1D vs 3D model for microbunched electron cooling

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Introduction

- Up to this point our, theoretical analysis of microbunched electron cooling (MBEC) has relied exclusively on a **quasi-1D, charged disk model** for the space charge interaction.
- According to this approximation:

✓ *the beam (hadron or electron) is subdivided into* ***thin, co-axial disks****.*

✓ *For purposes of simulation, each such disk is considered to be a separate macro-particle executing* ***1D motion****.*

✓ *The (longitudinal) interaction force between two disks can be calculated analytically by averaging the force between the constituent point charges.*

- The disk model is very useful as a starting point and quite versatile, allowing us to develop an **analytical theory** of MBEC and to perform **fast calculations** of the cooling timescales.

- However, even though it incorporates some 3D effects (such as the finite transverse size of the hadron/electron beams), it remains only an **approximate tool**:

✓ *important effects such as the **angular spread** of the beam and **betatron oscillations** are not taken into consideration.*

✓ *as a result, the **plasma oscillations** (crucial for the gain amplification) can only be analyzed in the 1D regime.*

✓ *Last but not least, beams are not composed of disks but of **point charges**, which interact via the Coulomb force (in the rest frame).*

3D model – simulation & theory

- In what follows, we present results from a realistic, **fully 3D model for MBEC** (with point charge macro-particles that are free to move in a 6D phase space). More details in EIC-ADD-TN-021.
- A two-track approach was followed:

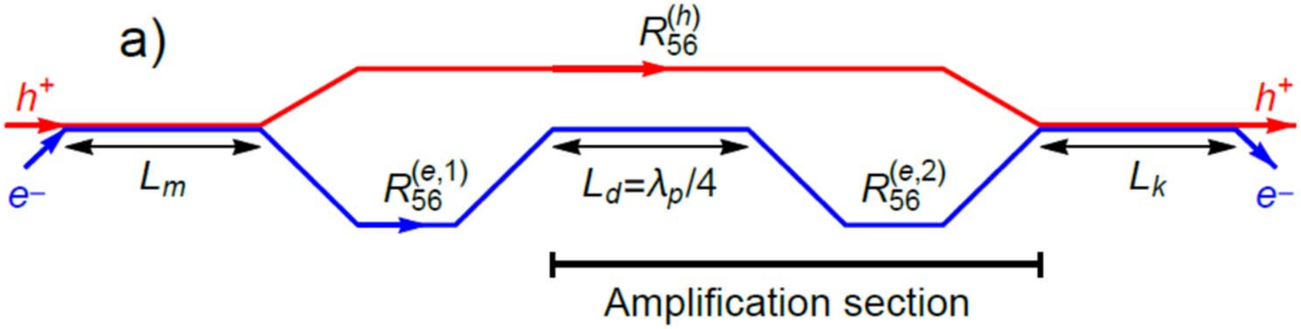
✓ *development of a* ***macro-particle simulation algorithm*** *(with tracking done in real space).*

✓ *development of a* ***frequency-domain, theoretical formalism*** *(starting from the linearized Vlasov equation).*

- Theory is used to benchmark the code, which can also deal with nonlinear (saturation) effects.

3D model – simulation & theory

- For simplification, we only consider one amplification stage.



- We also neglect plasma oscillations & beam size variation in the modulator & kicker sections.
- However, in the amplifications section, we rigorously include the e-beam angular spread & focusing in our treatment of plasma oscillations.

3D model - simulation & theory

- We seek to compare 3D theory and simulation by calculating a physically meaningful quantity.
- A good candidate is the 3D analogue to the **disk-to-disk wake** (strictly speaking a 1D quantity).
- First, one imagines immersing a thin disk of hadrons (point charges) into a "sea" of (point charge) electrons in the modulator. The disk is at $z = 0$.
- This creates a perturbation in the electron beam that can be propagated through the electron beam line.
- The perturbed e-beam acts back on a hadron disk ***displaced by z*** in the kicker. The average energy kick as a function of z gives the wake.
- This also allows for a direct comparison with the "old" 1D wake.

