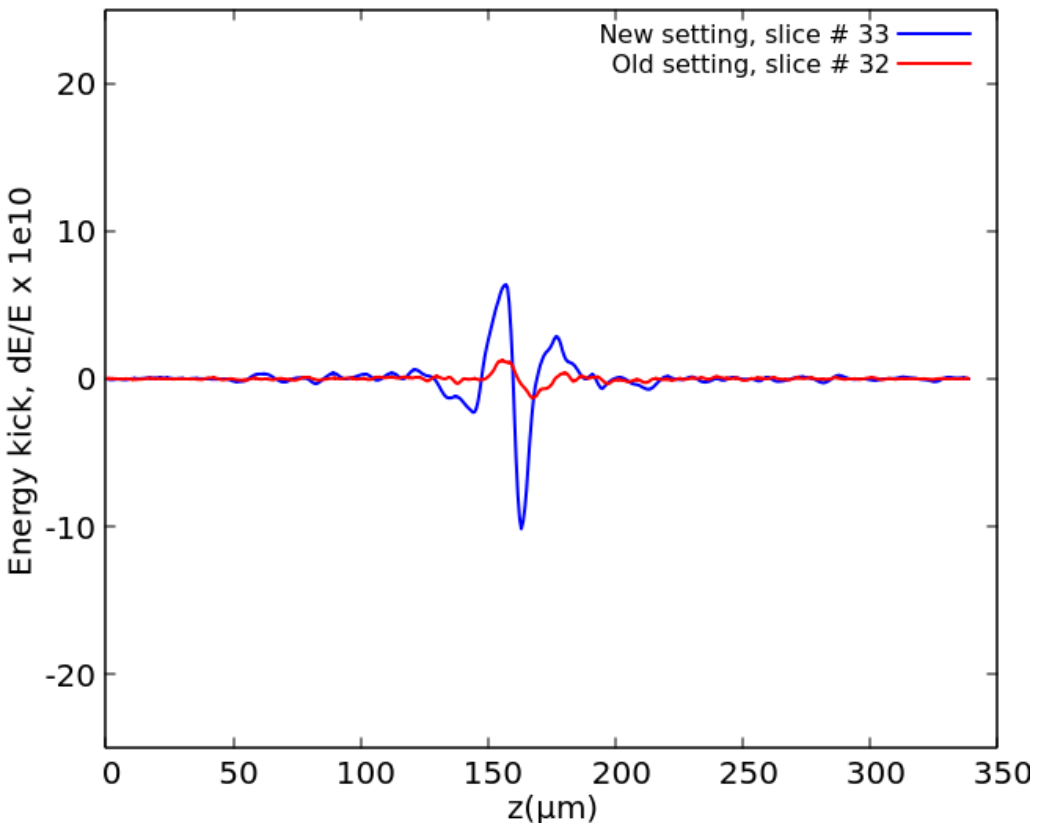
- The tracking code is updated so that the energy kick calculated from SPACE simulation is directly used with linear interpolation applied between data points.
- It has been implemented into the tracking code that ions sitting at different longitudinal slice of the electron bunch see different form of the cooling force. The cooling force for each slice of electrons depends on the local properties of the electrons and is provided by SPACE simulation.
- According to the simulation, cooling effect is very weak with the electron beam as obtained from the previous beam dynamic simulation, I.e. 7% of peak current increases over 40 minutes, even without energy jitter and other adverse effects.
- We need to improve electron beam quality so that we have 15 ps of duration with energy kick comparable to that of the best slice of the previous simulated electrons, I.e. slice 26.

Updates with new solenoid settings (comparison of cooling kicks)

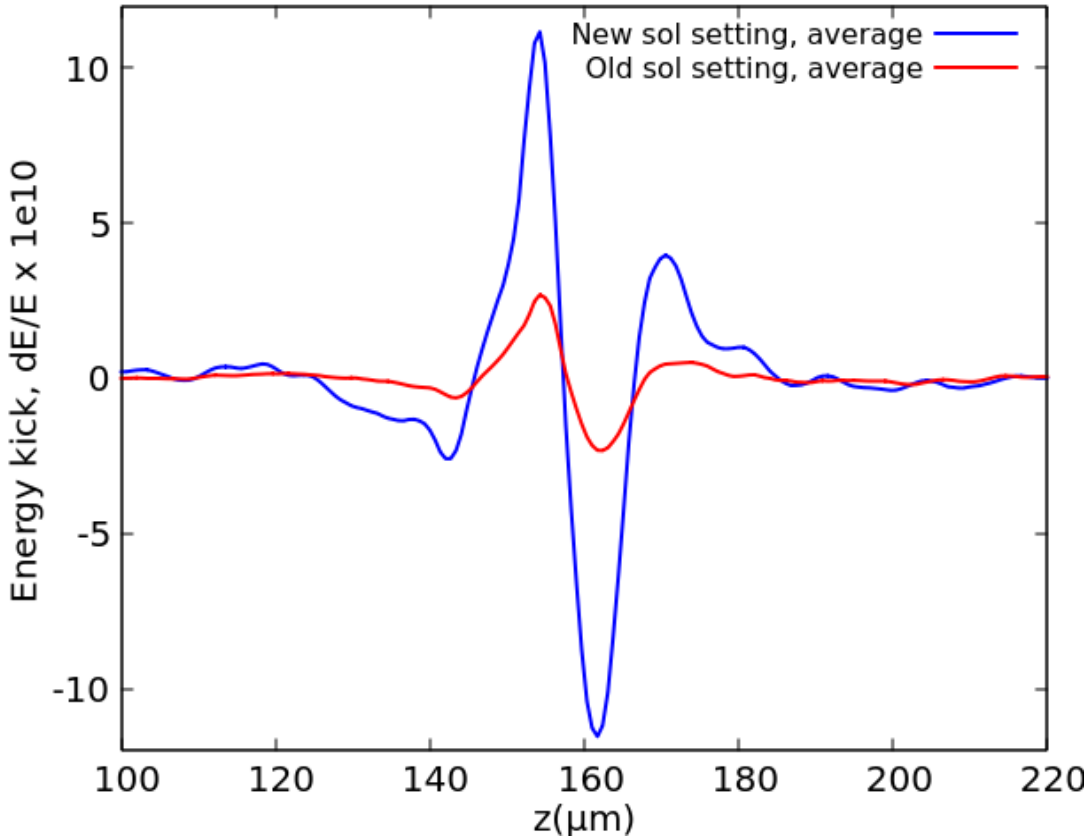
Best slice

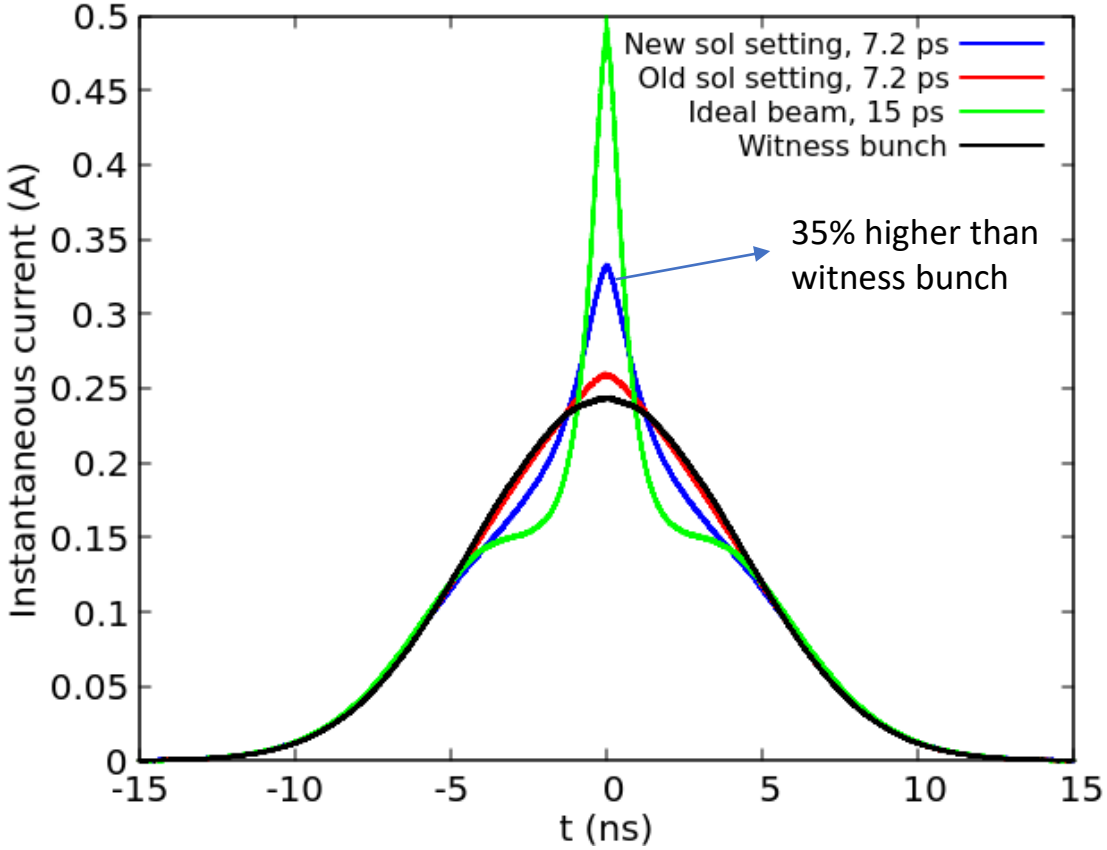


'Worst' slice



Updates with new solenoid settings





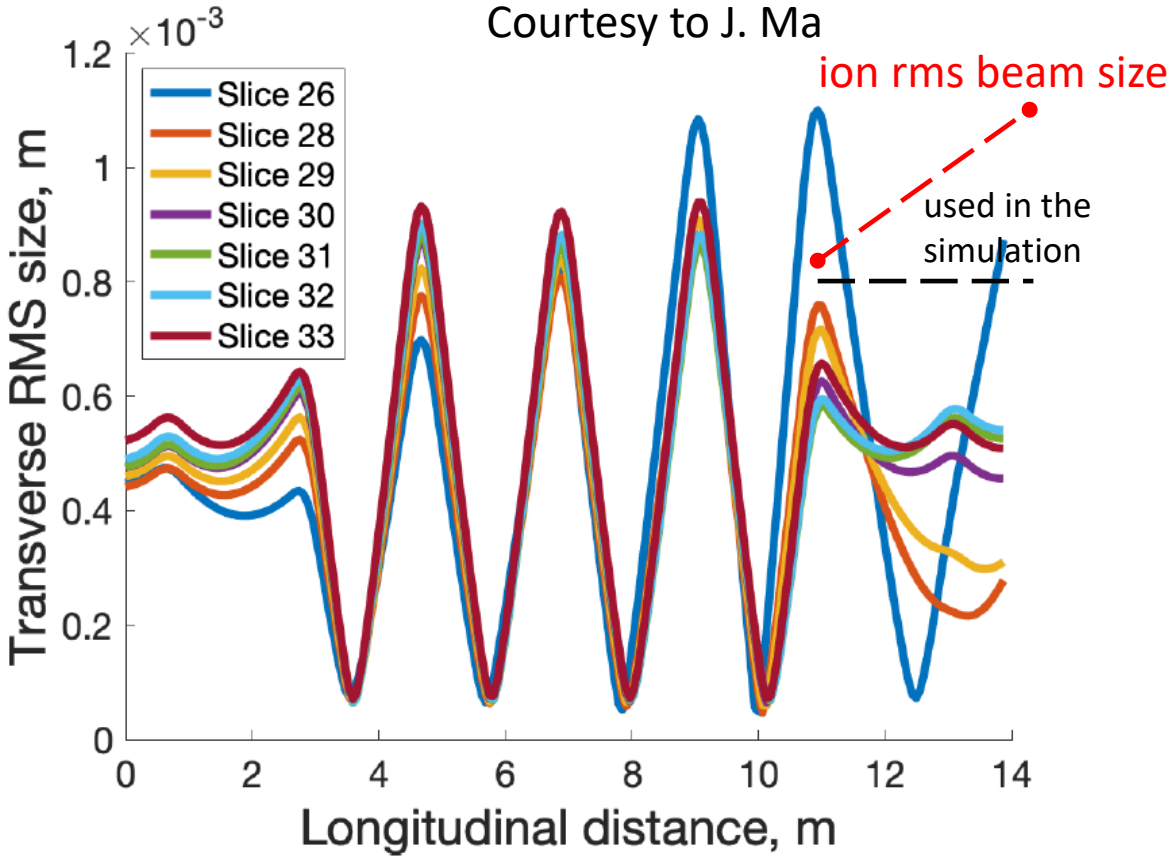
Latest SPACE simulation results for e beam size (old electron distribution)

$$\sigma_{ion} = \sqrt{\frac{2.5\mu m \cdot 7.5m}{28.5}} = 0.8mm$$

(*The beta function at kicker varies from 8.5 m to 14.3 m, corresponding to average ion beam size of 1 mm. To be corrected...)

- Poor spatial overlapping of the electrons with the ions can reduce the cooling effects.
- In order to properly study the effects, one need to obtain the dependance of the cooling energy kick on the transverse offset of the ion.
- As a start, I assumed Gaussian dependance in the simulation for now.

Courtesy to J. Ma



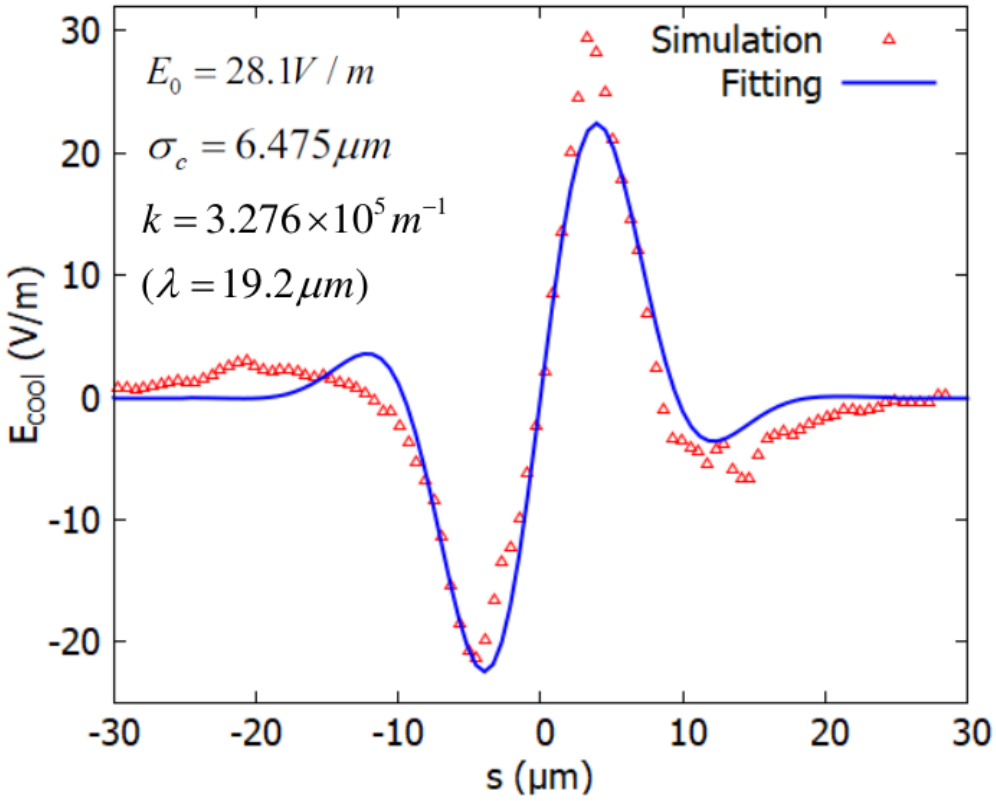
$$dE(z) \rightarrow dE(z)\exp\left(-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2}\right)$$

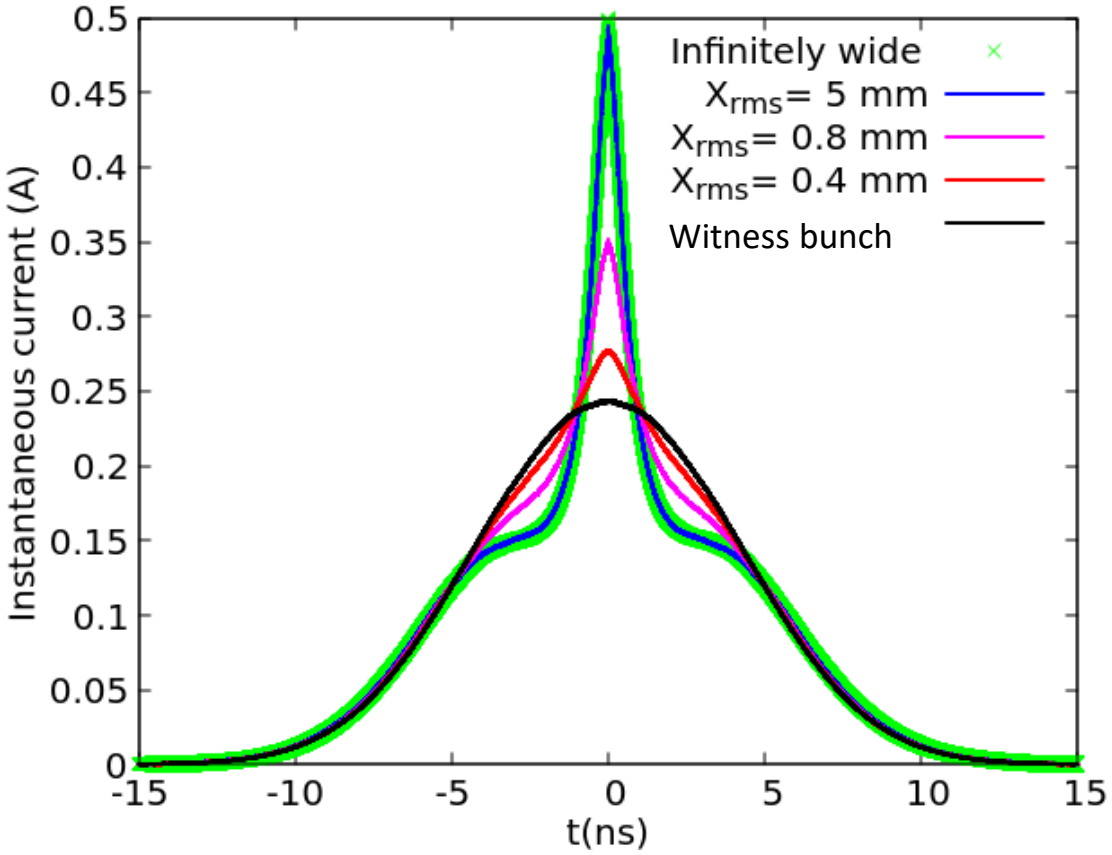
$\sigma_{x,y}$: rms width of the slice of electrons that the ion is overlapping with

