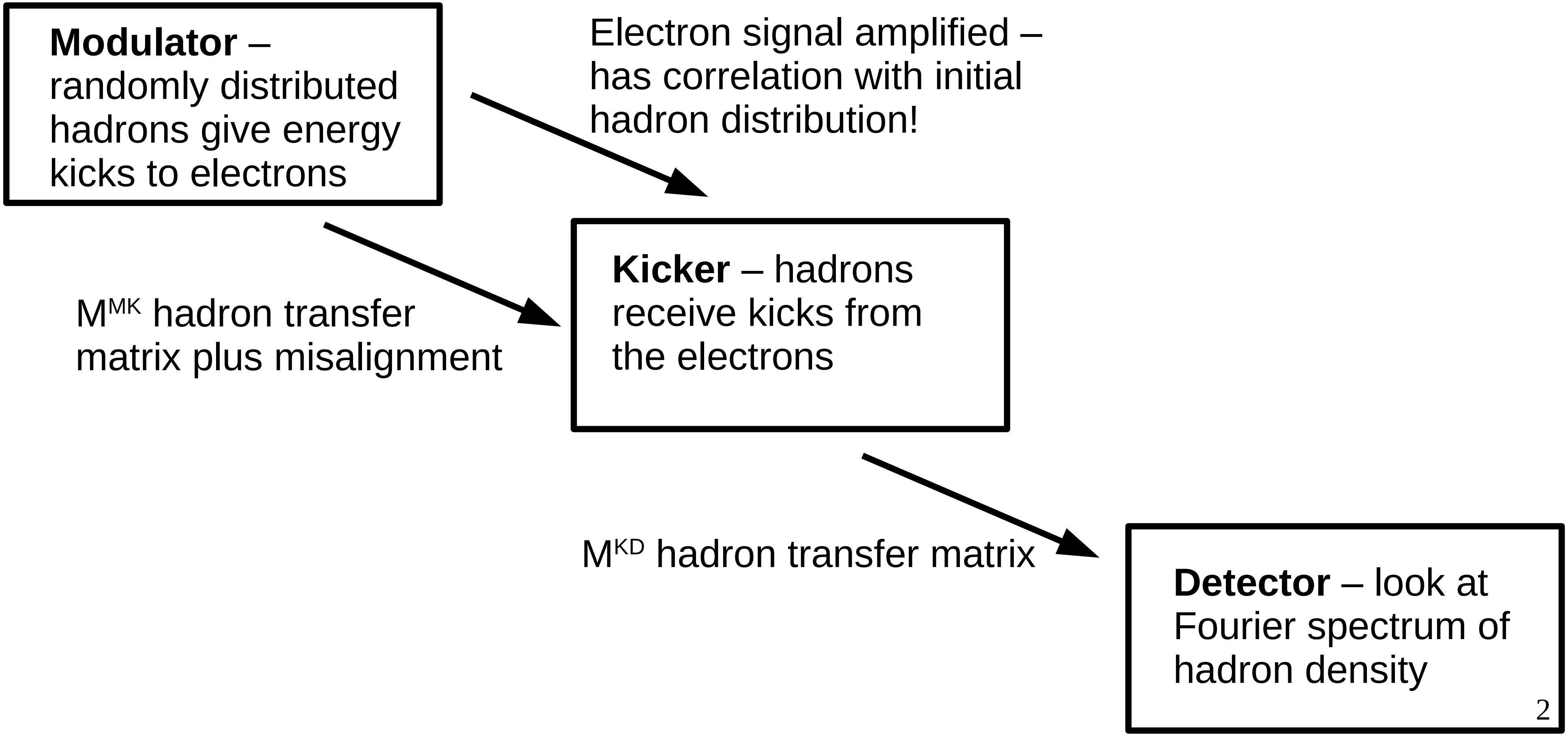
More Realistic Signal Suppression and Detection

- Full theory, to including transverse dynamics
- Simulation comparisons
- Optimal kicker $\rightarrow$ detector transfer matrix
- Dipole radiation as a diagnostic?
- Sensitivity to motion within dipole

Signal Suppression Schematic



Derivation

- Assuming random shot noise in the modulator, get particle positions in detector
- Obtain densities in Fourier space
- Perform integrals in terms of impedance function
- Plug in analytic expression for MBEC wake function
- Note on notation – repeated “u” indices are summed from 1-6, excluding 5
  – wake defined as fractional momentum kick to hadron

Derivation (cont.)

