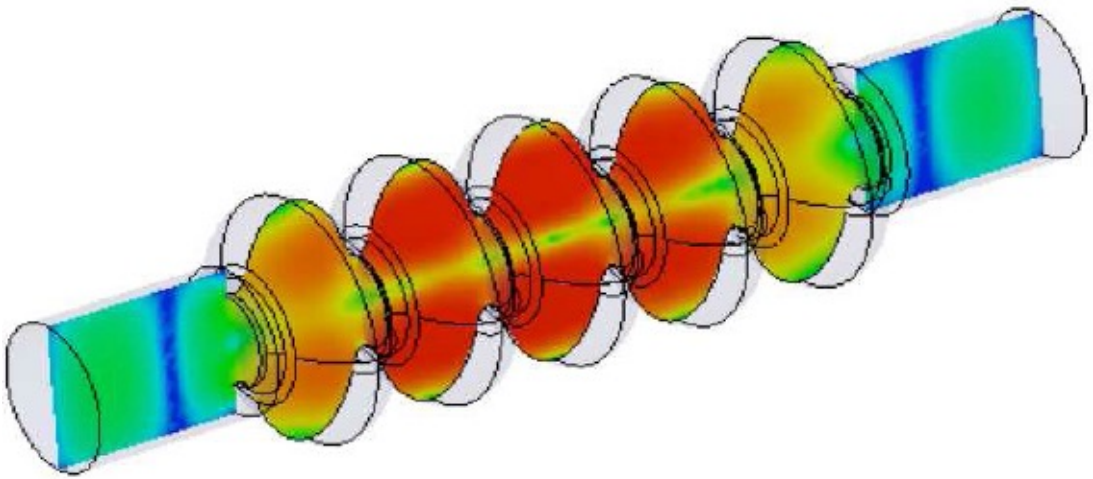
PHY 554

Fundamentals of Accelerator Physics

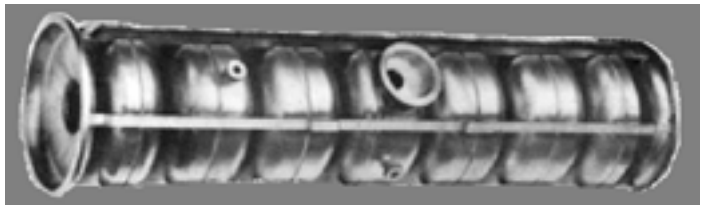
Lecture 16: SRF accelerators and ERLs

Jun Ma



SRF cavities for linacs and ERLs

TESLA / ILC / European XFEL 1.3 GHz cavity



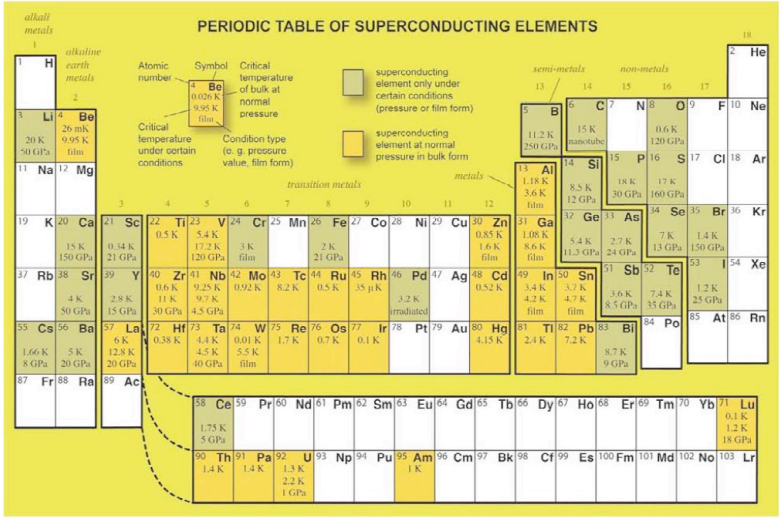
HEPL 1.3 GHz cavity

SNS 805 MHz cavities (***b*** = 0.61 and 0.81)

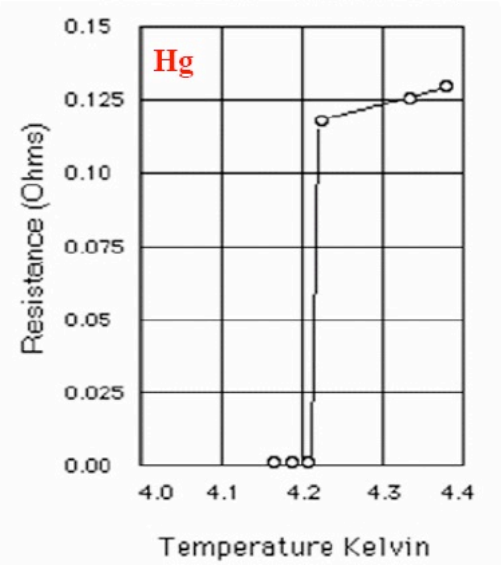
BNL-3 704 MHz cavity

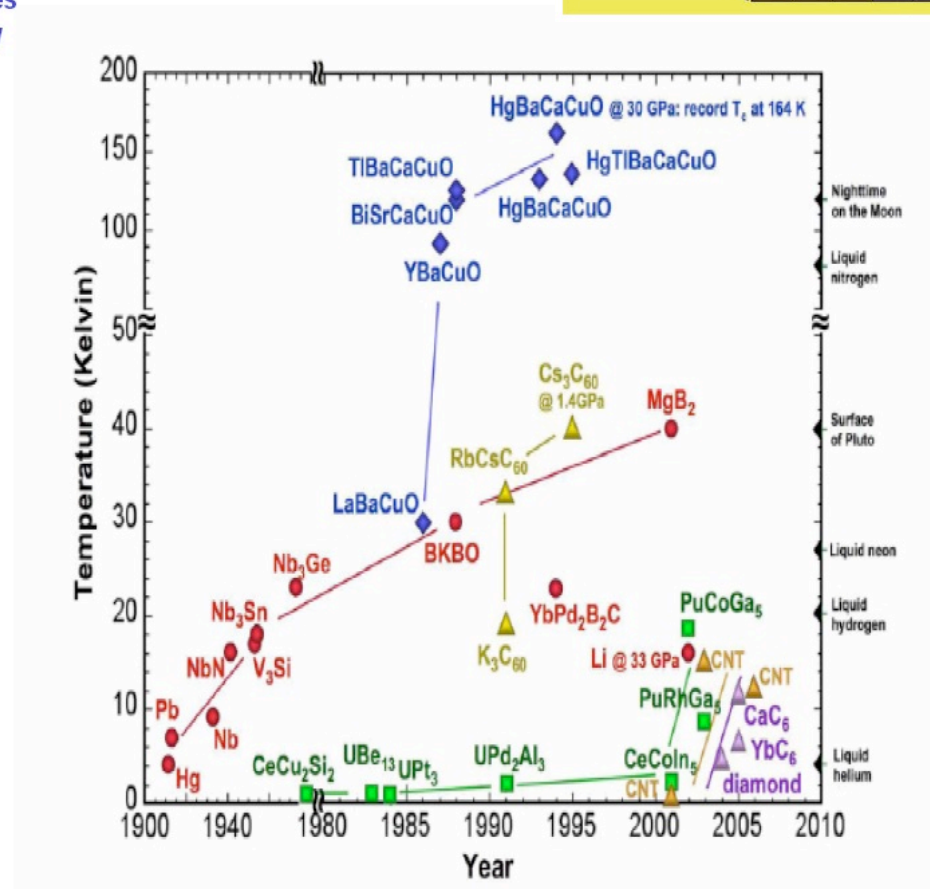
CEBAF Upgrade 1.5 GHz cavity

Discovery of superconductivity: April 8th of 1911



Discovered in 1911 by Heike Kamerlingh Onnes and Giles Holst after Onnes was able to liquify helium in 1908. Nobel prize in 1913





Simplified explanation for zero DC resistivity

- NC
  - Resistance to flow of electric current
  - Free electrons scatter off impurities, lattice vibrations (phonons)
- SC
  - Cooper pairs carry all the current
  - Cooper pairs do not scatter off impurities due to their coherent state
  - Some pairs are broken at $T > 0$ K due to phonon interaction
  - But super-current component has zero resistance

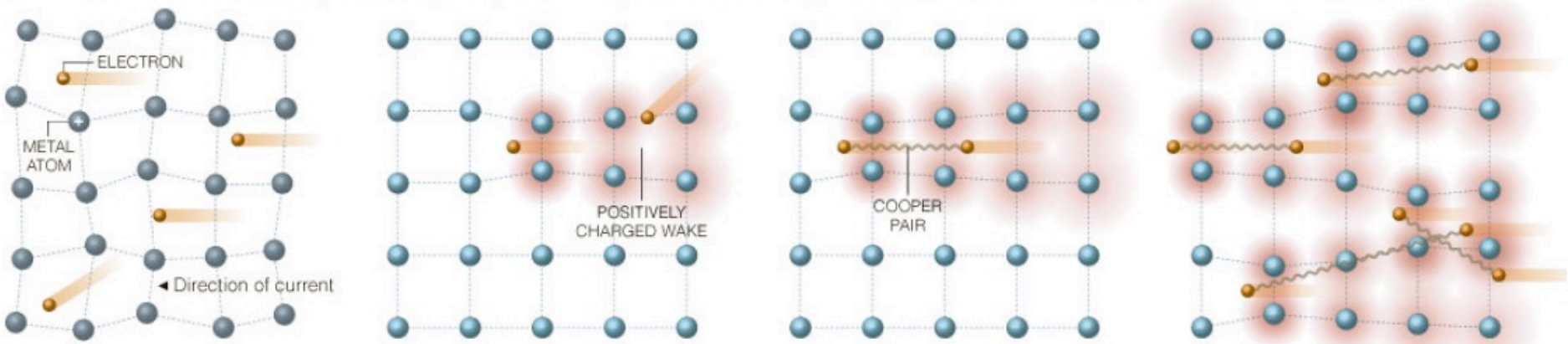
Microscopic theory of superconductivity

Bardeen-Cooper-Schrieffer (BCS) theory (1957).
Nobel prize in 1972

January 7, 2008

Low-Temperature Superconductivity

December was the 50th anniversary of the theory of superconductivity, the flow of electricity without resistance that can occur in some metals and ceramics.



