Today schedule:

1. Questions from the comp. simulation class
2. HW1 Review. Results of injector simulation .
   1. Emittance vs solenoid strength plots.
   2. Is minimum emittance location different for different solenoids field?

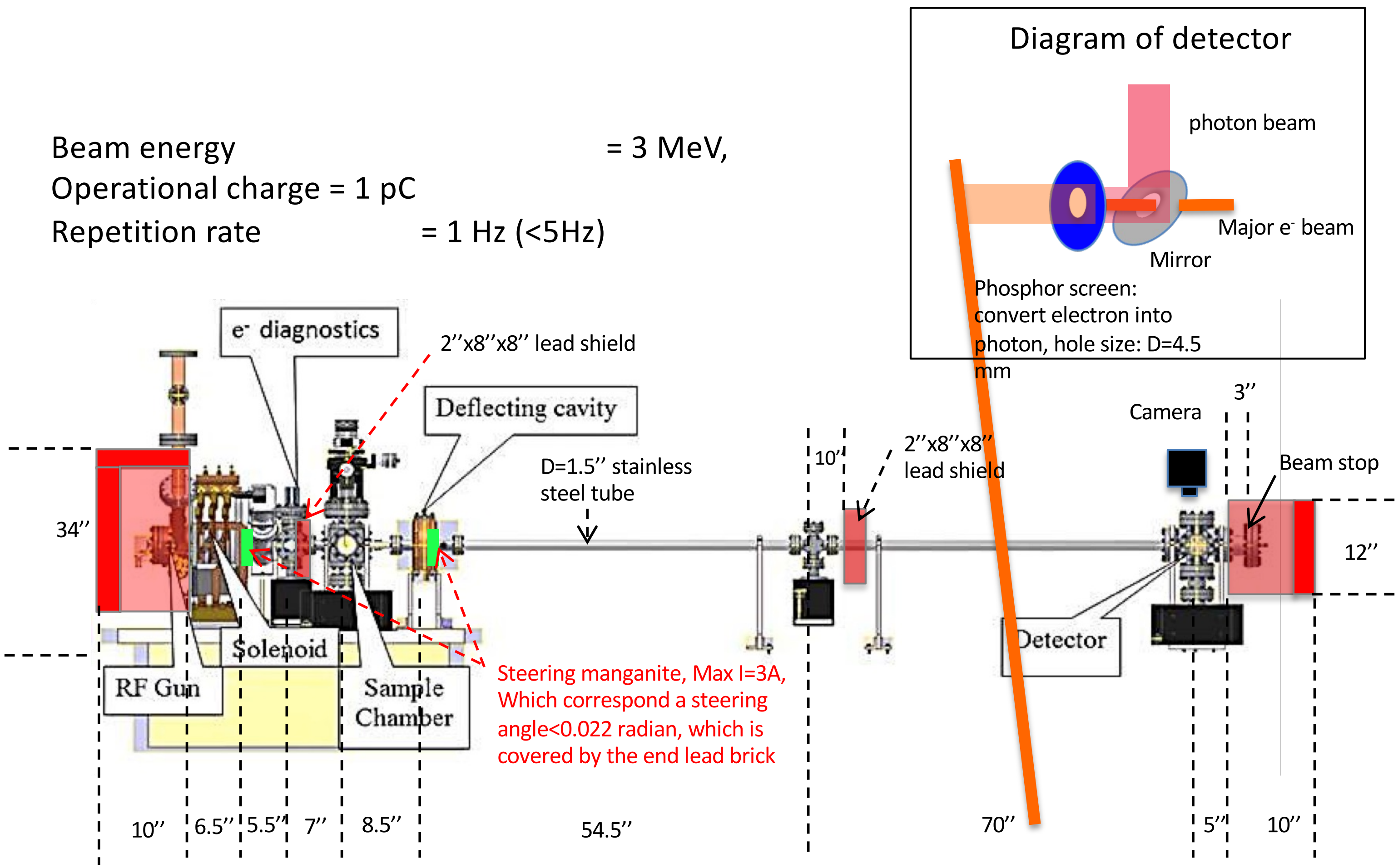
3. Q: UED beam line simulation. Do you have all necessary information?
4. RF acceleration (short lecture.)
5. Questions
6. Start Simulation linacs HW 2
7. If we have time today Beam transport components (or during the next comp. class)

Next week March 6 class will **take place at BNL:**

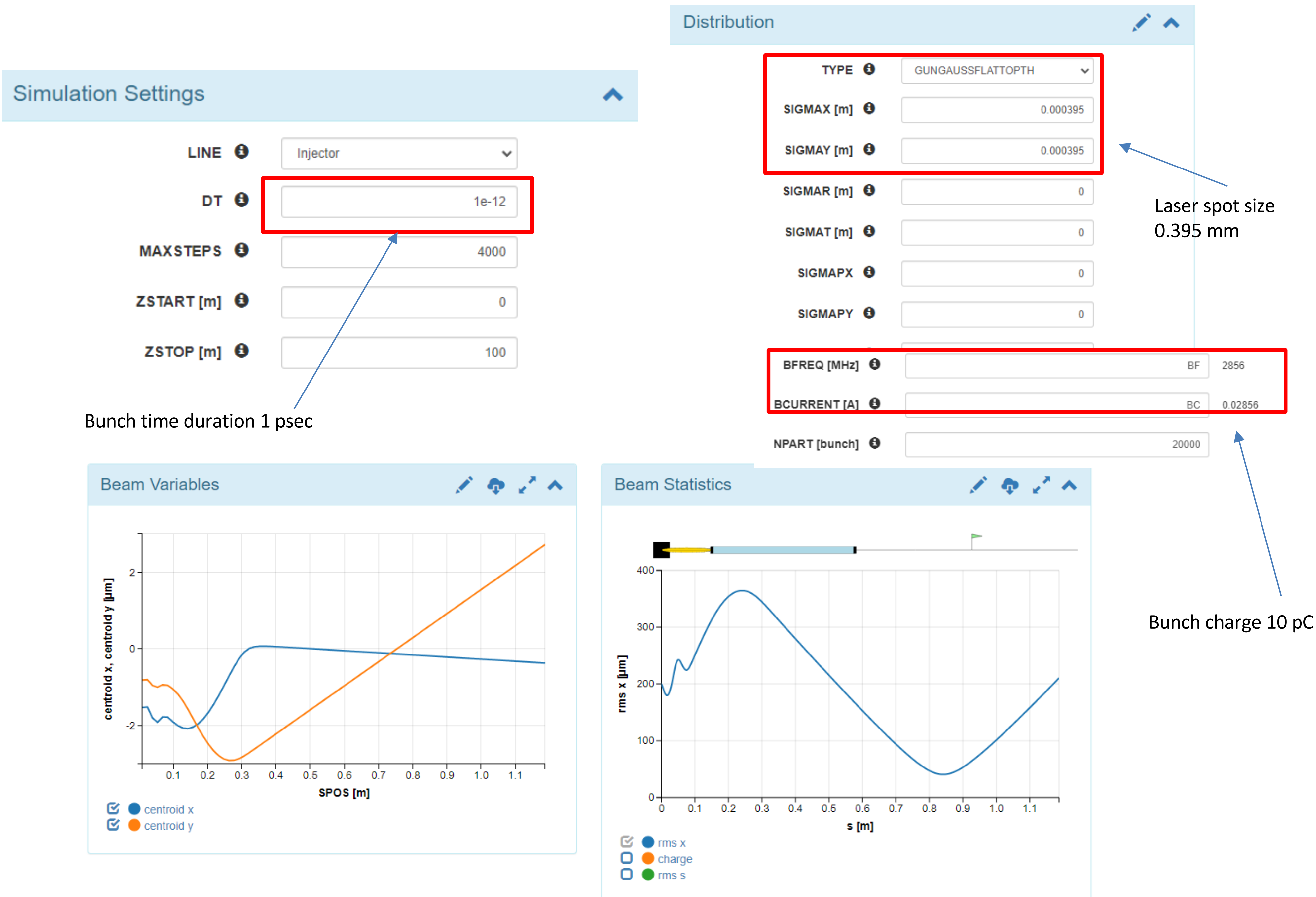
- TLD distribution , safety announcement , ATF tour
- UED QE measurements,
- Energy measurements using diffraction
- Comparison UED line simulation results with measurements
  - solenoid field requirements for smallest bunch size at the cathode for different bunch charges
- you will need to finish these simulation before the class