$$z_d^{(i)} = z_m^{(i)} + M_{5u}^{MD}\vec{x}_u^{(i)} + M_{56}^{KD}\sum_j w(z_m^{(i)} + M_{5u}^{MK}\vec{x}_u^{(i)} - z_m^{(j)} + \Delta z)$$

$$\rho(z) = \sum_i \delta(z - z_d^{(i)})$$

$$\tilde{\rho}(k) \equiv \int_{-L/2}^{L/2} e^{-ikz}\rho(z)dz = \sum_i e^{-ikz_d^{(i)}}$$

$$|\tilde{\rho}(k)|^2 = \sum_{i,a} e^{-ik[z_d^{(i)} - z_d^{(a)}]}$$

$$|\tilde{\rho}(k)|^2 = N + \sum_{i\neq a} e^{-ik[z_m^{(i)} - z_m^{(a)} + M_{5u}^{MD}(\vec{x}_u^{(i)} - \vec{x}_u^{(a)}) + M_{56}^{KD}\sum_j w(z_m^{(i)} - z_m^{(j)} + M_{5u}^{MK}\vec{x}_u^{(i)} + \Delta z) - M_{56}^{KD}\sum_j w(z_m^{(a)} - z_m^{(j)} + M_{5u}^{MK}\vec{x}_u^{(a)} + \Delta z)]}$$

$$|\tilde{\rho}(k)|^2 = N + \sum_{i\neq a} e^{-ik[z_m^{(i)} - z_m^{(a)} + M_{5u}^{MD}(\vec{x}_u^{(i)} - \vec{x}_u^{(a)})]}$$
$$\times\,(1 - ikM_{56}^{KD}\sum_j w(z_m^{(i)} - z_m^{(j)} + M_{5u}^{MK}\vec{x}_u^{(i)} + \Delta z) + ikM_{56}^{KD}\sum_j w(z_m^{(a)} - z_m^{(j)} + M_{5u}^{MK}\vec{x}_u^{(a)} + \Delta z))$$

Derivation (cont.)

$$\langle|\tilde{\rho}(k)|^2\rangle = N + \sum_{i\neq a}\langle e^{-ik[z_{ia}+M_{5u}^{MD}(\vec{x}_u^{(i)}-\vec{x}_u^{(a)})]}$$
$$\times\,(1 - ikM_{56}^{KD}w(M_{5u}^{MK}\vec{x}_u^{(i)}+\Delta z) + ikM_{56}^{KD}w(M_{5u}^{MK}\vec{x}_u^{(a)}+\Delta z)$$
$$- ikM_{56}^{KD}w(z_{ia}+M_{5u}^{MK}\vec{x}_u^{(i)}+\Delta z) + ikM_{56}^{KD}w(-z_{ia}+M_{5u}^{MK}\vec{x}_u^{(a)}+\Delta z))\rangle$$

11-D integrals such as (4th term):

$$-ikM_{56}^{KD}\int_{-L/2}^{L/2}dz_{ia}/L\int_{-\infty}^{\infty}d^5\vec{x}^{(i)}d^5\vec{x}^{(a)}\rho\big(\vec{x}^{(i)}\big)\rho\big(\vec{x}^{(a)}\big)e^{-ik[z_{ia}+M_{56}^{MD}(\vec{x}_u^{(i)}-\vec{x}_u^{(a)})]}$$
$$\times\,w(z_{ia}+M_{5u}^{MK}\vec{x}_u^{(i)}+\Delta z)$$

Derivation (cont.)

$$-ikM_{56}^{KD}\int_{-L/2}^{L/2}dz_{ia}/L\int_{-\infty}^{\infty}d^5\vec{x}^{(i)}d^5\vec{x}^{(a)}\rho\big(\vec{x}^{(i)}\big)\rho\big(\vec{x}^{(a)}\big)e^{-ik[z_{ia}+M_{56}^{MD}(\vec{x}_u^{(i)}-\vec{x}_u^{(a)})]}$$
$$\times\, w(z_{ia}+M_{5u}^{MK}\vec{x}_u^{(i)}+\Delta z)$$

Defining impedance: $Z(k)\equiv\int_{-L/2}^{L/2}e^{-ikz}w(z)dz$

$$\frac{-ikM_{56}^{KD}}{L}Z(k)e^{ik\Delta z}\int_{-\infty}^{\infty}d^5\vec{x}^{(i)}d^5\vec{x}^{(a)}\rho\big(\vec{x}^{(i)}\big)\rho\big(\vec{x}^{(a)}\big)e^{-ik[M_{56}^{MD}(\vec{x}_u^{(i)}-\vec{x}_u^{(a)})-M_{5u}^{MK}\vec{x}_u^{(i)}]}$$

5th term (using $Z(-k)=Z^*(k)$):

$$\frac{ikM_{56}^{KD}}{L}Z^*(k)e^{-ik\Delta z}\int_{-\infty}^{\infty}d^5\vec{x}^{(i)}d^5\vec{x}^{(a)}\rho\big(\vec{x}^{(i)}\big)\rho\big(\vec{x}^{(a)}\big)e^{-ik[M_{56}^{MK}\vec{x}_u^{(a)}+M_{5u}^{MD}(\vec{x}_u^{(i)}-\vec{x}_u^{(a)})]}$$

Derivation (cont.)

