Electron-ion longitudinal misalignment in CeC scheme

S. Seletskiy, W. Bergan, A. Fedotov, D. Kayran

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Model

- A long i-bunch with a short e-bunch at its center
- Variables for ion's motion (τ, δ), where: $\tau = \frac{\omega_s}{\eta}\frac{s}{\beta c}$
- The "wake" is given by:

$$w(z) = -V_0 \sin\left(2\pi \frac{z - z_1}{z_0}\right) \exp\left(-\frac{(z - z_1)^2}{\sigma_0^2}\right)$$

Parametrization is taken from ***S. Nagaitsev, V. Lebedev, G. Stupakov, E. Wang, W. Bergan, arXiv:2102.10239v1 [physics.acc-ph]***

$z = R_{56}\delta$ z_1 is the i-e longitudinal misalignment (an additional modulator-to-kicker pathlength difference)

The motion equations for an individual proton (an oscillator periodically experiencing the friction kick):

$$\begin{cases} \tau' &= \delta \\ \delta' &= -\tau + \alpha \mathbb{C}(\phi) F(\delta) + \sqrt{\alpha}\mathbb{C}(\phi) D \\ F(\delta) &= -\frac{V_0}{\beta^2 E_0} \sin\left(2\pi \frac{R_{56}\delta - z_1}{z_0}\right) \exp\left(-\frac{(R_{56}\delta - z_1)^2}{\sigma_0^2}\right) \\ D &= \frac{V_0}{\beta^2 E_0} \sum_i^{N_s} \left(e^{-a\varphi_i^2} \sin\varphi_i\right) \end{cases}$$

α is the number of times the proton "lands" on the e-bunch when $\tau \approx 0$

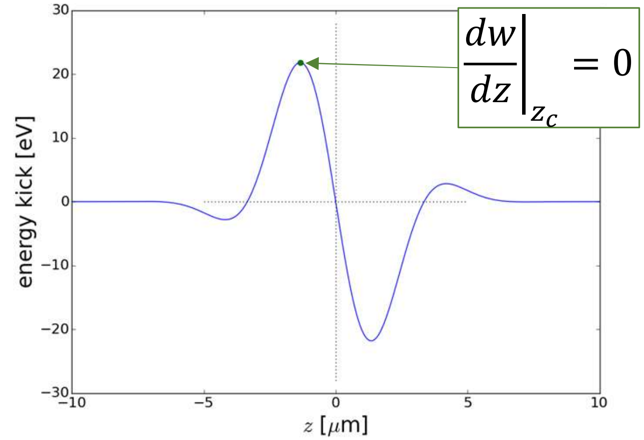
$$\mathbb{C}(\phi) = \sum_{n=0}^{\infty} \delta_D \left(\phi - \frac{\pi(2n+1)}{2}\right)$$

Let's consider two cases: $z_1 = \text{const}$ and z_1 is changing interaction to interaction

In this presentation we'll ignore the diffusive effects (term *D*). For proper treatment of the diffusion please see ***S. Seletskiy, A. Fedotov, D. Kayran, arXiv:2106.12617v1 [physics.acc-ph] 23 Jun 2021***

$z_1 = \text{const}$ (I)

- The friction kick is a non-monotonic function of δ
- Hence, when the constant misalignment z_1 is larger than the value (z_c) at which the first derivative of the kick changes sign, then the coherent excitations happen, and all the ions eventually perform oscillations with the same amplitude.
- In other words, the cooling disappears and instead the ions of small amplitudes get "heated up".