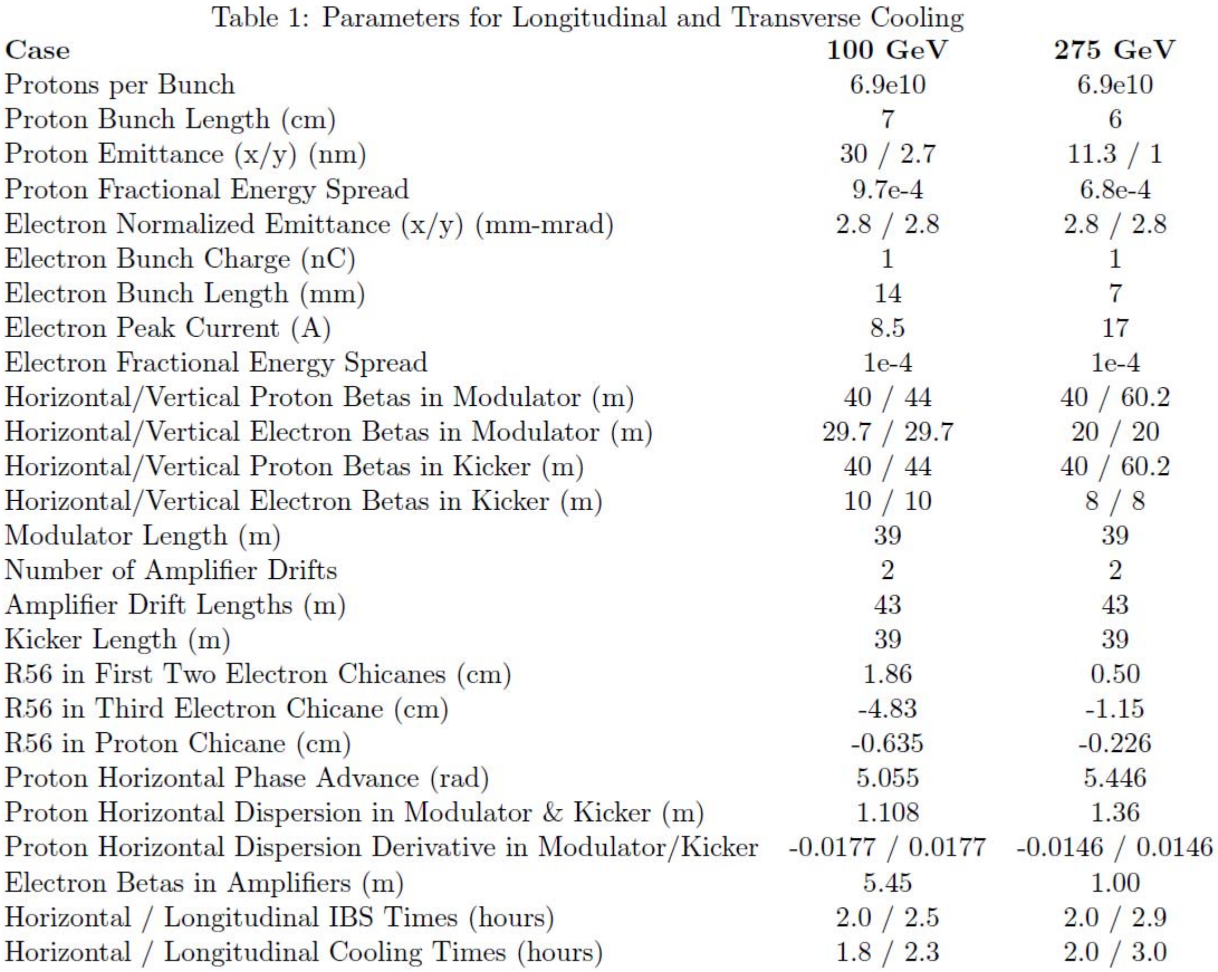


Table 1: Parameters for Longitudinal and Transverse Cooling

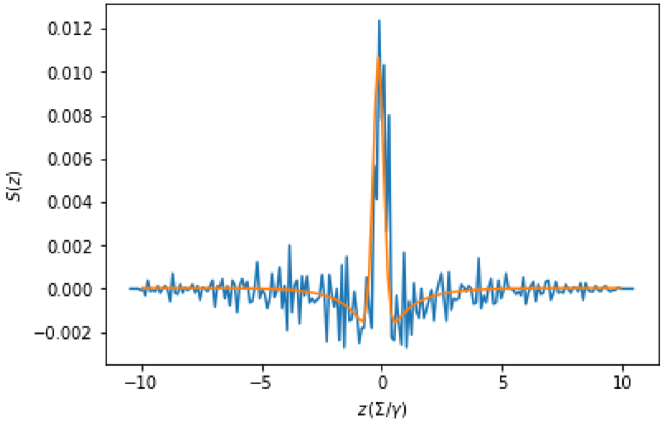
Case	**100 GeV**	**275 GeV**
Protons per Bunch	6.9e10	6.9e10
Proton Bunch Length (cm)	7	6
Proton Emittance (x/y) (nm)	30 / 2.7	11.3 / 1
Proton Fractional Energy Spread	9.7e-4	6.8e-4
Electron Normalized Emittance (x/y) (mm-mrad)	2.8 / 2.8	2.8 / 2.8
Electron Bunch Charge (nC)	1	1
Electron Bunch Length (mm)	14	7
Electron Peak Current (A)	8.5	17
Electron Fractional Energy Spread	1e-4	1e-4
Horizontal/Vertical Proton Betas in Modulator (m)	40 / 44	40 / 60.2
Horizontal/Vertical Electron Betas in Modulator (m)	29.7 / 29.7	20 / 20
Horizontal/Vertical Proton Betas in Kicker (m)	40 / 44	40 / 60.2
Horizontal/Vertical Electron Betas in Kicker (m)	10 / 10	8 / 8
Modulator Length (m)	39	39
Number of Amplifier Drifts	2	2