Influences due to finite transverse size of the electron bunch

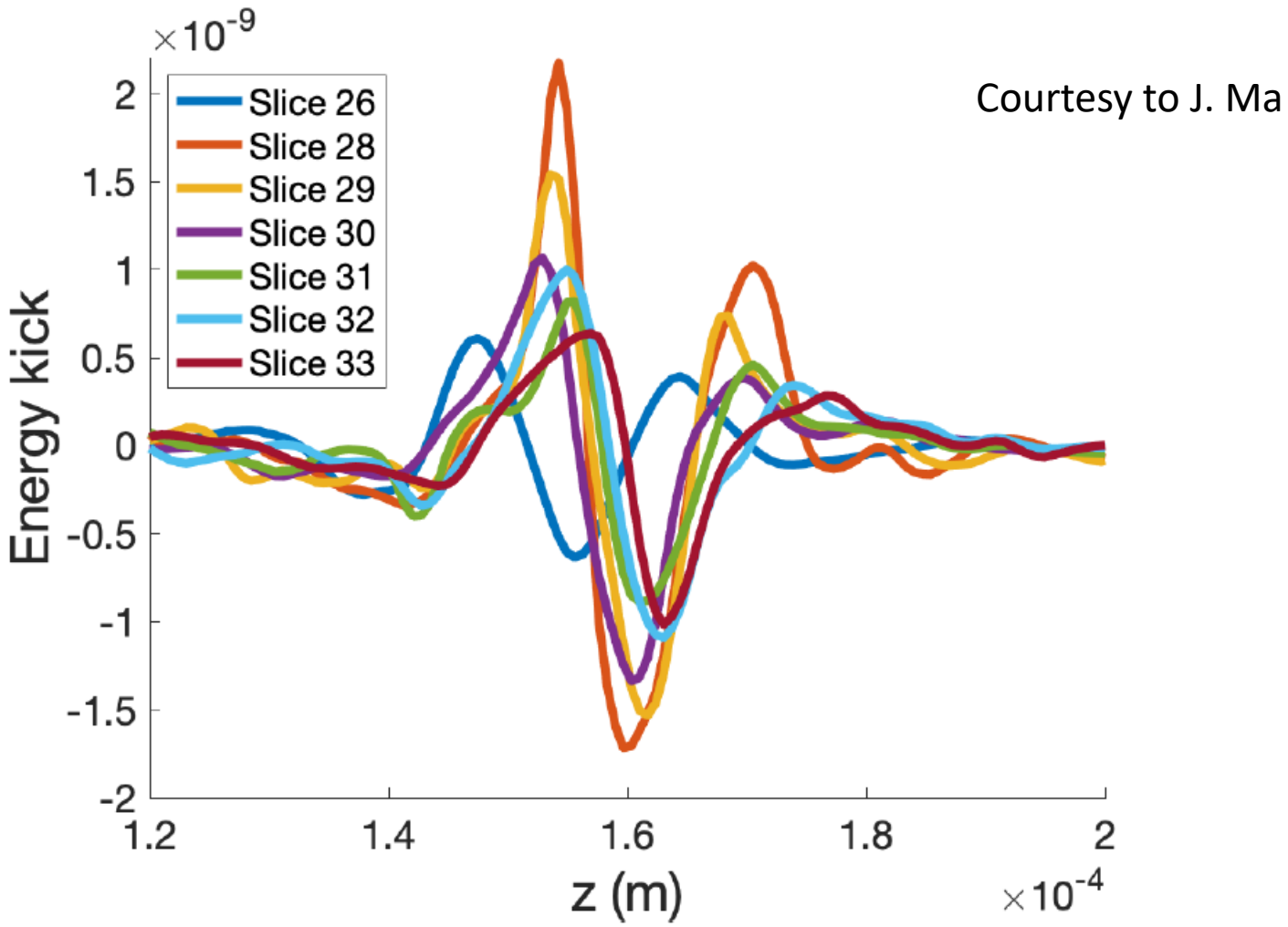
- Assuming the energy kick in CeC section has Gaussian dependance on the transverse offset of the ion with the R.M.S. width given by the R.M.S. size of the electron slice.

As a test, start from the ideal case...



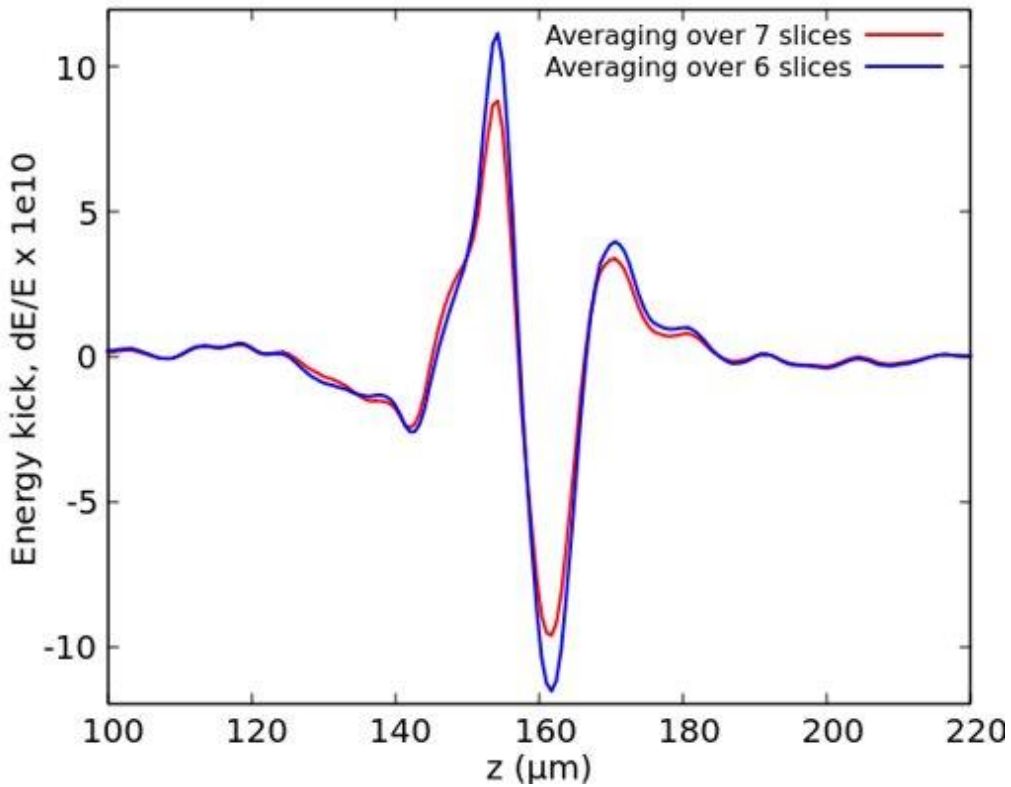


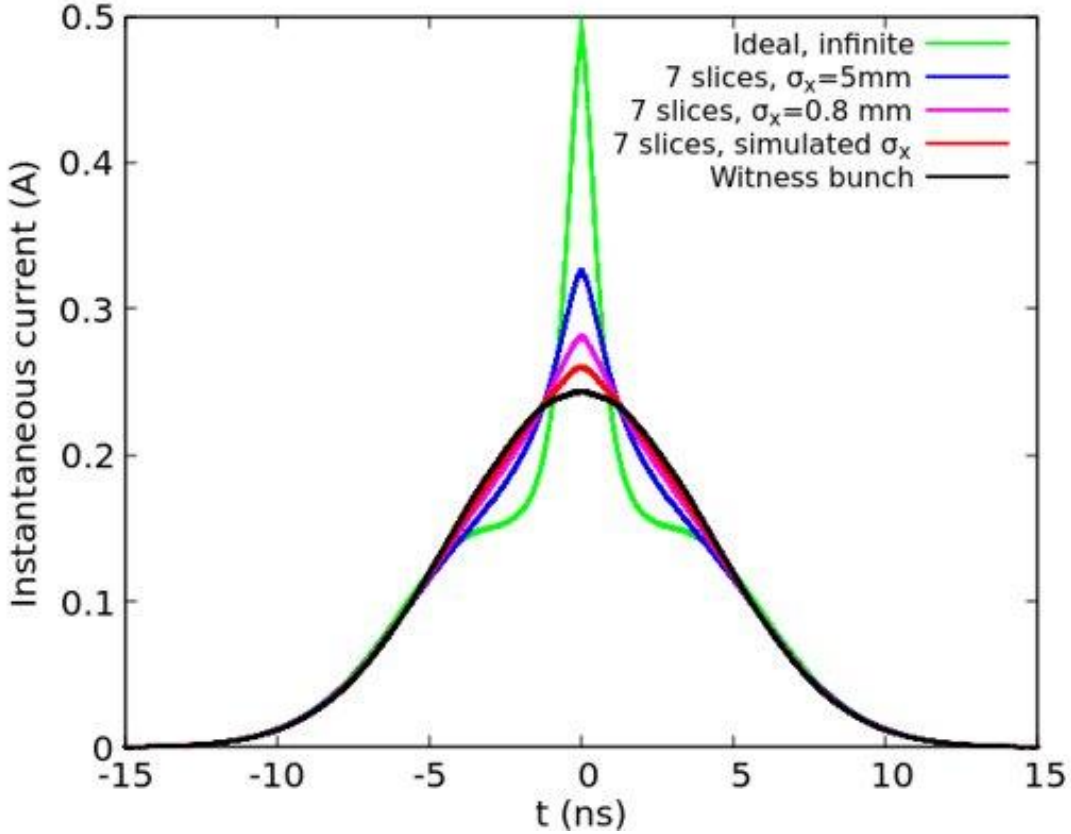
Then use the latest SPACE simulated energy kicks for 7 slices



Courtesy to J. Ma

Results for Solenoids Matched to e Slice with the Maximal Current





Summary on influences from transverse dependance of the cooling force

- If we assume Gaussian dependance of the cooling force on the transverse offset of the ion with the rms width given by that of the electron slices, the observable of the cooling effects, i.e. the enhancement of the peak current of the ion bunch w.r.t. the witness bunch, reduces by a factor of ~4 compared with the case when these effects are ignored.
- The significant reduction is related to the fact that the electron bunch is smaller than the ion bunch in the transverse plane (0.2-0.5 mm for the electrons vs 0.8 mm (should be 1 mm) for the ions). If we can make the transverse size of the electrons the same as that of the ions without reducing the cooling force at the center of the electron bunch, the cooling effect can be improved by a factor of 2.

Next step…

- Implement a better model for cooling dependance on the transverse offset: disk model and then SPACE simulation.

Disc model

$$\rho(r,\theta,z) = \rho_0 f_{//}(z) f_{\perp}(r)$$

$$\frac{1}{r}\left[\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\varphi(r,z)\right)\right] + \frac{\partial^2}{\partial z^2}\varphi(r,z) = \frac{\rho_0}{\varepsilon_0} f_{//}(z) f_{\perp}(r)$$

$$\frac{\partial^2}{\partial r^2}\phi + \frac{1}{r}\frac{\partial}{\partial r}\phi - k^2\phi = f(r)$$

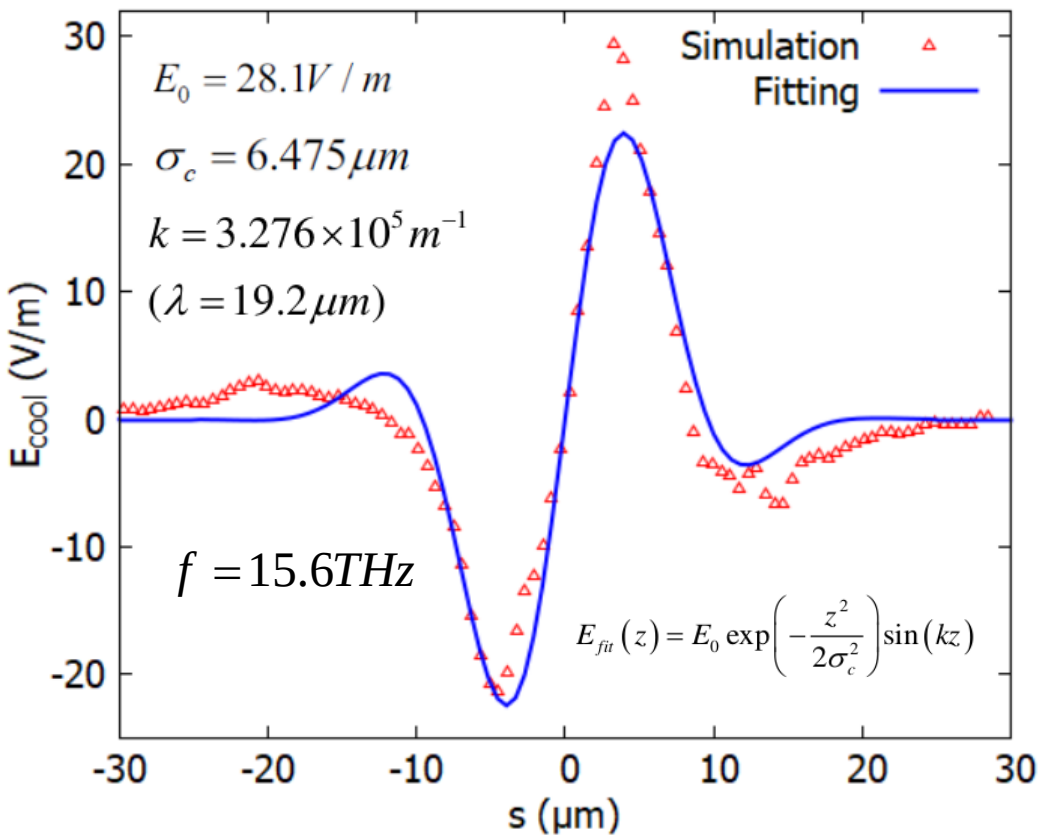
$$f(r) = \frac{\rho_0}{\varepsilon_0} f_{\perp}(r) \int_{-\infty}^{\infty} e^{-ikz} f_{//}(z)\,dz = \frac{\rho_0}{\varepsilon_0} f_{\perp}(r)\,\tilde{f}_{//}(k)$$

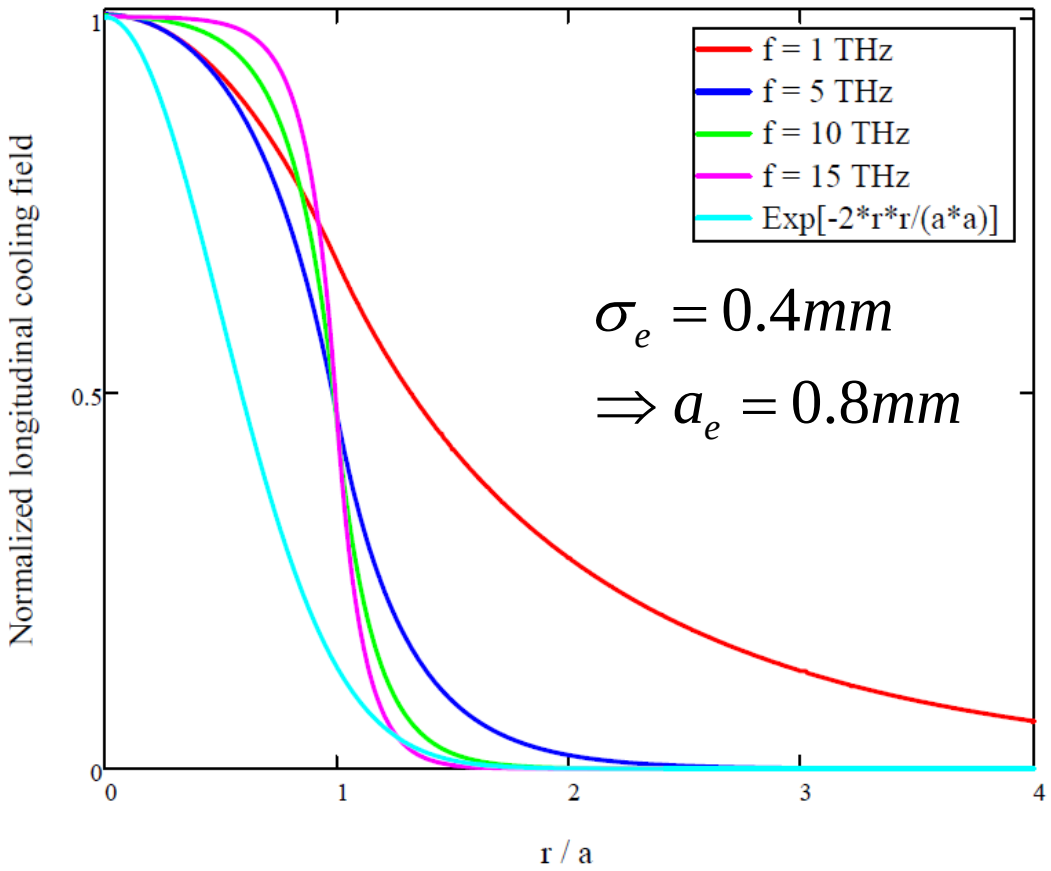
Flat disc $$f(r) = \frac{\rho_0}{\varepsilon_0}\tilde{f}_{//}(k)\frac{1}{\pi a^2}H(a-r)$$

$$E_z(r) = -ik\frac{\rho_0}{\pi\varepsilon_0}\tilde{f}_{//}(k)\left[I_0(kr)\int_{r/a}^{1}\eta H(1-\eta)K_0(ka\cdot\eta)d\eta + K_0(kr)\int_{0}^{r/a}\eta H(1-\eta)I_0(ka\cdot\eta)d\eta\right]$$

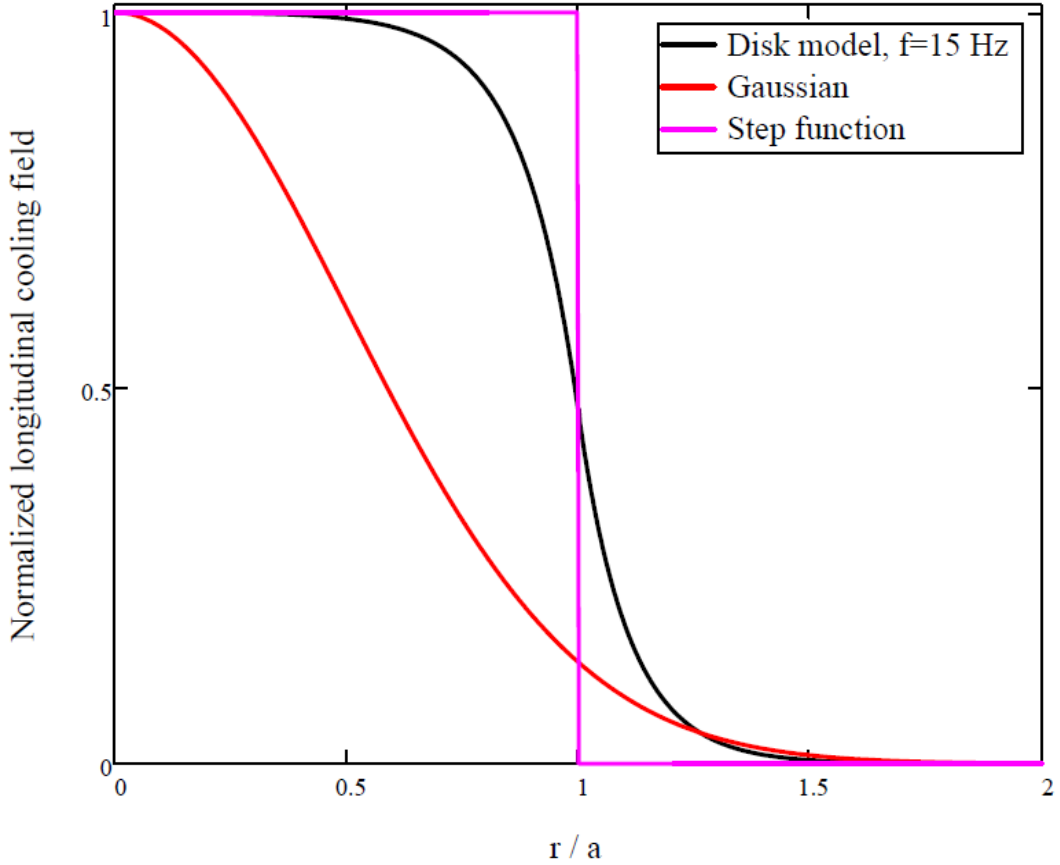
Dependance of Cooling Force on Transverse Offset: Disc Model (Flat Disc)