UED beamline layout

Beam energy = 3 MeV,
Operational charge = 1 pC
Repetition rate = 1 Hz (<5Hz)



https://www.sirepo.com/opal#/visualization/7Az3zlie



https://www.sirepo.com/opal#/visualization/7Az3zlie

Beam acceleration and transport

D. Kayran

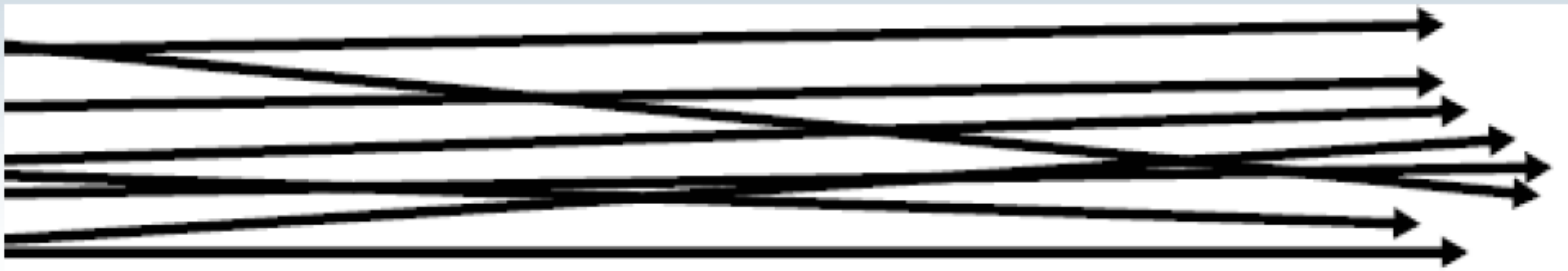
February 27, 2023

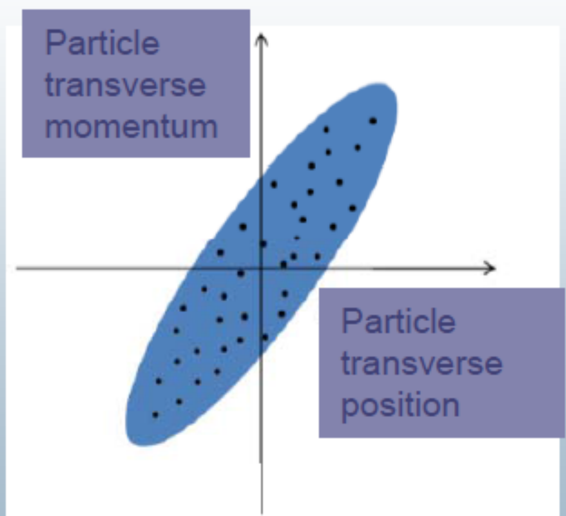
PHY 542 Fundamentals of Accelerator Physics and Technology with Simulations and Measurements Lab

Main accelerators components (continue)

- Source
- Beam Acceleration
- Beam transport

Defining beam quality

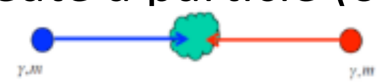




- Beam quality measures:
  - emittance (ε): volume of phase-space
  - Brightness (B): density of phase-space
- We desire high brightness & low emittance beams

Acceleration is needed!!

- In colliders: The minimum energy required to create a particle (or group of particle) with total mass M is: $E_{min} = Mc^2$



  - High energy colliding particles=>high energy center mass=> massive particles production (cross section σ)
  - luminosity:

$$L = f_c \frac{N_1 N_2}{A} \cong f_c \frac{N_1 N_2}{2\pi \sqrt{\beta_{x1}\varepsilon_{x1} + \beta_{x2}\varepsilon_{x2}} \sqrt{\beta_{y1}\varepsilon_{y1} + \beta_{y2}\varepsilon_{y2}}}$$

Numbers of events

$$N|_{A\to B} = \sigma_{A\to B} \cdot L$$

$$\varepsilon_{n,s} = \beta\gamma \sqrt{\langle s^2 \rangle \langle s'^2 \rangle - \langle ss' \rangle^2}$$

- Normalized emittance ~preserved during acceleration, geometrical emittance reduced ~$1/\gamma$.

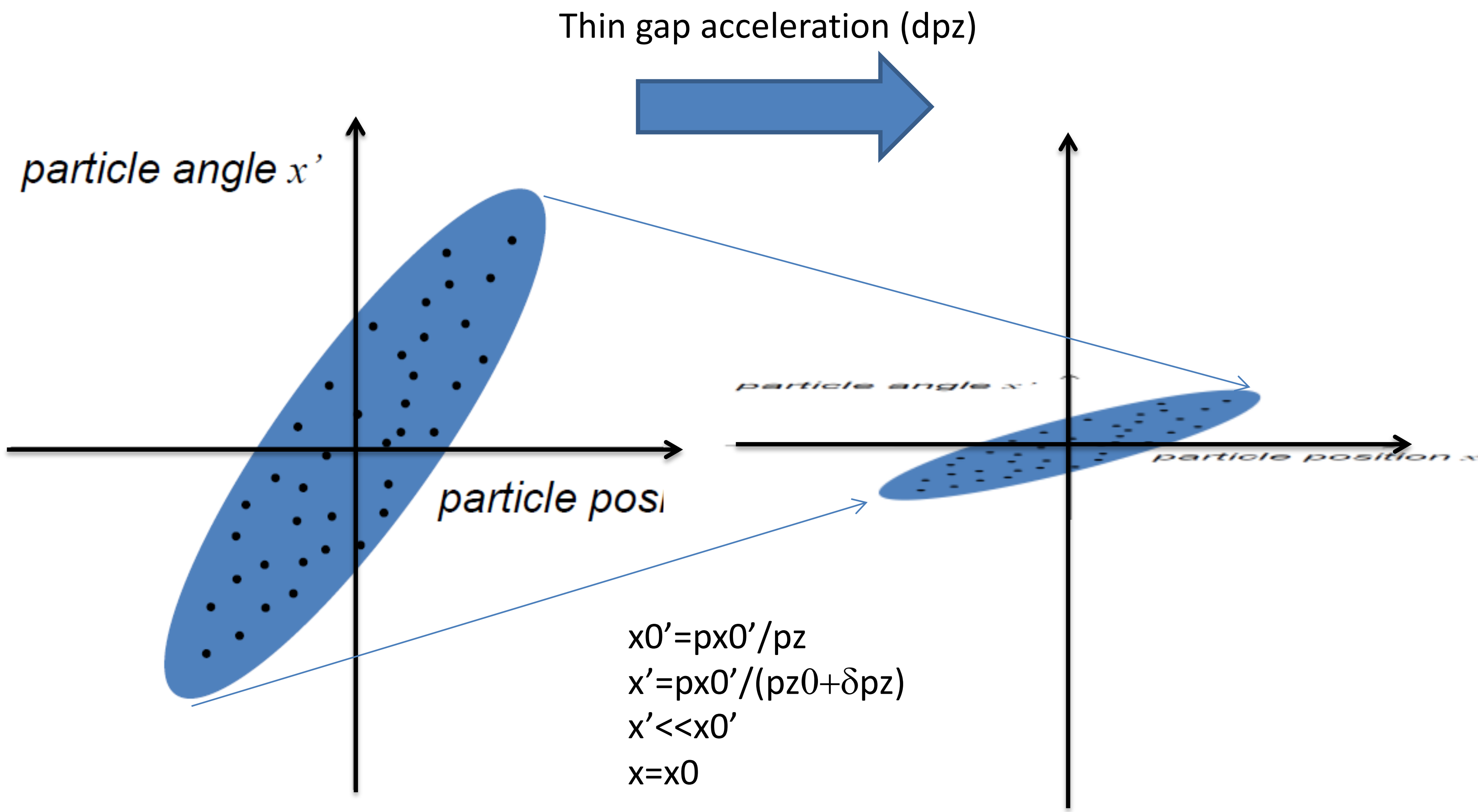
where s is either x or y.

The peak normalized rms brightness is given by

$$B_n = \frac{2I}{\varepsilon_{n,x}\varepsilon_{n,y}}$$

- In light source: Brightness B ~$1/\gamma^2$.

Geometrical emittance transformation



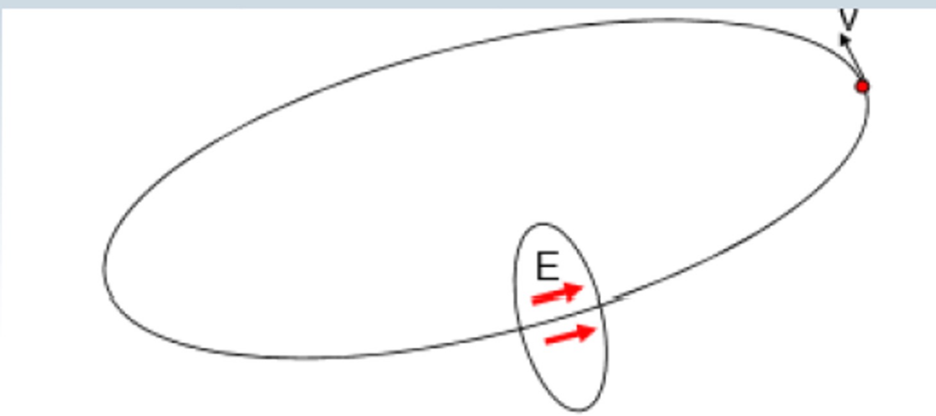
x0'=px0'/pz

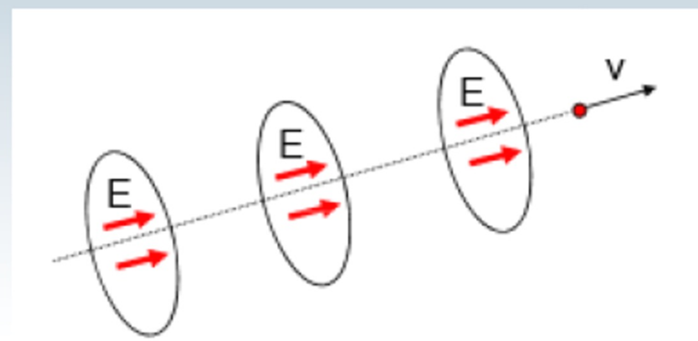
x'=px0'/(pz0+δpz)