This effect has been known in the electron coolers for more than 40 years - ***YA. S. Derbenev, A. N. Skrinsky, Particle Accelerators 1977, Vol. 8, pp. 1-20***

This effect appears in the coherent electron coolers too - ***S. Seletskiy, A. Fedotov, D. Kayran, arXiv:2106.12617v1 [physics.acc-ph] 23 Jun 2021***

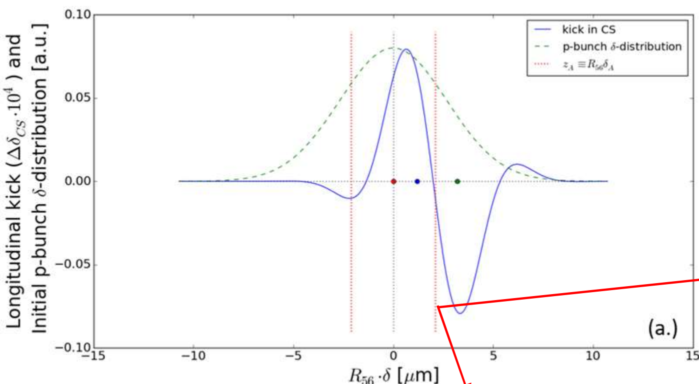
For more details about this effect see:

S. Seletskiy and A. Fedotov, "Effects of coherent offset of velocity distribution in electron coolers on ion dynamics", BNL-220641-2020-TECH, Nov. 2020

S. Seletskiy, A. Fedotov, D. Kayran, "Coherent excitations and circular attractors in cooled ion bunches", TUXA04, IPAC 2021

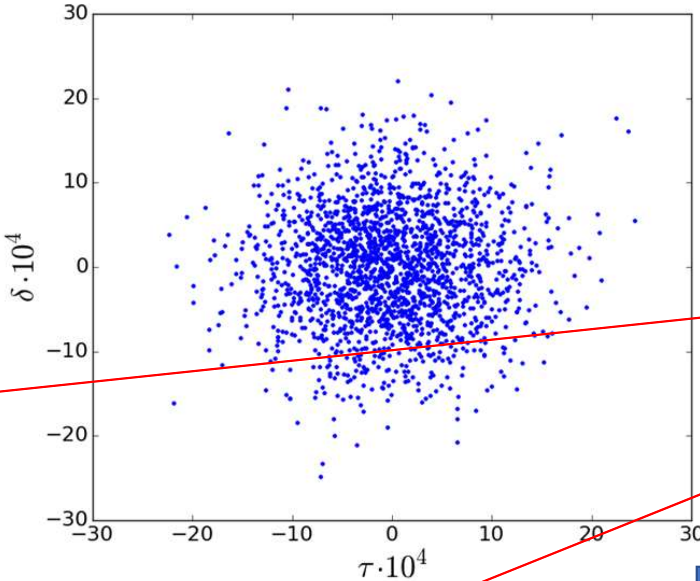
G. Stupakov, "MBEC cooling with a shifted wake", June 2021