Amplifier Drift Lengths (m)	43	43
Kicker Length (m)	39	39
R56 in First Two Electron Chicanes (cm)	1.86	0.50
R56 in Third Electron Chicane (cm)	-4.83	-1.15
R56 in Proton Chicane (cm)	-0.635	-0.226
Proton Horizontal Phase Advance (rad)	5.055	5.446
Proton Horizontal Dispersion in Modulator & Kicker (m)	1.108	1.36
Proton Horizontal Dispersion Derivative in Modulator/Kicker	-0.0177 / 0.0177	-0.0146 / 0.0146
Electron Betas in Amplifiers (m)	5.45	1.00
Horizontal / Longitudinal IBS Times (hours)	2.0 / 2.5	2.0 / 2.9
Horizontal / Longitudinal Cooling Times (hours)	1.8 / 2.3	2.0 / 3.0

- **Use the evolving SHC parameters to check theory & simulation.**
- **Table from W. Bergan (July 30 2021)**
- **One modification: make M & K symmetrical in terms of beam parameters.**

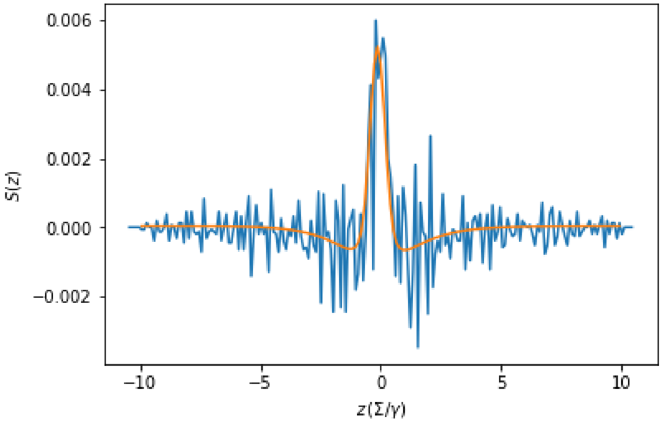
Theory vs simulation $- S = \delta n / n_0$