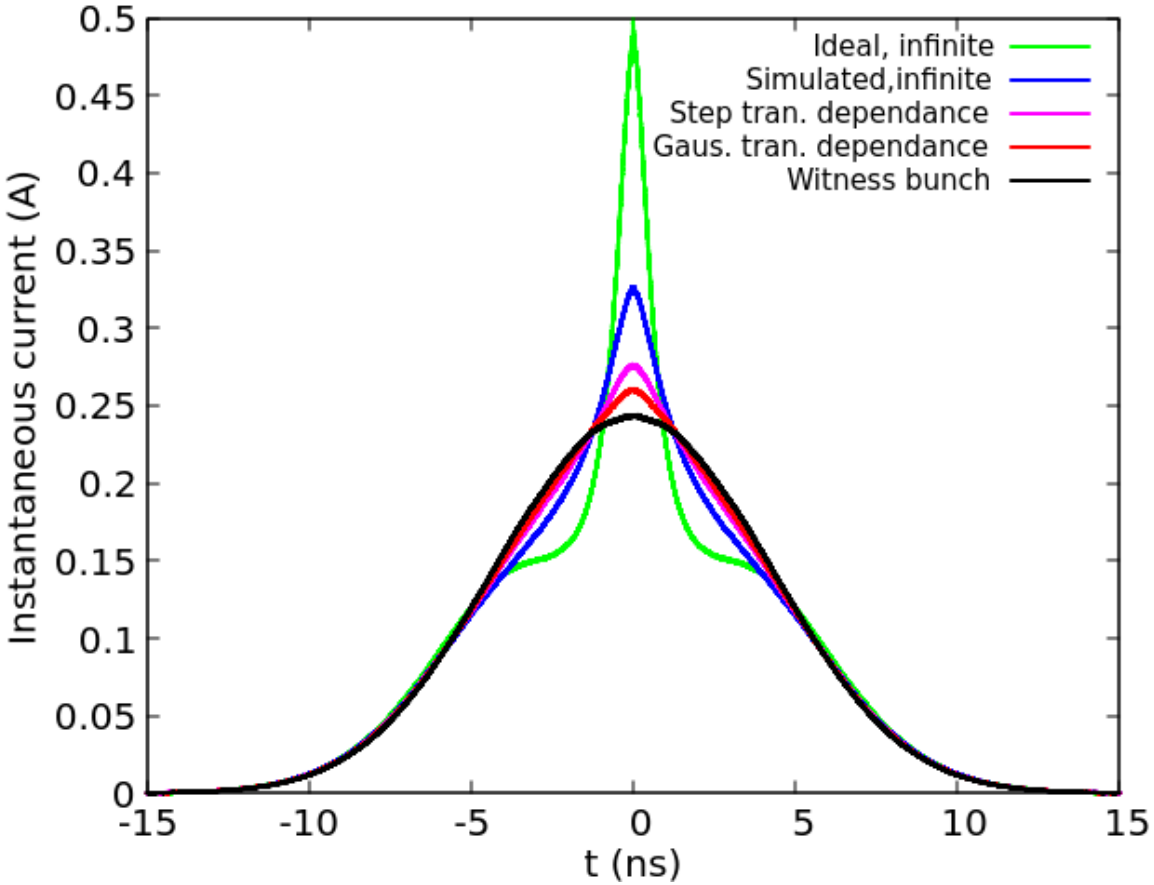
$$E_z(r) = -ik\frac{\rho_0}{\pi\varepsilon_0}\tilde{f}_{//}(k)\left[I_0(kr)\int_{r/a}^{1}\eta H(1-\eta)K_0(ka\cdot\eta)d\eta + K_0(kr)\int_{0}^{r/a}\eta H(1-\eta)I_0(ka\cdot\eta)d\eta\right]$$





Dependance of cooling force on transverse offset: step function





Disc model, Gauss disk

$$\rho(r,\theta,z)=\rho_0 f_{//}(z)\, f_{\perp}(r)$$

$$\frac{1}{r}\left[\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\varphi(r,z)\right)\right]+\frac{\partial^2}{\partial z^2}\varphi(r,z)=\frac{\rho_0}{\varepsilon_0} f_{//}(z)\, f_{\perp}(r)$$

$$\frac{\partial^2}{\partial r^2}\phi+\frac{1}{r}\frac{\partial}{\partial r}\phi-k^2\phi=f(r)$$

$$f(r)=\frac{\rho_0}{\varepsilon_0} f_{\perp}(r)\int_{-\infty}^{\infty} e^{-ikz} f_{//}(z)\,dz=\frac{\rho_0}{\varepsilon_0} f_{\perp}(r)\,\tilde{f}_{//}(k)$$

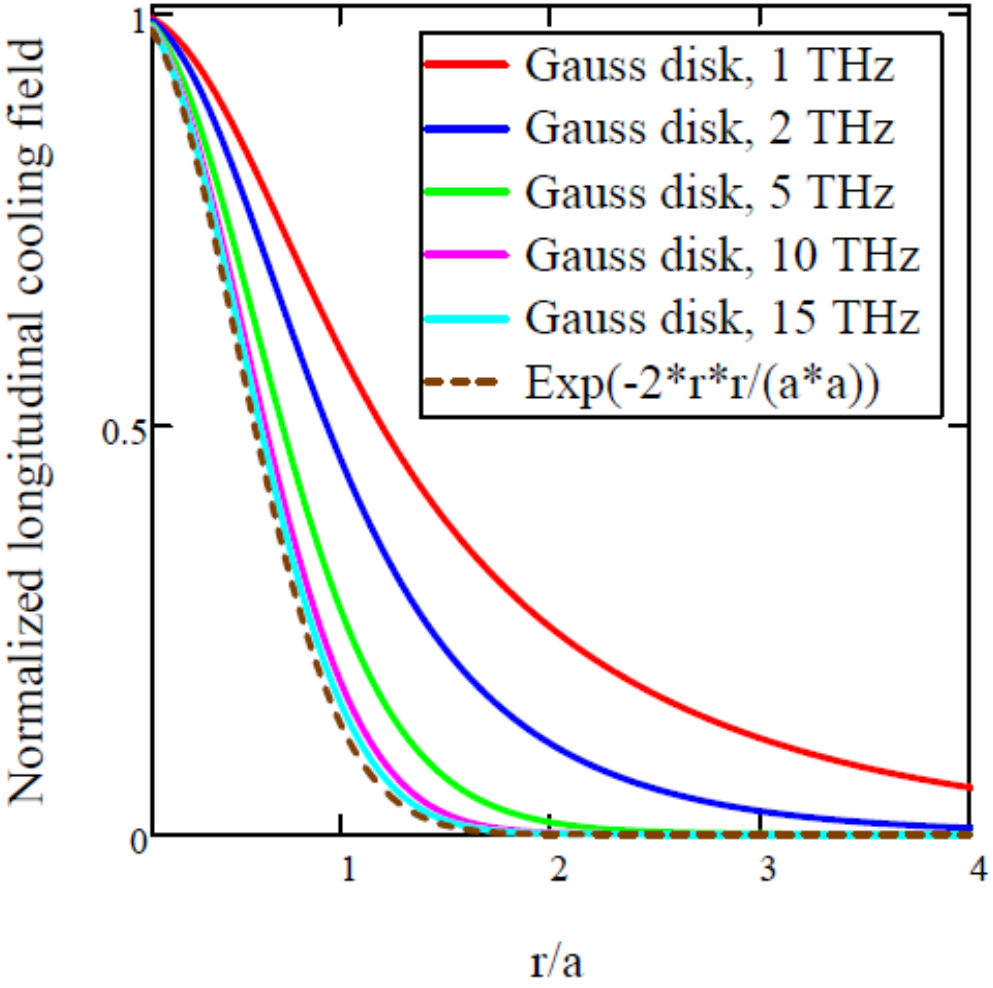
$$f(r)=\frac{\rho_0}{\varepsilon_0}\tilde{f}_{//}(k)\frac{1}{2\pi\sigma_r^2}\exp\left(-\frac{r^2}{2\sigma_r^2}\right) \qquad \sigma_x=\sigma_y=\sigma_r$$

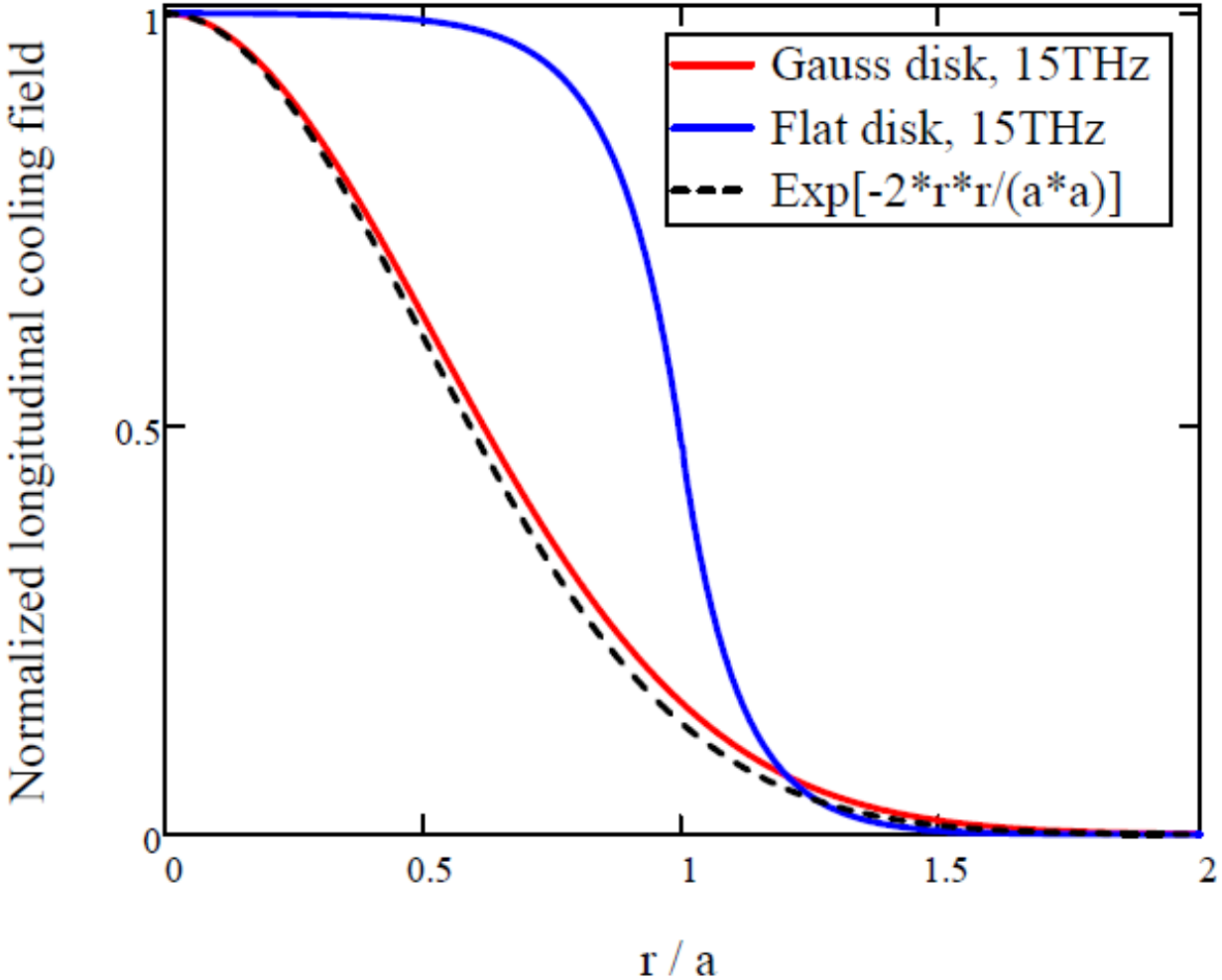
$$\tilde{E}_z(r,k)=-ik\frac{\rho_0\tilde{f}_{//}(k)}{2\pi\varepsilon_0\sigma_r^2}\left\{I_0(kr)\int_{\infty}^{r}\xi K_0(k\xi)\mathrm{e}^{-\frac{\xi^2}{2\sigma_r^2}}\,d\xi-K_0(kr)\int_{0}^{r}\xi I_0(k\xi)\mathrm{e}^{-\frac{\xi^2}{2\sigma_r^2}}\,d\xi\right\} \qquad \bar{r}\equiv\frac{r}{2\sigma_r}$$

$$=-ik\frac{2\rho_0\tilde{f}_{//}(k)}{\pi\varepsilon_0}\left\{I_0(\bar{k}\bar{r})\int_{\infty}^{\bar{r}}\bar{\xi}K_0(\bar{k}\bar{\xi})\mathrm{e}^{-2\bar{\xi}^2}\,d\bar{\xi}-K_0(\bar{k}\bar{r})\int_{0}^{\bar{r}}\bar{\xi}I_0(\bar{k}\bar{\xi})\mathrm{e}^{-2\bar{\xi}^2}\,d\bar{\xi}\right\} \qquad \bar{k}\equiv 2k\sigma_r$$

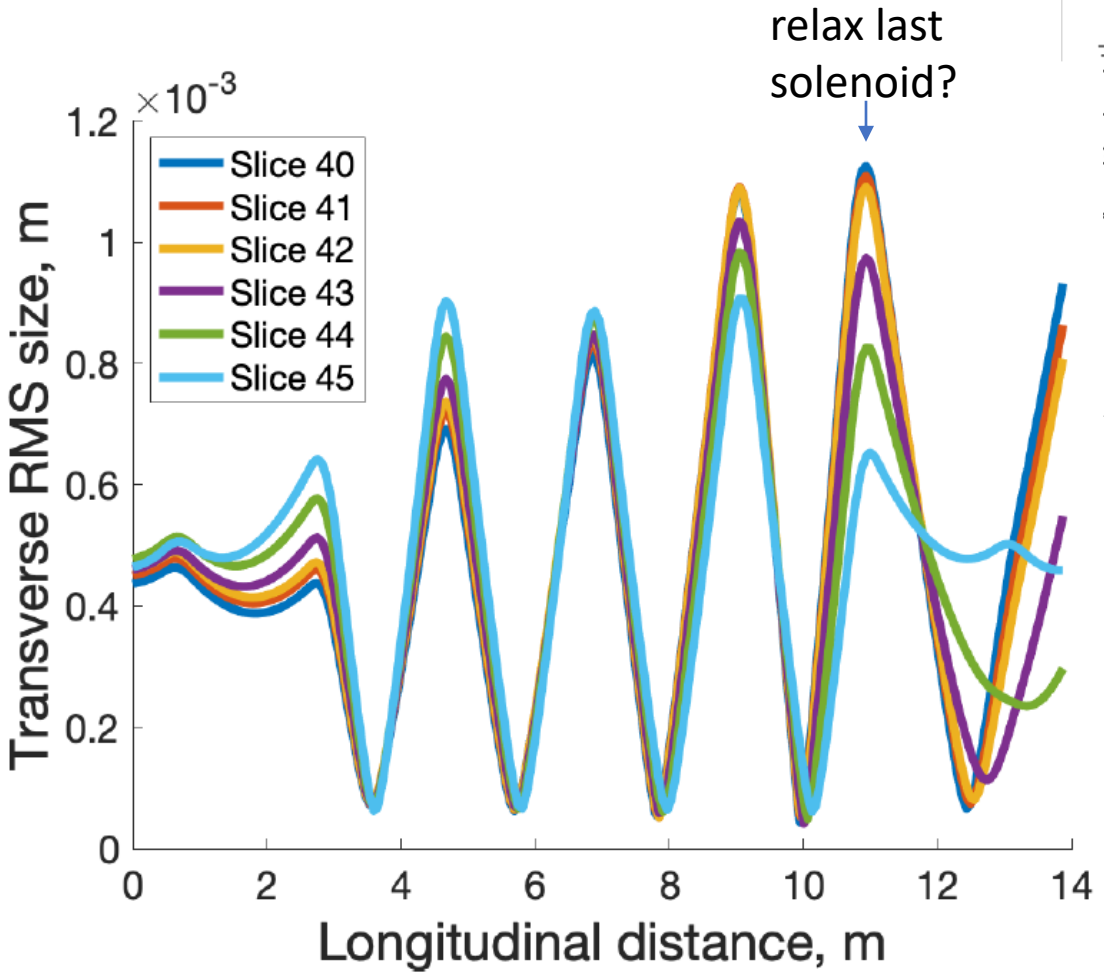
Disc Model, Gauss disk

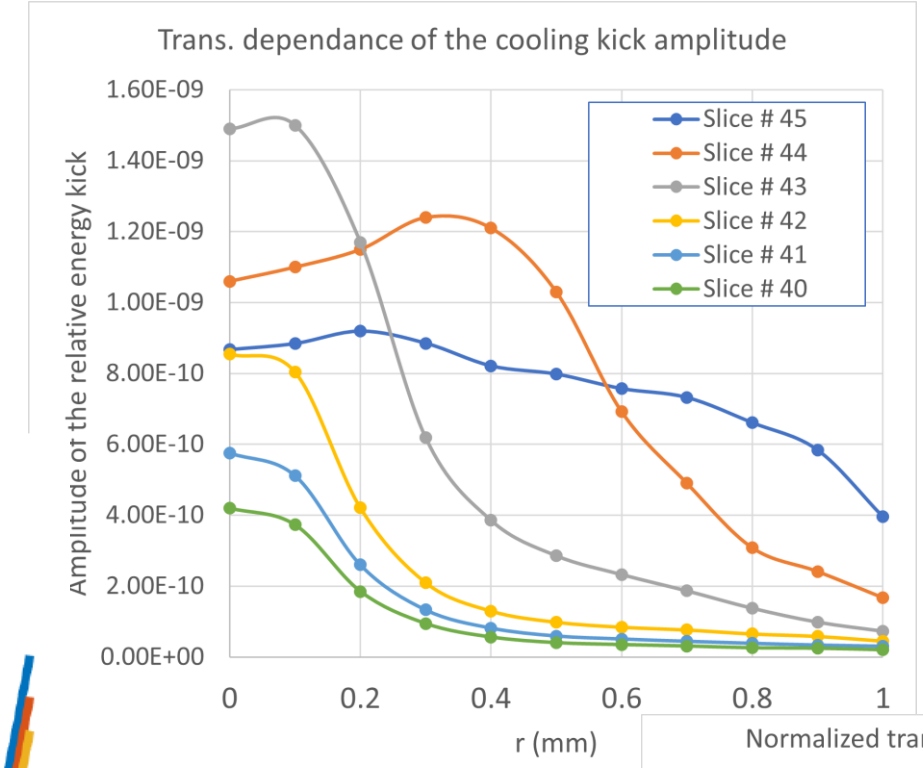
To compare Gaussian disk with flat disk, we take $\sigma_r = a/2$ with a being the radius of a flat disk. By doing this, the two disk models have the same rms transverse spatial size.