ELECTRICAL RESISTANCE
Electrons carrying an electrical current through a metal wire typically encounter resistance, which is caused by collisions and scattering as the particles move through the vibrating lattice of metal atoms.

CRITICAL TEMPERATURE
As the metal is cooled to low temperatures, the lattice vibration slows. A moving electron attracts nearby metal atoms, which create a positively charged wake behind the electron. This wake can attract another nearby electron.

COOPER PAIRS
The two electrons form a weak bond, called a Cooper pair, which encounters less resistance than two electrons moving separately. When more Cooper pairs form, they behave in the same way.

SUPERCONDUCTIVITY
If a pair is scattered by an impurity, it will quickly get back in step with other pairs. This allows the electrons to flow undisturbed through the lattice of metal atoms. With no resistance, the current may persist for years.

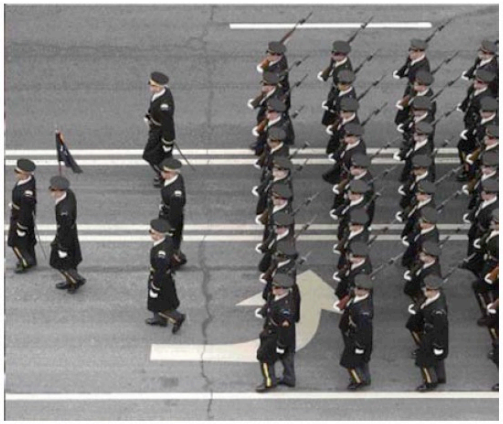
Sources: Oak Ridge National Laboratory; Philip W. Phillips

JONATHAN CORUM/THE NEW YORK TIMES

BCS "theory"

What is the phase coherence?

Incoherent (normal) crowd:
each electron for itself



Phase-coherent (superconducting) condensate
of electrons

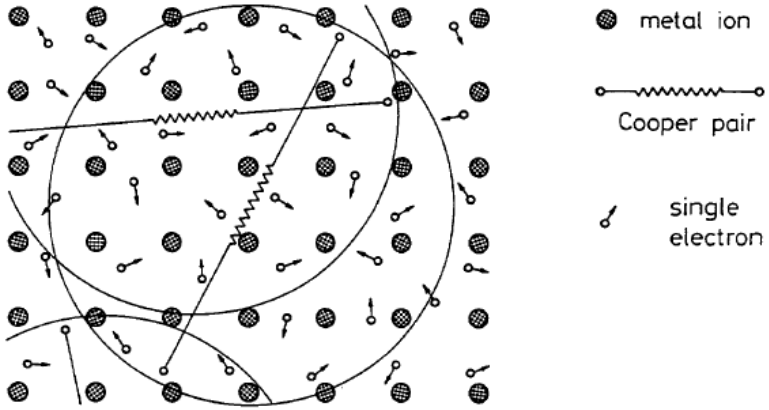


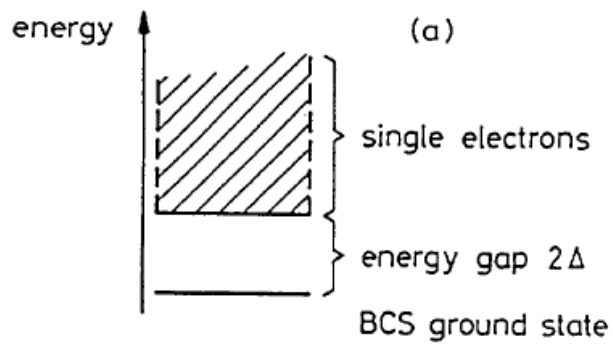
Figure 22: Cooper pairs and single electrons in the crystal lattice of a superconductor. (After Essmann and Träuble [12]).

- Attraction between electrons with antiparallel momenta k and spins due to exchange of lattice vibration quanta (phonons)
- Instability of the normal Fermi surface due to bound states of electron (Cooper) pairs
- Bose condensation of overlapping Cooper pairs in a coherent superconducting state.
- Scattering on electrons does not cause the electric resistance because it would break the Cooper pair

The strong overlap of many Cooper pairs results in the macroscopic phase coherence

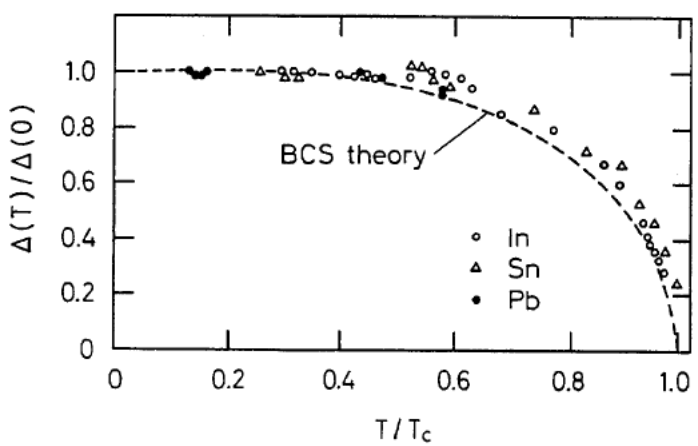
BCS theory

- The BCS ground state is characterized by the macroscopic wave function and a ground state energy that is separated from the energy levels of unpaired electrons by an energy gap. In order to break a pair an energy of 2Δ is needed:



$$n_{\mathrm{n}} \propto \exp\left(-\frac{\Delta}{k_B T}\right)$$

- Temperature dependence of the energy gap according to BCS theory in comparison with experimental data:



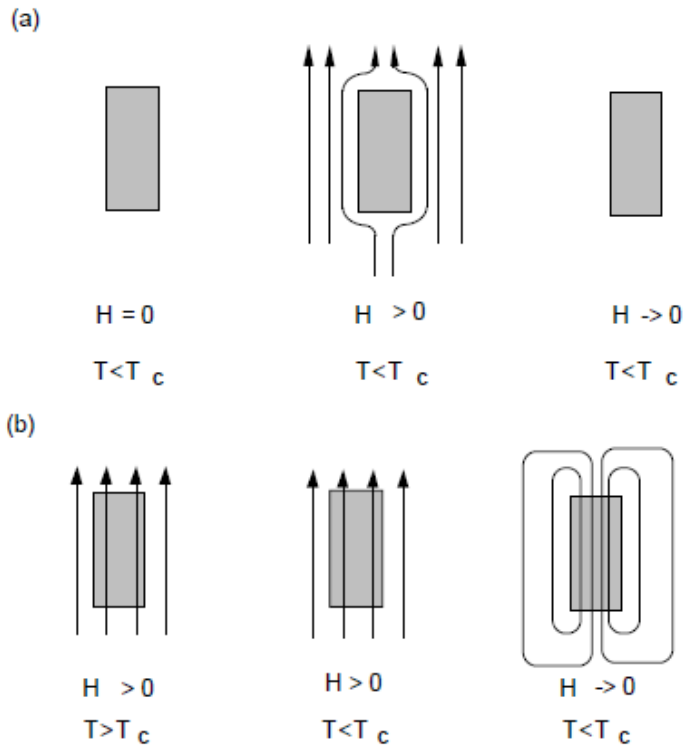
element	Sn	In	Tl	Ta	Nb	Hg	Pb
$\Delta(0)/k_B T_c$	1.75	1.8	1.8	1.75	1.75	2.3	2.15

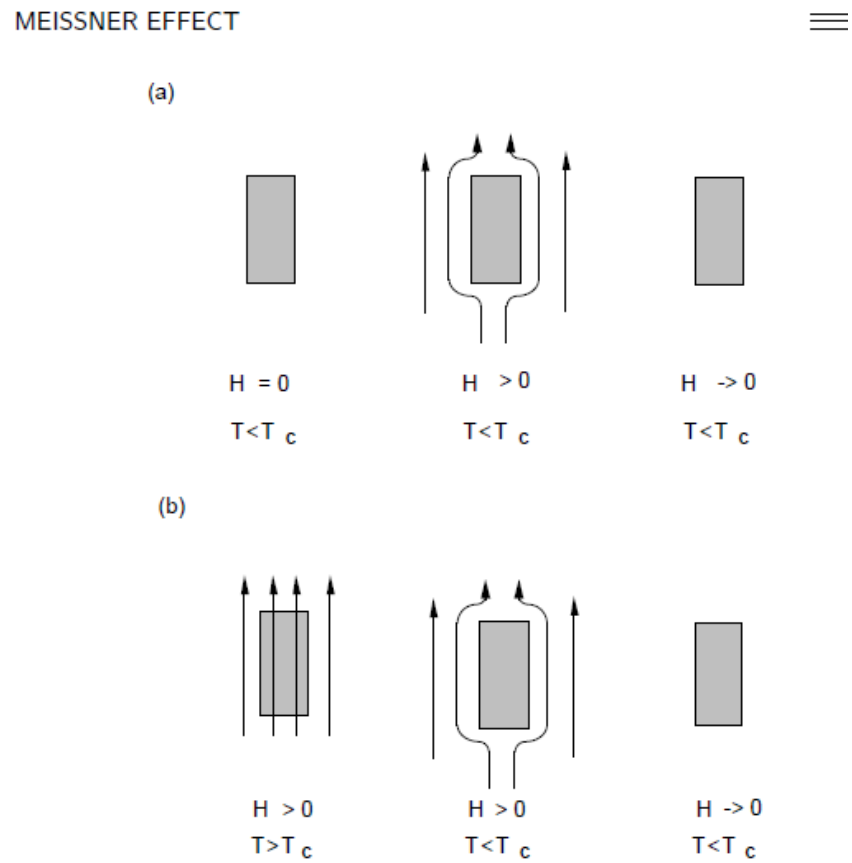
Remarkable prediction!