x'<<x0'

x=x0

Types of accelerators

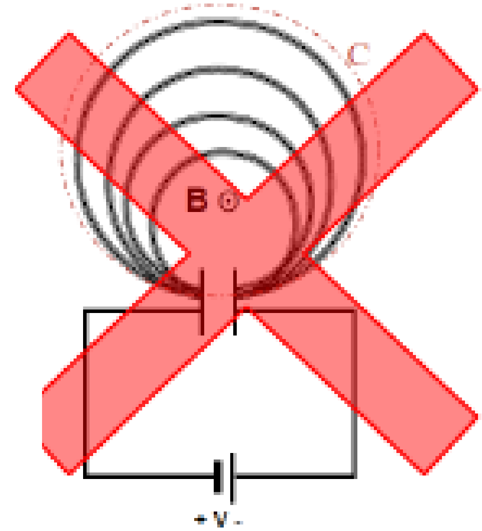




- Circular accelerators
  - Repeated passage of beams via a series of cavities
  - Suitable for heavy particles, i.e. protons
- Linear accelerators
  - Particles pass only once through each cavity
  - Suitable for light particles, i.e. electrons

Acceleration

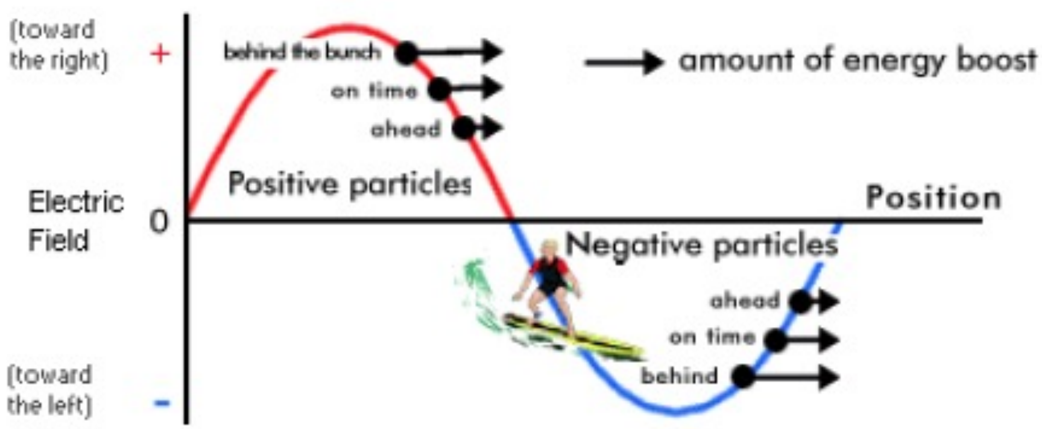
$$\frac{d\vec{p}}{dt} = q\left(\vec{E} + \frac{\vec{\mathrm{v}}}{c} \times \vec{B}\right); \;\; \frac{dE}{dt} = q\left(\vec{E} \cdot \vec{\mathrm{v}}\right);$$



- Single pass acceleration
- Limited by maximum voltage per until discharge. ~1.5 MV in air

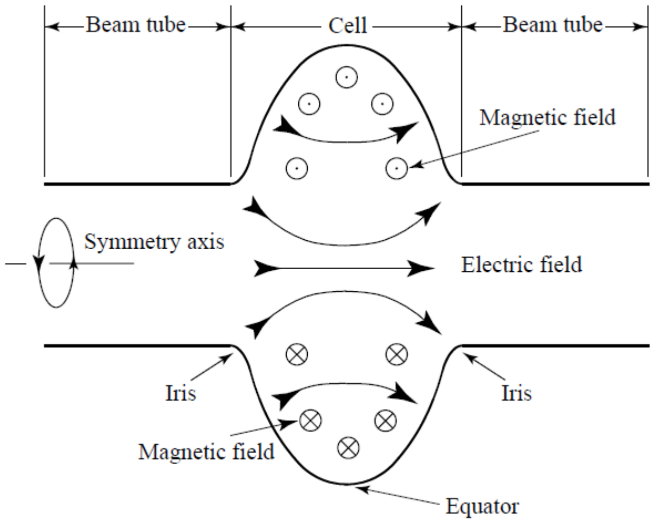
$$\Delta E = e\oint \vec{E} \cdot d\vec{l} = -\frac{e}{c}\frac{\partial}{\partial t}\left(\int \vec{H} \cdot d\vec{s}\right)$$

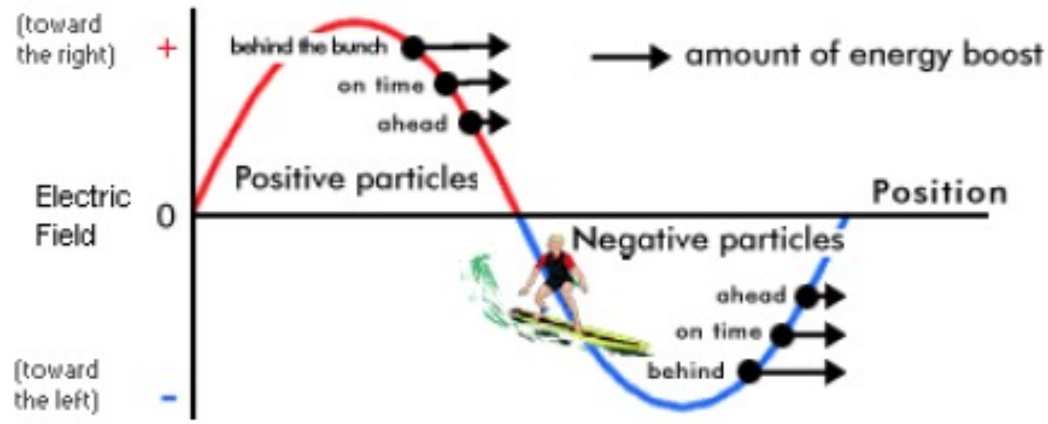
Maxwell equation prohibits multiple acceleration is DC electric field:



- In RF cavities energy gain depends on the phase.
- The main purpose of using RF cavities in accelerators is to add (remove) energy to charged-particle beams at a fast acceleration rate

RF Field acceleration:





The RF field must be synchronous (correct phase relation) with the beam for a sustained energy transfer.

$$E_z(z,t) = E(z)\cos\left(\omega t - \int_0^z k(z)\,dz + \phi\right),$$

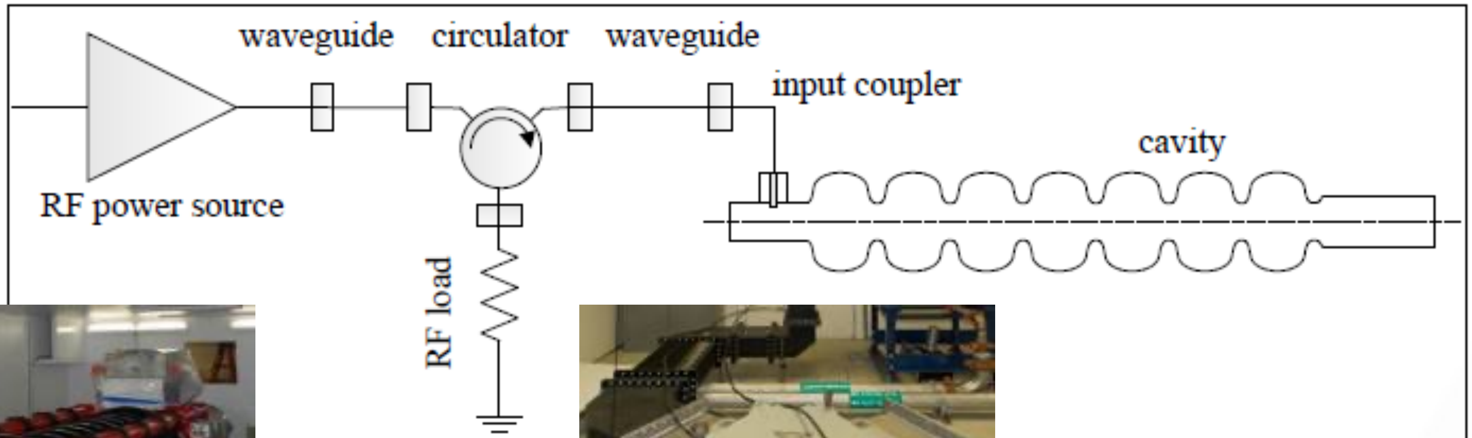
For efficient particle acceleration, the phase velocity of the wave must closely match the beam velocity. If we consider a particle of charge *q* moving along *+z* direction with a velocity at each instant of time equal the phase velocity of the traveling wave, then the electric force on the particle is given by