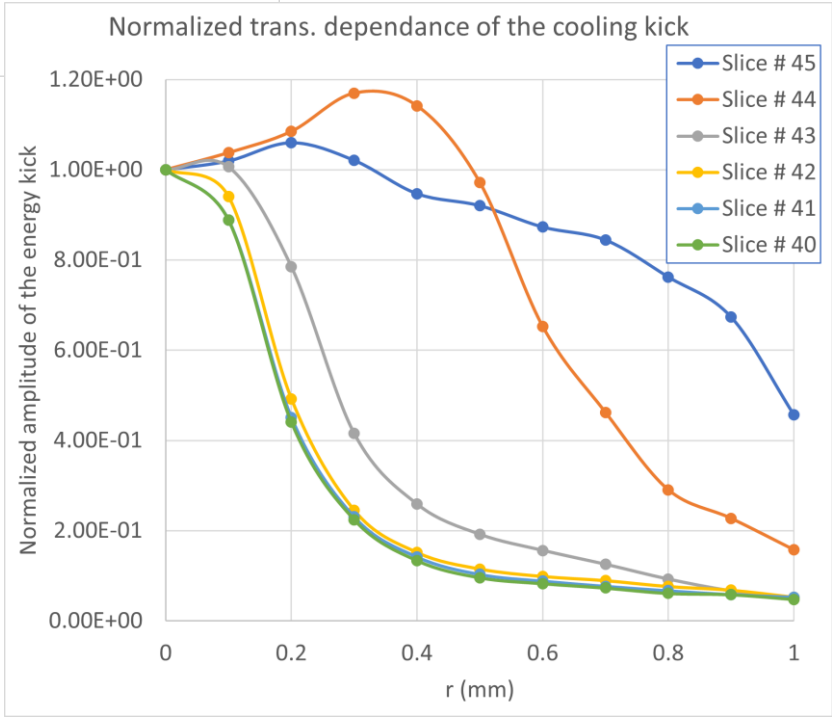


Simulated dependance for time-varying transverse profile



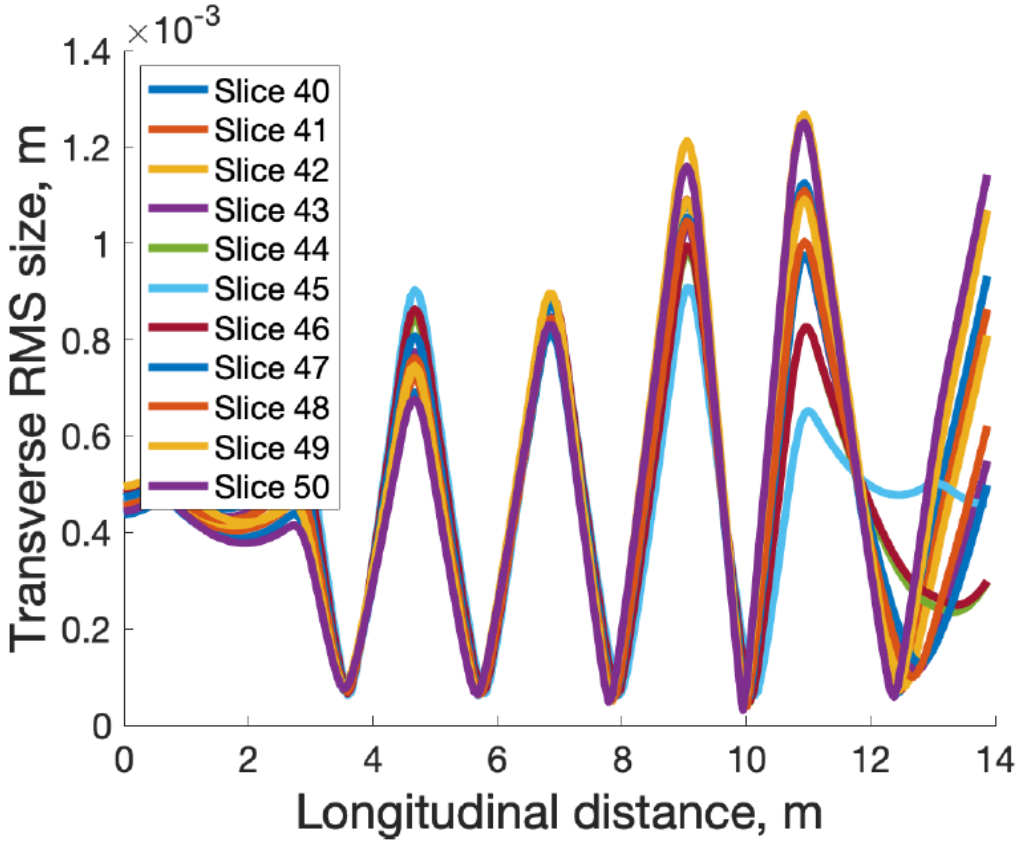


Going through an additional waist can reduce the integrated energy kick significantly except for the ions with $r < 0.25\sigma_r$.

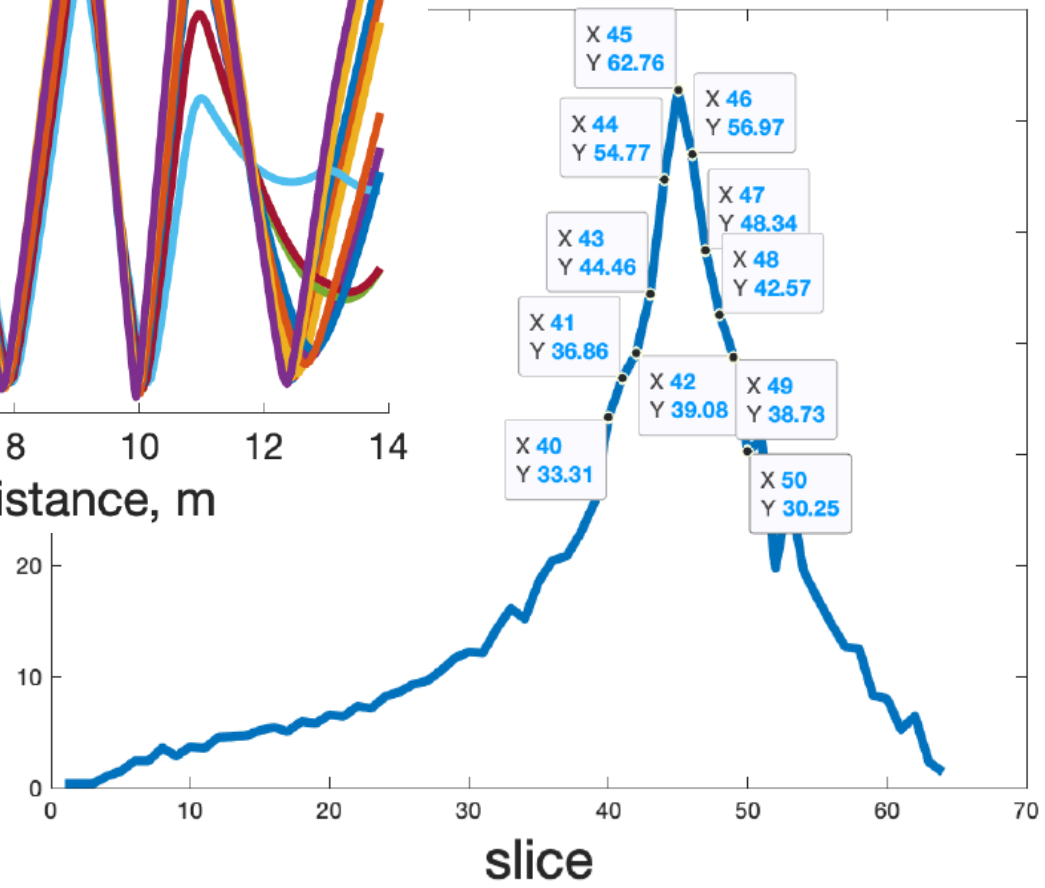


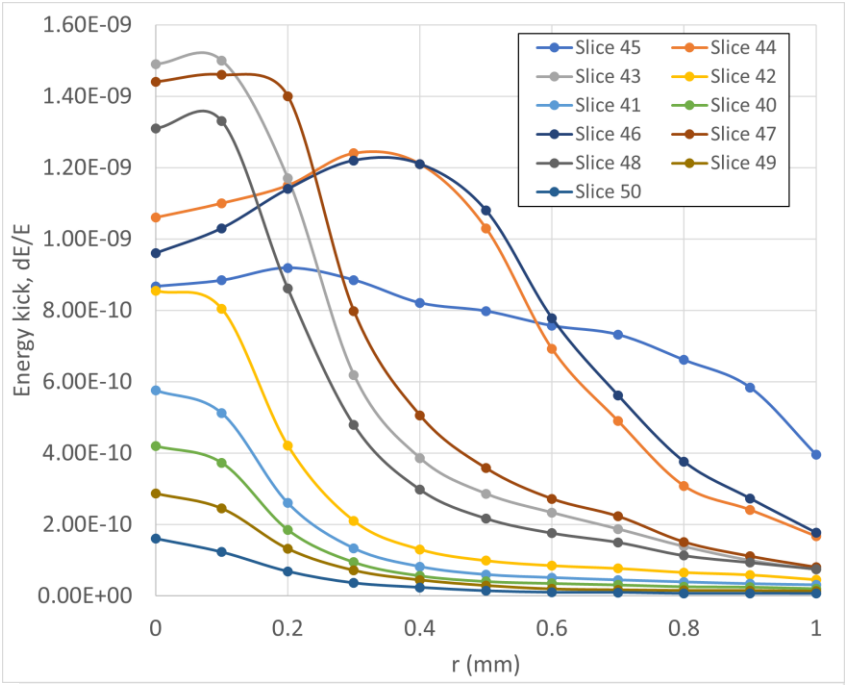
All simulated slices

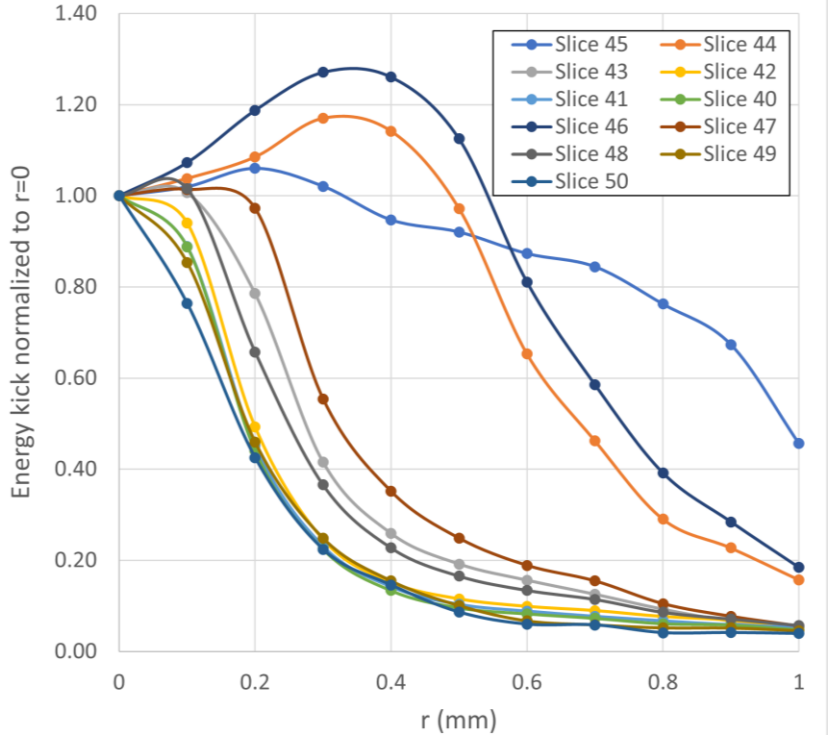
Amplitude of the energy kick, i.e. $(dE_{MAX} - dE_{MIN})/(2E_0)$



Peak current

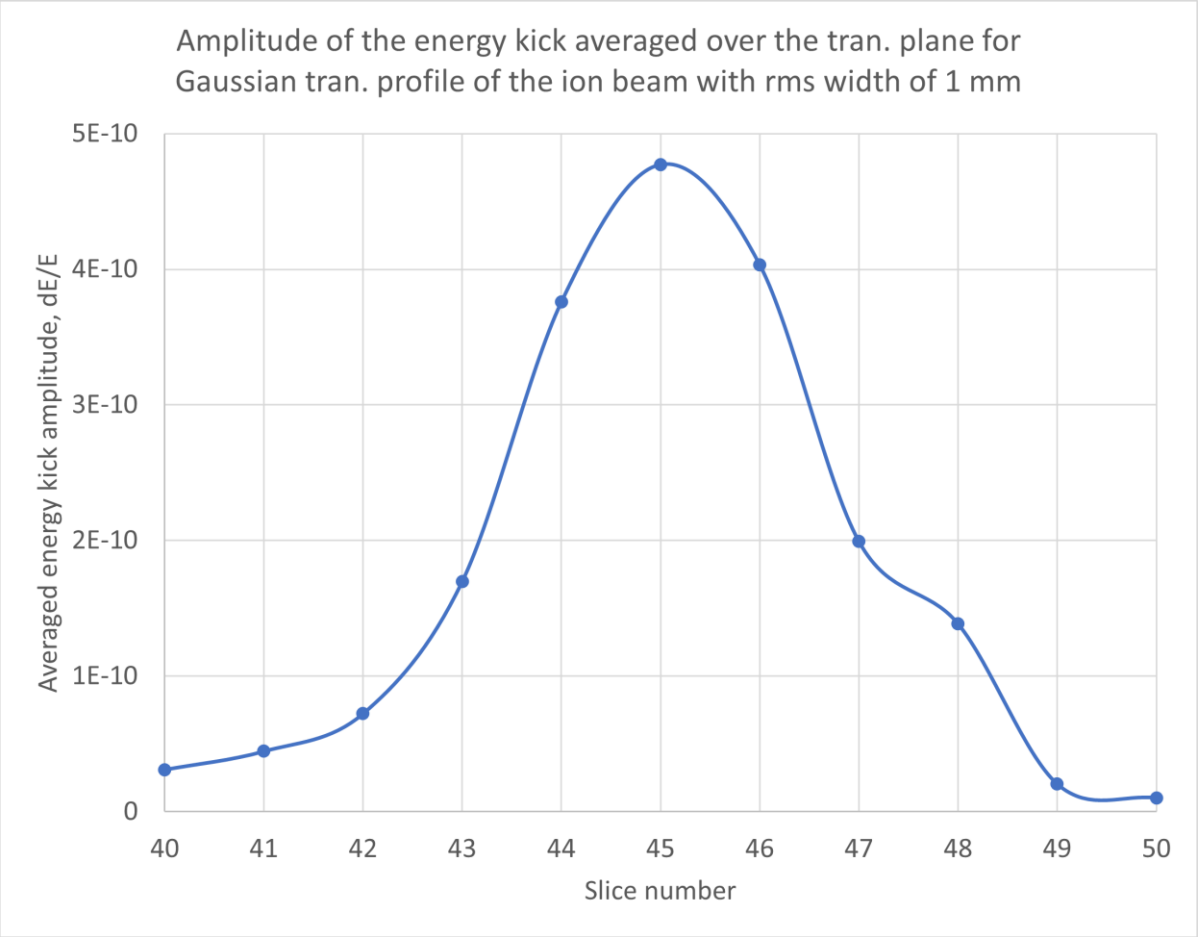




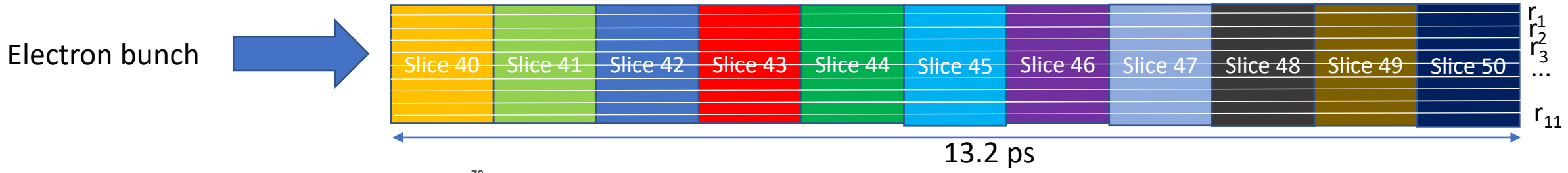


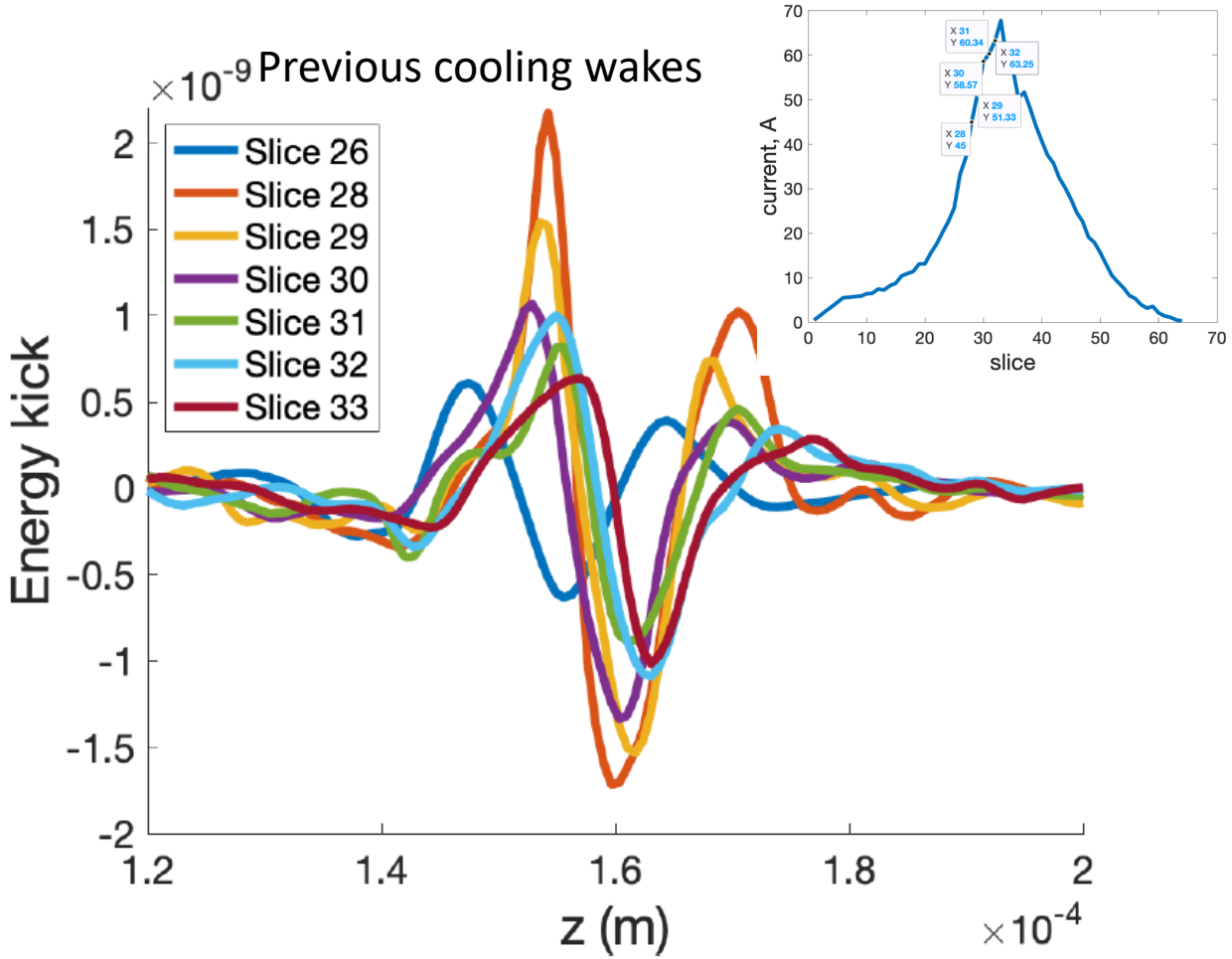
Averaged over the transverse profile of the ion bunch with RMS size of 1mm