Meissner effect
perfect conductor ≠ SC

Inside a perfect conductor $\partial \mathbf{B}/\partial t = 0$ In a superconductor (see next)

But **B** = constant is allowed.



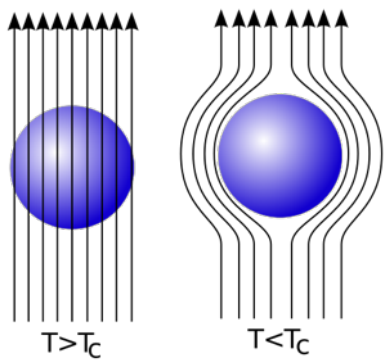


Superconducting state

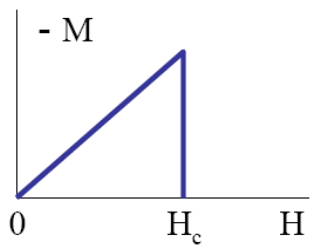
- The superconducting state is characterized by the critical temperature T_c and field H_c

$$H_c(T) = H_c(0)\cdot\left[1-\left(\frac{T}{T_c}\right)^2\right]$$

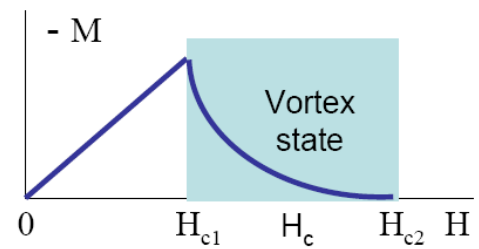
- The external field is expelled from a superconductor if $H_{ext} < H_c$ for Type I superconductors.
- For Type II superconductors the external field will partially penetrate for $H_{ext} > H_{c1}$ and will completely penetrate at H_{c2}



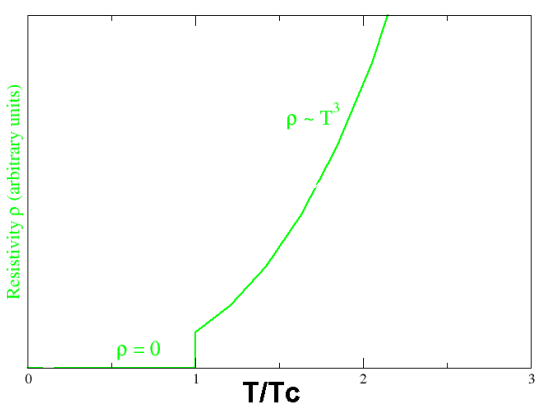
Superconductor in Meissner state = ideal diamagnetic



Complete Meissner effect in type-I superconductors



High-field partial Meissner effect in type-II superconductors

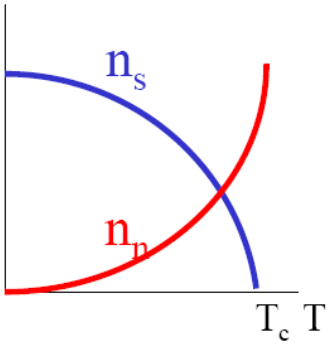


Type-I: Meissner state $B = H + M = 0$ for $H < H_c$; normal state at $H > H_c$

Type-II: Meissner state $B = H + M = 0$ for $H < H_{c1}$; partial flux penetration for $H_{c1} < H < H_{c2}$; normal state for $H > H_{c2}$

Two fluid model & AC fields

- Two-fluid model: coexisting SC and N "liquids" with the densities $n_s(T) + n_n(T) = n$.
- Electric field E accelerates only the SC component, the N component is short circuited.
- Second Newton law for the SC component: $m d\mathbf{v}_s/dt = e\mathbf{E}$ yields the first London equation:



$$n_n \propto \exp\left(-\frac{\Delta}{k_B T}\right)$$

Two fluid model considers both superconducting and normal conducting components:

- At $0 < T < T_c$ not all electrons are bonded into Cooper pairs. The density of *unpaired,* "normal" electrons is given by the Boltzmann factor

$$n_n \propto \exp\left(-\frac{\Delta}{k_B T}\right)$$

  where 2D is the energy gap around Fermi level between the ground state and the excited state.

- Cooper pairs move without resistance, and thus dissipate no power. In DC case the lossless Cooper pairs short out the field, hence the normal electrons are not accelerated and the SC is lossless even for $T > 0$ K.

Superconducting part of AC current

$$\vec{F} = m\vec{a}$$

- The Cooper pairs are electrons and do have an inertial mass
- They cannot follow an AC electromagnetic fields instantly and do not shield it perfectly.
- A residual EM field will acts on the unpaired electrons causing power dissipation.

First London equation

$$\vec{F} = -e\vec{\mathbf{E}} = m\vec{a}$$

$$\vec{j}_s = -en_s\vec{\mathrm{v}} \Rightarrow \frac{\partial \vec{j}_s}{\partial t} = -en_s\vec{a}$$

$$\Rightarrow i\omega\vec{j}_s = n_s\frac{e^2\vec{\mathbf{E}}}{m}$$

$$\sigma_s = \frac{n_s e^2}{m\omega} > 0$$

$$\vec{j}_s = -i\sigma_s\vec{E}$$

- Using Maxwell equation $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$ we obtain

$$\frac{\partial}{\partial t}\left(\frac{m}{n_s e^2}\nabla \times \mathbf{J}_s + \mathbf{B}\right) = 0$$

Second London equation

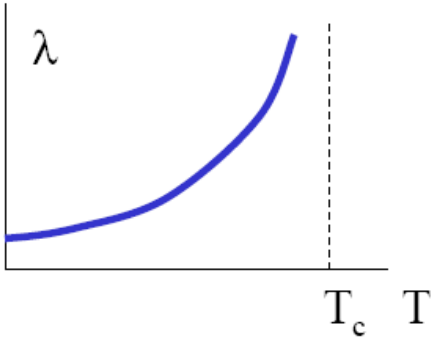
- The Meissner effect requires $\vec{\mathbf{B}} = -\frac{m}{n_s e^2}\nabla \times \vec{\mathbf{J}}_s$

London penetration depth

- Using the Maxwell equations, $\nabla\times \mathbf{E} = -\mu_0\partial_t\mathbf{H}$ and $\nabla\times \mathbf{H} = \mathbf{J}_s$ we obtain the second London equation:

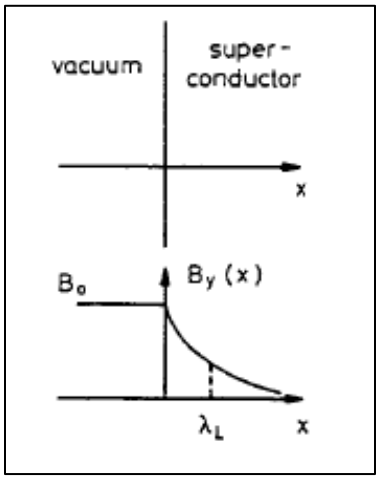
$$\lambda^2\nabla\mathbf{H} - \mathbf{H} = 0$$

- London penetration depth: $$\lambda = \left(\frac{m}{e^2 n_s(T)\mu_0}\right)^{1/2}$$



- It is important to understand that this equation is not valid in a normal conductor. The depth is frequency independent!
- If we consider a simple geometry, a boundary between a superconductor and vacuum, then the solution is

$$B_y(x) = B_0 \exp\left(-x/\lambda_L\right)$$



- Magnetic field does not stop abruptly, but penetrates into the material with exponential attenuation. The penetration depth is quite small, 20 – 50 nm.
- According to BCS theory not single electrons, but pairs are carriers of the super-current. However, the penetration depth remains unchanged: 2e/2m=e/m.

AC current in two-fluid model

- To calculate the surface impedance of a superconductor we take into account both the "superconducting" electrons n_s and "normal" electrons n_n in the two-fluid model
- There is no scattering, thus $\vec{j}_{s,n} = -n_{s,n} e \vec{\mathbf{v}}_{s,n}$ and we already got this of n_s

$$\boxed{m\frac{\partial \vec{\mathbf{v}}_s}{\partial t} = -e\vec{\mathbf{E}} \;\Rightarrow\; \frac{\partial \mathbf{j}_s}{\partial t} = \frac{n_s e^2}{m}\vec{\mathbf{E}}}$$

- Or in an AC field

$$\vec{j}_s = -i\frac{n_s e^2}{m\omega}\vec{\mathbf{E}} = -i\sigma_s \vec{\mathbf{E}} \;\text{ or }\; \vec{j}_s = \frac{-i}{\omega\mu_0\lambda_L^2}\mathbf{E}$$

- The total current is simply a sum of currents due to two "fluids":

$$\vec{j} = \vec{j}_n + \vec{j}_s = \left(\sigma_n - i\sigma_s\right)\vec{\mathbf{E}}$$

- Thus one can apply the same treatment to a superconductor as was used for a normal conductor before with the substitution of the newly obtained conductivity.

$$\sigma_s = \frac{n_s e^2}{m\omega}$$

Surface impedance of superconductors

- We expect the real part of the surface resistance to drop exponentially below T_c.
- The surface impedance

$$Z_s = \sqrt{\frac{\omega\mu_0}{2\sigma}}(1+i) \Rightarrow \sqrt{\frac{\omega\mu_0}{2(\sigma_n - i\sigma_s)}}(1+i)$$

- The penetration depth

$$\delta = \frac{1}{\sqrt{\pi f \mu_0 \sigma}} \Rightarrow \frac{1}{\sqrt{\pi f \mu_0 (\sigma_n - i\sigma_s)}}$$