100 GeV parameters – Re2 = 1.86 cm

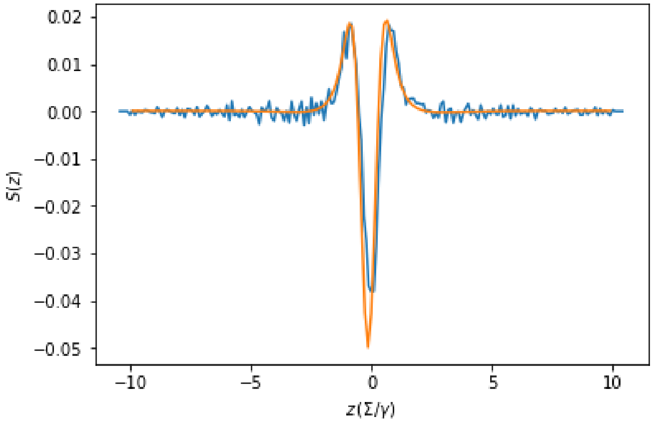
- Focus on segment of the beam
- Periodic boundary conditions



a) After the first chicane



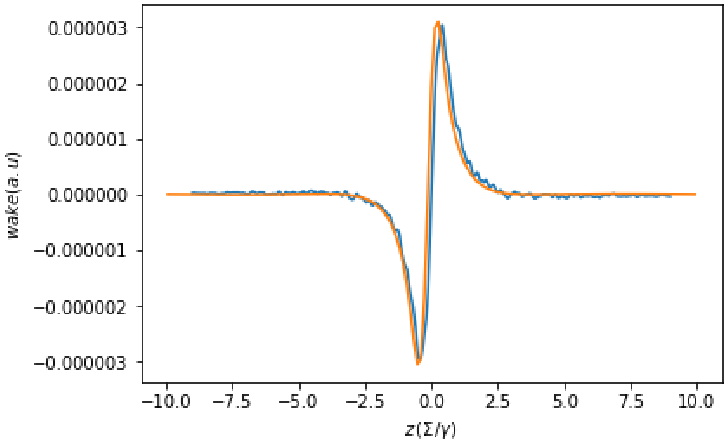
b) After the drift



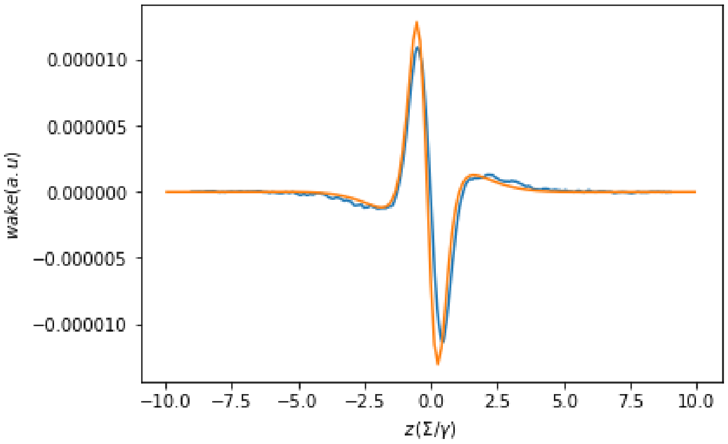
c) After the second chicane

Theory vs simulation – wake comparison

100 GeV parameters – Re2 = 1.86 cm



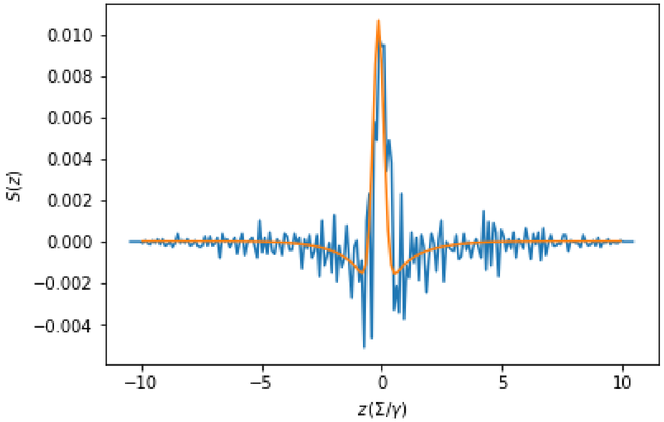
a) Without amplification