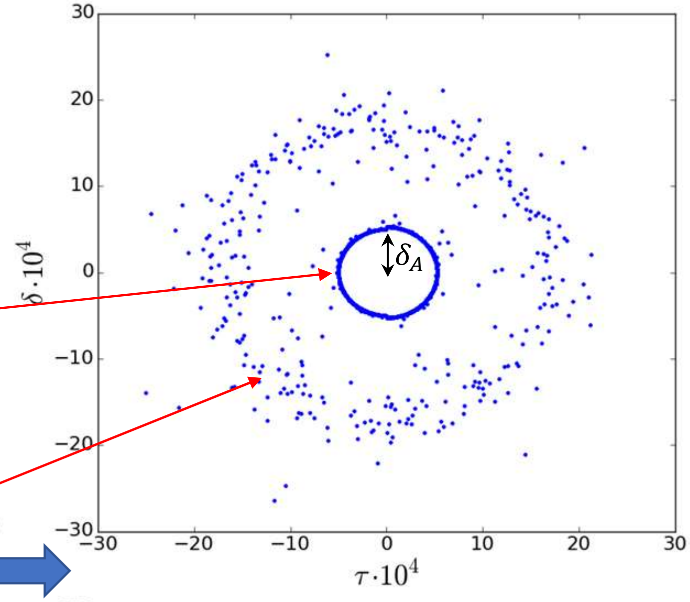
$z_1 = \text{const (II)}$

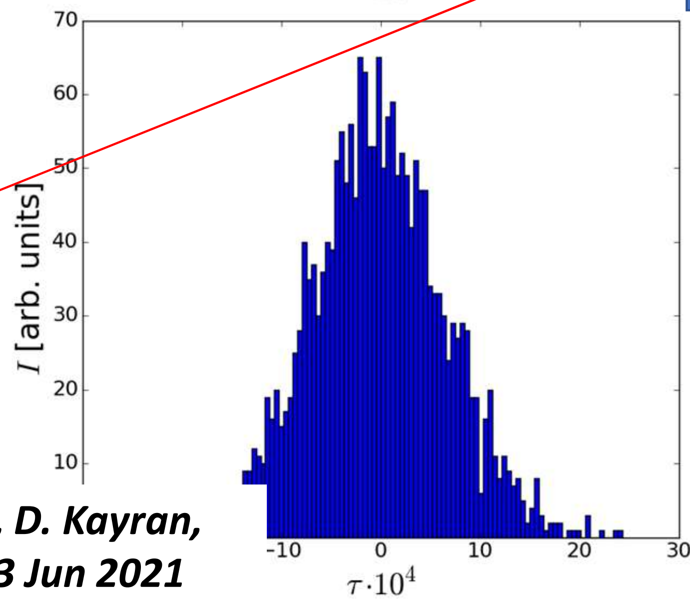


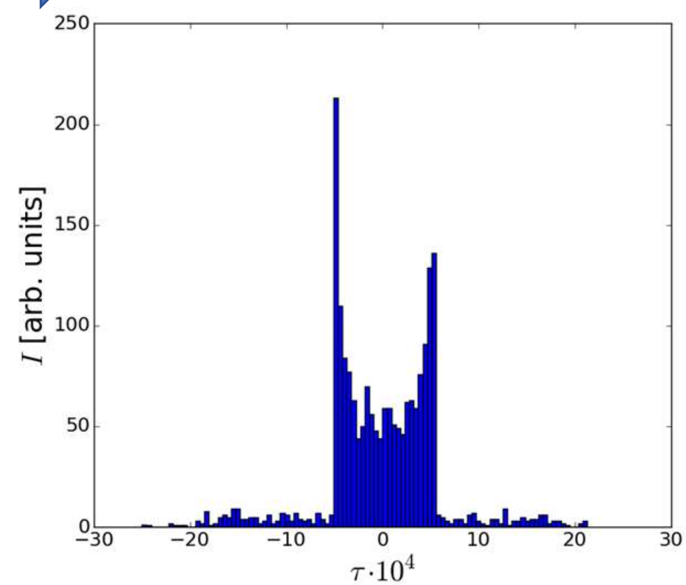
When $z_1 \geq z_c$ the circular attractor appears in the phase space. It stops the cooling for all the protons with $\delta < \delta_A$. As a matter of fact, it "heats" all the small amplitude protons up.

A second ("weak") attractor in the phase space, exists because the wake has an actual "heating" part and because in our model the protons interact with the e-bunch only when the longitudinal proton velocity is at its extremum.









For details see: ***S. Seletskiy, A. Fedotov, D. Kayran, arXiv:2106.12617v1 [physics.acc-ph] 23 Jun 2021***

$z_1 = \text{const}$ (III)

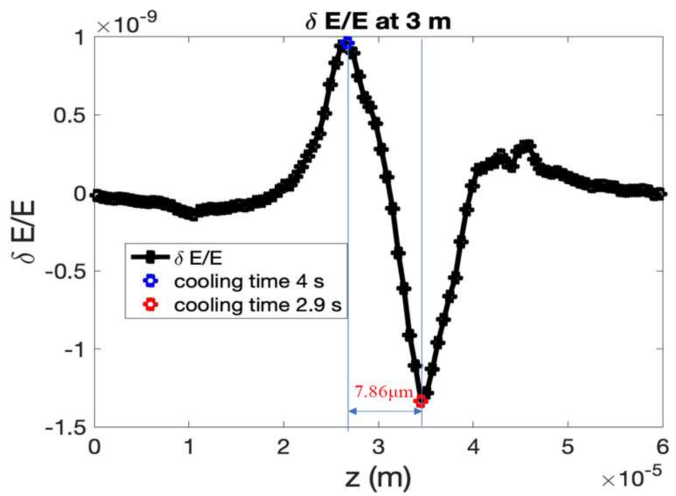
- Since we have a simple analytical expression approximating the wake, we can write a useful formula.