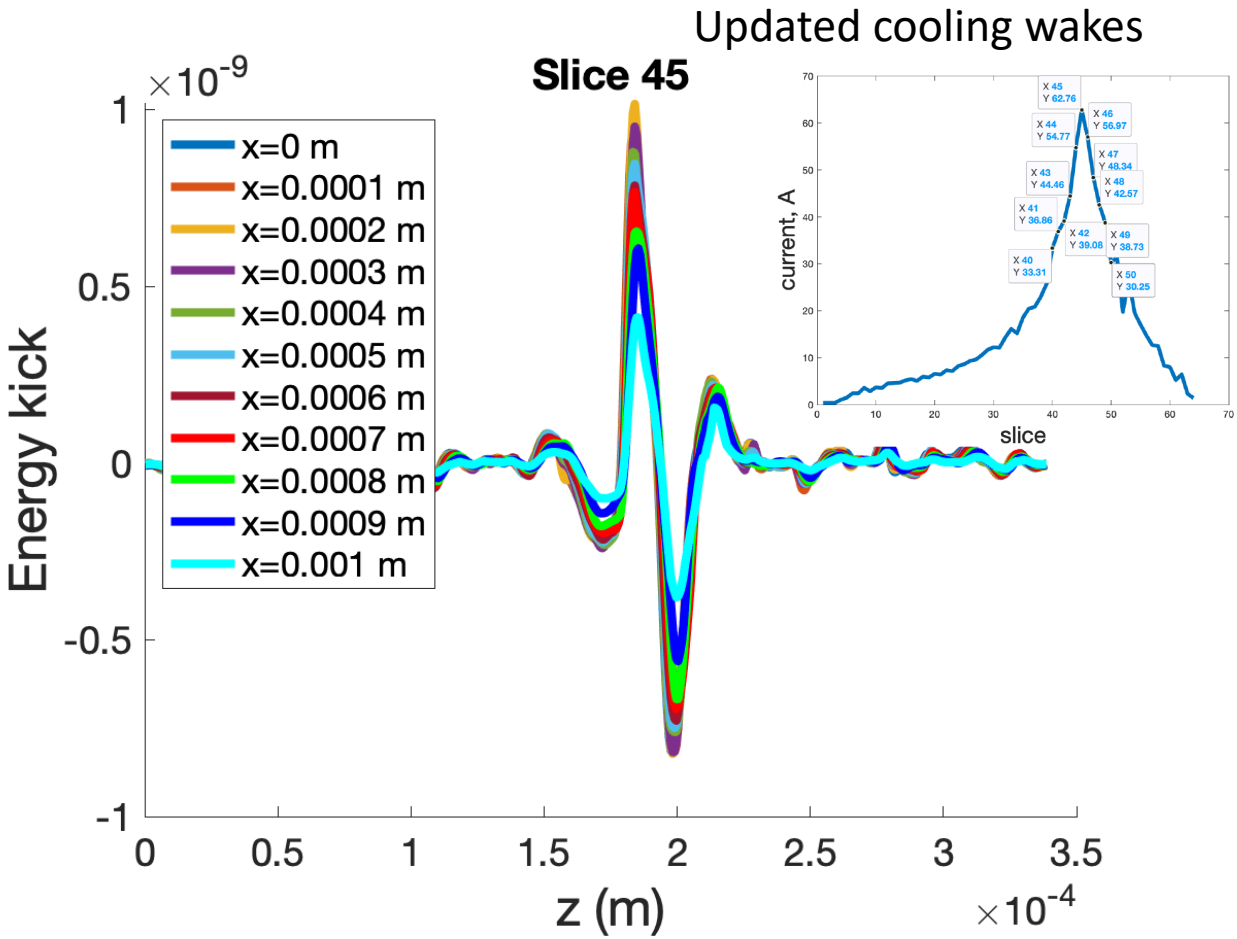
$$E_{i_{slice}} = 2\pi \sum_{j=1}^{10} \frac{E_{i_{slice}}\left(r_j\right) + E_{i_{slice}}\left(r_{j+1}\right)}{2} \frac{1}{2\pi\sigma_{ion}^2} \exp\left[-\frac{1}{2\sigma_{ion}^2}\left(\frac{r_j + r_{j+1}}{2}\right)^2\right] \cdot \left(\frac{r_j + r_{j+1}}{2}\right) \cdot \Delta r \qquad \Delta r = r_{j+1} - r_j$$



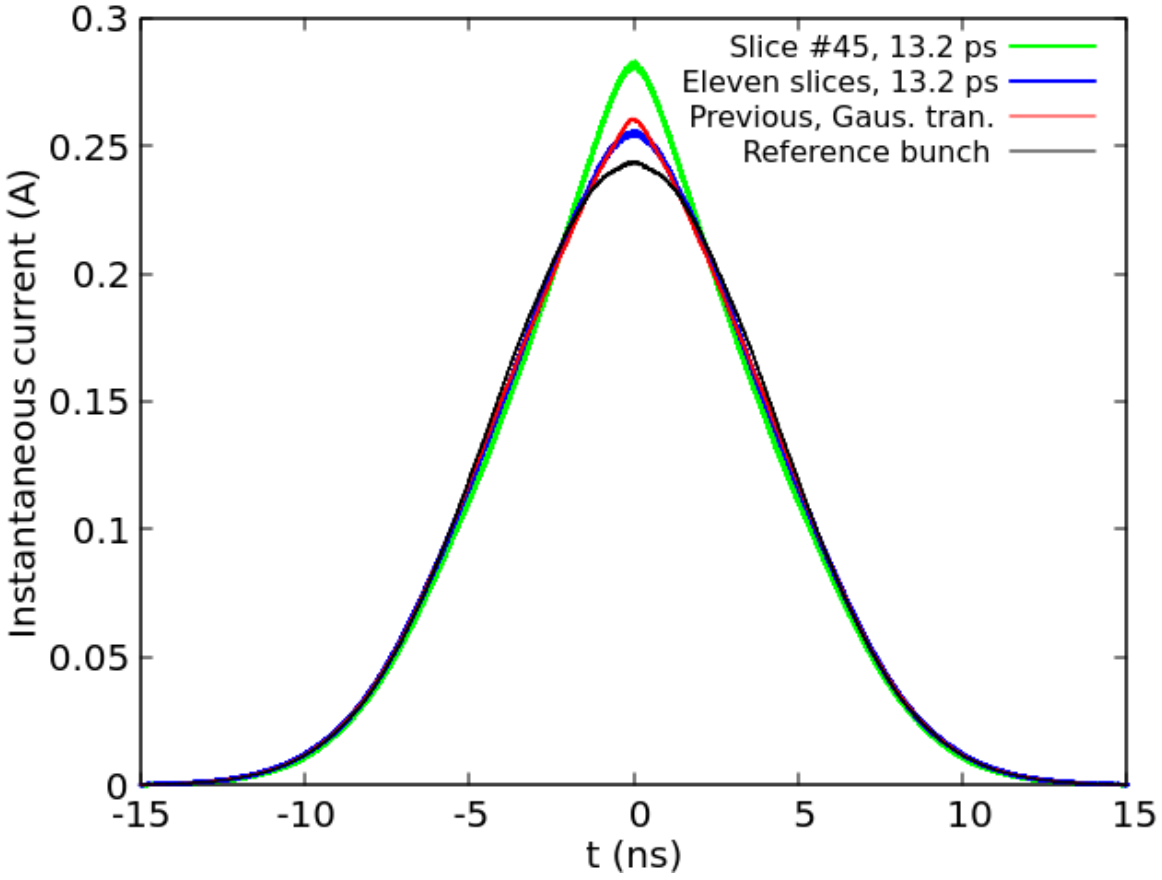
What has been done differently in the ion tracking code…