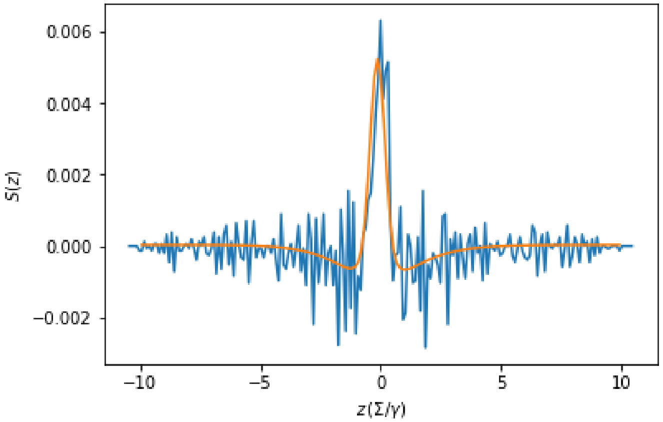
b) With amplification

Theory vs simulation – $S = \delta n/n_0$

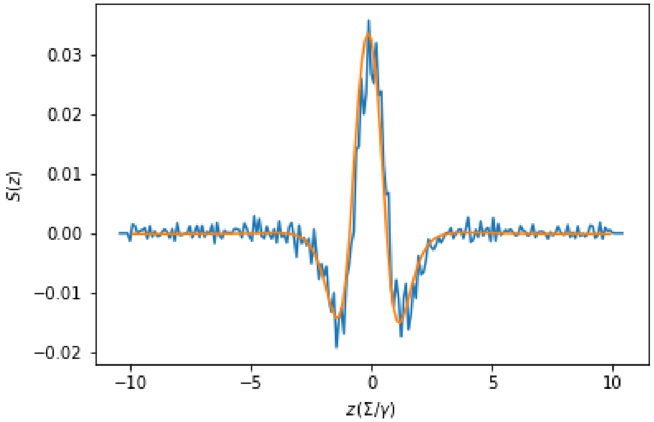
100 GeV parameters – Re2 = -4.83 cm



a) After the first chicane



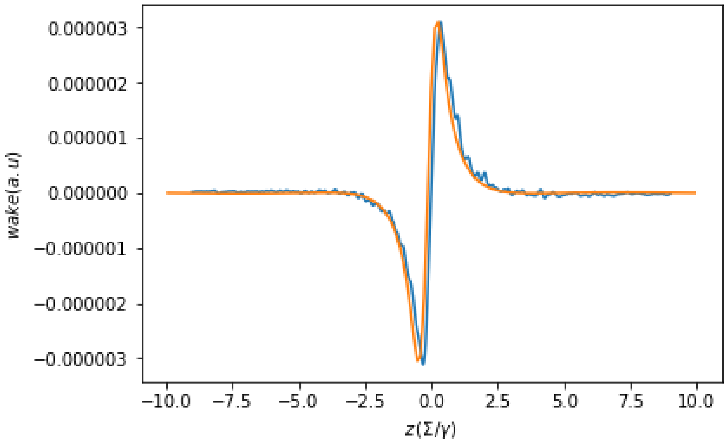
b) After the drift



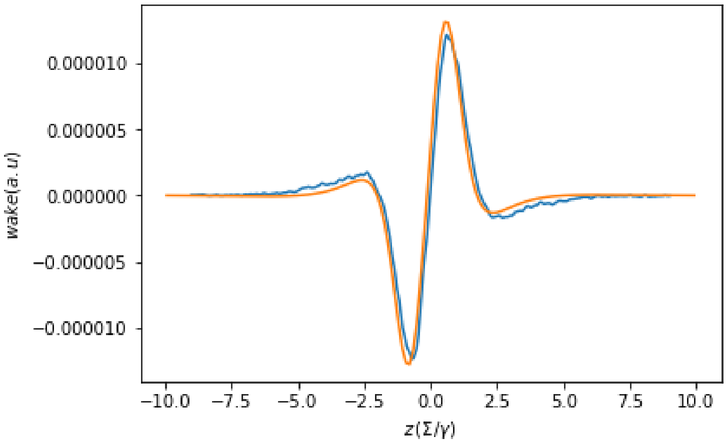
c) After the second chicane

Theory vs simulation – wake comparison

100 GeV parameters – Re2 = -4.83 cm