$$\langle|\tilde{\rho}(k)|^2\rangle = N+2N\frac{(N-1)}{L}kM_{56}^{KD}[Re(Z(k))\sin(k\Delta z)+Im(Z(k))\cos(k\Delta z)]$$

$$\times\, e^{-\frac{k^2}{2}\sum_{u\neq 5}\sigma_{\tilde{x}_u}^2\left[\left(\tilde{M}_{5u}^{MD}\right)^2+\left(\tilde{M}_{5u}^{MK}-\tilde{M}_{5u}^{MD}\right)^2\right]}$$

$$\sigma_{\tilde{x}} \equiv \sqrt{\epsilon_x\beta_x}$$

$$\sigma_{\tilde{x}'} \equiv \sqrt{\epsilon_x/\beta_x}$$

$$\sigma_{\tilde{y}} \equiv \sqrt{\epsilon_y\beta_y}$$

$$\sigma_{\tilde{y}'} \equiv \sqrt{\epsilon_y/\beta_y}$$

$$\sigma_{\tilde{\delta}} \equiv \sigma_\delta$$

$$\tilde{M}_{51} \equiv M_{51} - \frac{\alpha_x}{\beta_x}M_{52}$$

$$\tilde{M}_{52} \equiv M_{52}$$

$$\tilde{M}_{53} \equiv M_{53} - \frac{\alpha_y}{\beta_y}M_{54}$$

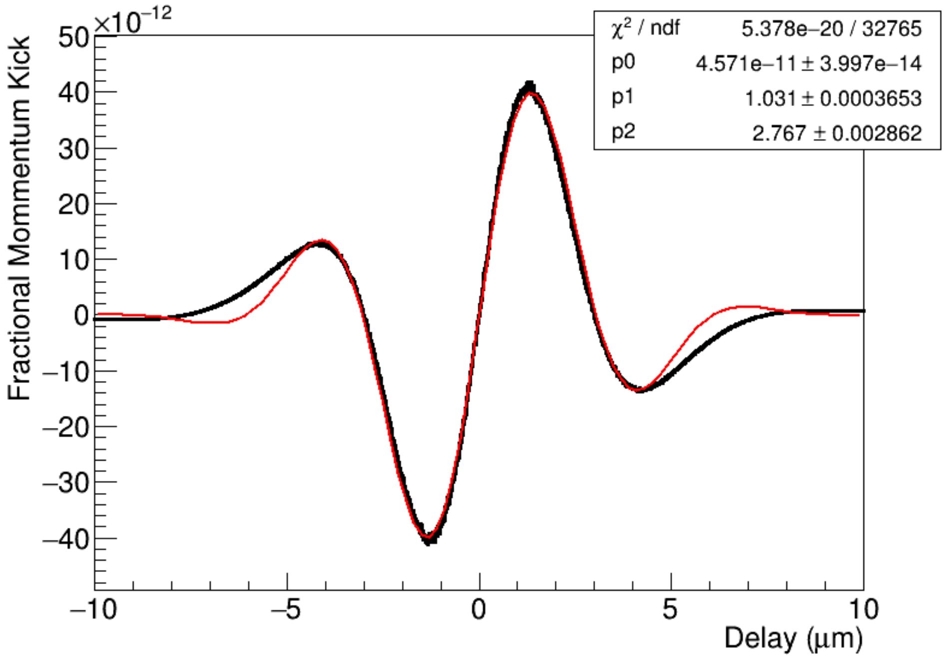
$$\tilde{M}_{54} \equiv M_{54}$$

$$\tilde{M}_{56} \equiv D_xM_{51}+D_x'M_{52}+D_yM_{53}+D_y'M_{54}+M_{56}$$

$$\frac{\Delta\langle|\tilde{\rho}(k)|^2\rangle}{\langle|\tilde{\rho}(k)|^2\rangle} = 2nkM_{56}^{KD}[Re(Z(k))\sin(k\Delta z)+Im(Z(k))\cos(k\Delta z)]$$

$$\times\, e^{-\frac{k^2}{2}\sum_{u\neq 5}\sigma_{\tilde{x}_u}^2\left[\left(\tilde{M}_{5u}^{MD}\right)^2+\left(\tilde{M}_{5u}^{MK}-\tilde{M}_{5u}^{MD}\right)^2\right]}$$

Analytic Wake Model (Saturation On)



$$w(z) = A\sin(\kappa z)e^{-z^2/2\lambda^2}$$

$$A = 4.571 \times 10^{-11}$$

$$\kappa = 1.031/\mu m$$

$$\lambda = 2.767\mu m$$

$$Z(k) = \frac{Ai}{2}\sqrt{2\pi}\lambda\left[e^{(k+\kappa)^2/2\lambda^2} - e^{(k-\kappa)^2/2\lambda^2}\right]$$

See: S. Nagaitsev et al, "Cooling and diffusion rates in coherent electron cooling concepts", FERMILAB-CONF-21-054-AD, `https://lss.fnal.gov/archive/2021/conf/fermilab-conf-21-054-ad.pdf` (2021).