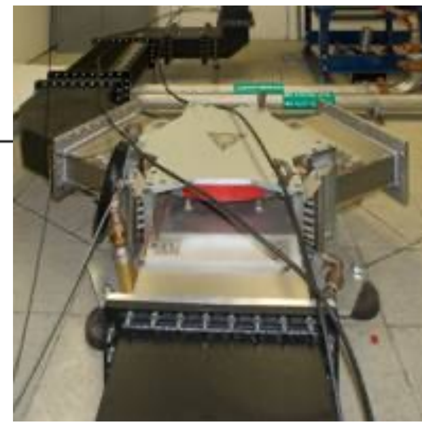
$$F_z = q\,E(z)\cos\phi$$

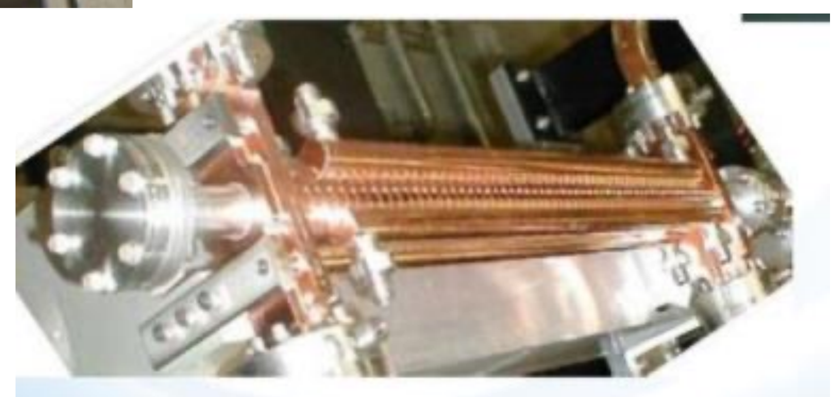
Energy gain

$$\Delta\mathcal{E} = q\int_{-L/2}^{L/2} E(0,z)\cos\left[\omega t(z) + \phi\right] dz$$