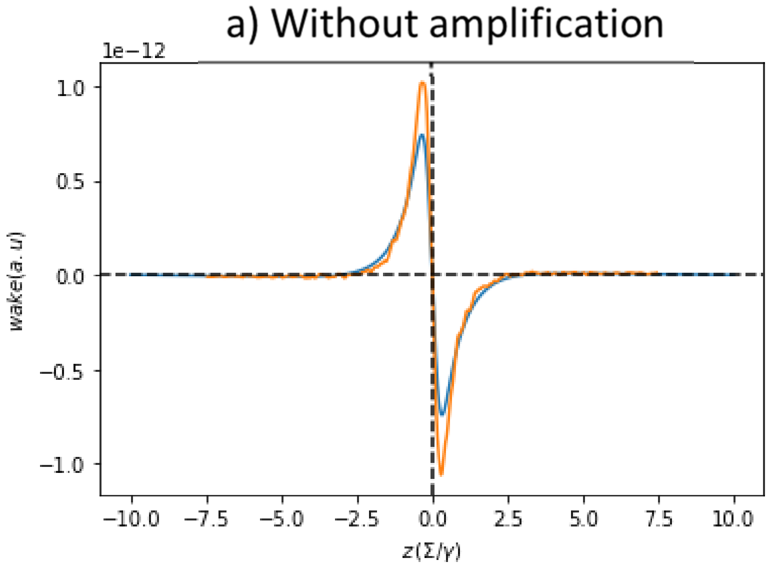


a) Without amplification

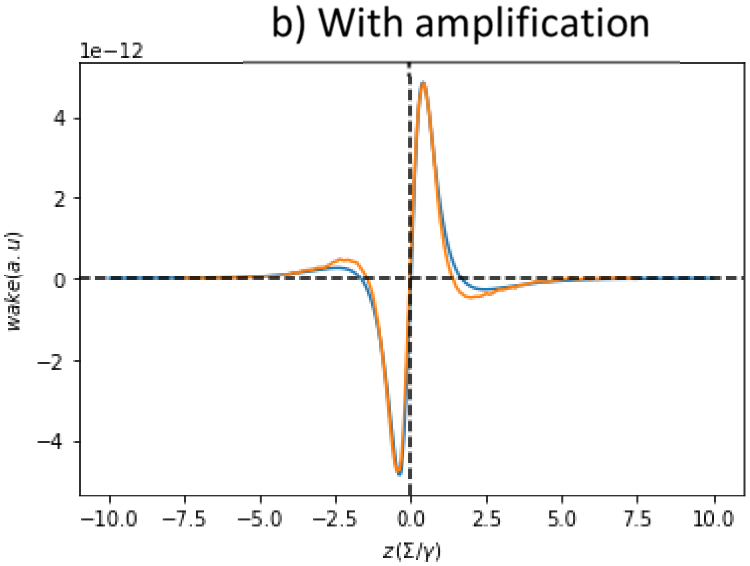


b) With amplification

3D vs disk model – 100 GeV/Re1=1.86 cm


a) Without amplification

disk model wakes by W. Bergan

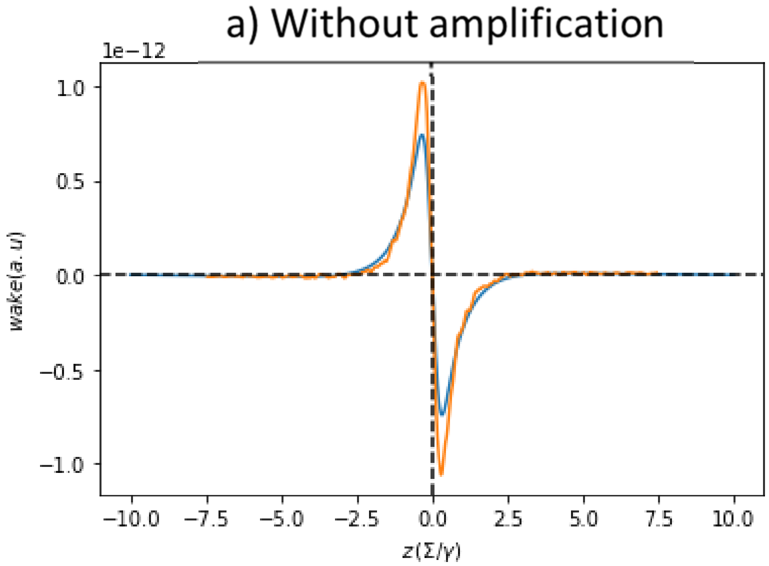

b) With amplification

(re-scaled wakes)

Compared to the disk model, the 3D calculation shows:

- *40% stronger wake without amplification*
- *30% smaller gain factor →* ***30% weaker 2-stage wake***