Signal Suppression for Our Wake

$$\frac{\Delta\langle|\tilde{\rho}(k)|^2\rangle}{\langle|\tilde{\rho}(k)|^2\rangle} = nkM_{56}^{KD}A\sqrt{2\pi}\lambda\cos(k\Delta z)e^{-\frac{k^2}{2}\sum_{u\neq 5}\sigma_{\tilde{x}_u}^2\left[\left(\tilde{M}_{5u}^{MD}\right)^2+\left(\tilde{M}_{5u}^{MK}-\tilde{M}_{5u}^{MD}\right)^2\right]}$$

$$\times\left[e^{-(k+\kappa)^2\lambda^2/2}-e^{-(k-\kappa)^2\lambda^2/2}\right]$$

$$\sigma_{\tilde{x}} \equiv \sqrt{\epsilon_x\beta_x} \qquad \tilde{M}_{51} \equiv M_{51} - \frac{\alpha_x}{\beta_x}M_{52}$$

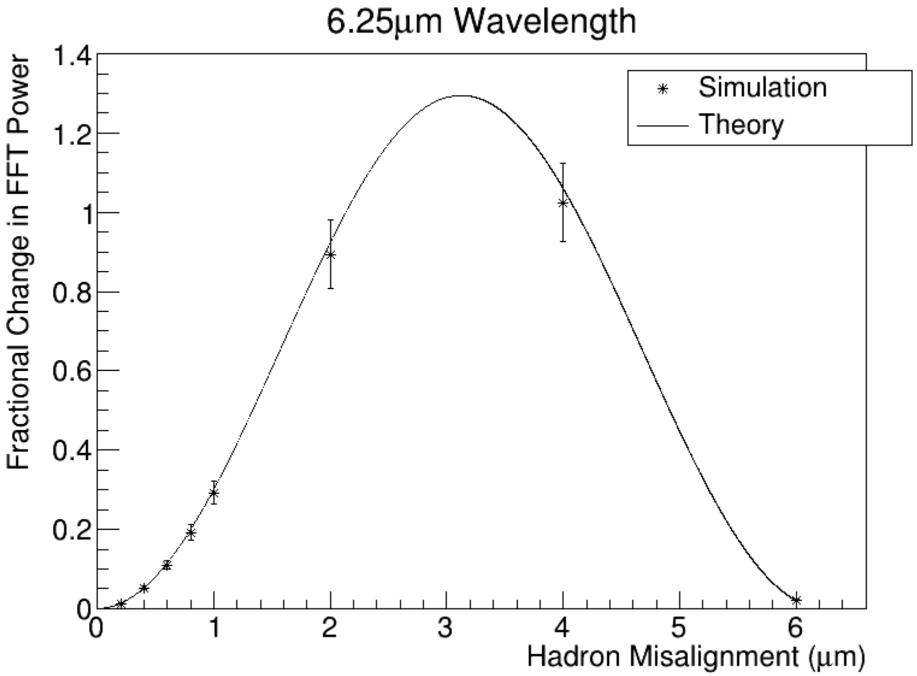
$$\sigma_{\tilde{x}'} \equiv \sqrt{\epsilon_x/\beta_x} \qquad \tilde{M}_{52} \equiv M_{52}$$

$$\sigma_{\tilde{y}} \equiv \sqrt{\epsilon_y\beta_y} \qquad \tilde{M}_{53} \equiv M_{53} - \frac{\alpha_y}{\beta_y}M_{54}$$

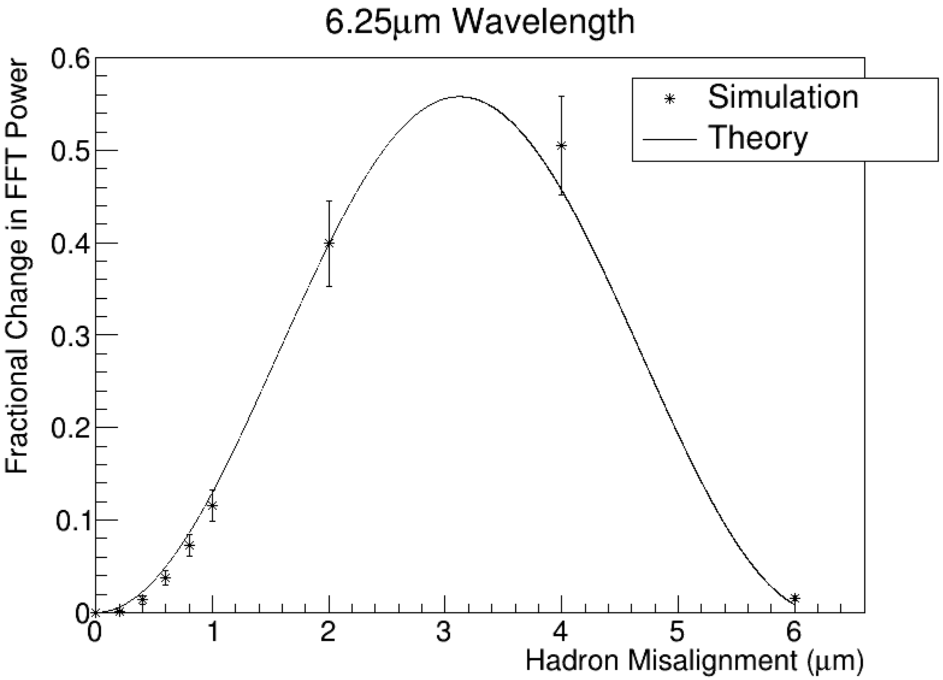
$$\sigma_{\tilde{y}'} \equiv \sqrt{\epsilon_y/\beta_y} \qquad \tilde{M}_{54} \equiv M_{54}$$