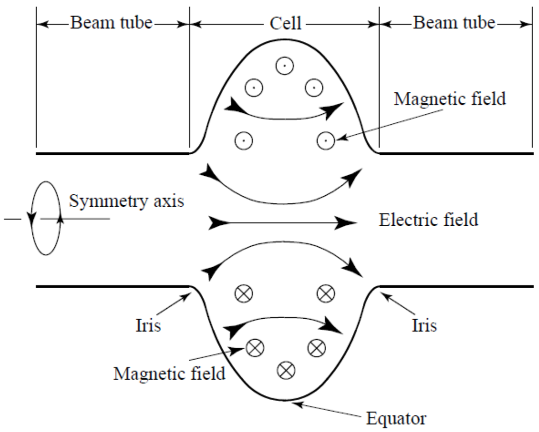
RF Cavity connected to RF power source



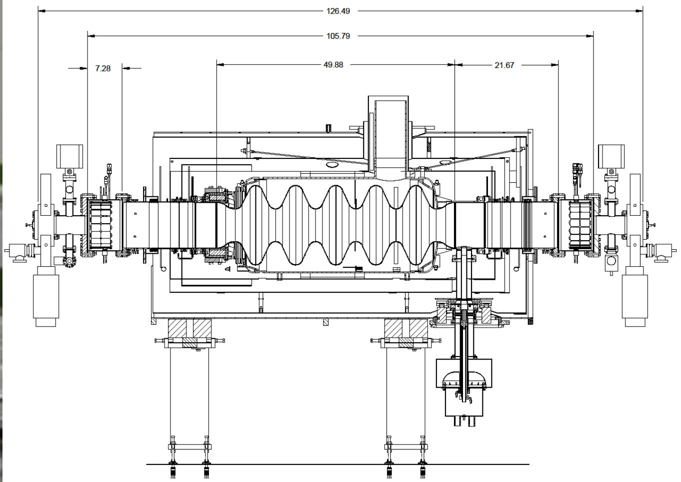




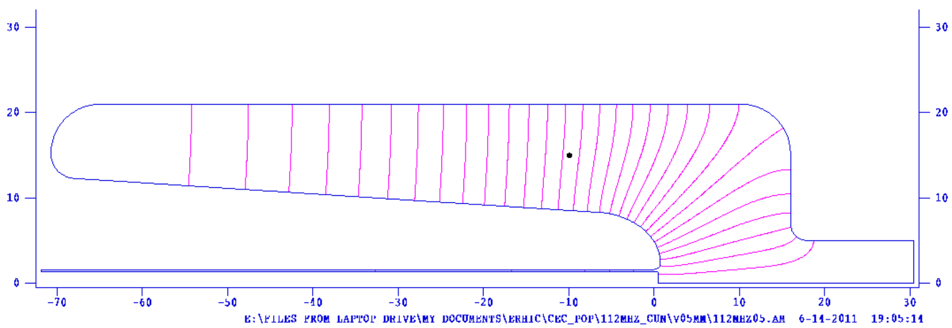
RF cavities



Typical Single cell

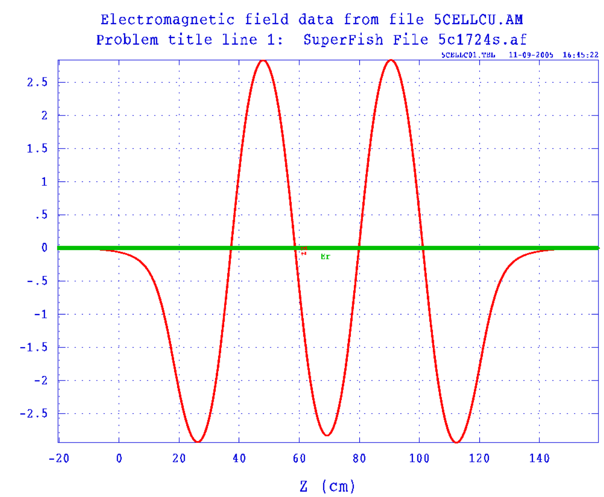


BNL ERL: 5Cell cavity 704MHz



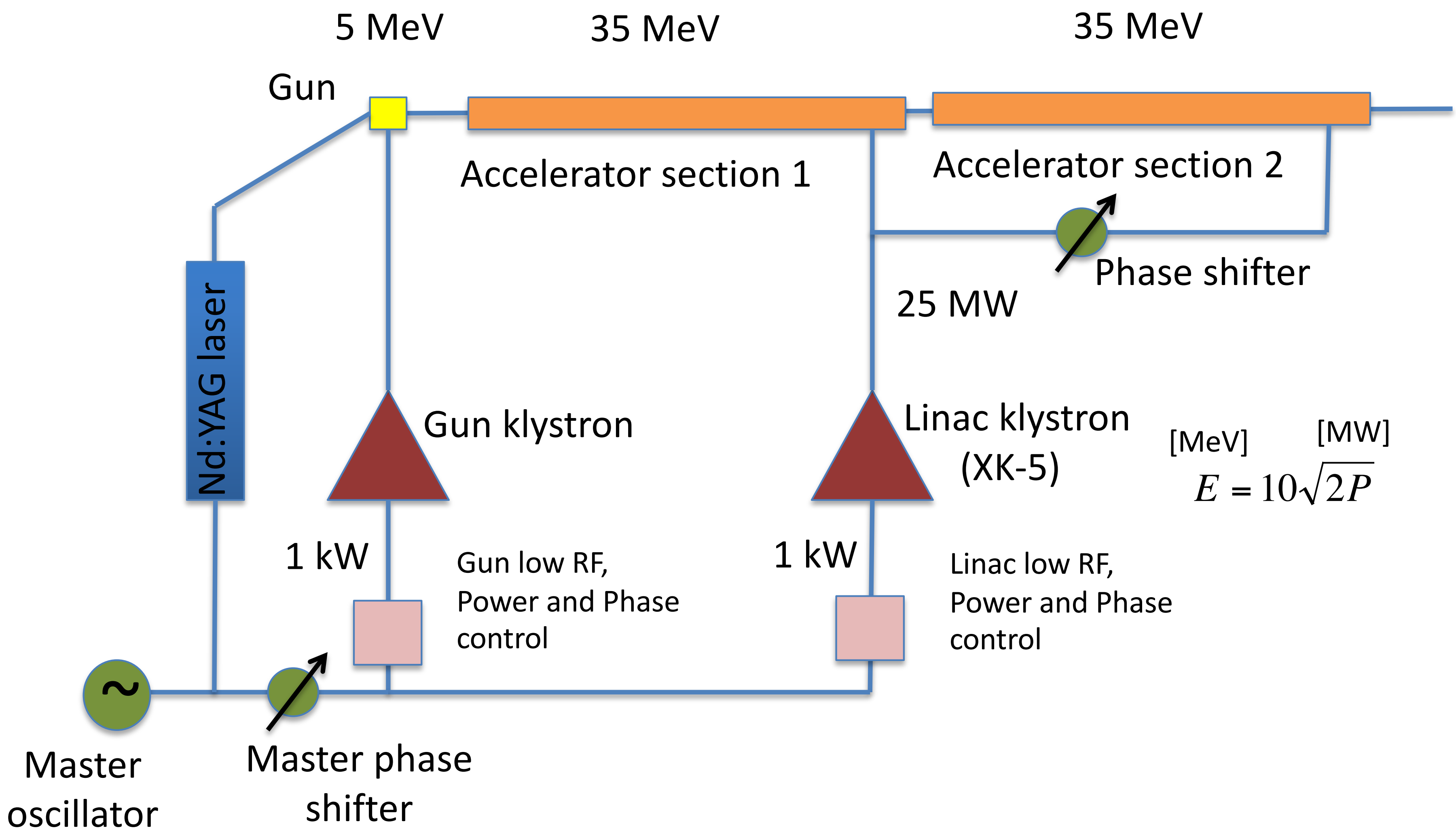
Quarter-wave 112MHz resonator

SRF GUN at CEC-X

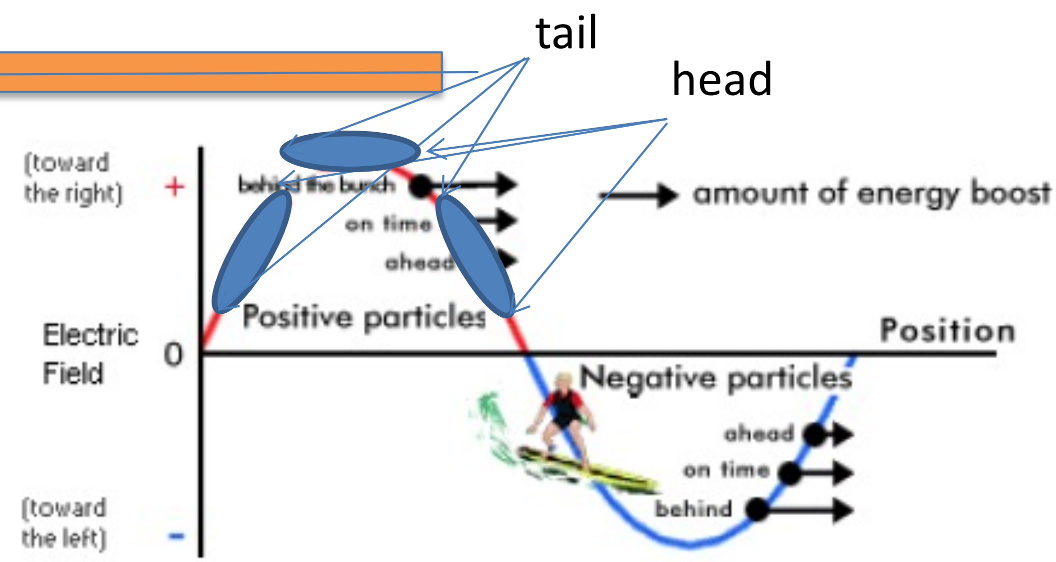


SRF LINAC at CEC-X

ATF accelerator system



Multi linacs acceleration



$E=E_{inj}+Elinac1+Elinac2$

$Elinac1=eU1\cos(phi1)$

$Elinac2=eU2\cos(phi2)$

If there is enough voltage provided by one linac.
The final energy can be reached by combination different phases.

For ATF:
U1=U2=36MV, Einj=5MeV
Efinal=35MeV

Why one operation could be better then others?

phi1	phi2
65.4	65.4
65.4	-65.4
0.0	99.6
0.0	-99.6
90.0	33.6
-90.0	-33.6
33.6	90.0
-33.6	-90.0

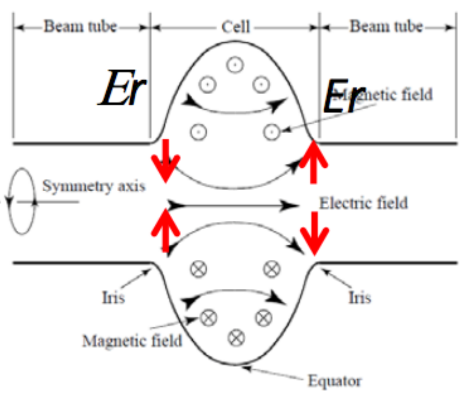
Few things to remember

- Space charge force depends on energy
  - Higher energy => less space charge effects

$$\sigma_x''(\zeta,s) + \kappa_\beta^2 \cdot \sigma_x(\zeta,s) = \frac{r_e \lambda(\zeta)}{2\gamma^3 \sigma_x(\zeta,s)} + \frac{\varepsilon_{n,x}^2}{2\gamma \sigma_x^{\,3}(\zeta,s)}$$

- Focusing due to entrance and exit of RF field
  - More energy gain => stronger focusing

Ein



Eout

$$\Delta p_r = \frac{e}{c} \int E_r dz$$

$$r'_{in} = \frac{\Delta p_{in\_r}}{p_{in\_z}} \qquad r'_{out} = \frac{\Delta p_{out\_r}}{p_{out\_z}}$$

$$\Delta p_{in\_r} \sim -\Delta p_{out\_r}$$

Eout= Ein+ΔE

Entrance kick is larger then exit kick

PHY 542
COMPUTATIONAL EXERCISE – RF Linac

Exercise: RF linac accelaration

1. Open file *ATF_LINAC.in*. Find acceleration linac line description. There are two linacs. Make sure that the both cavities gradient is sufficient to accelerate e-beam on 36 MeV by each cavity. Change adjust maximum gradient (maxE parameter).
   (*hint: set acceleration phase to 0 in both linacs and run ASTRA for this project*)
2. Search for optimum linac set points for fix energy gain 30 MeV. Set up linac acceleration gradient 16 MV/m. Set the same phase for both linac to accelerate 15 MeV each. (phi=65 deg). Find final energy spread and emittance.
3. Repeat step 2 for different linac phases:
   a. Linac Phase1=65 LinacPhase2=-65
   b. Linac_Phase1=34 Linac Phase2=90
   c. Linac Phase1=90 LinacPhase2=34 (have you got the same energy?)
   d. Linac Phase1=0 LinacPhase2=100
4. What linacs phase settings provide minimum **emittance**?
5. What linacs phase settings provide minimum **energy spread**?

Same exercise without space charge:

6. Try turn off space charge and repeat steps 2-5.
7. Why final emittance is different without space charge?

Main accelerators components

- Source
- Beam Acceleration
- Beam transport

- Each particle is defined in 6-D space (coordinates and momentums) $\vec{x} = (x, p_x, y, p_y, z, p_z)$

- In accelerators physics is more convenient to use reference particle and paraxial approximation pz>>px,py then:

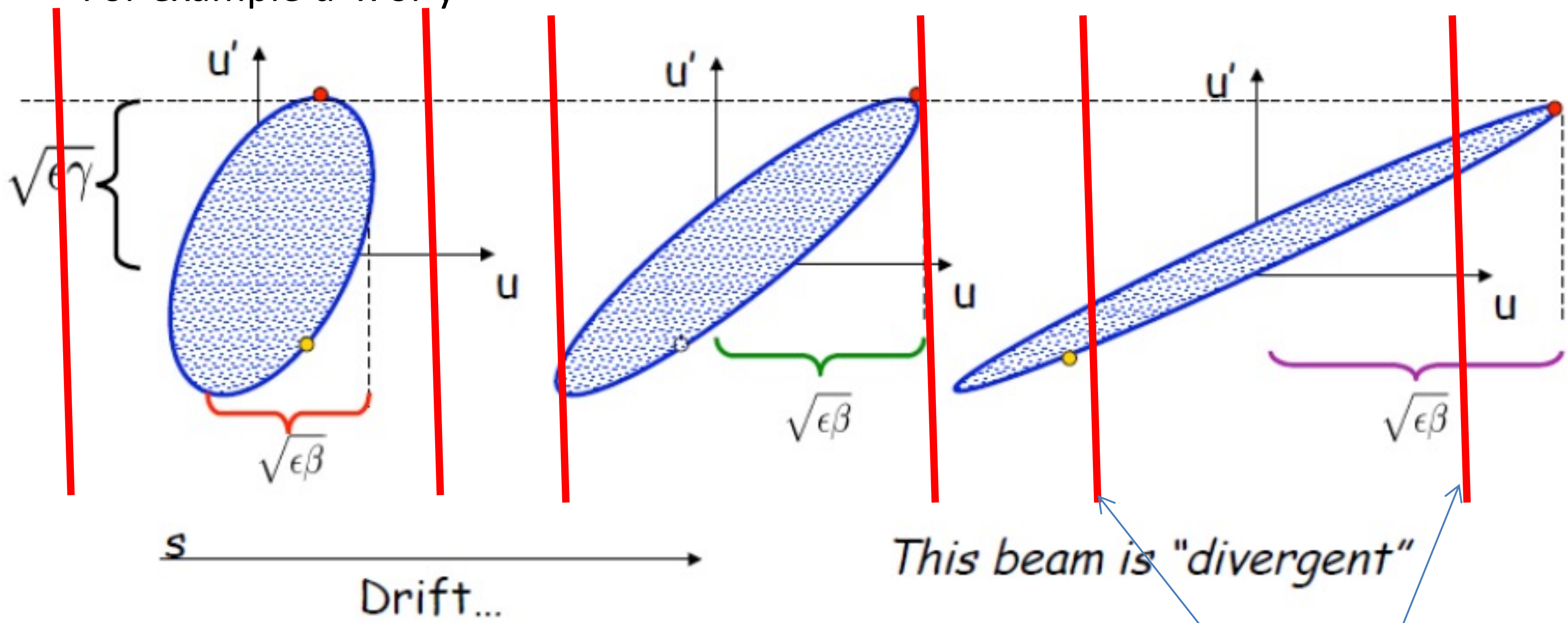
$$\vec{x} = (x, x', y, y', c\Delta t, \Delta E/E)$$

$$x' = \frac{p_x}{p_z} \quad y' = \frac{p_y}{p_z}$$

- $\Delta t, \Delta E$-it s time and energy difference energy from reference particle.