- Note that $1/\omega$ is of the order of 100 ps whereas the relaxation time for normal conducting electrons if of the order of 10 fs. Also, $n_s >> n_n$ for $T << T_c$, hence $\sigma_n << \sigma_s$.
- Then

$$\delta \approx (1+i)\lambda_L\left(1 + i\frac{\sigma_n}{2\sigma_s}\right) \quad \text{and} \quad H_y = H_0 e^{-x/\lambda_L} e^{-ix\sigma_n/2\sigma_s\lambda_L}$$

- The fields decay rapidly, but now over the London penetration depth, which is much shorter than the skin depth of a normal conductor.
- For the impedance we get

$$Z_s \approx \sqrt{\frac{\omega\mu_0}{\sigma_s}}\left(\frac{\sigma_n}{2\sigma_s} + i\right) \quad X_s = \omega\mu_0\lambda_L \quad R_s = \frac{1}{2}\sigma_n\omega^2\mu_0^2\lambda_L^3$$

BCS surface resistivity

- Let us take a closer look at the surface impedance

$$Z_s \approx \sqrt{\frac{\omega\mu_0}{\sigma_s}}\left(\frac{\sigma_n}{2\sigma_s}+i\right) \quad X_s = \omega\mu_0\lambda_L \quad R_s = \frac{1}{2}\sigma_n\omega^2\mu_0^2\lambda_L^3$$

- One can easily show that $X_s >> R_s \rightarrow$ the superconductor is mostly reactive.
- The surface <u>resistivity</u> is proportional to the <u>conductivity</u> of the normal fluid! That is if the normal-state resistivity is low, the superconductor is more lossy.
  - *Analogy: a parallel circuit of a resistor and a reactive element driven by a current source. Observation: lower Q for cavities made of higher purity Nb.*
- While this explanation works for all practical purposes, it is a simplification.
- For real materials instead of the London penetration depth we should use an effective penetration depth, which is

$$\lambda = \lambda_L\sqrt{\frac{\xi_0}{\xi}} \ ,$$

where ξ_0 and ξ are the coherence lengths of the pure and real materials respectively.

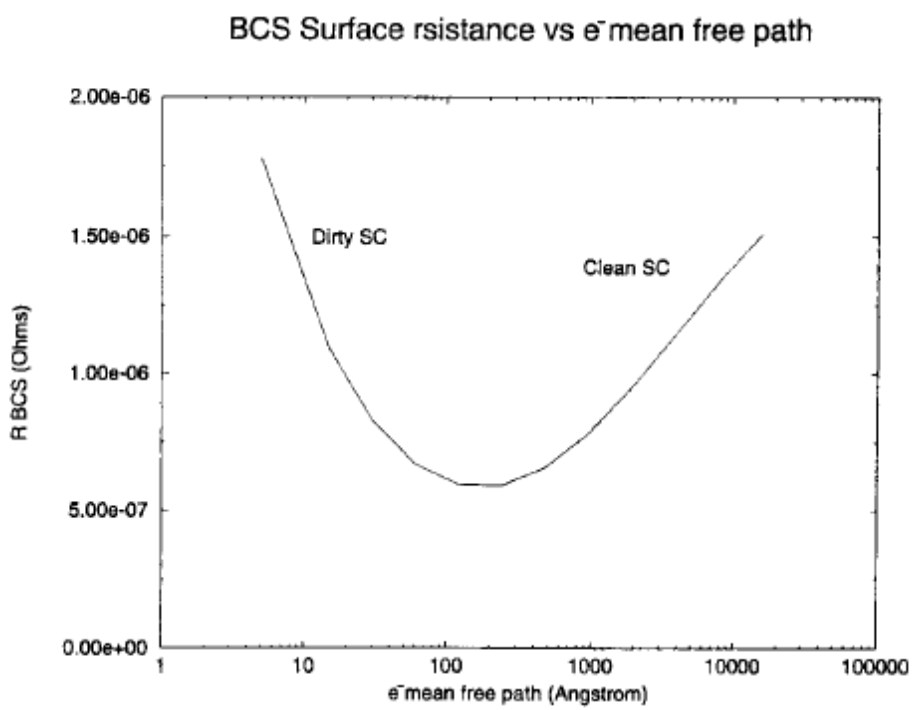
- In the real material the coherence length is given by

$$\xi^{-1} = \xi_0^{-1} + l^{-1},$$

where l is the electron mean free path.

BCS surface resistivity (2)

- Let us now consider two extremes
  1. For clean superconductors, $l >> \xi_0$, thus $R_{BCS} \sim l$. For very clean materials the equation is not valid anymore and BCS theory predicts roughly constant surface resistivity.
  2. For dirty superconductors, $l << \xi_0$, thus $\xi \cong l$, and we get $R_{BCS} \sim l^{-1/2}$.
- Between the clean and dirty limits R_{BCS} reaches a minimum, when the coherence length and mean free path are approximately equal



BCS surface resistivity *vs.* T

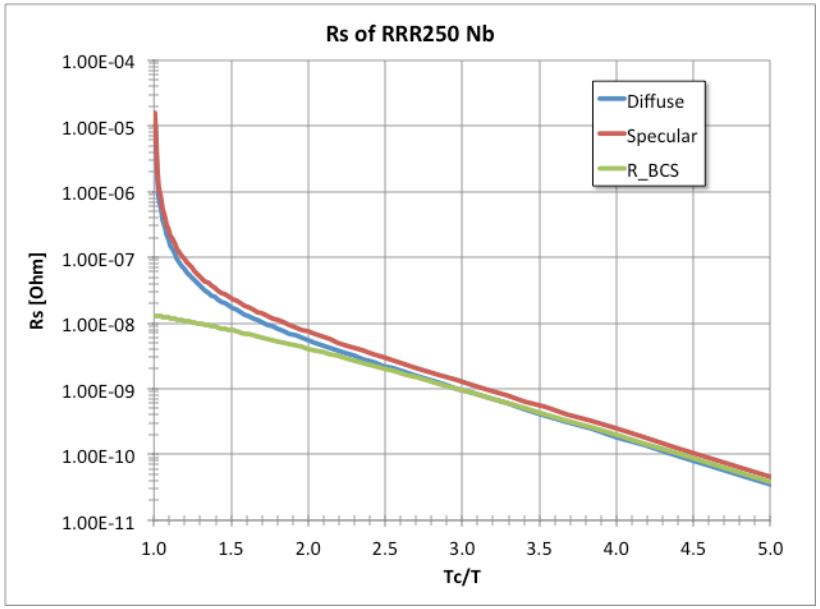
- Calculation of surface resistivity must take into account numerous parameters. Mattis and Bardeen developed theory based on BCS, which predicts

$$R_{BCS} = A\frac{\omega^2}{T}e^{-\left(\frac{\Delta}{k_B T_c}\right)\frac{T_c}{T}},$$

where A is the material constant.

- While for low frequencies (≤ 500 MHz) it may be efficient to operate at 4.2 K (liquid helium at atmospheric pressure), higher frequency structures favor lower operating temperatures (typically superfluid LHe at 2 K, below the lambda point, 2.172 K).
- Approximate expression for Nb:

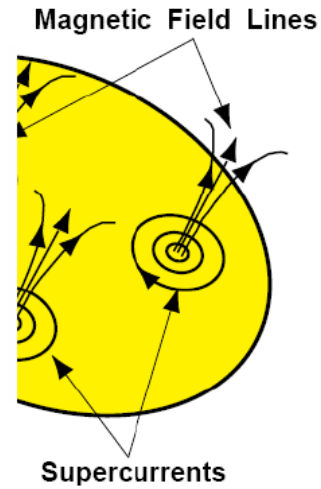
$$R_{BCS} \approx 2\times10^{-4}\left(\frac{f[\mathrm{MHz}]}{1500}\right)^2\frac{1}{T}e^{\left(\frac{-17.67}{T}\right)}[\mathrm{Ohm}]$$



- Above ~$T_c/2$, this formula is not valid and one have to perform more complicated calculations. The plots show comparison of the surface resistivity calculated using the formula with more precise calculation using Halbritter's program **SRIMP**.
- In this program the Nb mean free path (in Angstroms) is assumed to be approximately **60 × RRR**.

Trapped magnetic flux

- Ideally, if the external magnetic fields is less than H_{c1}, the DC flux will be expelled due to Meissner effect. In reality, there are lattice defects and other inhomogeneities, where the flux lines may be "pinned" and trapped within material.



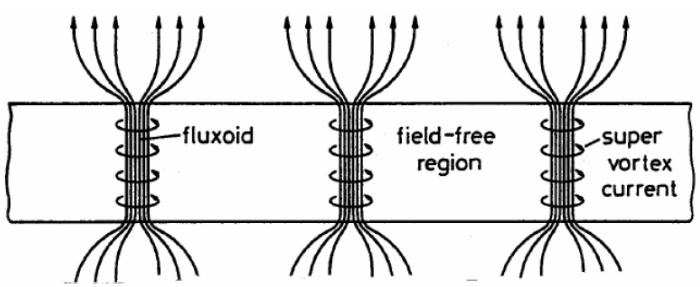
- The resulting contribution to the residual resistance

$$R_{mag} = \frac{H_{ext}}{2H_{c2}} R_n$$

- For high purity (RRR=300) Nb one gets

$$R_{mag} = 0.3(n\Omega) H_{ext}(mOe)\sqrt{f(GHz)}$$

- Earth's field is 0.5 G, which produces residual resistivity of 150 nOhm at 1 GHz and $Q_0 < 2\text{x}10^9$
- Hence one needs magnetic shielding around the cavity to reach quality factor in the 10^{10} range.
- Usually the goal is to have residual magnetic field of less than 10 mG.