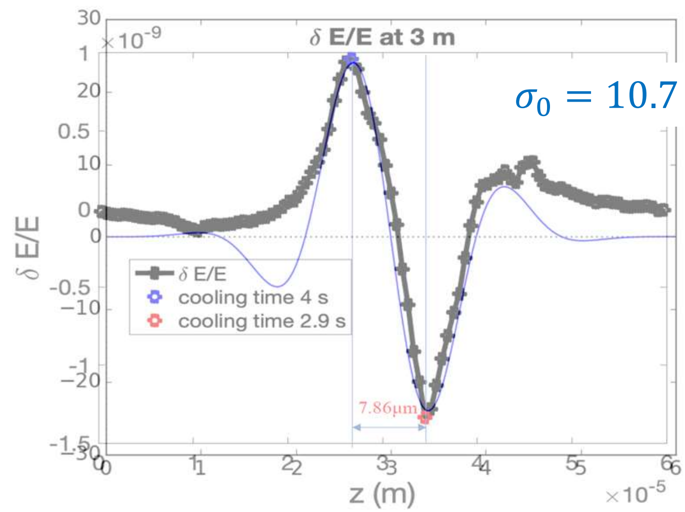
The critical value of z_1 (z_c) can be found from:

$$\frac{z_1}{\sigma_0^2} = \frac{\pi}{z_0} \cos\left(\frac{2\pi}{z_0} z_1\right)$$

For the EIC CeC: $\sigma_0 = 3\ \mu\text{m}$, $z_0 = 6.7\ \mu\text{m}$ ⇨ $z_1 \leq 1.3\ \mu m$

For the PoP CeC:





$\sigma_0 = 10.7\ \mu m$, $z_0 = 18\ \mu m$

$z_1 \leq 3.9\ \mu\text{m}$

V. N. Litvinenko, "Effect of energy jitter on CeC Cooling", Joint CeC meeting, July 23, 2021

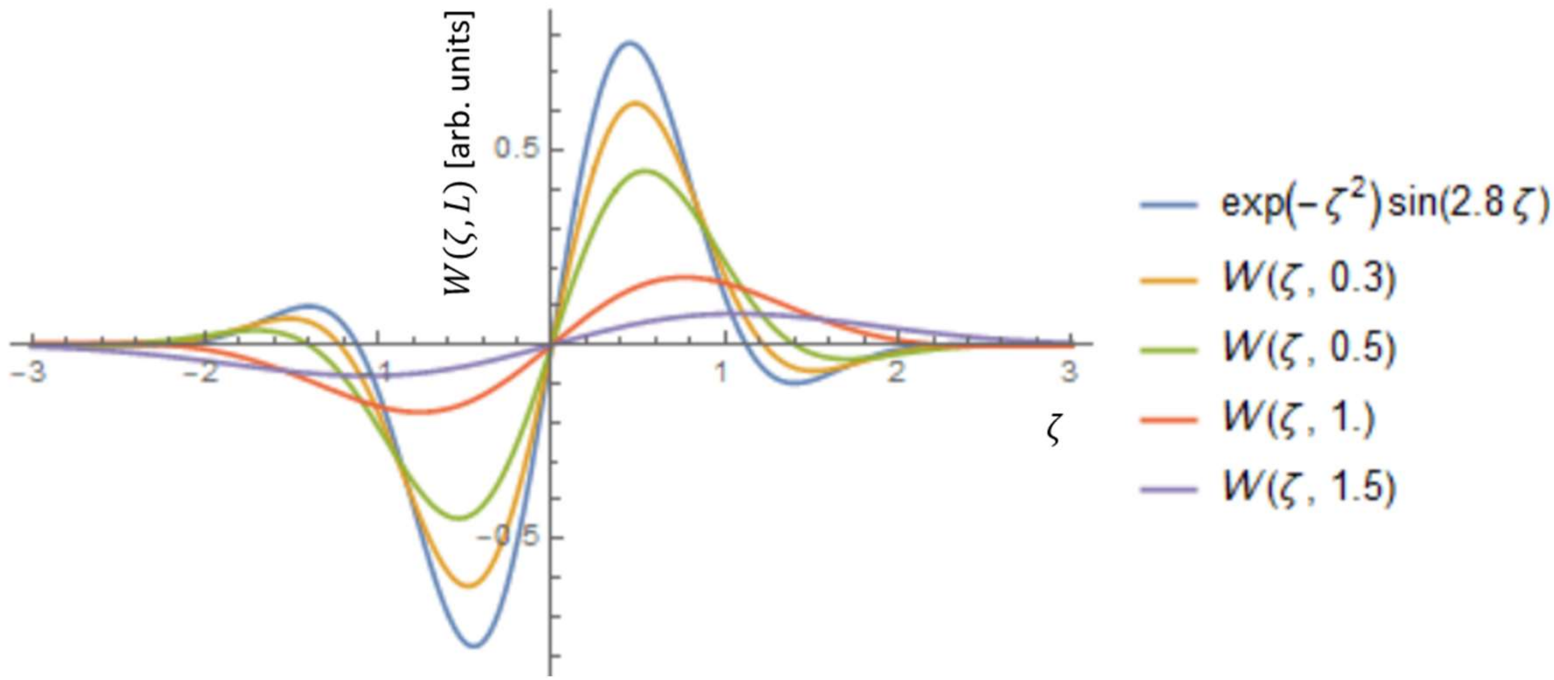
Longitudinal jitter

- Since the electron bunch length is much smaller than the length of the p-bunch, and since the e-bunch is longitudinally placed at the center of the p-bunch, each proton interacts with the electrons when its synchrotron phase $\phi \approx \frac{\pi}{2} \text{ or } \frac{3\pi}{2}$, that is when $|\delta| \approx \max$ for the particular proton.
- We will also assume that the cooling is a much slower process than the jitter. That is, we will assume that δ doesn't change much from e-p interaction-to-interaction while z_1 "jumps" each time the proton sees an electron bunch.
- Effective average cooling wake is given by: $$W \equiv \langle w \rangle = \int_{-\infty}^{\infty} w(z, z_1) f(z_1) dz_1$$
- Here $f(z_1)$ is a probability density function, and we will consider two cases:
  - The noise has a normal distribution: $f_n = \frac{1}{\sqrt{\pi} L\sigma_0} \exp\left(-\frac{z_1^2}{L^2\sigma_0^2}\right)$
  - The noise has a uniform distribution: $z_1 \in [-L\sigma_0, L\sigma_0]$ $\quad f_u = \frac{1}{2L\sigma_0}$
- For convenience, we will use: $\zeta \equiv \frac{z}{\sigma_0} \quad \zeta_1 \equiv \frac{z_1}{\sigma_0} \quad k \equiv 2\pi \frac{\sigma_0}{z_0}$