$$\sigma_{\tilde{\delta}} \equiv \sigma_\delta \qquad \tilde{M}_{56} \equiv D_xM_{51} + D_x'M_{52} + D_yM_{53} + D_y'M_{54} + M_{56}$$

Comparison with Simulation (K→D Transfer Matrix Inverse of M→K) (Normalized Relative to No-Offset Case)



Linearized Simulation



Full Simulation
(Saturation included)

Optimal Transfer Matrix

- With current modulator → kicker transfer matrix, gradient-descent optimization of signal suppression formula gives maximum (in absolute value sense):

$$\frac{\Delta\left\langle|\tilde{\rho}(k)|^2\right\rangle}{\left\langle|\tilde{\rho}(k)|^2\right\rangle} = -0.22\cos(k\Delta z)$$

with parmeters:

$$M_{51}^{KD} = -8.44 \times 10^{-4} \quad M_{52}^{KD} = -3.00 \times 10^{-2} m \quad M_{56}^{KD} = 2.36 \times 10^{-3} m \quad k = 1.04 \times 10^{6}/m$$

For comparison, inverse-matrix plots from last slide had used:

$$M_{51}^{KD} = -7.93 \times 10^{-4},\ M_{52}^{KD} = -2.86 \times 10^{-2} m,\ M_{56}^{KD} = 2.26 \times 10^{-3} m,\ k = 1.06 \times 10^{6}/m$$

Detection with Dipole Radiation

Can We Make Hadron Signal Frequency the Critical Frequency?

$$\omega_c = \frac{3\gamma^3 c}{2\rho}$$

(See Jackson E&M, 3rd ed., eqtn. 14.81)

- 275GeV protons
- Gamma of 293
- Omega of ck = 3.11e14/s

$$B = \frac{E_0}{ec\rho} = \frac{2E_0\omega_c}{3\gamma^3 ec^2} = 25.2T$$

Not realistic!

Dipole Radiation Estimates

$$\frac{d^2I}{d\omega d\Omega} = \frac{e^2}{4\pi\epsilon_0 3\pi^2 c}\left(\frac{\omega\rho}{c}\right)^2\left(\frac{1}{\gamma^2}+\theta^2\right)^2\left[K_{2/3}^2(\xi)+\frac{\theta^2}{(1/\gamma^2)+\theta^2}K_{1/3}^2(\xi)\right]$$

$$\xi = \frac{\omega\rho}{3c}\left(\frac{1}{\gamma^2}+\theta^2\right)^{3/2}$$

(See Jackson E&M, 3rd ed., eqtn. 14.76 and 14.79)

- 3.782 T magnets → 243m bending radius (CDR, pg 196)
- Gamma of 293
- Omega of ck = 3.11e14/s
- Look at 0 angle (on-axis)

$$\frac{d^2I}{d\omega d\Omega} = 1.42\times 10^{-34}\frac{J}{\frac{1}{s}sr}$$

Dipole Radiation, Continued

$$\frac{d^2 I}{d\omega d\Omega} = 1.42 \times 10^{-34} \frac{J}{\frac{1}{s} sr}$$

- Assume frequency bandwidth of ω/10
- Intensity 10m downstream is 4.5e-23 J/m^2 per hadron per revolution
- With 7mm/6cm of h+ bunch overlapping with electrons, core has 8.1e9 h+
- Get 3.6e-13 J/m^2 per bunch per revolution
- With 1160 bunches and revolution time of 13μs, have 3.3e-5 W/m^2 intensity from core protons

