PHY 554. Homework 5.

(a) The closed orbit of a three-bump system is

$$y(s) = \frac{\sqrt{\beta}}{2\sin\pi\nu}\sum_{i=1}^{3}\sqrt{\beta_i}\theta_i\cos(\pi\nu - |\psi - \psi_i|).$$

Using the condition $y(s_3) = y'(s_3) = 0$, Then

$$\begin{cases} \sqrt{\beta_1}\theta_1\cos(\pi\nu + \psi_{13}) + \sqrt{\beta_2}\theta_2\cos(\pi\nu + \psi_{23}) + \sqrt{\beta_3}\theta_3\cos\pi\nu = 0 \\ \sqrt{\beta_1}\theta_1\sin(\pi\nu + \psi_{13}) + \sqrt{\beta_2}\theta_2\sin(\pi\nu + \psi_{23}) + \sqrt{\beta_3}\theta_3\sin\pi\nu = 0 \end{cases}$$

where $\psi_{13} = \psi_1 - \psi_3$ and $\psi_{23} = \psi_2 - \psi_3$, we find

$$\theta_2 = -\theta_1\sqrt{\frac{\beta_1}{\beta_2}}\frac{\sin\psi_{13}}{\sin\psi_{23}}, \qquad \theta_3 = \theta_1\sqrt{\frac{\beta_1}{\beta_3}}\frac{\sin\psi_{12}}{\sin\psi_{23}}.$$

(b) When $\psi_{31} = n\pi$, we find $\theta_2 = 0$, i.e. only two steering dipoles are needed for a local bump. Since $\psi_{32} = \psi_{31} - \psi_{21} = n\pi - \psi_{21}$, we have $\sin\psi_{32} = (-1)^{n-1}\sin\psi_{31}$, and $\theta_3 = (-1)^{n-1}\sqrt{\beta_1/\beta_2}\,\theta_1$.