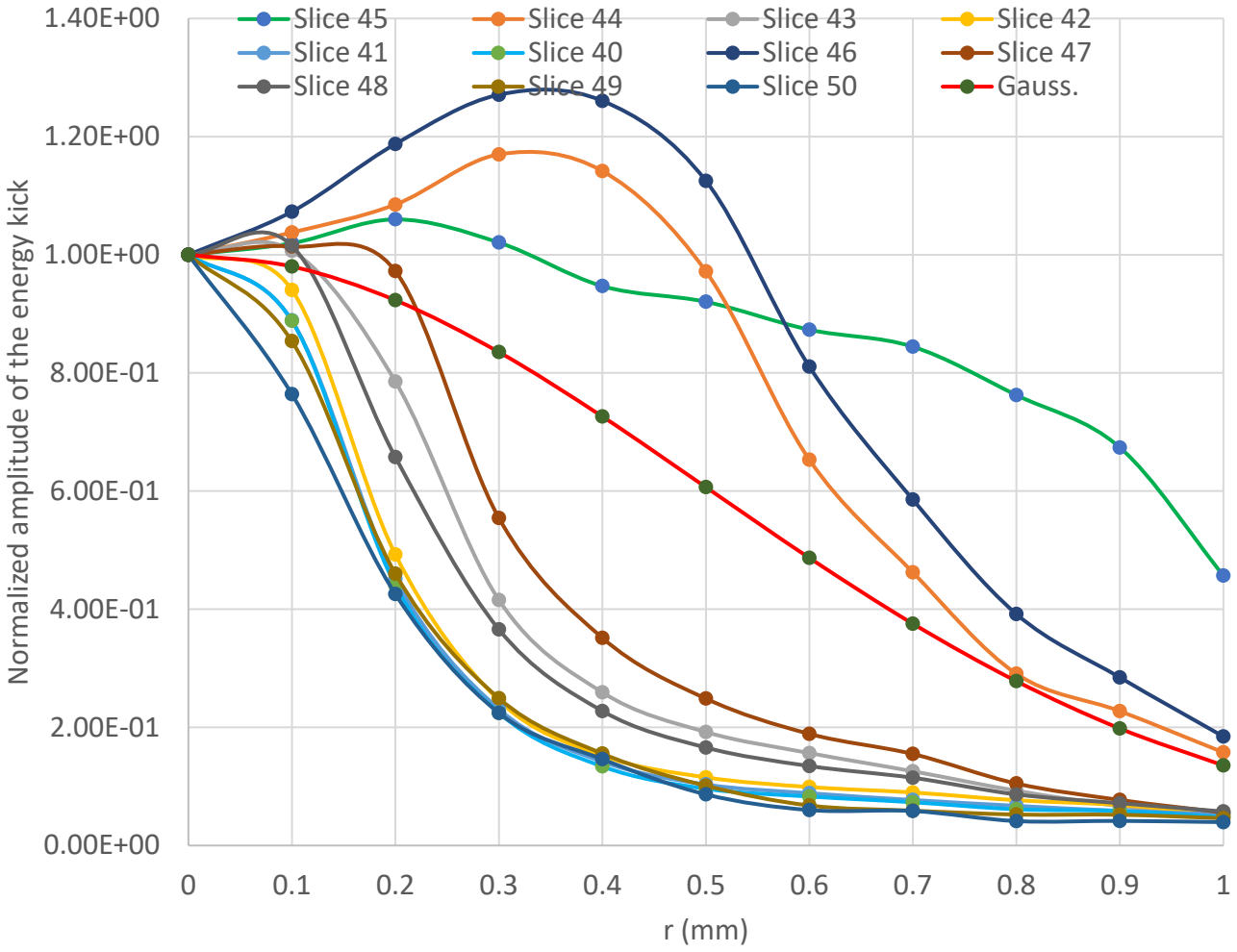






Results from ion tracking with 2D interpolation

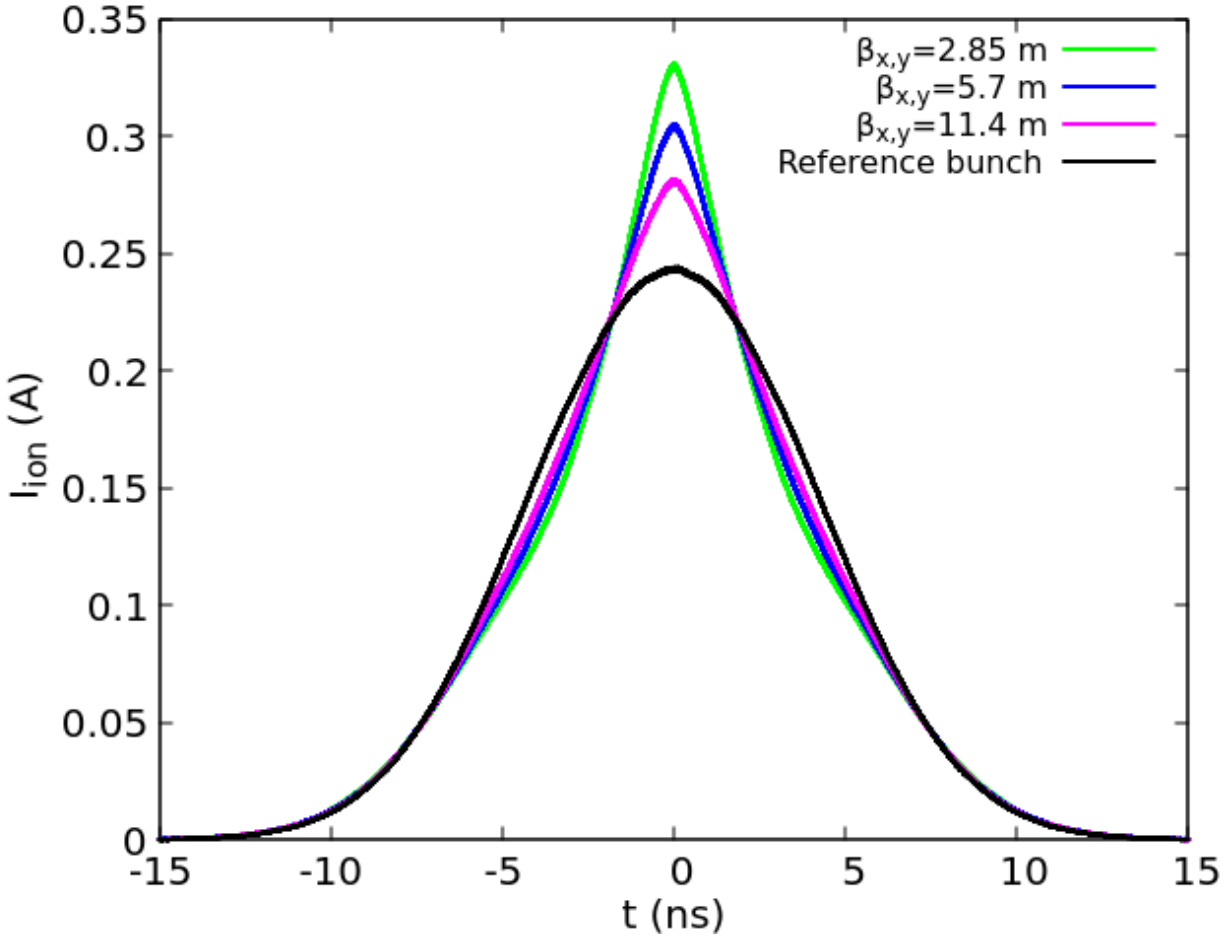


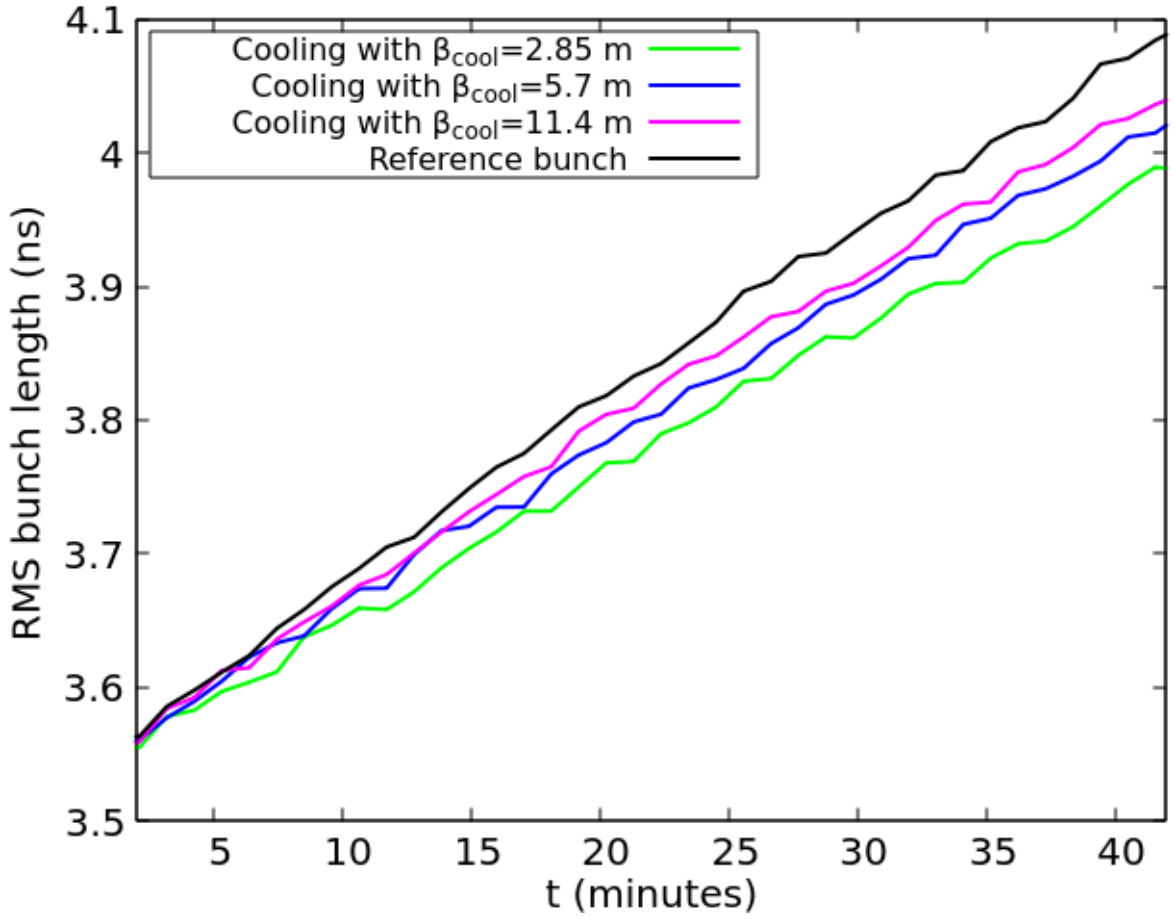


NEXT STEPS

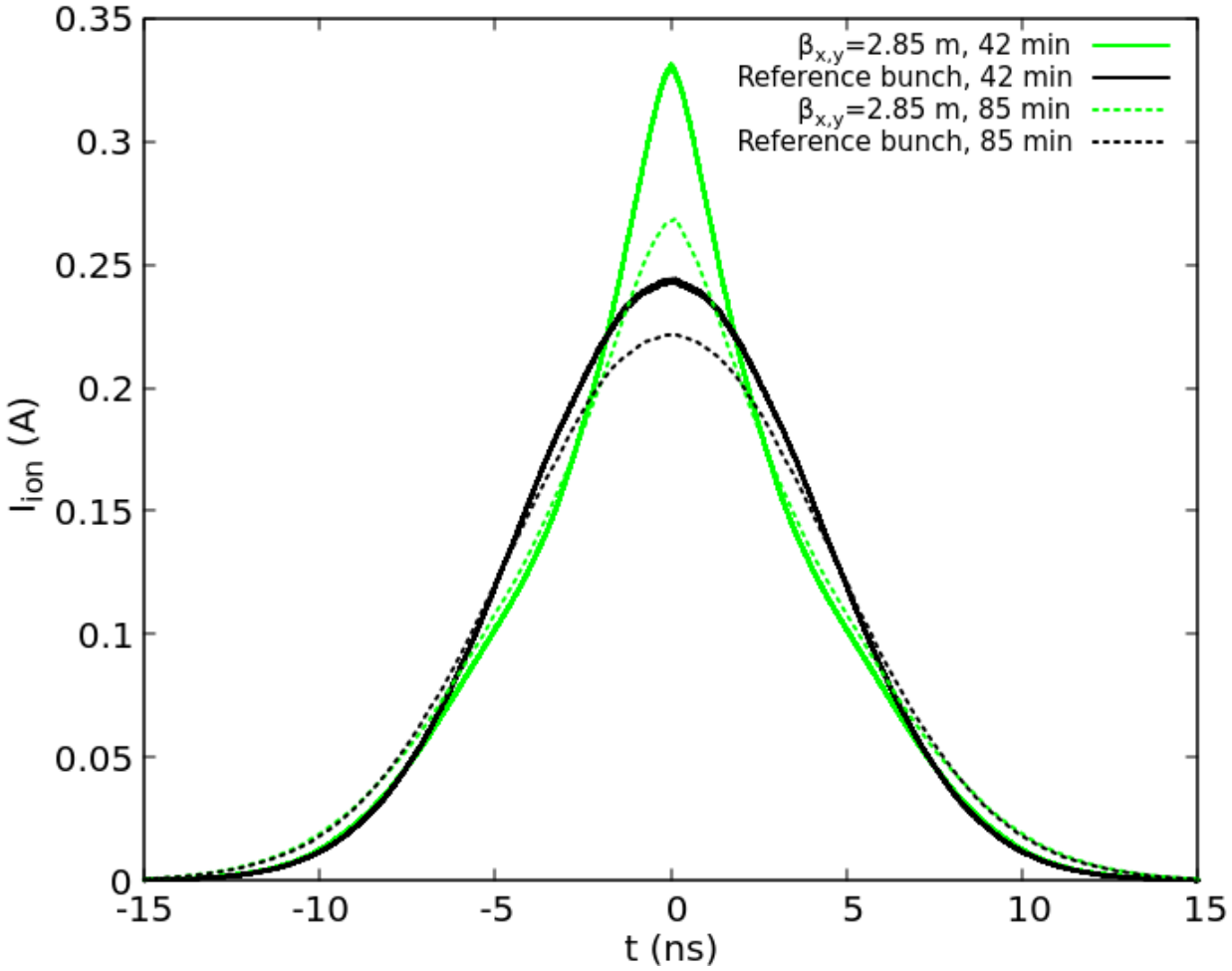
- Continue optimizing electron bunch for both high gain at bunch center and better uniformity over the whole bunch; If possible, try to increase beta function at modulator and kicker (Plasma phase advance at modulator is 1.1 rad with rms beam size of 1mm and 50A of peak current). (Yichao and Jun)
- Investigate how the cooling performance varies with the emittance of the ion bunch and explore possibilities of reduce transverse emittance of the RHIC ion beam. (Gang)
- Introduce transverse offset of the ion at both the modulator and the kicker section. Obtain cooling wakes for ions with various transverse offset through the cooling section (Jun). Investigate how the cooling performance is affected by the transverse offset (Gang).
- Explore possibilities of unsymmetric IR2 at the modulator and kicker to optimize cooling performance. (Gang, Jun and Yichao)

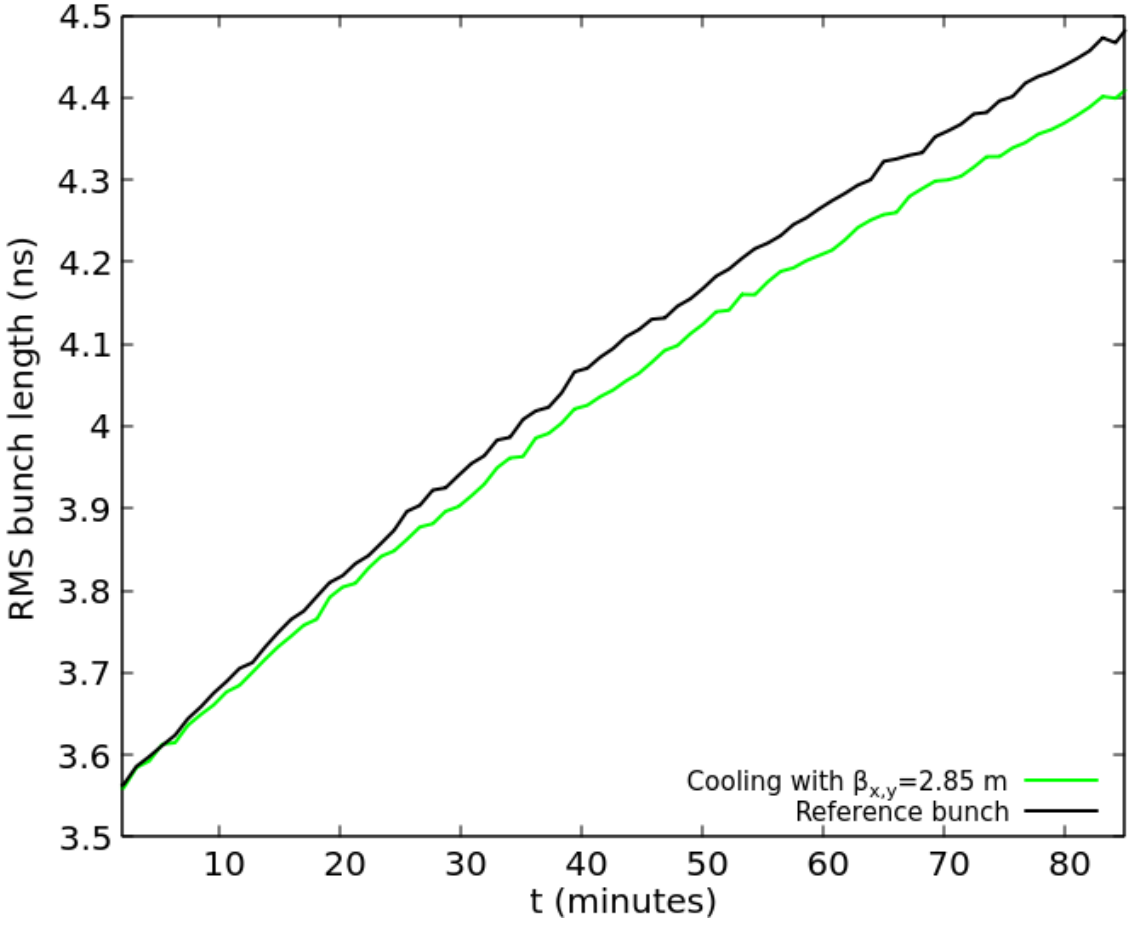
Influence of β_{ion} at the kicker section on the cooling performance
Wakes from slice #45 with 13.2 ps of duration



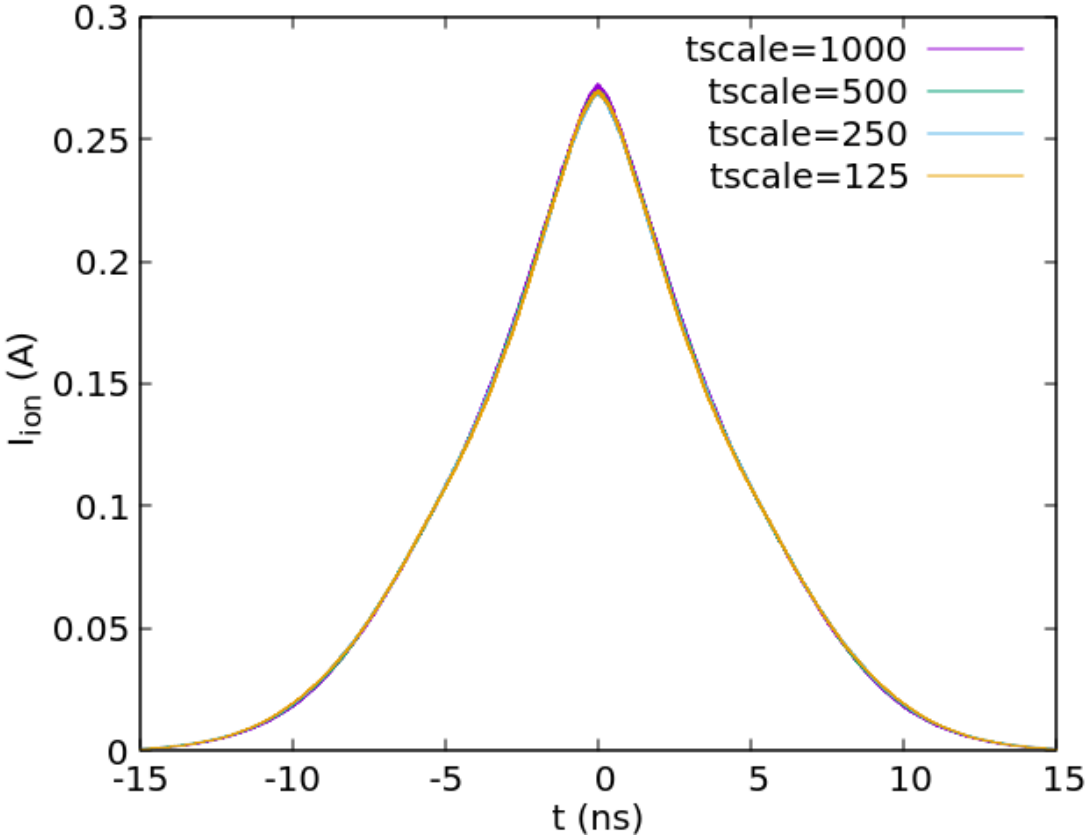


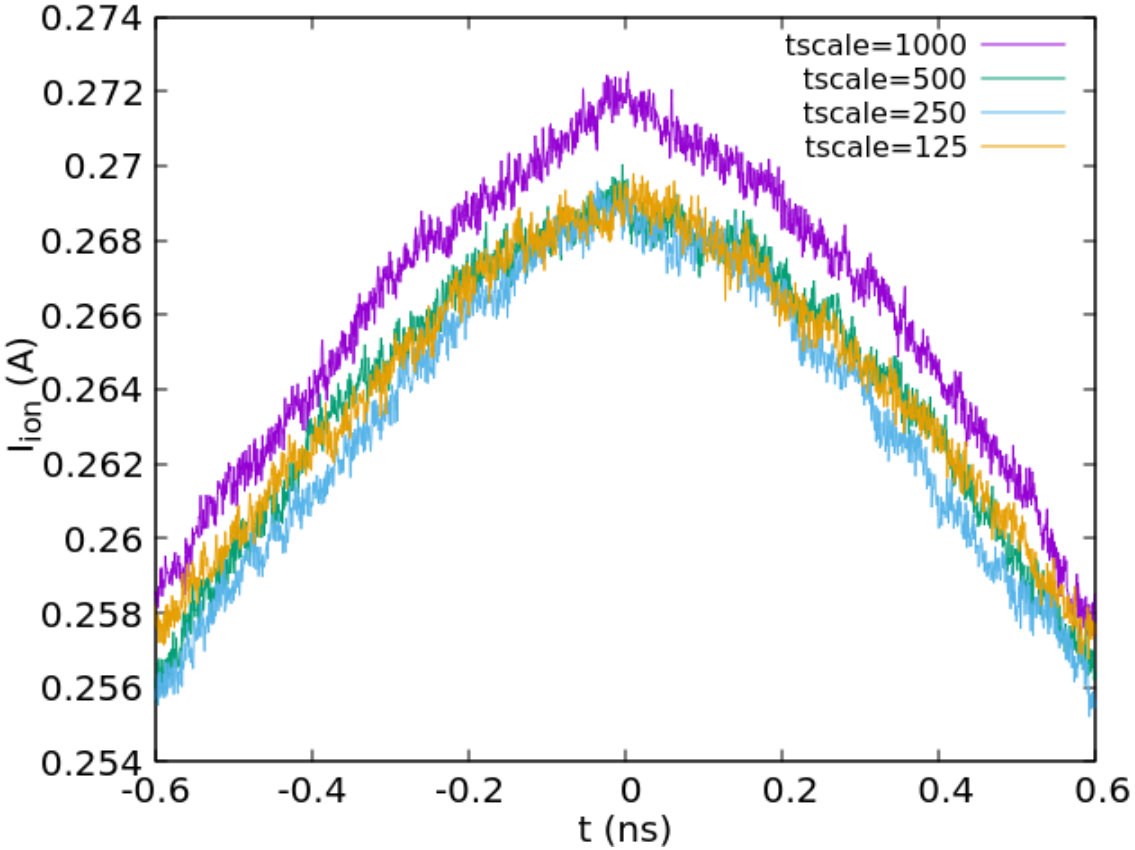
Cooling longer with 2.85 m of beta function
(Incorrect, 11.4 meters of beta function was used for 85 min results)