Beam phase space modification drift space only

If there is no coupling between X and Y we can work with 2D phase spaces
For example u=x or y

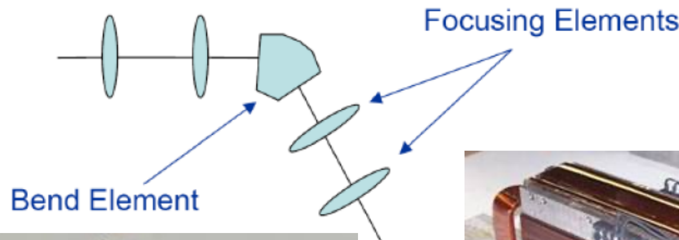


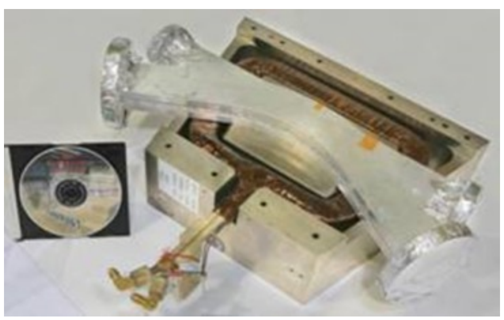
Eventually beam spreads out and hits the aperture
Focusing is needed.

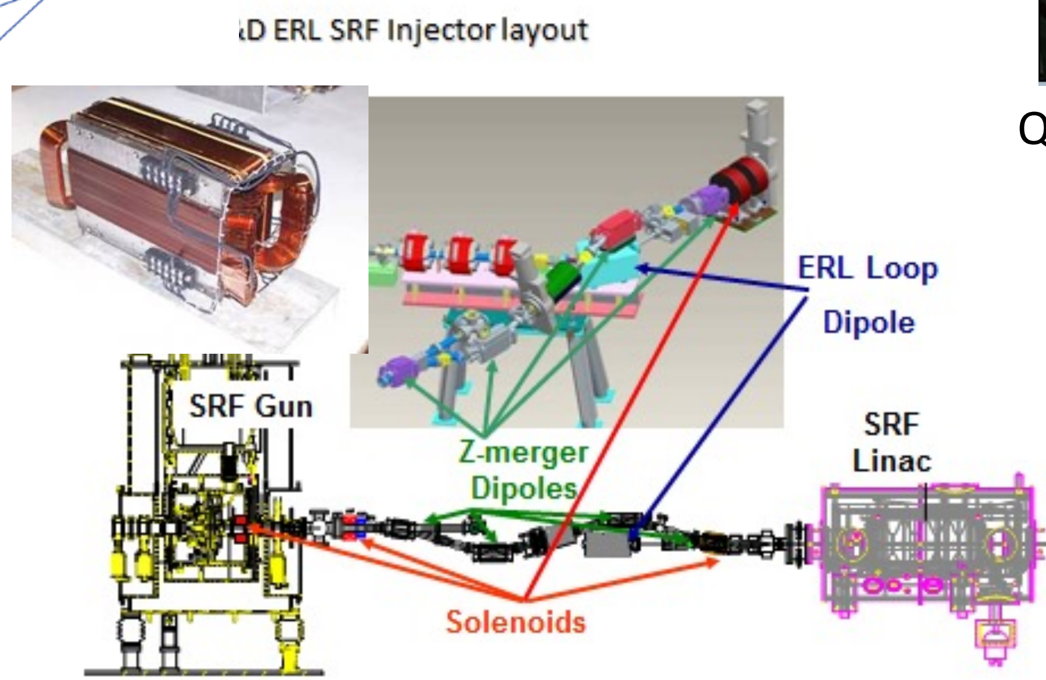
Vacuum pipe aperture
radius=a (+/- a)

Magnetic lattice

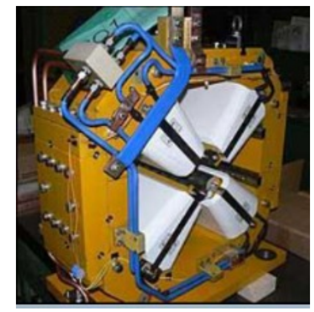
- Usually the set of different kind of magnets is needed in order to successfully propagate charge beam through the system.





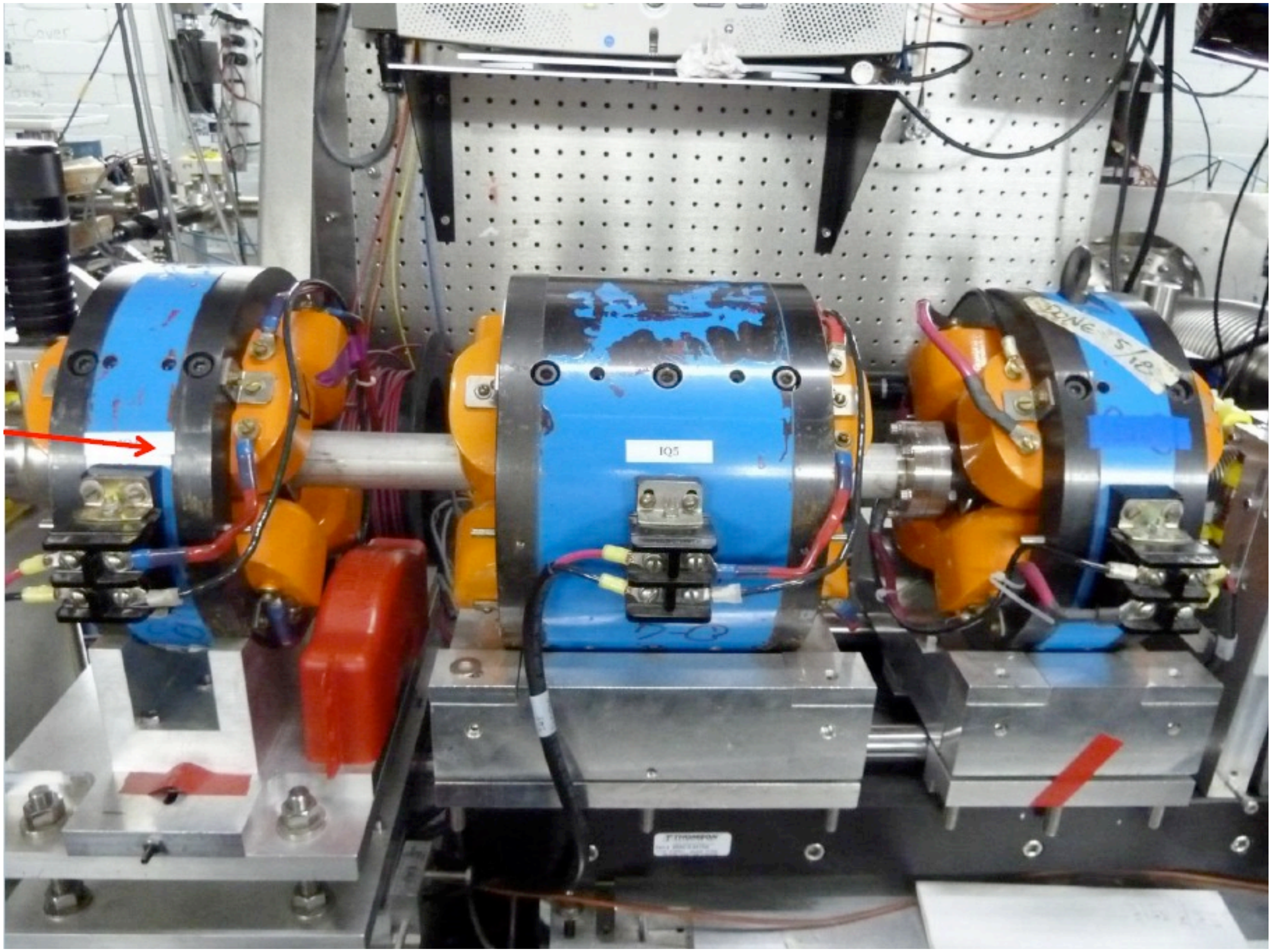


Dipoles



Quadrupoles

ATF qudrupoles



How can we say that these are quadrupoles?

Why magnetic field not electric field?