Black-body Background Radiation

- 3.3e-5 W/m^2 on-axis radiation intensity from core protons
- If integrate over detector with radius 3.4mm (1/(10 gamma) angular acceptance), a intensity of **~3.1e-5 W/m^2** 10 m downstream – total power of 1.1nW = 35 billion photons/second at detector

$$\frac{dP}{d\omega} = \frac{\hbar}{4\pi^2 c^2} \frac{\omega^3}{e^{\hbar\omega/k_B T} - 1}$$

$$P = \sigma T^4$$

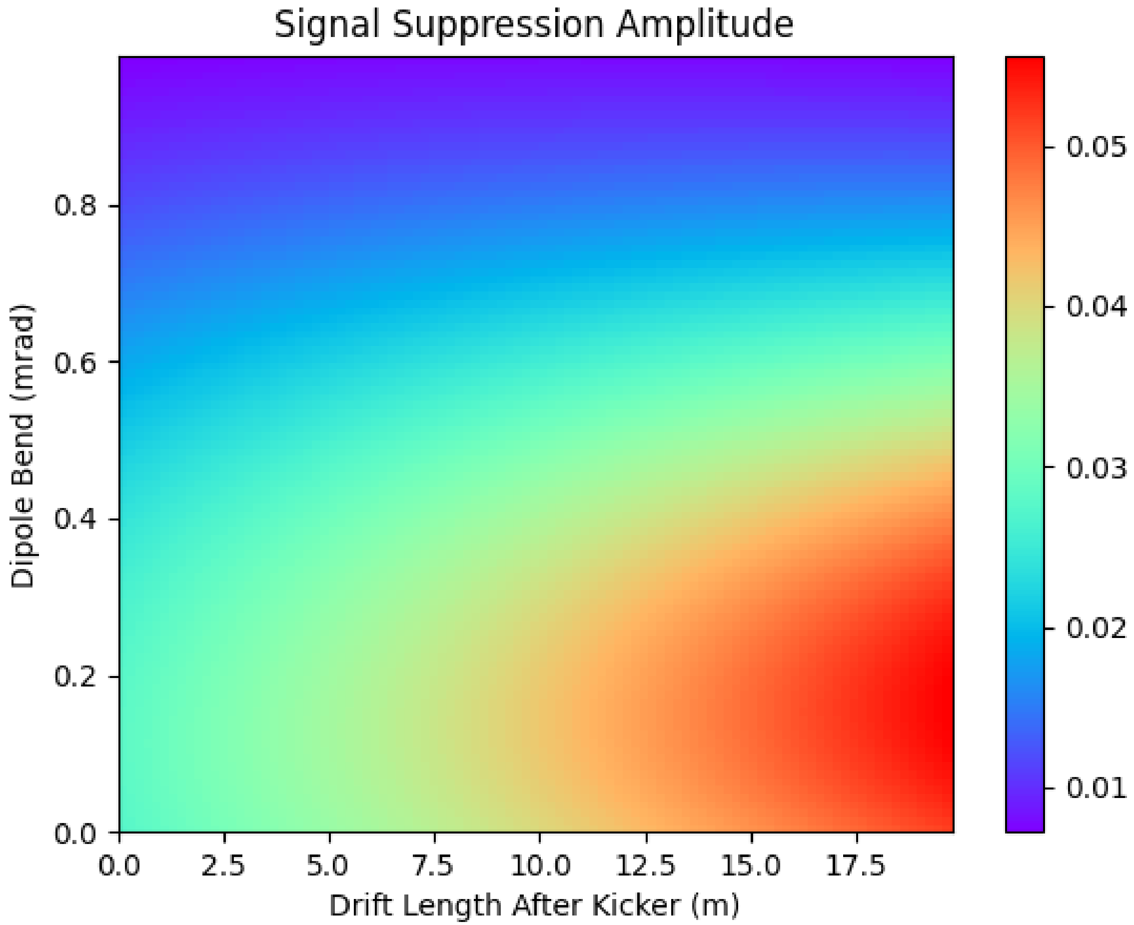
See, eg, J. P. Sethna, “Statistical Mechanics: Entropy, Order Parameters, and Complexity”, eqtn. 7.80 and 7.82

- At 300K, intensity of background in 10% bandwidth is 10 W/m^2 – swamps any signal
- At 77.29K (LN2), have 1.7e-9 W/m^2 within 10% bandwidth – much less than signal

  But, total power over all frequencies is 2 W – again much larger than signal
- At 4.22K (LHe), total power over all frequencies is 1.8e-5 W – comparable to signal

Drift and Dipole to Generate R56?