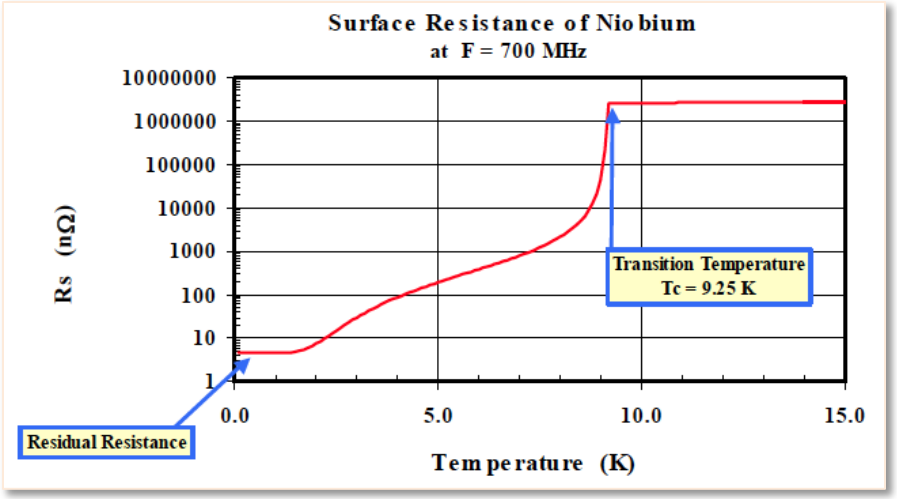
Residual surface resistivity

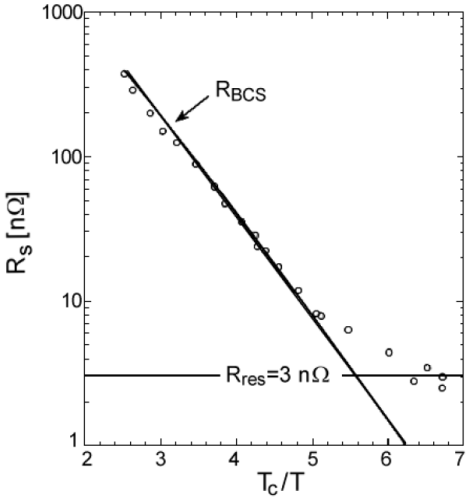
- At low temperatures the measured surface resistivity is larger than predicted by theory:

$$R_s = R_{BCS}(T) + R_{res}$$

where R_{res} is the temperature independent residual resistivity.
It can be as low as 1 nOhm, but typically is ~10 nOhm.

- Characteristics:
  - no strong temperature dependence
  - no clear frequency dependence
  - can be localized
  - not always reproducible
- Causes for this are:
  - magnetic flux trapped in at cool-down
  - dielectric surface contaminations (chemical residues, dust, adsorbents)
  - NC defects & inclusions
  - surface imperfections
  - hydrogen precipitates





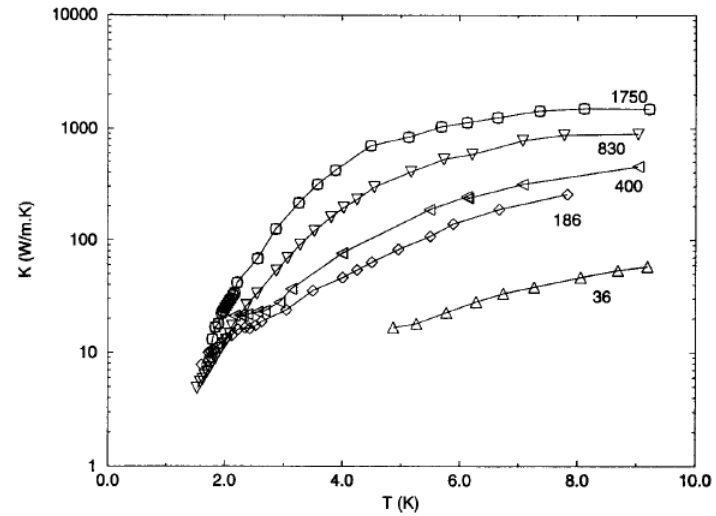
RRR

- Residual Resistivity Ratio (RRR) is a measure of material purity and is defined as the ratio of the resistivity at 273 K (or at 300 K) to that at 4.2 K in normal state.
- High purity materials have better thermal conductivity, hence better handling of RF losses.
- The ideal RRR of niobium due to phonon scattering is 35,000. Typical "reactor grade" Nb has RRR ≈ 30. Nb sheets used in cavity fabrication have RRR ≥ 200.

$$\lambda(4.2K) \approx 0.25 \cdot RRR \quad [W/(m \cdot K)]$$

$$RRR = \left(\sum_i f_i / r_i \right)^{-1}$$

where f_i denote the fractional contents of impurity i (measured in weight ppm) and the r_i the corresponding resistivity coefficients, which are listed in the table below.



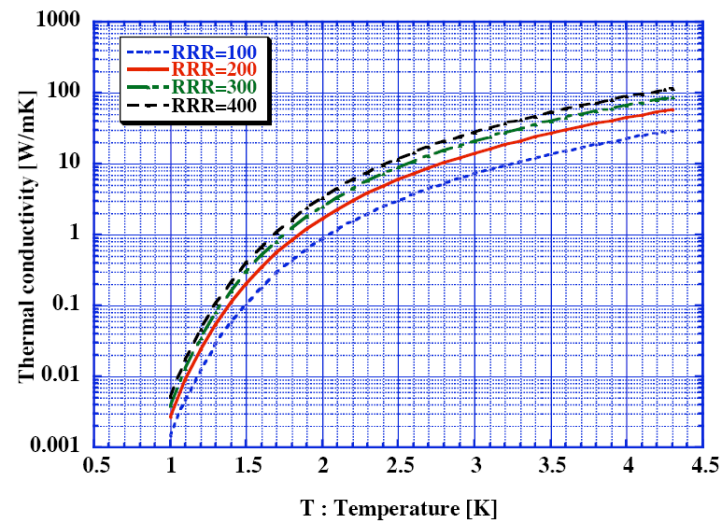
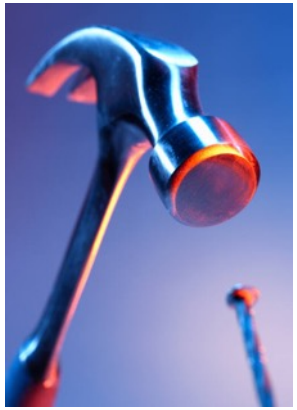


Table II Weight factor r_i of some impurities (see equation (4))

Impurity atom i	N	O	C	H	Ta
r_i in 10^4 wt. ppm	0.44	0.58	0.47	0.36	111

Worth remembering…



- The superconducting state is characterized by the critical temperature and magnetic field.
- There are Type I and Type II superconductors.
- Two-fluid model and BCS theory explain surface resistivity of superconductors.
- Nb is a material of choice in either bulk form or as a film on a copper substrate.
- Other materials are being investigated.
- At low temperatures residual resistivity limits performance of superconducting cavities.
- There are several phenomena responsible for the deviation of "real world" losses from theoretical predictions.
- Material quality (impurities, mechanical damage) plays important role.
- Performance of SC cavities is dependent on the quality of a thin surface layer.

Main non-trivial/nonlinear effects and the limits of SRF linacs?

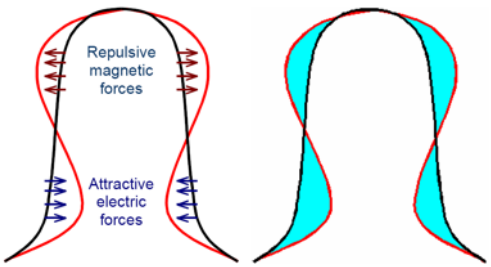
With SR cavities capable of $Q_o \sim 10^{10}$, 850 MHz SRF cavity can have bandwidth of the resonance bandwidth of 0.1 Hz (e.g. it would ring for about 10 seconds without external RF source!).
While being the result of excellent conductivity, it makes cavity susceptible to small mechanical size change – 1 nanometer change in a cavity ~ 1 meter in size could cause ~ 10 Hz change – e.g. 100-fold the bandwidth, and take it completely out of the resonance…

Low level RF system (and cavity tuning system) is used to keep cavity both at resonance, stable and under control. In addition, depending on the application, the cavity Q is reduced to by using strong external coupling. For ERLs it is typical to have : $Q_{ext} \sim 10^8$. It turns bandwidth into a measurable few Hz range.

Side note: if mechanical hand watch would have $Q=10^{10}$,
it would not require rewinding for about 300 years… and it would be a really good astronomical instrument.

Ponderomotive effects: radiation pressure

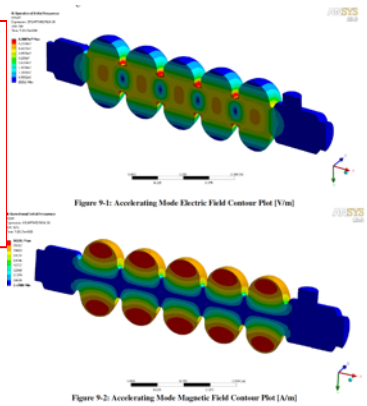
- Ponderomotive effects are nothing else but changes of the cavity shape and its frequency caused by the electromagnetic field (radiation) pressure:



$$P_{Rad} = \frac{1}{4}\left(\mu_0 H^2 - \varepsilon_0 E^2\right)$$

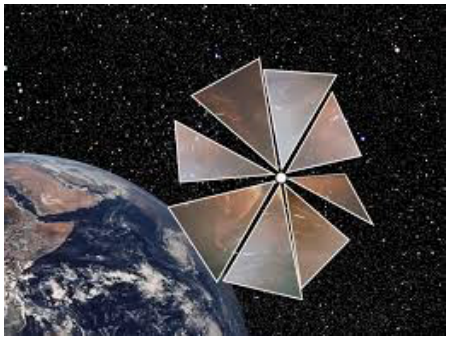
Typical SRF linac pressure

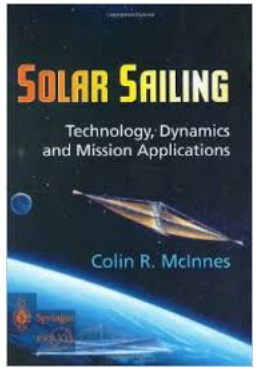
~ 100- 1000 N/m^2



  - Static Lorentz detuning (CW operation)
  - Dynamic Lorentz detuning (pulsed operation)