Noise with a normal distribution

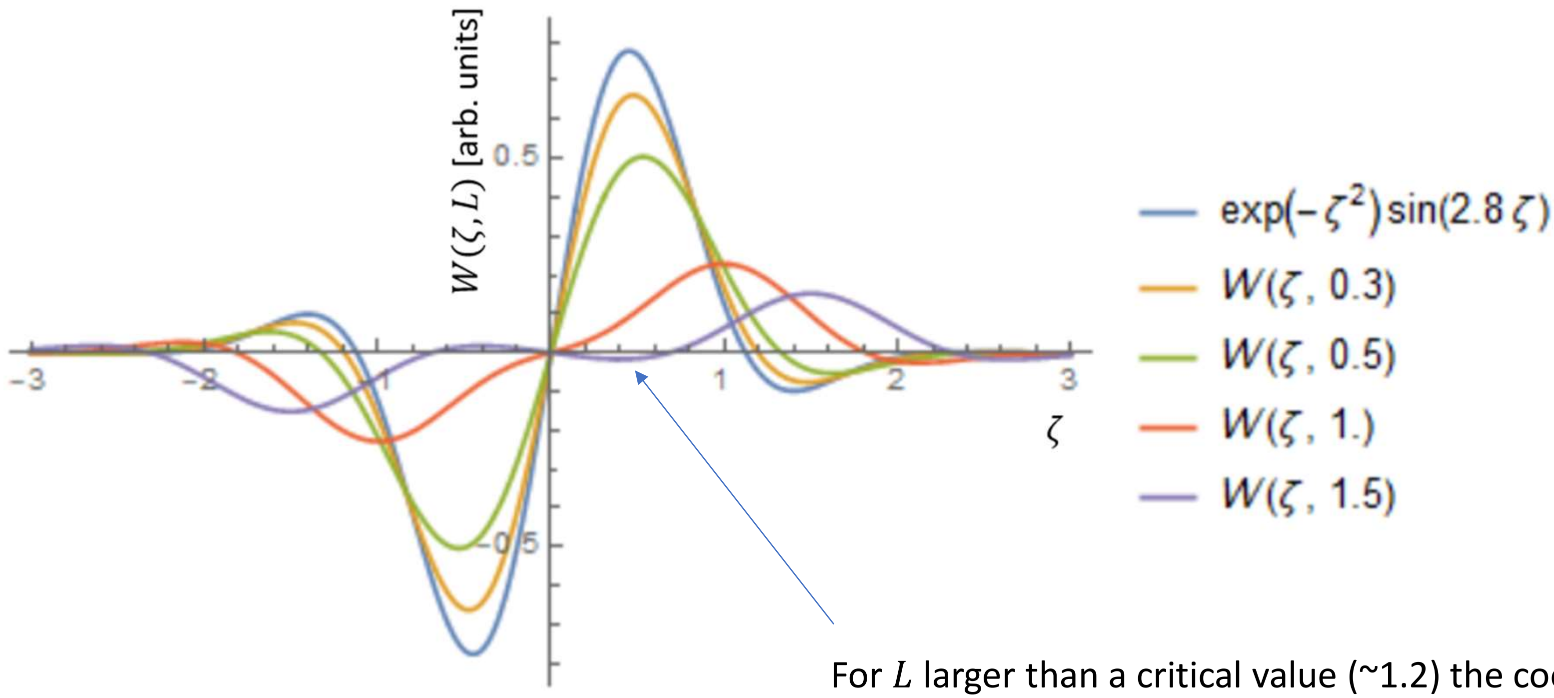
The noise has a normal distribution with PDF $f(\zeta_1) = \frac{1}{\sqrt{\pi}L} e^{-\frac{{\zeta_1}^2}{L^2}}$

$$W(\zeta, L) = \frac{V_0}{\sqrt{\pi}L} \int_{-\infty}^{\infty} \sin(k(\zeta - \zeta_1)) \exp\left(-(\zeta - \zeta_1)^2 - \zeta_1^2/L^2\right) d\zeta_1$$



Uniformly distributed noise

$$W(\zeta, L) = \frac{V_0}{2L} \int_{-L}^{L} \sin(k(\zeta - \zeta_1)) \exp\left(-(\zeta - \zeta_1)^2\right) d\zeta_1$$