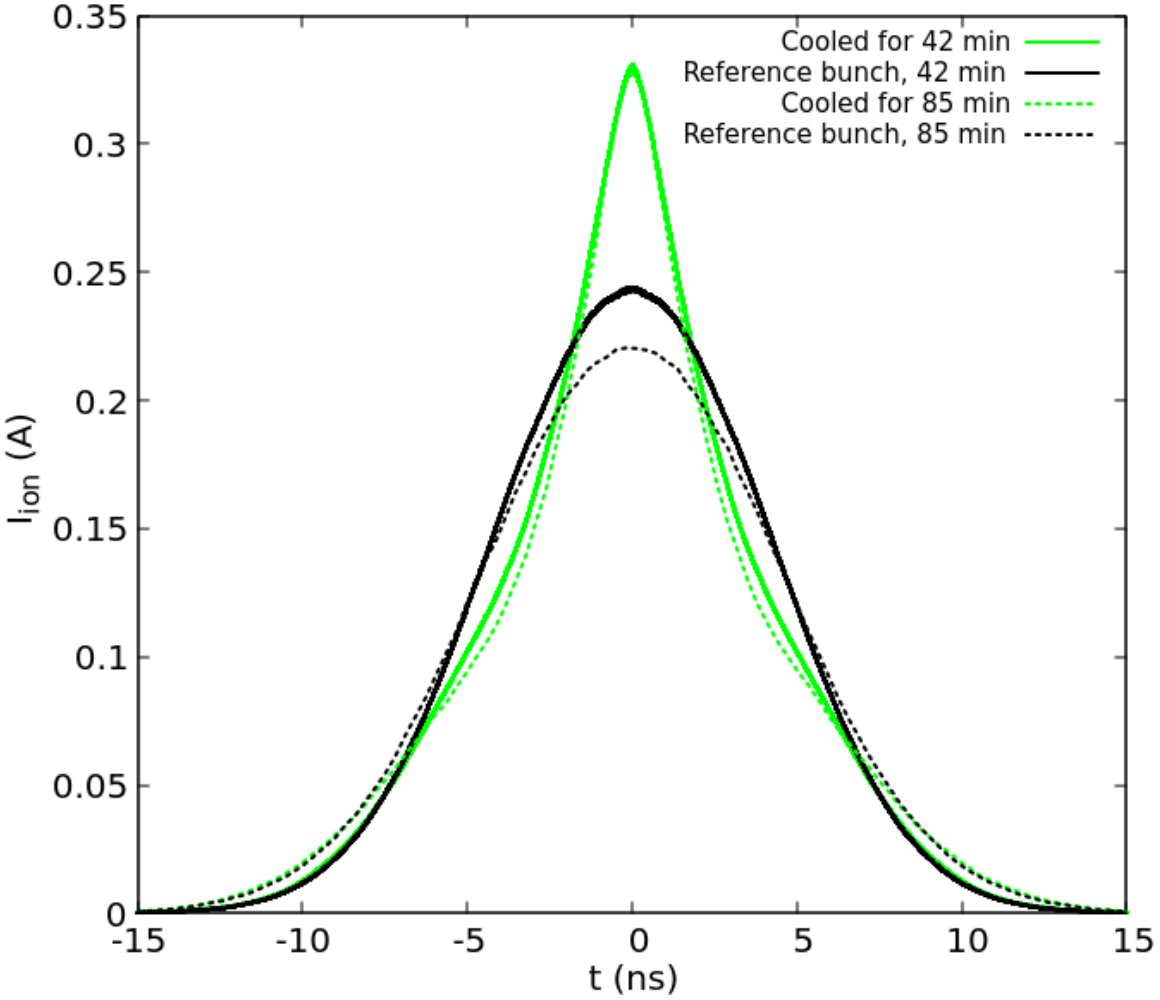


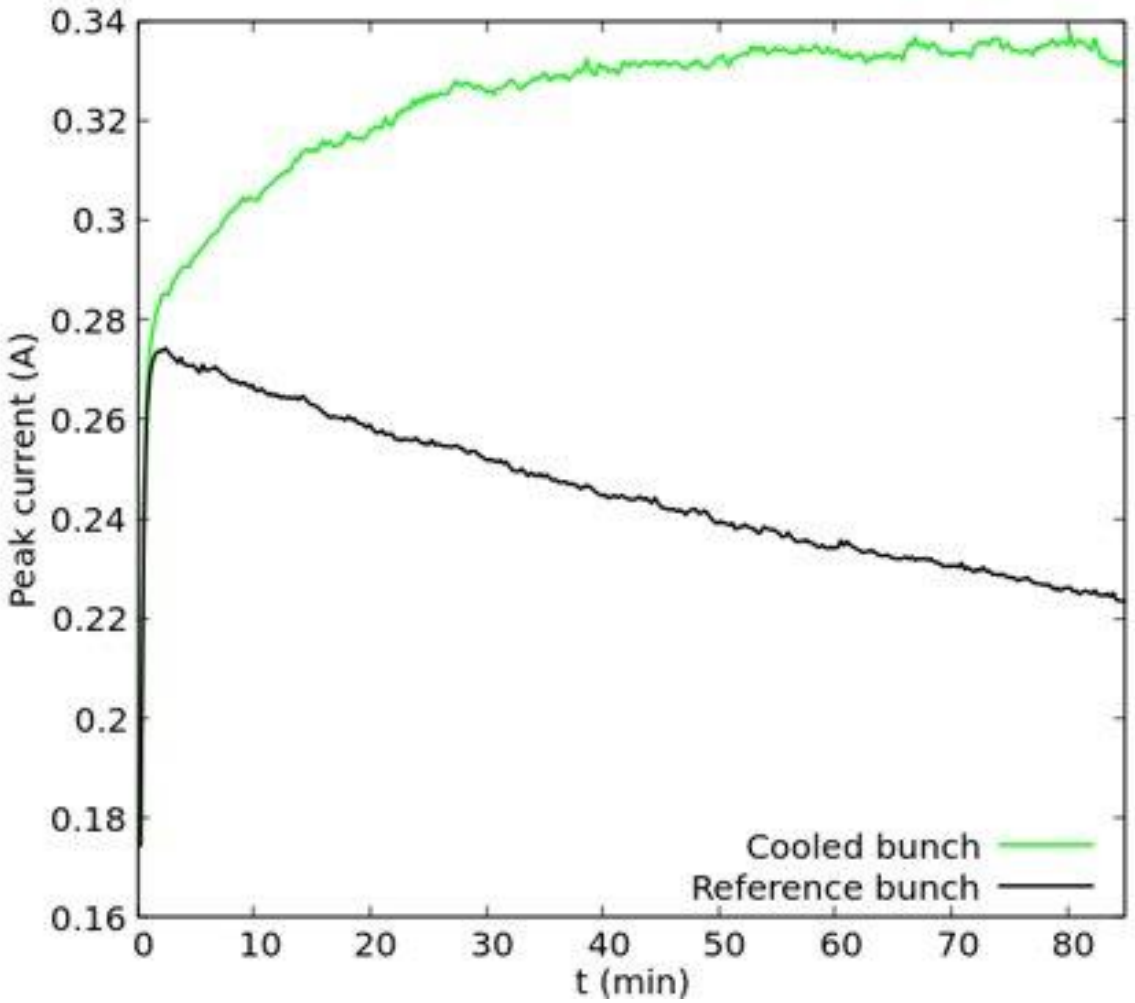
Check convergence with tscale



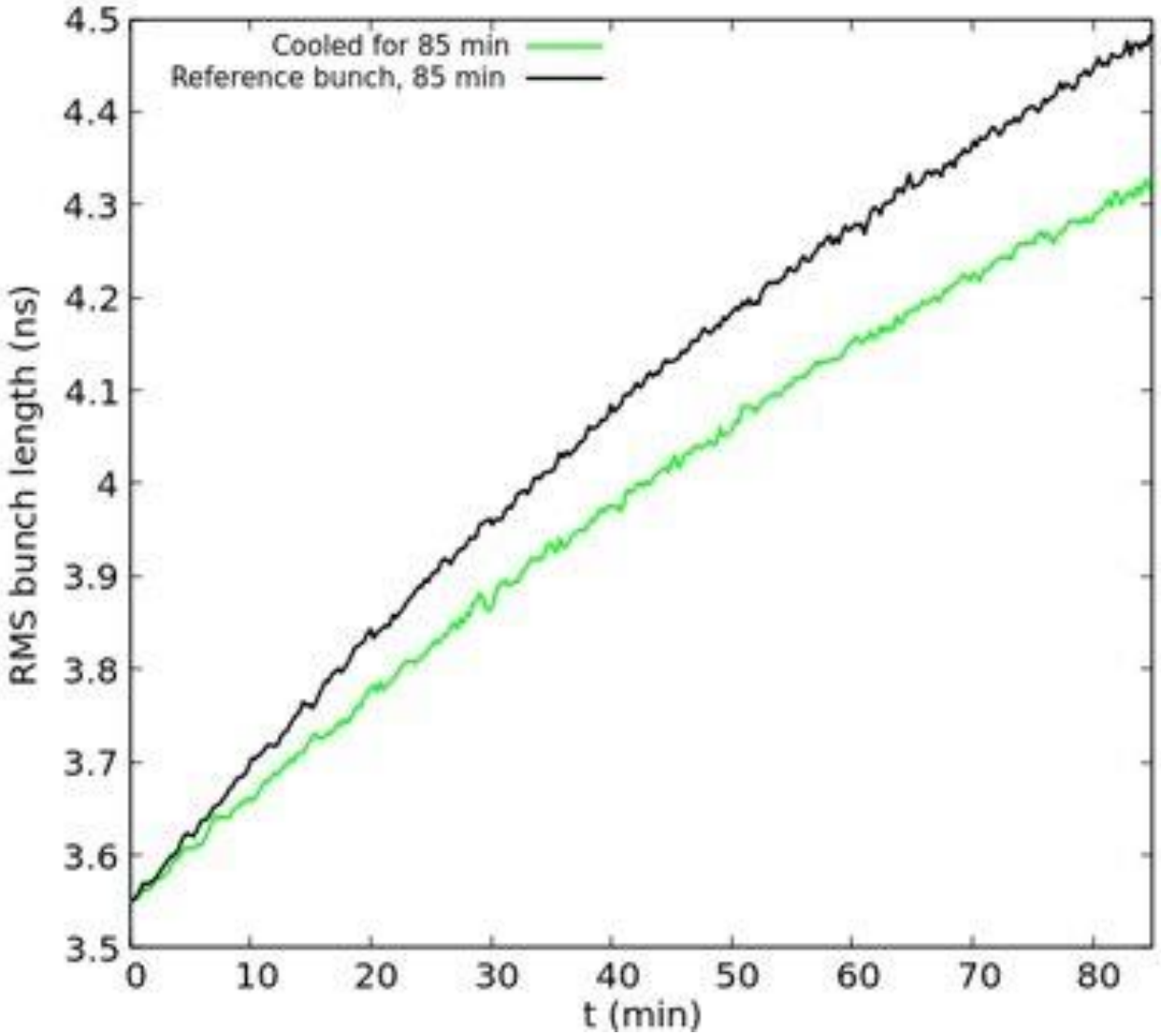


Cooling longer with 2.85 m of beta function (corrected)

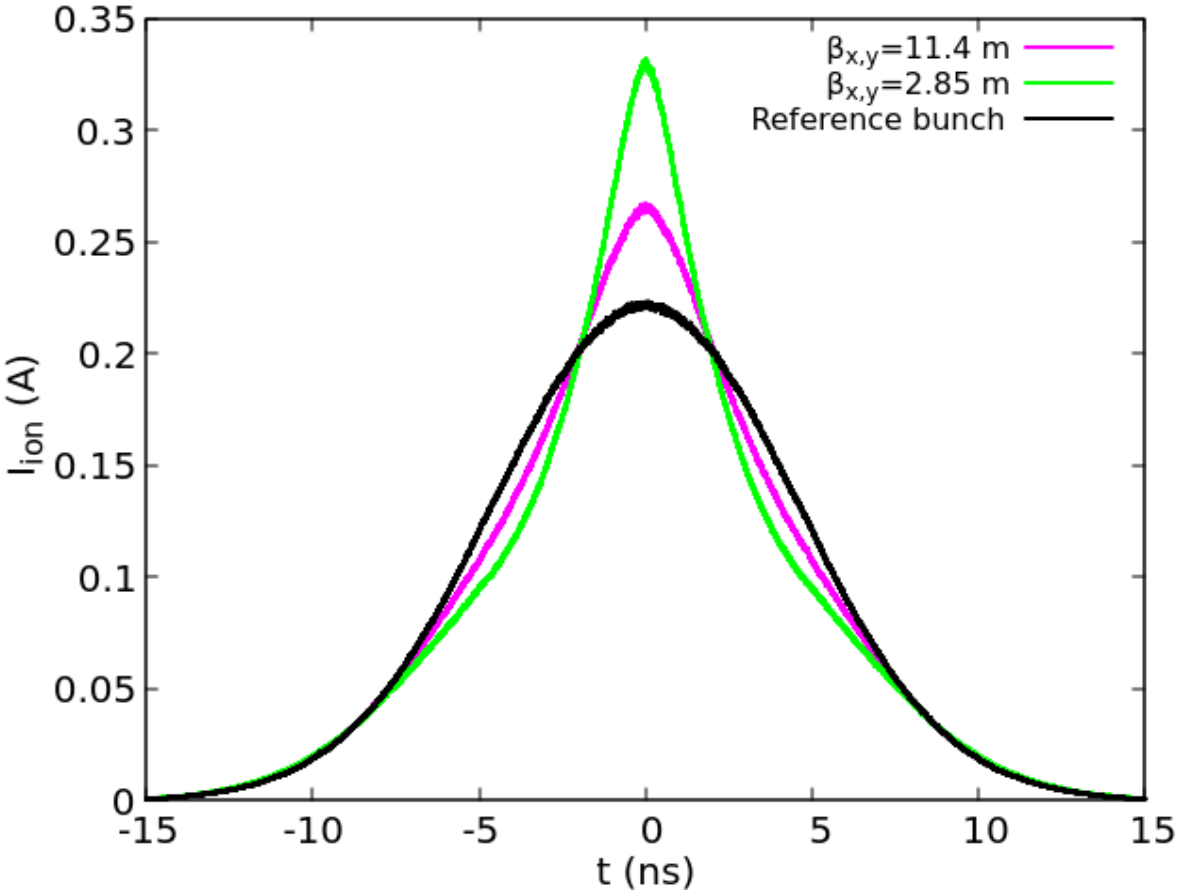


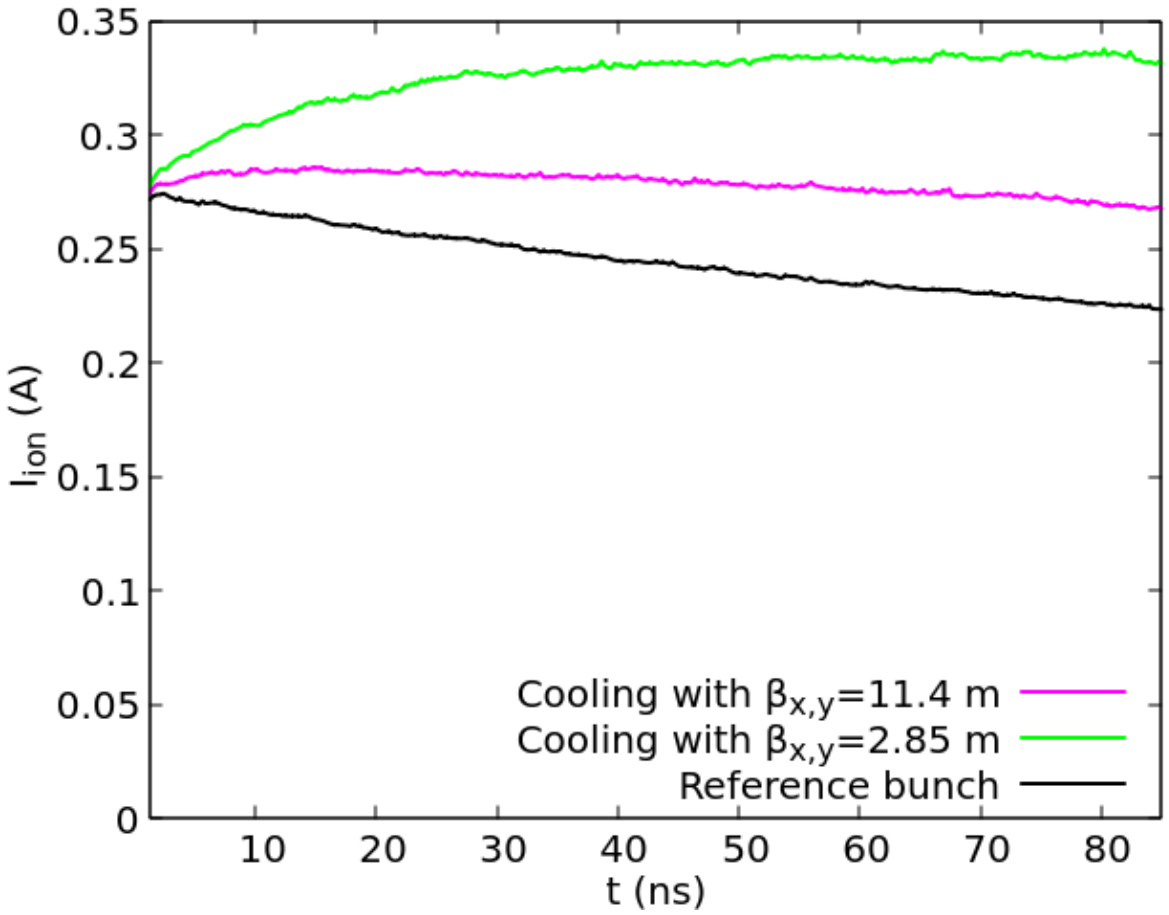


Cooling with 2.85 m of beta function
Evolution of RMS bunch length

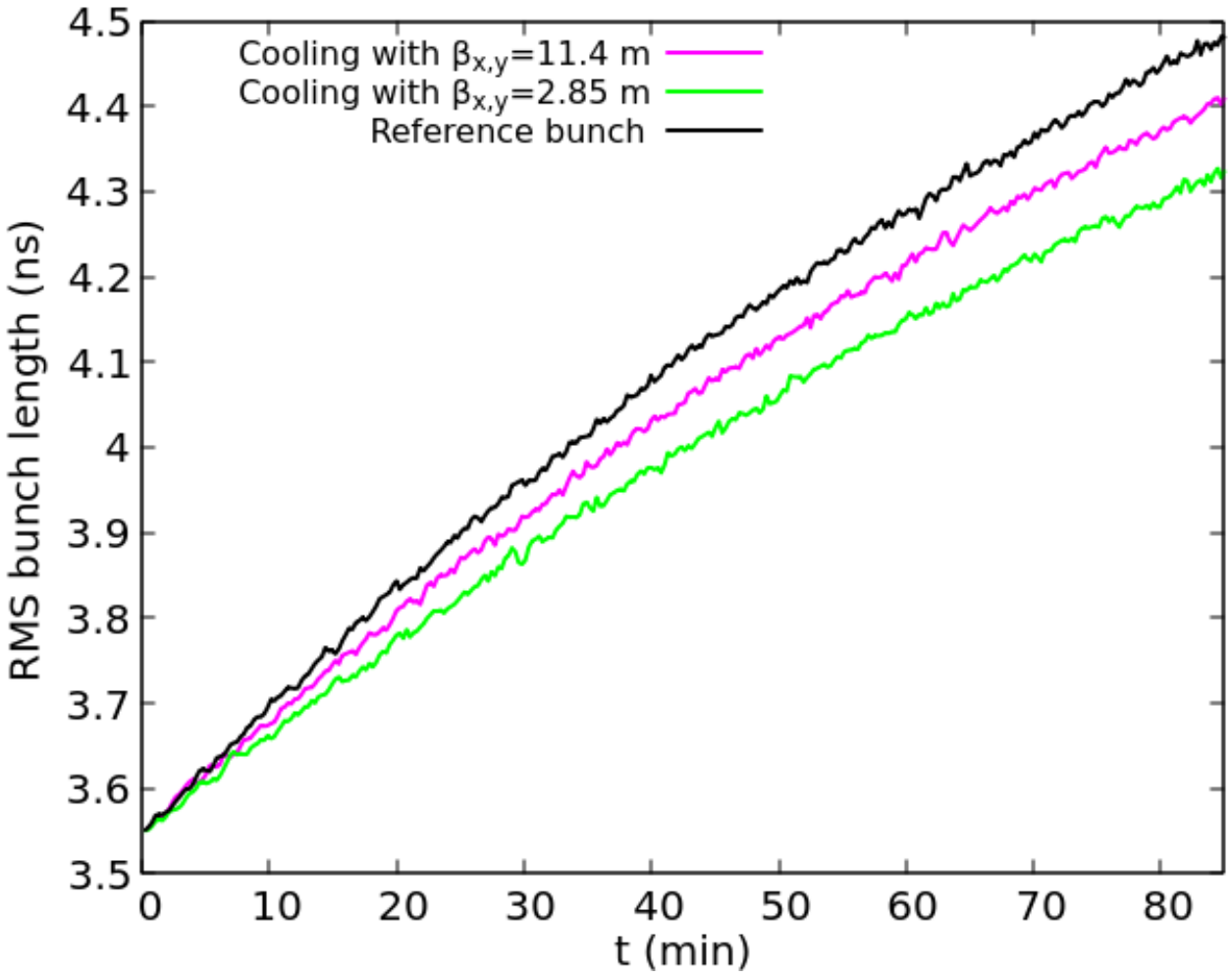


Influence of reducing Ion beam beta function by a factor of 4 at the cooling section