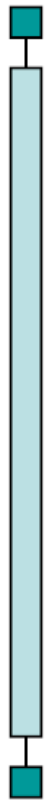
Solar sailing… again



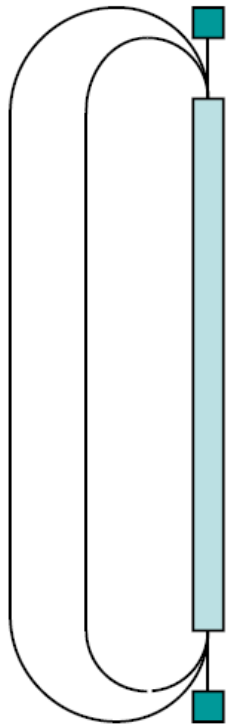


- This effect is called **Lorentz de-tuning** and should be taken into account in the RF control system to make it stable

Types of SRF accelerators



Linac



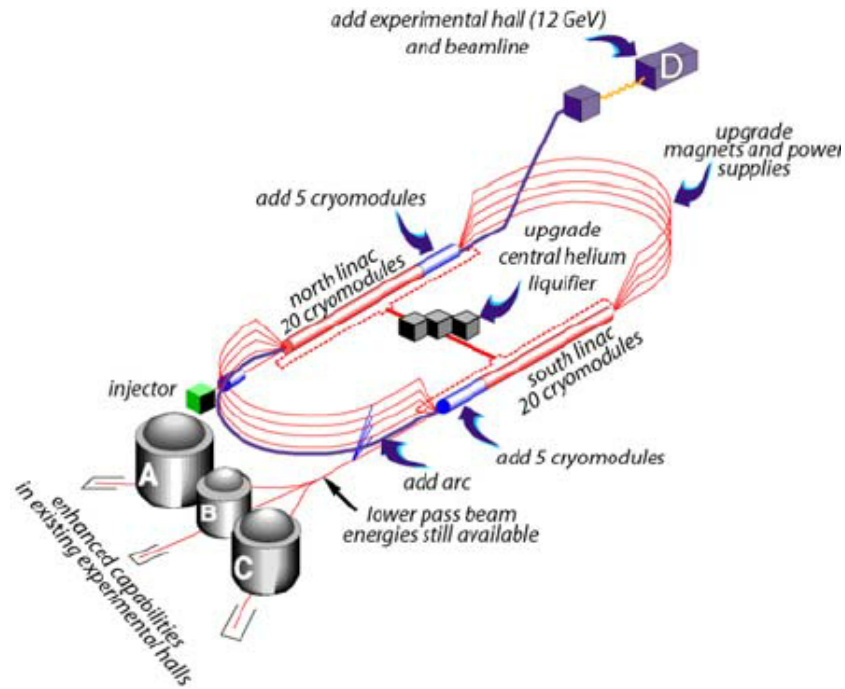
Recirculating Linac

RF Installation

Beam injector and dump

Beamline

12 GeV RLA at JLAB, Newport News, VA



Beam power in linacs and recirculating linacs is simply limited by the power: P=V*I

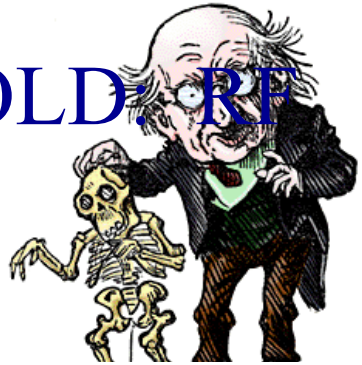
Recycling! ERL: Perpetum Mobile of Accelerators

In eRHIC ERL 20GeV, 500mA beam will have reactive power of 10 GW !

Regular linac - RF transmitter alone will cost $5/W -> $50,000M

Hence - ENERGY RECOVERY IS THE MUST

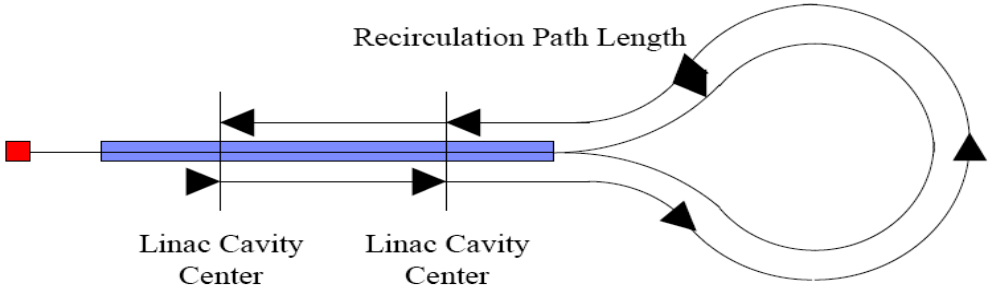
NEW is ONLY a WELL FORGOTEN OLD: RF Energy Recovery Linacs

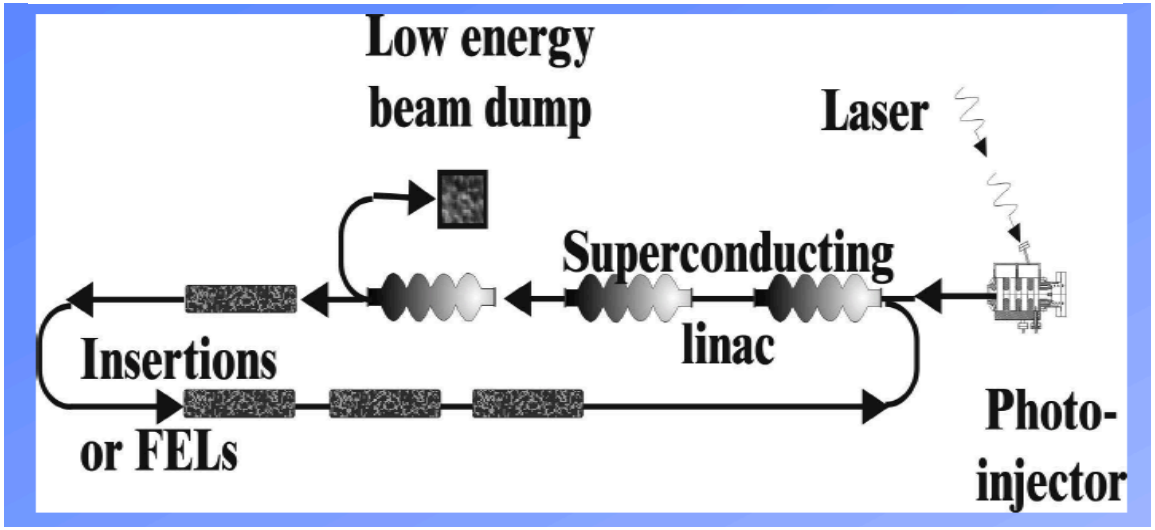


INVENTOR:
M. Tigner(1965) -
Nuovo Cimento 37 1228

RF ERL suggested

followed by Stanford,
BINP, Jefferson Lab,
JAERI, BNL,Cornell,
LBNL, Daresbury
and more ...



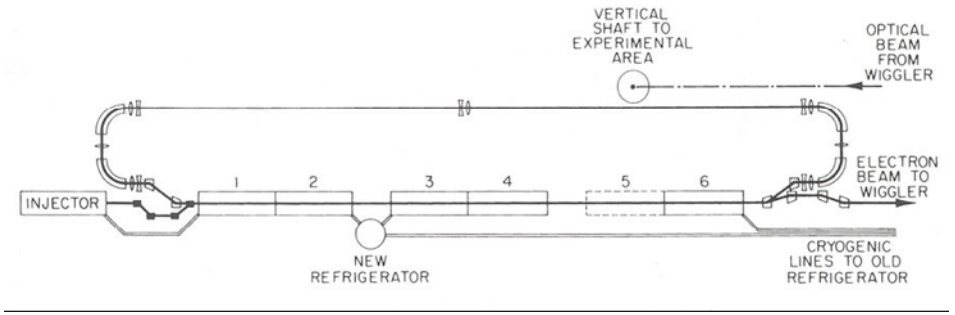


First Tests

S.O. Schreiber and E.A. Heighway (Chalk River)*Double Pass Linear Accelerator,* IEEE NS-22 (1975) (3) 1060-1064
D.W. Feldman et al, (LANL) *Energy Recovery in the LANL FEL* NIM A259 (1987) 26-30
T.I. Smith et al, (Stanford University) *Development of the SCA/FEL* ...NIM A259 (1987) 1-7

- Same-cell energy recovery was first (?) demonstrated in a superconducting linac at the SCA/FEL in July 1986
- Beam was injected at 5 MeV into a ~50 MeV linac (up to 95 MeV in 2 passes),
- 150 µA average current (12.5 pC per bunch at 11.8 MHz)

T.Smith et all, Stanford U



MUSL Univ of Illinois, 1977
SCA, Stanford, 1986
S-DALINAC, 1990
CEBAR, 1995
IR FEL Jlab, 1999
JAERI, 2002