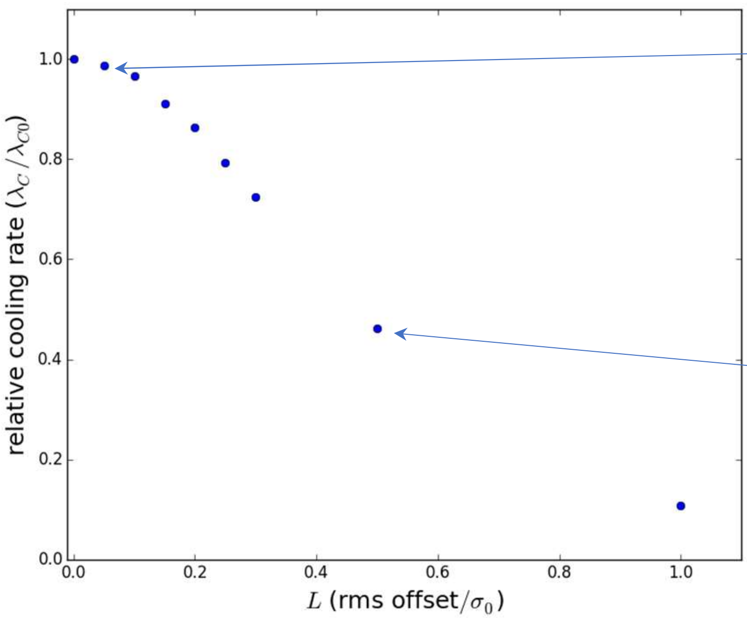
For L larger than a critical value (~1.2) the cooling turns into a heating.
For a noise “weaker” than the critical noise the cooling is simply suppressed.

Jitter effect on the cooling rate

$$\lambda_C = \frac{8\sqrt{\pi \ln 2} V_0 \sigma_{es}}{T_r E_0 \sigma_{ps}} \frac{R_{56}\sigma_0^3}{z_0(2R_{56}^2\sigma_\delta^2 + \sigma_0^2)^{3/2}} \exp\left(-\frac{2(\pi R_{56}\sigma_0\sigma_\delta)^2}{z_0^2(2R_{56}^2\sigma_\delta^2 + \sigma_0^2)}\right)$$

For details see: ***S. Seletskiy, A. Fedotov, D. Kayran, arXiv:2106.12617v1 [physics.acc-ph] 23 Jun 2021***

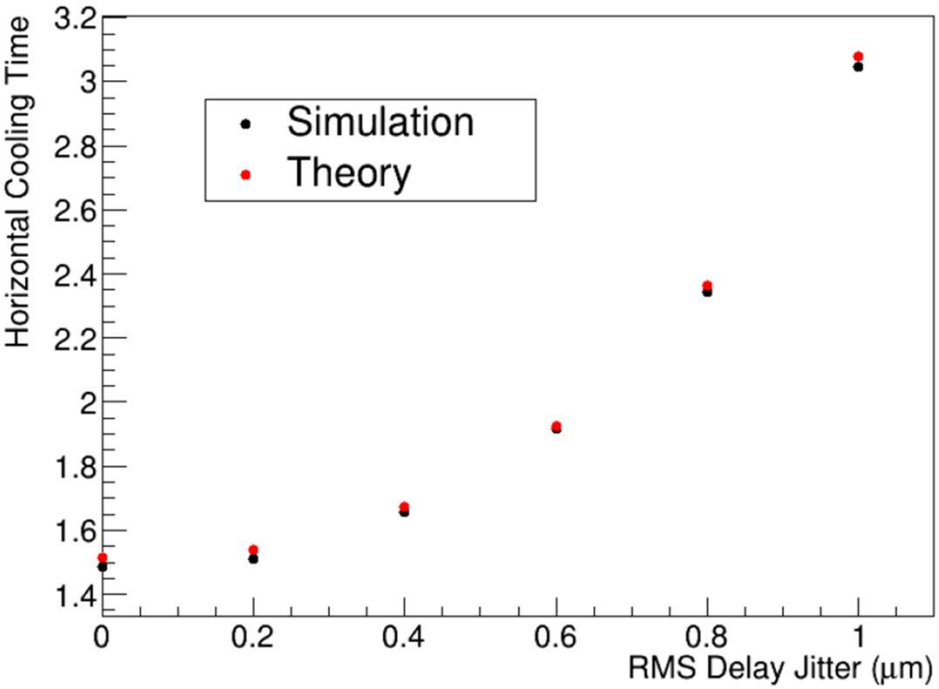
Rough scaling: $\lambda \propto V_0/z_0$