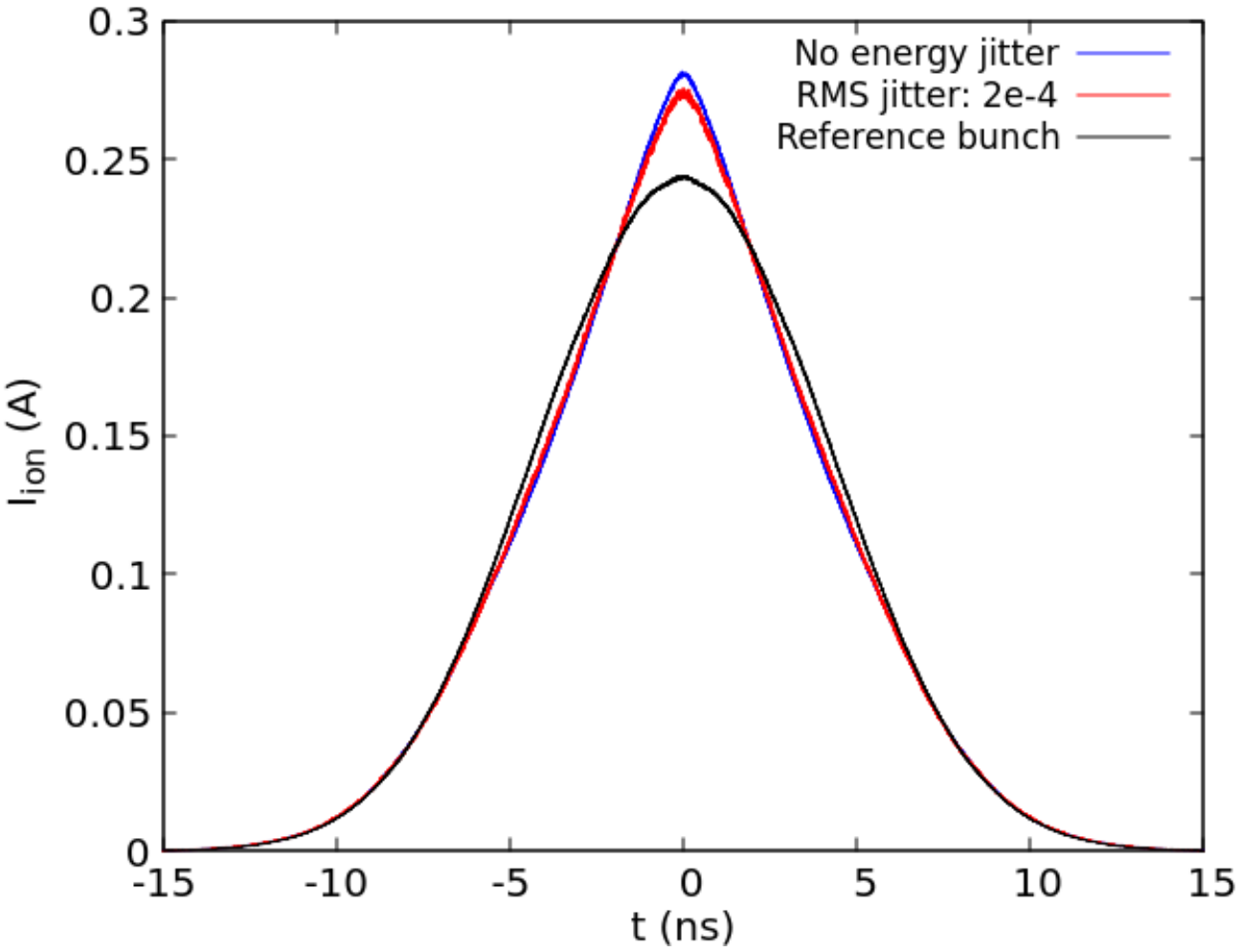




RMS bunch length



Add energy jitter (Gaussian jitter)



PCA dependance on emittance

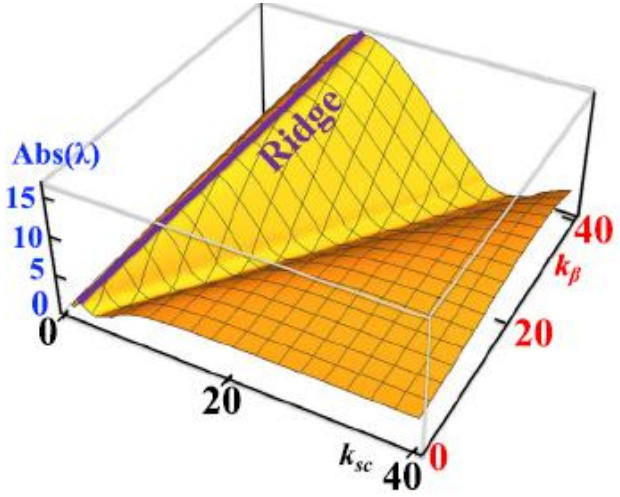


FIG. 3. 3D plot of the absolute value of the growth rate per cell: $\lambda = \max(|\mathrm{Re}\lambda_1|, |\mathrm{Re}\lambda_2|)$. There is a clearly identifiable ridge along the $k_\beta \cong 3 \cdot (k_{sc} - 1.2)$ line, where growth rates peak.

$$\frac{d^2\hat{a}}{d\hat{s}^2} - {k_{sc}}^2\hat{a}^{-1} - {k_\beta}^2\hat{a}^{-3} = 0;$$

$$\frac{d^2}{d\hat{s}^2}\tilde{g}_k + 2\frac{k_{sc}^2}{\hat{a}(\hat{s})^2}\cdot\tilde{g}_k = 0; \qquad \hat{a} = \frac{a}{a_o} \geq 1;$$

$$\hat{s} = \frac{s}{l}; \qquad \hat{s} \in \{-1, 1\},$$

$$k_{sc} = \sqrt{\frac{2}{{\beta_o}^3{\gamma_o}^3}\frac{I_o}{I_A}\frac{l^2}{{a_o}^2}}; \qquad k_\beta = \frac{\varepsilon l}{{a_o}^2}.$$

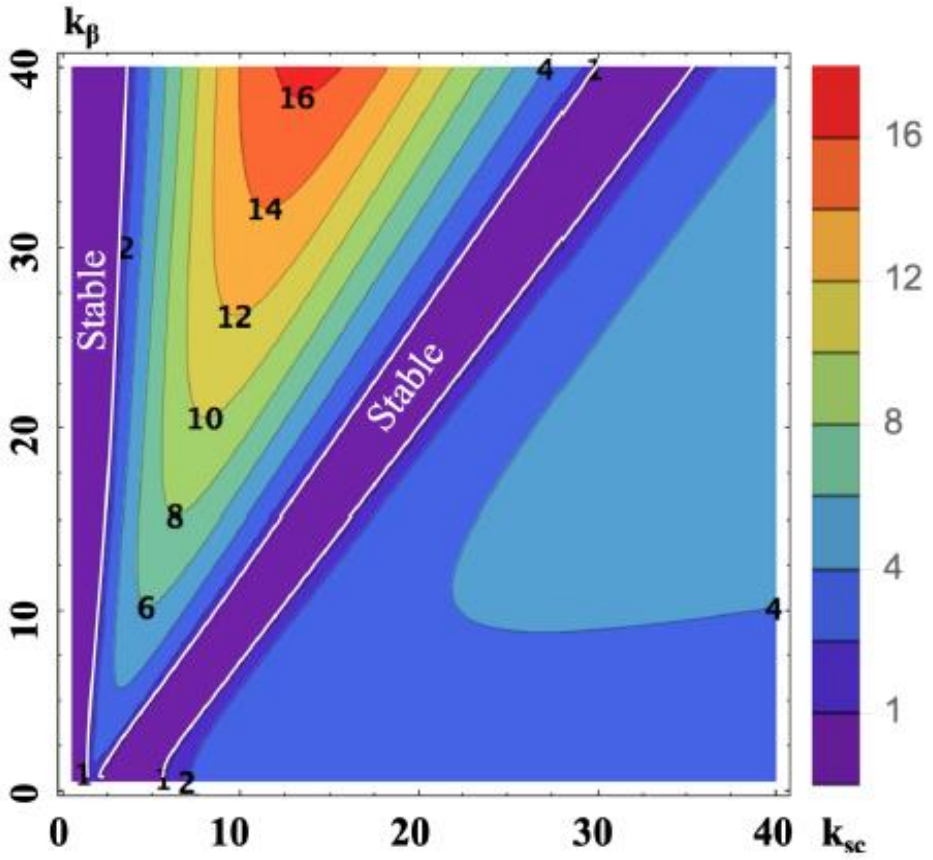


FIG. 4. Contour plots of the absolute value of the cell's growth rate: $\lambda = \max(|\mathrm{Re}\lambda_1|, |\mathrm{Re}\lambda_2|)$. The purple area highlighted by white lines shows the areas of the stable oscillations $|\lambda_{1,2}| = 1$. Outside of these areas, the oscillations grow exponentially.

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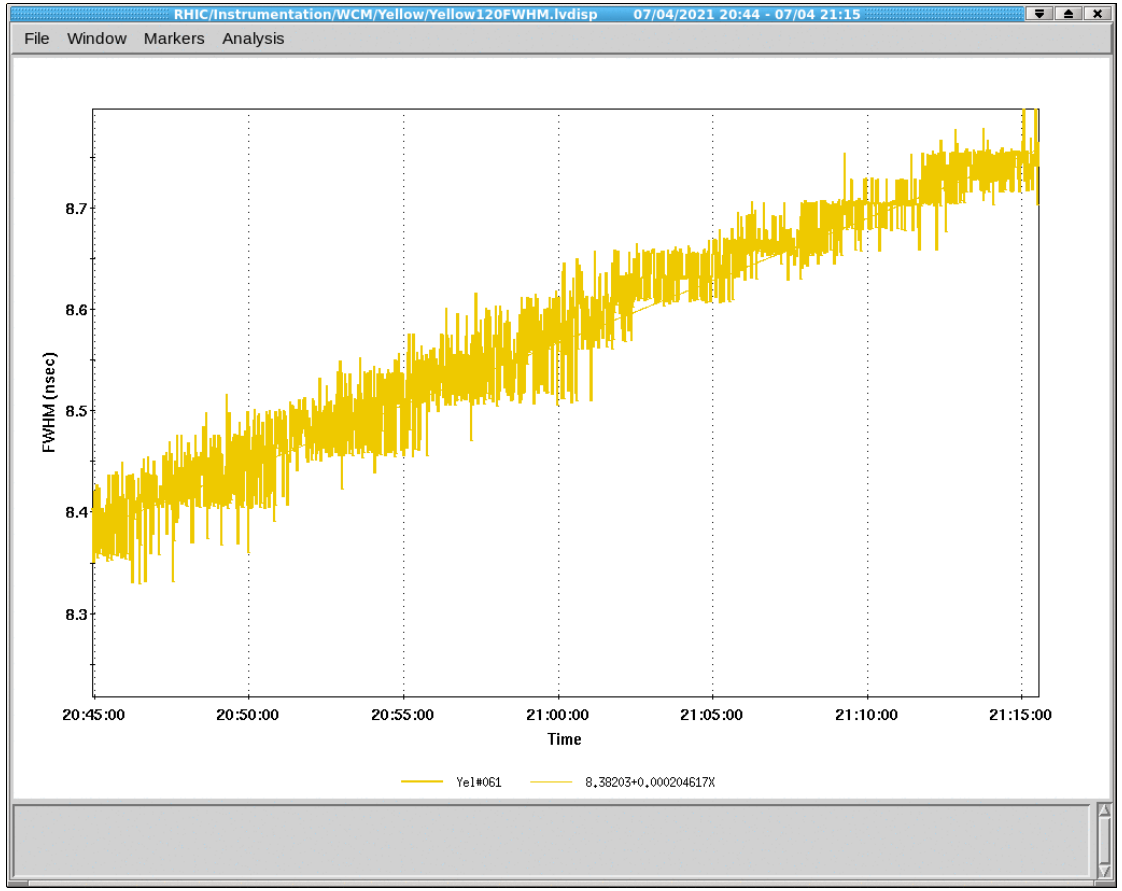
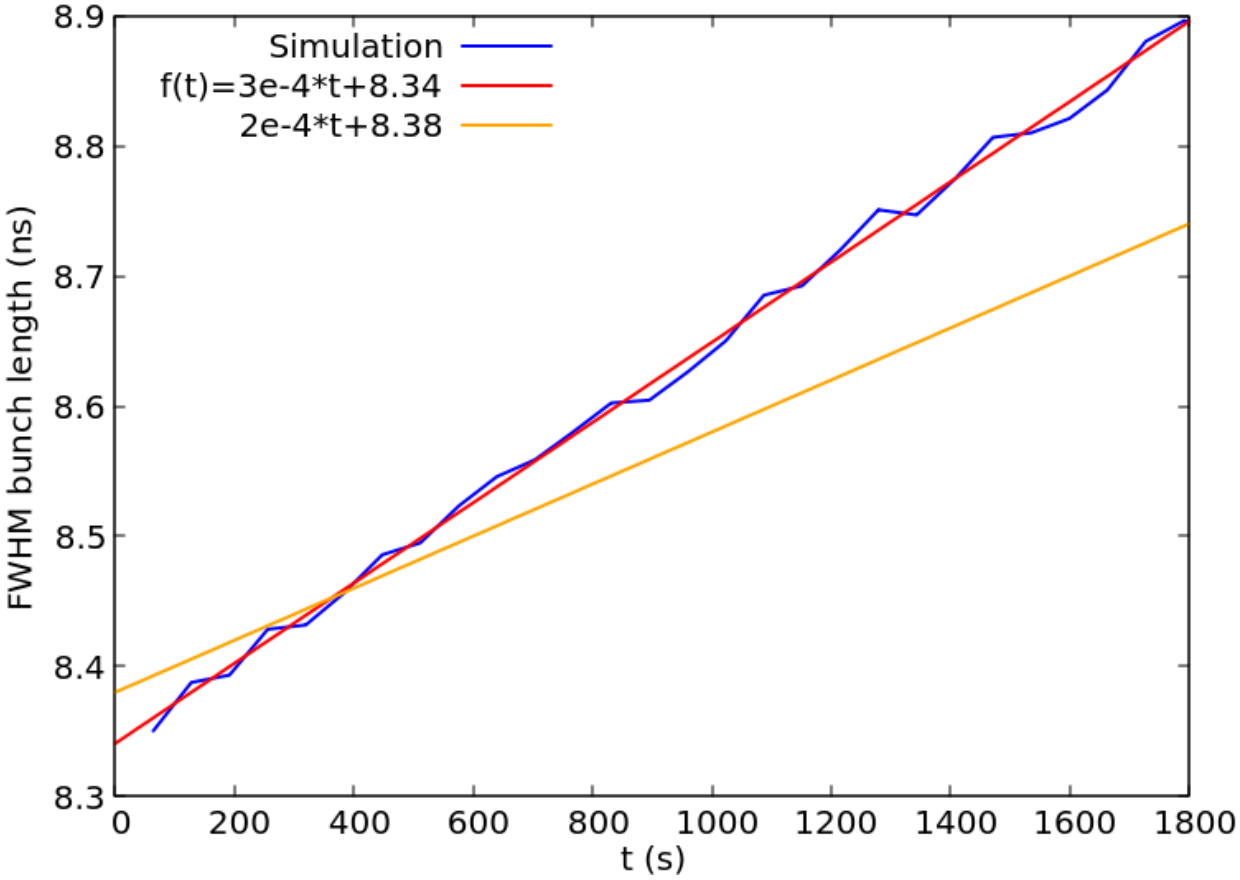
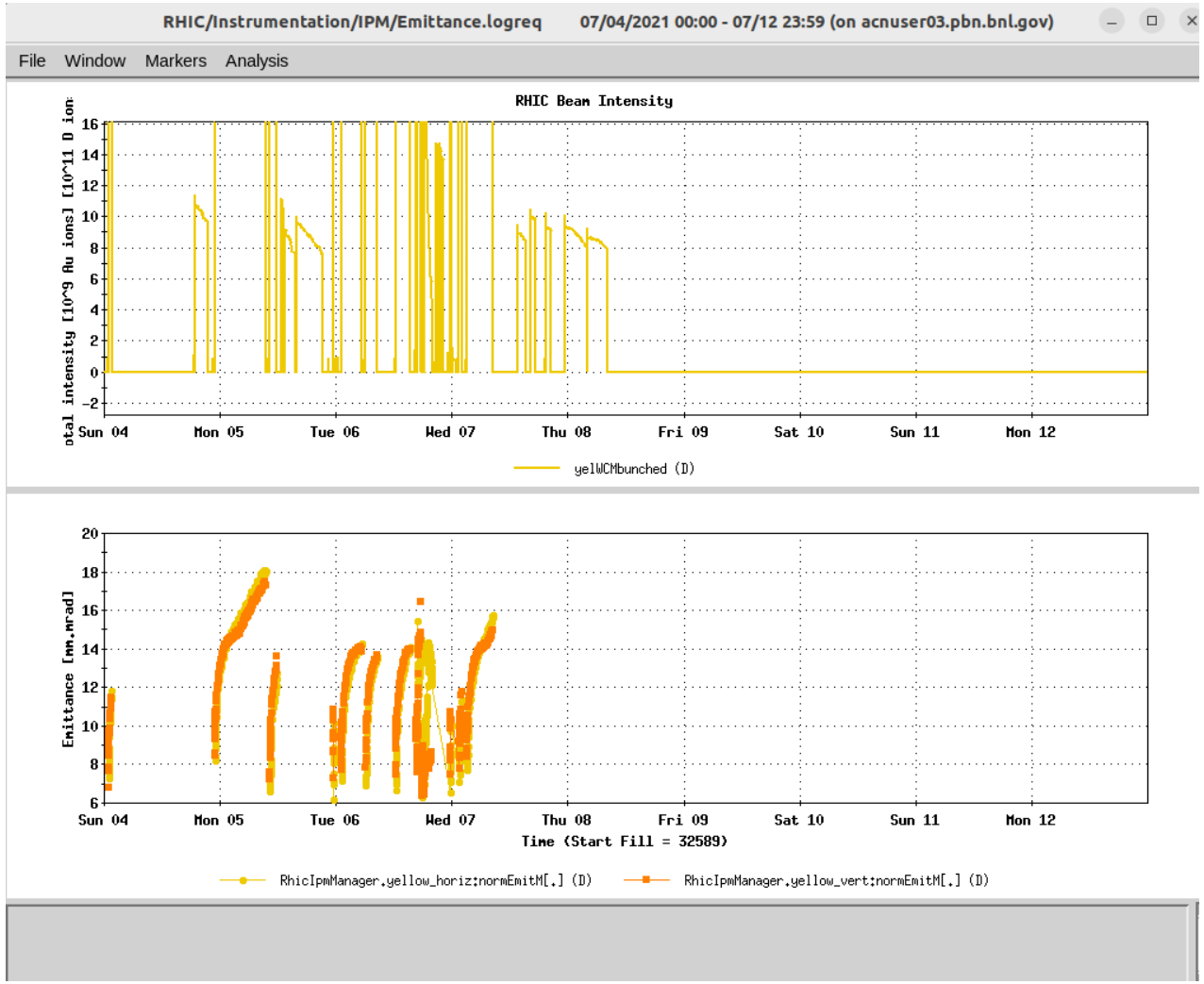
Check IBS routine with measurement in FY 21 7/4/2021 20:44





I can't find any IPM emittance measurements during CeC dedicated time in Run 21, but the measurement for regular RHIC store looks smaller than 15 mm.mrad



Ion beam size

$$\beta(s) = \beta^* + \frac{s^2}{\beta^*}$$

$$s_1 = 4m$$

$$s_2 = 7m$$

$$\bar{\beta} = \beta^* + \frac{s_1^{\ 2} + s_2^{\ 2}}{2\beta^*} \geq 2\sqrt{\beta^* \frac{s_1^{\ 2} + s_2^{\ 2}}{2\beta^*}} = \sqrt{2\left(s_1^{\ 2} + s_2^{\ 2}\right)}$$

The minimal average beta happens at $\beta^* = \sqrt{\frac{s_1^{\ 2} + s_2^{\ 2}}{2}}$.