D.W. Feldman et al,
Energy Recovery in the Los Alamos FEL

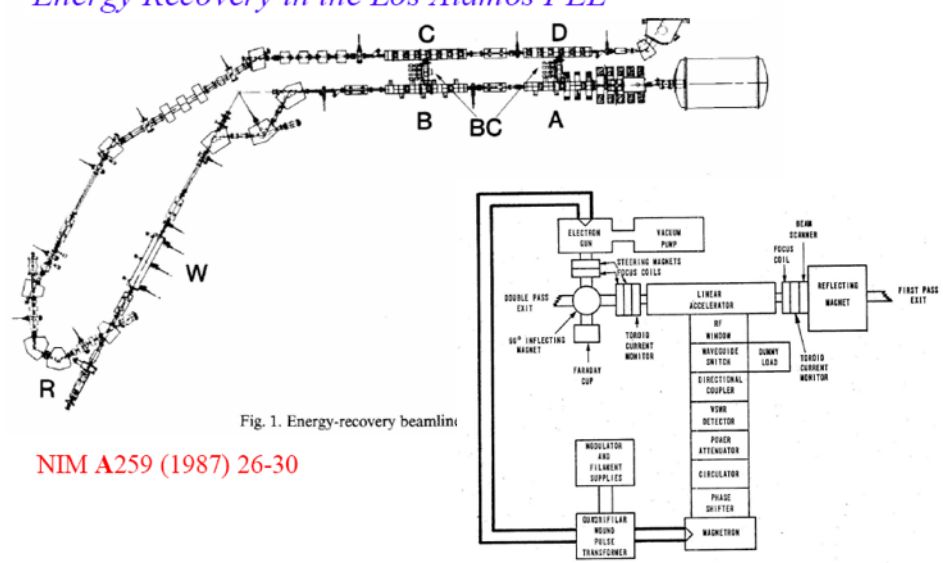
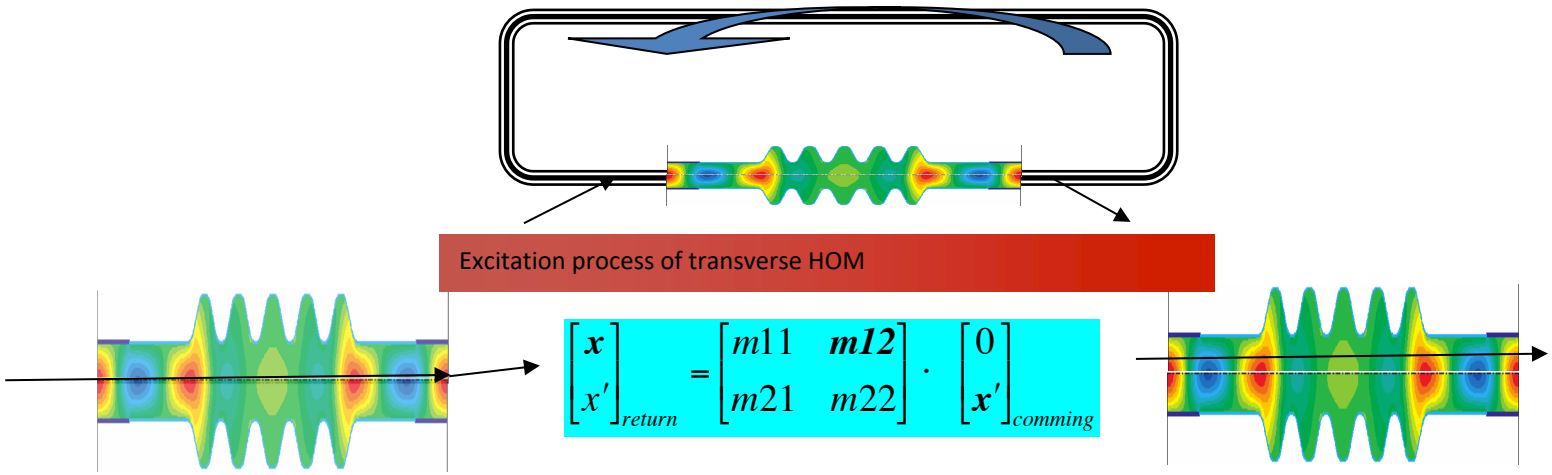


Fig. 1. Energy-recovery beamlin

NIM A259 (1987) 26-30

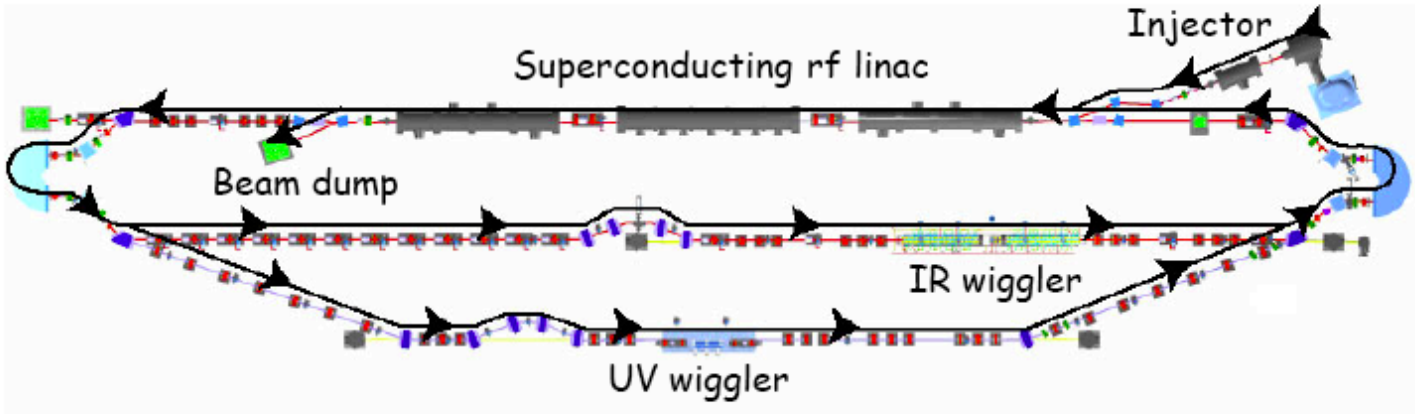
Transverse Beam-Break-Up instability in RLAs and ERLS



$$I_{th}^{(1)} = \frac{-2 p_r c}{e(R/Q)_m Q_m k_m M_{ij} \sin(\omega_m t_r + l\pi/2)}$$

JLAb: 160 MeV ERL

JLab 10kW IR FEL and 1 kW UV FEL

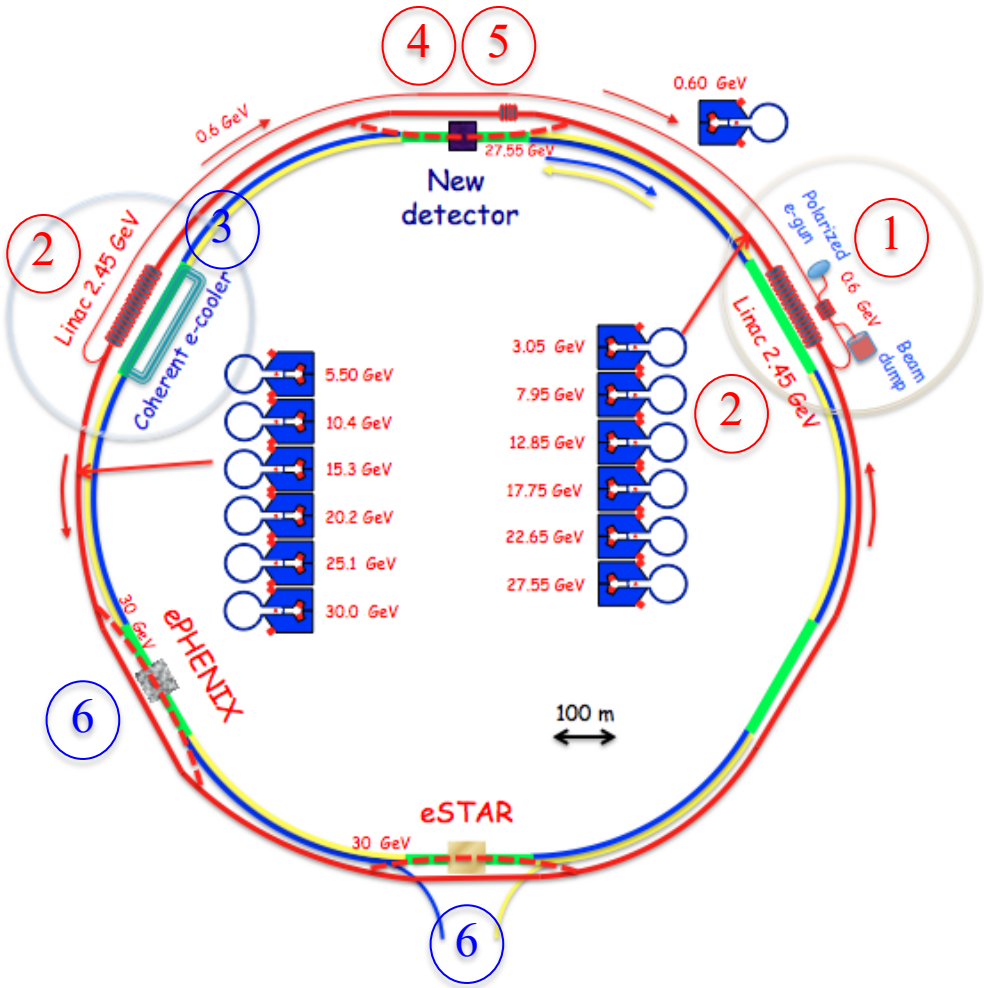


Output Light Parameters	IR	UV
Wavelength range (microns)	1.5 - 14	0.25 - 1
Bunch Length (FWHM psec)	0.2 - 2	0.2 - 2
Laser power / pulse (microJoules)	100 - 300	25
Laser power (kW)	>10	> 1
Rep. Rate (cw operation, MHz)	4.7 – 75	4.7 – 75

