Homework 8. Due October 11

Problem 1. 2x5 points. Find not-trivial solution for building an unit 2x2 transport matrix out of repeating cells:

$$M^4 = I; M \neq I$$

(a) show that one of the solutions $trace(M) = 0$; Hint: used $M^2 = -I$;

(b) for a "symmetric" FODO cell and finite length equally strong quadrupoles $K_F = -K_D = K; l_F = l_D = L;\ l_1 = l_2 = l$ write the condition that $M_x^4 = M_y^4 = I$, e.g. the 4x4 transport matrix is unit.

Solution

Ignoring trivial solution and complications imposed by $M^2 = I$

$$M^4 = I \to M^2 = \pm I \;\; pick \;\; M^2 = -I; \;\; ad - bc = 1$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a^2 + bc & b(a+d) \\ c(a+d) & a^2 + bc \end{bmatrix}\begin{bmatrix} a(a+d)-1 & b(a+d) \\ c(a+d) & d(a+d)-1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

has obvious solution

$$TraceM = a + d = 0$$

Using previous problem we can write

$$Trace[M_{x,y}] = 0$$

$$\left(\cosh\varphi + l\omega\sinh\varphi\right)\left(\cos\varphi - l\omega\sin\varphi\right) - \omega\sin\varphi\left(\frac{\sinh\varphi}{\omega} + l\cosh\varphi\right)$$

$$+\omega\sinh\varphi\left(\frac{\sin\varphi}{\omega} + l\cos\varphi\right) + \cos\varphi\cosh\varphi = 0$$

This a transcendental equation, which has solution which can be found numerically