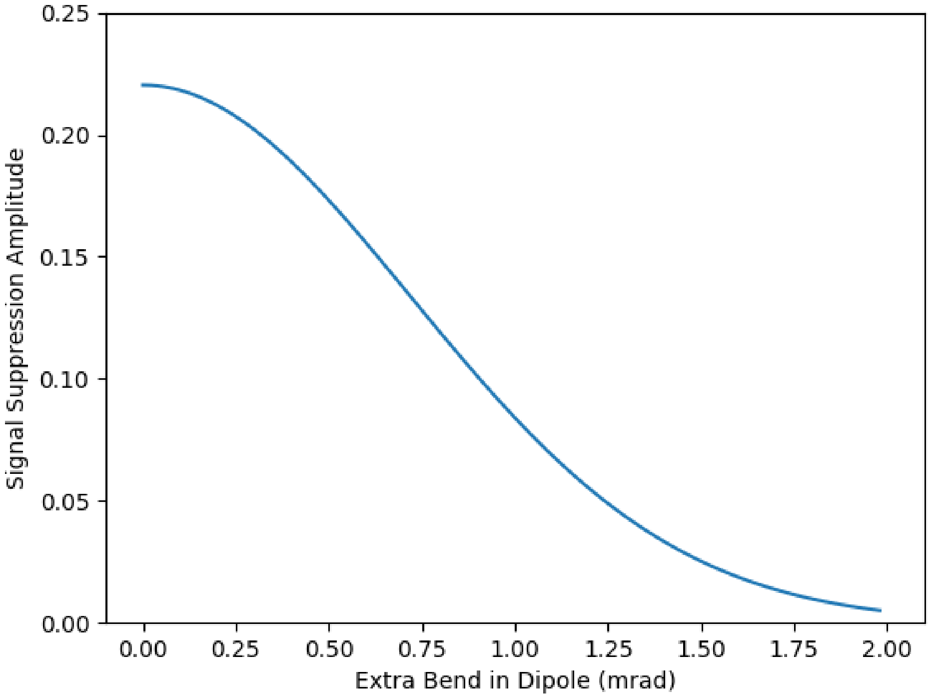
- Need to also generate proper transfer matrix from kicker to detector
- What can be achieved by a drift and dipole?
- Use dipole of strength given previously, scan bend angle within dipole and drift length before (plus $L_K/2$ unavoidable drift from kicker)
- Ignore edge focusing of dipole
- Amplitudes of signal suppression defined by -A cos(kΔz)



Need large drift, and little bending

Note $\frac{1}{10\gamma} \approx 0.3mrad$

Sensitivity to Extra Bend in Dipole (Optimal Transfer Matrix)



Note: $\frac{1}{10\gamma} \approx 0.3mrad$

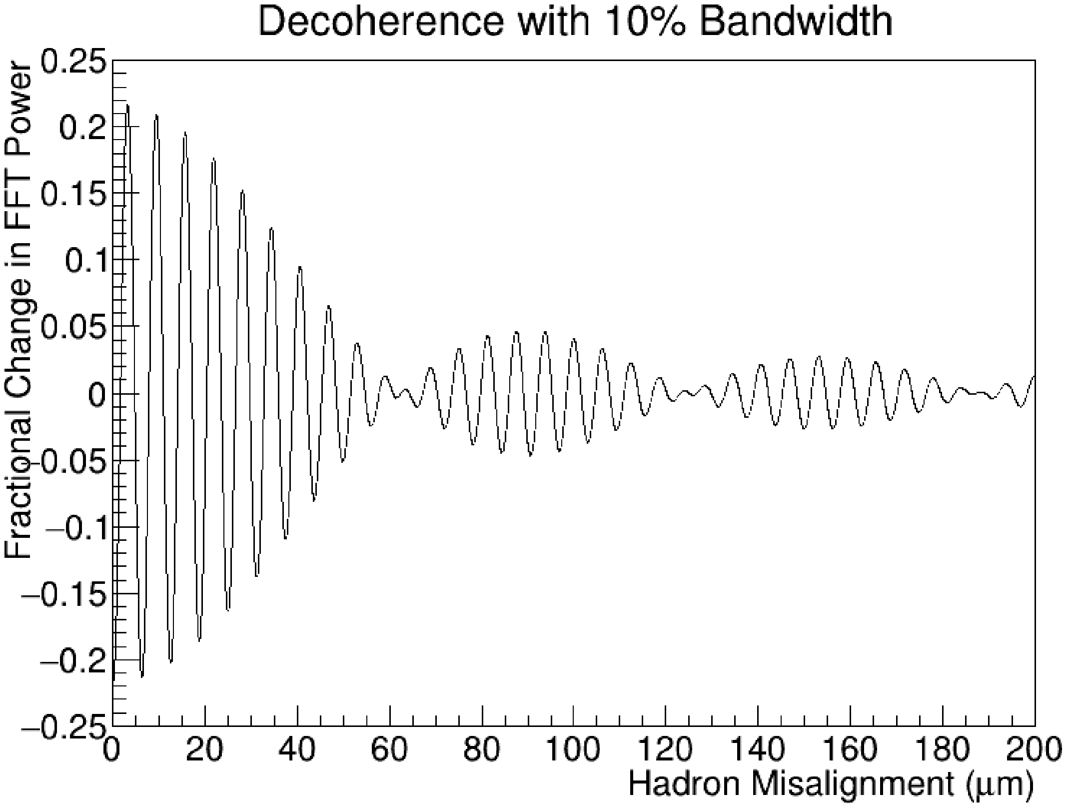
Conclusions

- Can get optimal signal suppression amplitude of ~22%
- Can get ~1/10 of optimal power with only a short drift and dipole
- Operation at critical frequency not possible with a pure dipole
- Cryogenic detector likely needed so that we aren't swamped by thermal noise
- Signal output is sensitive to where we observe in dipole, on scale comparable to radiation opening angle, leading to up to a few percent reduction in signal