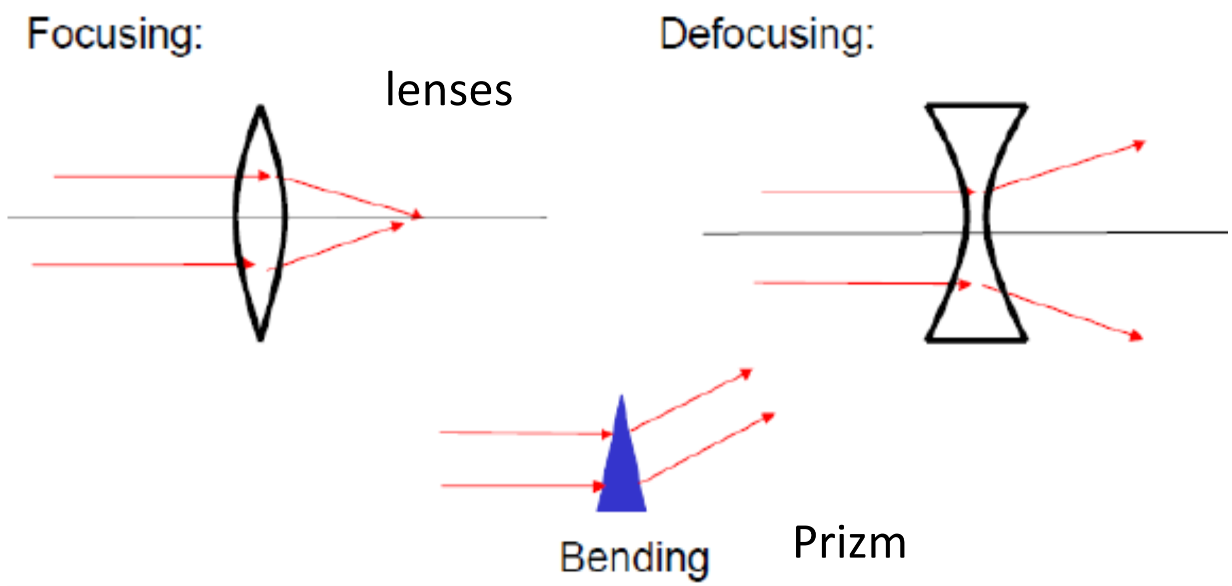
Ratio of magnetic and electric forces

$$F = q(E + v \times B) \Rightarrow \frac{F_M}{F_E} = \frac{vB}{E} \Rightarrow \frac{F_M}{F_E} = 1 \Rightarrow E = vB$$

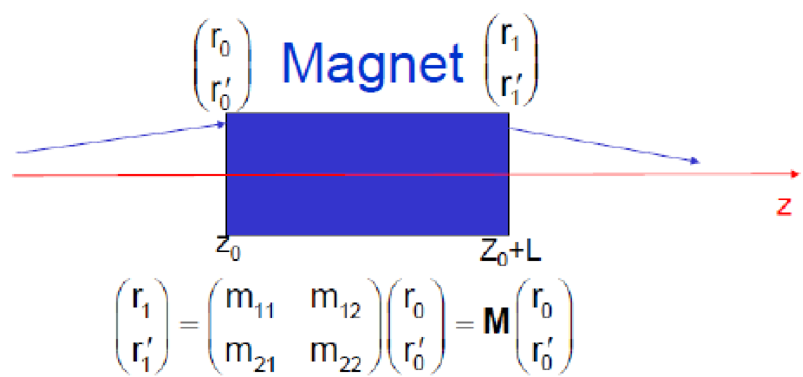
- For ultra-reletivistic particles v~c
  - B=1T is equivalent to E=300MV/m!!!!
- For low energy (v=0.01c)
  - B=1T is equivalent of E=3MV/m

- Electrostatic accelerators existed but the use of such systems are very limited of low energy!!

The light optics similarity

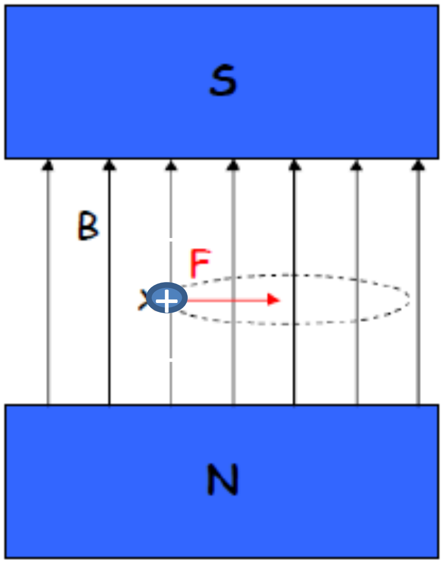


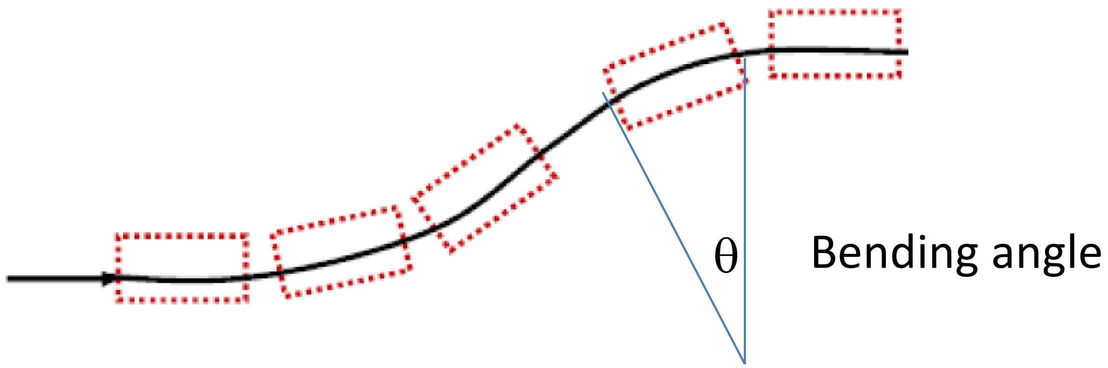
The same matrix formalism can be adopted in first order and linear approximation.
Vectors (x,x') and (y, y')



Bending magnets

- A dipole magnet with constant magnetic field
- Positive particle coming in the screen will bend to the right.
- Using combination of the dipoles one could create any kind of transport lines.



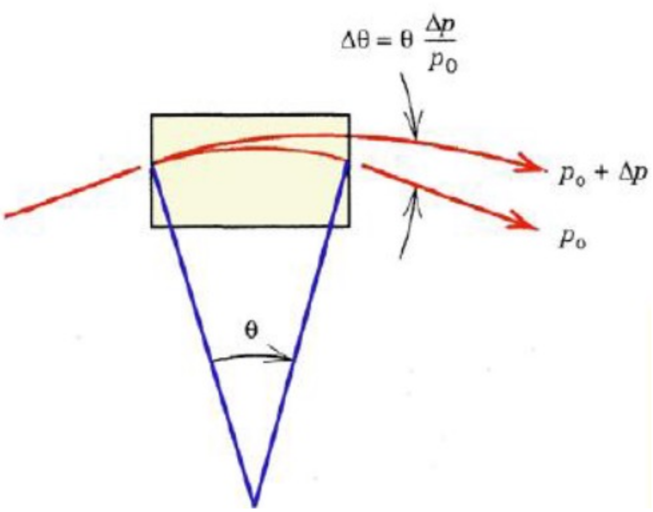


$$\theta = \frac{e}{p_0}\int_{s_1}^{s_2} B dl = \frac{e}{B\rho}\int_{s_1}^{s_2} B dl$$

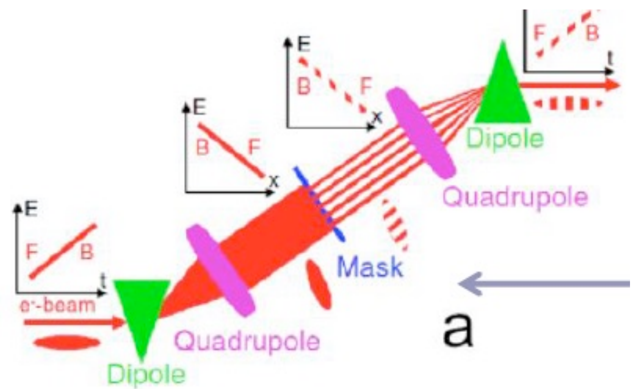
Where, p_0 is the momentum and $B\rho = p_0/e$ is the <u>momentum 'rigidity'</u> of the beam.

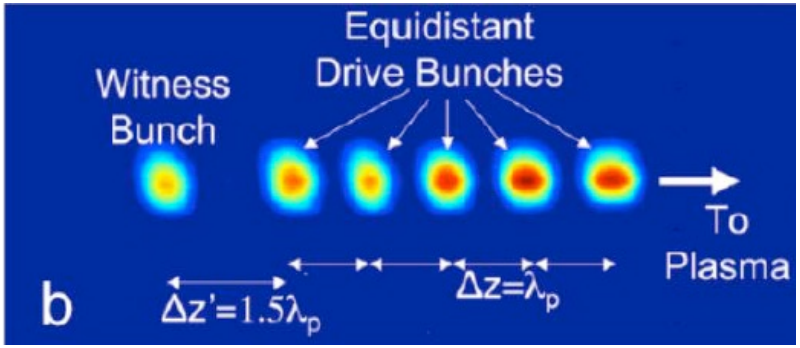
Dispersion

- Particle with different momentum will be bend on different angle
- Can cause beam quality degradation but also used for some experiments.

