Homework 6
Due: *Monday, April 11, 2020*

1. <u>Momentum and γ</u>: (<u>Not Graded</u>) Show that

$$\left(\vec{\nabla}\vec{p}\right) \cdot \vec{v} = mc^2\nabla\gamma$$

2. Derive Eqn 11 in the "Laser and Beam Coupling to Plasma" notes. You can start from Eqn 9

$$\partial_t^2\vec{p} + c^2\vec{\nabla}\times\vec{\nabla}\times\vec{p} + \left[\omega_p^2 + \frac{1}{m}\partial_t\vec{\nabla}\cdot\vec{p} + c^2\nabla^2\gamma\right]\frac{\vec{p}}{\gamma} + mc^2\partial_t\vec{\nabla}\gamma = 0 \qquad (11)$$

$$\frac{\partial\vec{p}}{\partial t} = -e\vec{E} - mc^2\vec{\nabla}\gamma \qquad (9)$$

<u>Jupyter Notebook Homework</u>

3. Initialize an asymmetric particle beam in the Panofsky-Wenzel theorem notebook to see if this theorem is satisfied. Submit your simulation parameters and a screenshot for $\frac{\partial E_z}{\partial x}$ and $\frac{\partial F_x}{\partial \xi}$.

4. For a bi-Gaussian beam, calculate expression for the peak Λ. Run several simulations in the PWFA blowout regime notebook with difference Λ to verify the relationship between Λ and the blowout radius, $r_{b,max}$. Submit a plot of $r_{b,max}$ vs $2\sqrt{\Lambda}$. The density profile of a bi-Gaussian beam is

$$n_b = n_{b0}\exp\left(-\frac{r^2}{2\sigma_r^2} - \frac{(\xi-\xi_0)^2}{2\sigma_z^2}\right)$$

5. <u>Accelerating Field in Blowout Regime</u>: Consider the electron beam at the SLAC National Laboratory; The beam contained $N = 1.8\times10^{10}$ electrons at 42 GeV, had a bunch length of $\sigma_z/c = 50$ fs, and was focused down to $\sigma_r = 10\ \mu m$. Calculate the peak electric field expected at the experimental density of 2.7×10^{17} cm^{-3}. Is your calculation consistent with the 50 GeV/m observed in Blumenfeld 2007 paper (see reference below. The pdf is provided in the Assignments section of Blackboard)?

You can use the back of the envelope calculation given in the notes to find Λ or use the Jupyter notebook in question 4. Note,

$$n_{b0} = \frac{N}{(2\pi)^{3/2}\sigma_r^2\sigma_z}$$

Reference: I. Blumenfeld, et al. Energy doubling of 42 GeV electrons in a metre-scale plasma wakefield accelerator. *Nature*, *445*(7129), 741–744 (2007). https://doi.org/10.1038/nature05538