Backup Slides

- Decoherence of signal suppression
- Undulator radiation
- Parameter table

Decoherence of Signal Suppression



$$\langle \cos(k\Delta z) \rangle = \frac{1}{\Delta k} \int_{k}^{k+\Delta k} \cos(k'\Delta z) dk' = \frac{\sin((k+\Delta k)\Delta z) - \sin(k\Delta z)}{\Delta k \Delta z}$$

Undulator Radiation?

$$K = \frac{eB\lambda_u}{2\pi mc}$$

$$\lambda = \frac{\lambda_u}{2\gamma^2}(1 + K^2/2)$$

- For simplicity, say undulator field is also 3.782T
  Then, K = 0.20, need undulator period of 1.02m

Undulator Radiation?

$$\frac{dP}{d\omega\Omega} = \hbar\omega\alpha N^2\gamma^2\frac{1}{\omega}\frac{I}{e}\frac{K^2}{(1+K^2/2)^2}\left[J_0\left(\frac{K^2}{4(1+K^2/2)}\right) - J_1\left(\frac{K^2}{4(1+K^2/2)}\right)\right]^2$$

(Assuming operation at first harmonic – see lecture 12 of https://www.slac.stanford.edu/~xiahuang/USPAS_2017.htm)

- For N=1 undulator period, 10% bandwidth, detector 10m from undulator, we get ~5.5e-4W/m^2 intensity on-axis from core protons

  (cf. 3.3e-5 W/m^2 for dipole radiation)
- Does not change any of our conclusions about thermal noise – still lose to 10W/m^2 thermal noise at 300K unless we have ~100 periods (impossible in arcs)
- Undulator likely not worth the extra effort (not considered: beam evolution within undulator also makes this tricky, as seen in dipole case)

Case	100 GeV	275 GeV
Geometry		
Modulator Length (m)	45.420181	45.420181
Kicker Length (m)	45.420181	45.420181
Number of Amplifier Drifts	2	2
Amplifier Drift Lengths (m)	36.579819	36.579819
Proton Parameters		
Protons per Bunch	6.9e10	6.9e10
Proton Bunch Length (cm)	7	6
Proton Fractional Energy Spread	9.7e-4	6.8e-4
Proton Emittance (x/y) (nm)	30 / 2.7	11.3 / 1
Horizontal/Vertical Proton Betas in Modulator (m)	20.96904 / 55.920402	39 / 39
Horizontal/Vertical Proton Dispersion in Modulator (m)	0.848258 / 0.235044	1 / 0
Horizontal/Vertical Proton Dispersion Derivative in Modulator (m)	-0.023971 / 0	-0.023 / 0
Horizontal/Vertical Proton Betas in Kicker (m)	20.96904 / 55.920402	39 / 39
Horizontal/Vertical Proton Dispersion in Kicker (m)	0.848258 / 0.235044	1 / 0
Horizontal/Vertical Proton Dispersion Derivative in Kicker (m)	0.023971 / 0	0.023 / 0
Proton Horizontal/Vertical Phase Advance (rad)	5.145 / 4.9	4.7895 / 4.7895
R56 in Proton Chicane (mm)	-6	-2.25956
Electron Parameters		
Electron Bunch Charge (nC)	1	1
Electron Bunch Length (mm)	15	7
Electron Peak Current (A)	~ 8	~ 17
Electron Fractional Slice Energy Spread	1e-4	1e-4
Electron Normalized Emittance (x/y) (mm-mrad)	2.8 / 2.8	2.8 / 2.8
Horizontal/Vertical Electron Betas in Modulator (m)	60.043513 / 84.017684	40 / 20
Horizontal/Vertical Electron Betas in Kicker (m)	10.006454 / 13.977728	4 / 4
Horizontal/ Vertical Electron Betas in Amplifiers (m)	1.537031 / 1.537031	1 / 1
R56 in First Electron Chicane (mm)	15.786	5.0
R56 in Second Electron Chicane (mm)	15.786	5.0
R56 in Third Electron Chicane (mm)	-42.573	-11.45
Cooling Times		
Horizontal/Vertical/Longitudinal IBS Times (hours)	2.0 / 4.0 / 2.5	2.0 / 5? / 2.9
Horizontal/Vertical/Longitudinal Cooling Times (hours)	1.7 / 3.8 / 2.2	1.8 / ∞ / 2.8