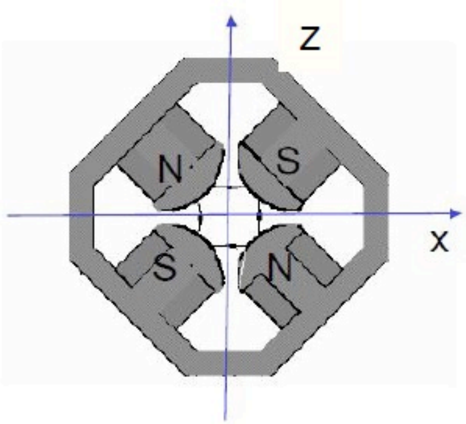


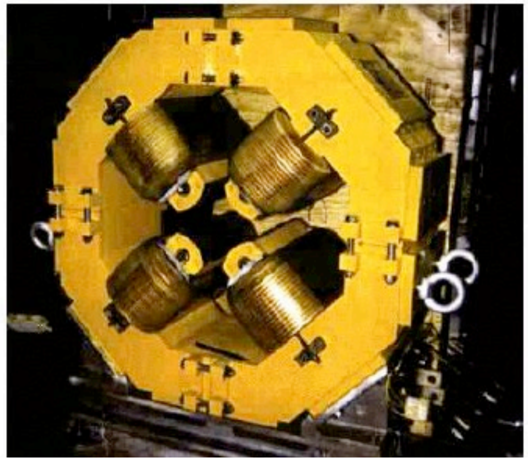
- Mask at ATF

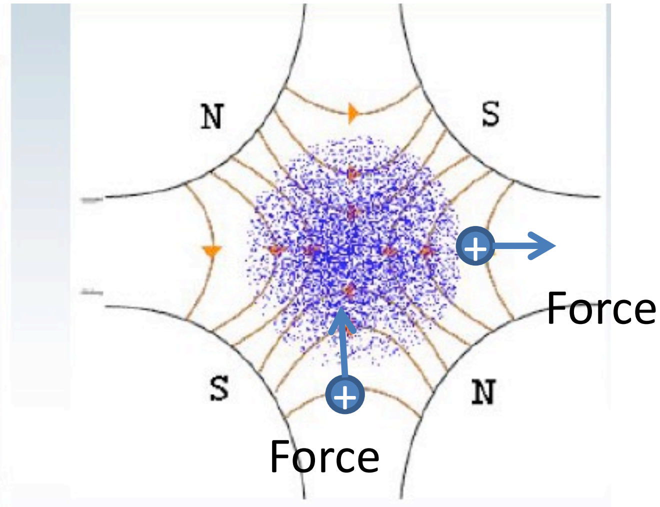




Quadrupoles



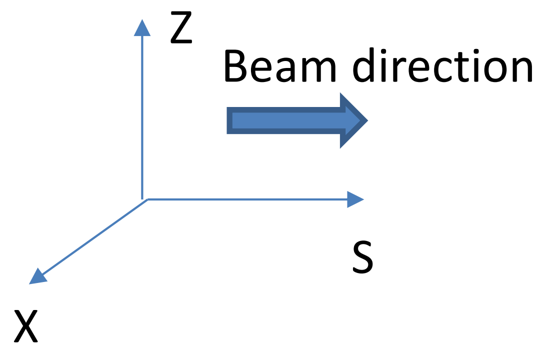




$$B = B_1\left(z\hat{x} + x\hat{z}\right)$$

- Due to special field symmetry focus beam in one direction but defocus in other.
- Particles moving at axis are not experienced any force.

Sometimes in accel. physics especially in circular accel. (X,Z, S) coordinates are used



Quadrupoles(cont.)

- Particle displaced by (x,z) from the center

$$\boxed{B = B_1 (z\hat{x} + x\hat{z})}$$

$$\vec{F} = evB_1\hat{s} \times (z\hat{x} + x\hat{z}) = -evB_1 z\hat{z} + evB_1 x\hat{x}$$

the equations of motion become:

$$\frac{1}{v^2}\frac{d^2x}{dt^2} = \frac{eB_1}{\gamma m v}x, \quad \frac{1}{v^2}\frac{d^2z}{dt^2} = -\frac{eB_1}{\gamma m v}z$$

or $$\frac{d^2x}{ds^2} = x'' = \kappa x \quad \frac{d^2z}{ds^2} = -\kappa z$$ where $$\kappa = \frac{eB_1}{\gamma m v}$$

When matrix transformation from entrance to exit of quadrupole :

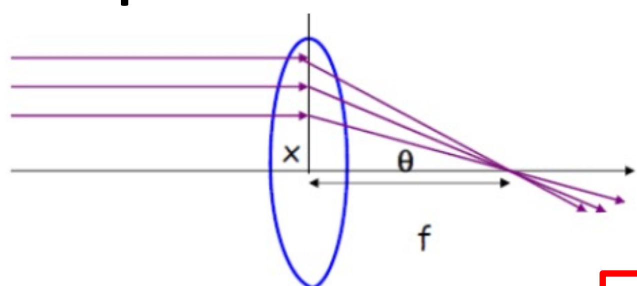
$$\begin{pmatrix} x \\ x' \end{pmatrix} = \begin{pmatrix} \cos\sqrt{\kappa}L & \frac{1}{\sqrt{\kappa}}\sin\sqrt{\kappa}L \\ -\sqrt{\kappa}\sin\sqrt{\kappa}L & \cos\sqrt{\kappa}L \end{pmatrix} \begin{pmatrix} x_0 \\ x0' \end{pmatrix} \quad \begin{pmatrix} z \\ z' \end{pmatrix} = \begin{pmatrix} \cosh\sqrt{\kappa}L & \frac{1}{\sqrt{\kappa}}\sinh\sqrt{\kappa}L \\ \sqrt{\kappa}\sinh\sqrt{\kappa}L & \cosh\sqrt{\kappa}L \end{pmatrix} \begin{pmatrix} z_0 \\ z0' \end{pmatrix}$$

Thin lens approximation

- For thin lens when K<<1/L^2

$$\begin{pmatrix} \cos(\sqrt{K}L) & \frac{1}{\sqrt{K}}\sin(\sqrt{K}L) \\ -\sqrt{K}\sin(\sqrt{K}L) & \cos(\sqrt{K}L) \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 \\ -KL & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{F} & 1 \end{pmatrix}$$

- If the quadrupole is thin enough, the particles coordinate doesn't change while momentum change. The quad works almost as a optical lens…



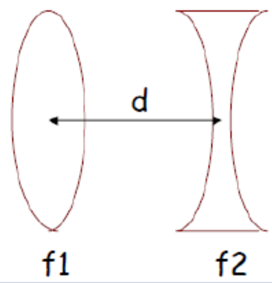
$$\Delta x' = \frac{x}{f}$$

- With only one difference:

It Focus in the one plane and defocus in the other plane

Focus the beam in both directions.

- Using doublets
- Using optical analogy one can calculate



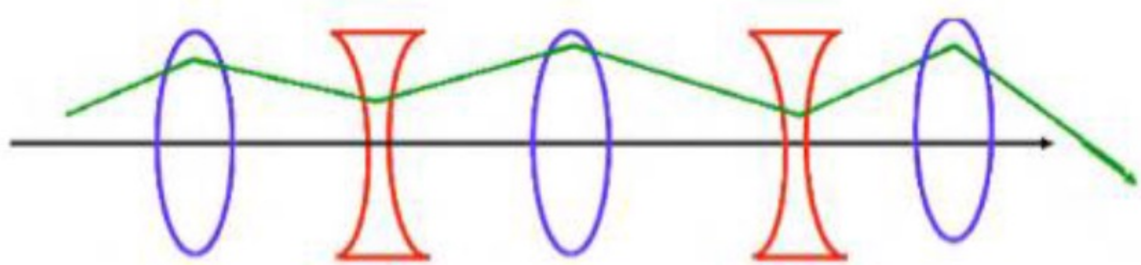
The combined f is:

$$\frac{1}{f_{combined}} = \frac{1}{f1} + \frac{1}{f2} - \frac{d}{f1f2}$$

What if f1 = -f2?

$$f_{combined} = \frac{f1^2}{d}$$

- A quadrupole doublet is focusing in both planes.
- Strong focusing by sets of quadrupole doublets with alternative gradient. Could keep beam inside vacuum chamber.



Solenoid

- A solenoid is a set of helical coils.
- Typically, solenoid radius is smaller than the its length.
- Magnetic field is generated along the axis line.
- Solenoid couples X and Y motion.
- Solenoid produced focusing in both direction

$$1/f = e\int Bz^2 dz/(2pc)^2$$

- Solenoids are preferable at low energy