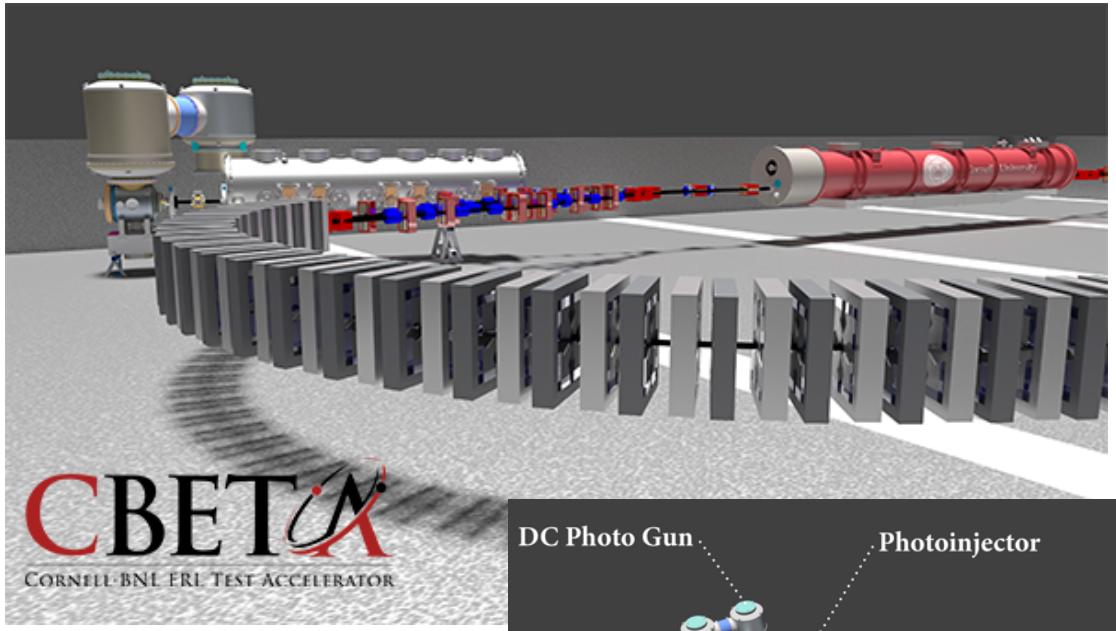
Electron Beam Parameters	IR	UV
Energy (MeV)	80-200	200
Accelerator frequency (MHz)	1500	1500
Charge per bunch (pC)	135	135
Average current (mA)	10	5
Peak Current (A)	270	270
Beam Power (kW)	2000	1000
Energy Spread (%)	0.50	0.13
Normalized emittance (mm-mrad)	<30	<11
Induced energy spread (full)	10%	5%

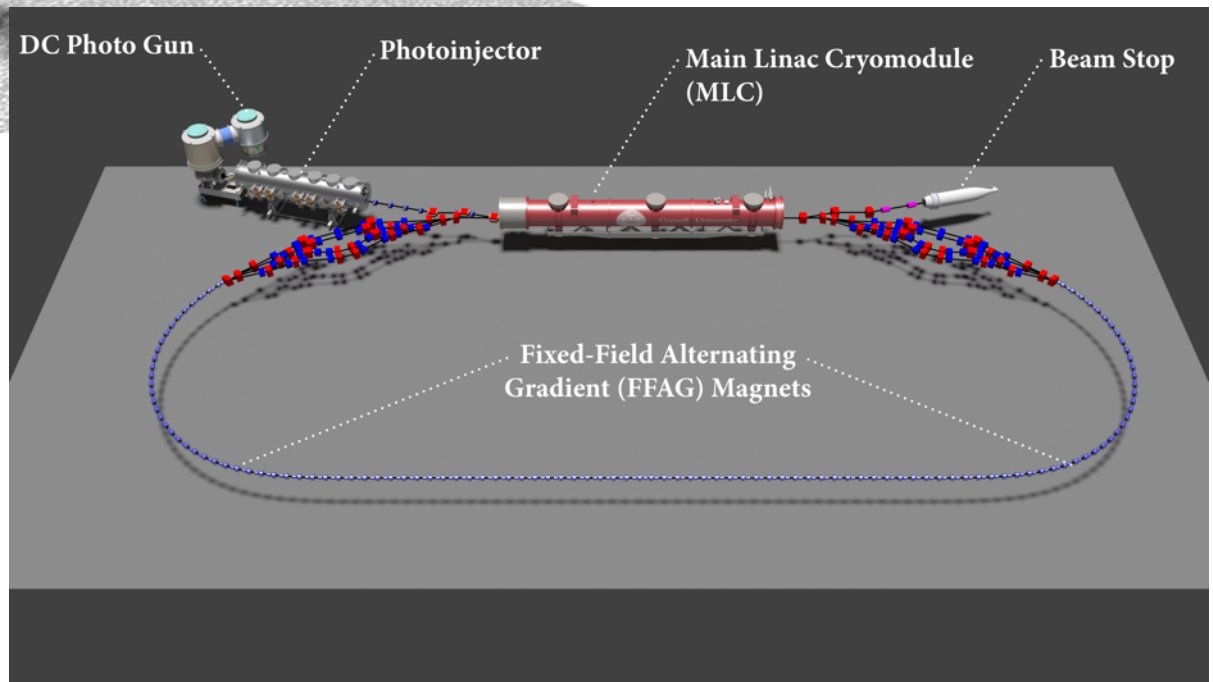
S. Benson et al, *High power lasing in the IR upgrade at Jefferson Lab*, 2004 FEL Conference Proceedings, 229-232.

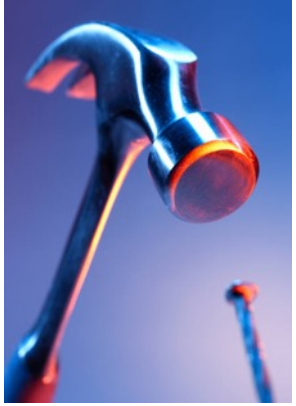
Project of ERL based electron-ion collider - eRHIC



C-BETA project: Cornell U & BNL







RF accelerators

- RF accelerators are working horse in most of modern high energy facilities
- Variety of RF accelerators is rather large
- There are superconducting and normal conduction RF cavities
- Superconducting RF cavities can have quality factor a million times higher than that of best room-temperature Cu cavities.
- There is a number of critical parameters characterizing accelerating cavities:

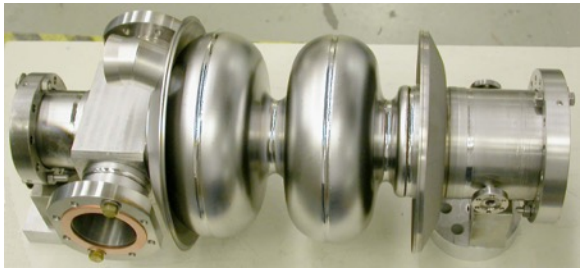
$$V_{rf}\,,\ E_{peak}\,,\ H_{peak},\ R_s,\ Q_0,\ Q_{ext},\ R/Q,\ G,\ R_{sh}\ldots$$

- In a multi-cell cavity every eigen mode splits into a pass-band. The number of modes in each pass-band is equal to the number of cavity cells.
- Coaxial lines and rectangular waveguides are commonly used in RF systems for power delivery to cavities
- Energy-recovery linacs represent a new and promising direction for very high power energy efficient accelerators

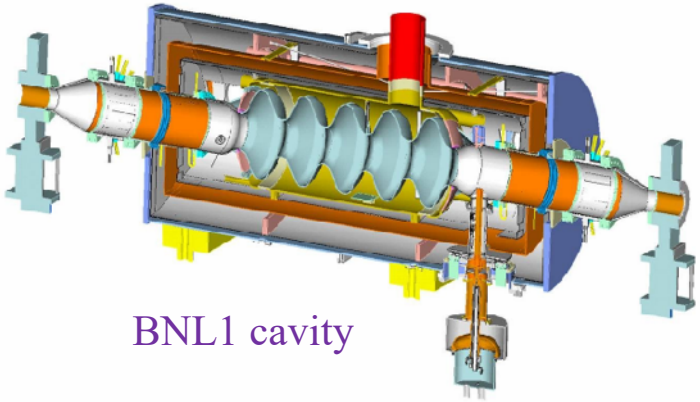
Typical requirements

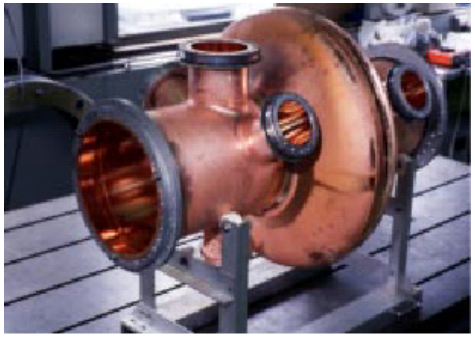
	Examples	Accelerating gradient	RF power	HOM damping
Pulsed linacs	SNS, XFEL, ILC	High (> 20 MV/m)	High peak (> 250 kW), low average (~ 5 kW)	Moderate ($Q = 10^4 \ldots 10^6$)
Low-current CW linacs	CEBAF, JLab FEL, ELBE	Moderate to low (8…20 MV/m)	Low (5…15 kW)	Relaxed
High-current ERLs	Cornell ERL, BERLinPro, Coherent electron cooler for RHIC, eRHIC ERLs	Moderate (15…20 MV/m)	Low (few kW)	Strong ($Q = 10^2 \ldots 10^4$)
High-current ERL injectors	Cornell ERL injector, JLab FEL 100 mA injector, injectors for BNL ERLs	Moderate to low (5…15 MV/m)	High (50…500 kW)	Strong ($Q = 10^2 \ldots 10^4$)
High-current storage rings	CESR, KEKB, LHC, RHIC, light sources (CLS, TLS, BEPC-II, SOLEIL, DIAMOND, SRRF, NSLS-II, TPS, PLS-II)	Low (5…10 MV/m)	High (up to 400 kW)	Strong ($Q \sim 10^2$)

Also: SRF guns, crab and deflecting cavities, harmonic cavities.



Cornell ERL injector cavity



BNL1 cavity

LHC cavity