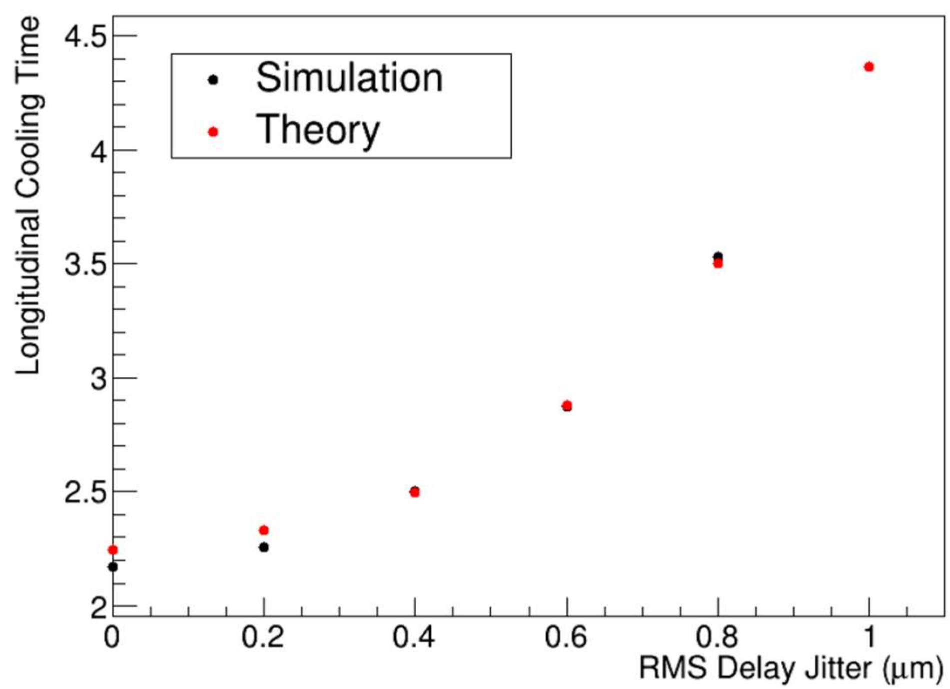


For the EIC CeC we can't afford loosing in the cooling rate, hence noise must be ~$0.1\sigma_0$ or 0.3 um

For PoP CeC we probably still can detect two times weaker cooling, san't we?
So, one probably can allow the rms value of the noise to be ~$0.5\sigma_0$ or 5.3 um

Jitter effect was confirmed by simulations





Simulations – original simulated wake with Gaussian random delay of electron beam
Theory – parametrized wake (analytic formula) convolved with the Gaussian PDF

Tolerances

- For PoP CeC both the "jittery" z_1 and z_1 =const must be less than **4 um** (approximately) or **13 fs**
- It can be caused by:
  - Either e-beam or i-beam energy error $\Delta\delta$: $\Delta\delta \cdot R_{56} < 4\ \mu$m, $R_{56} \approx \frac{12\ \mathrm{m}}{\gamma^2} \approx 1.4$ cm ⇨ $\Delta\delta < 2.9 \cdot 10^{-4}$
  - e-beam pathlength error due to an average angular misalignment of e-i trajectories ($\Delta\theta$): $\Delta\theta^2 \cdot \frac{12\ \mathrm{m}}{2} < 4\ \mu$m ⇨ $\Delta\theta < 0.8$ mrad
- How a requirement of ~½ of 13 fs (**~7 fs**) to the modulator-kicker time jitter can be satisfied?