3D vs disk model – 100 GeV/Re1=-4.83 cm


a) Without amplification

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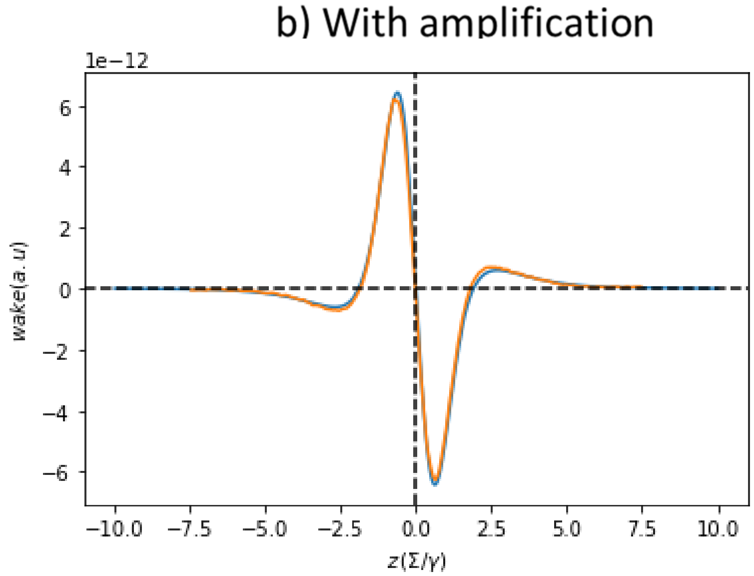
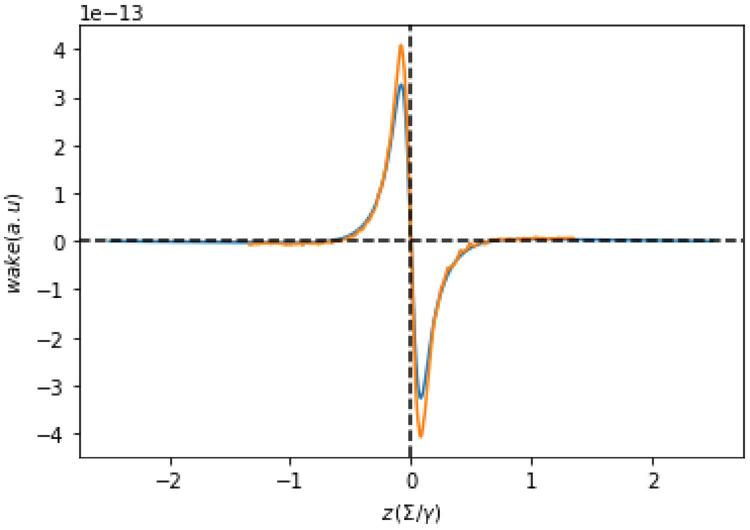

b) With amplification

Compared to the disk model, the 3D calculation shows:

- *40% stronger wake without amplification*
- *33% smaller gain factor →* ***33% weaker 2-stage wake***

3D vs disk model – 275 GeV/Re1=0.50 cm

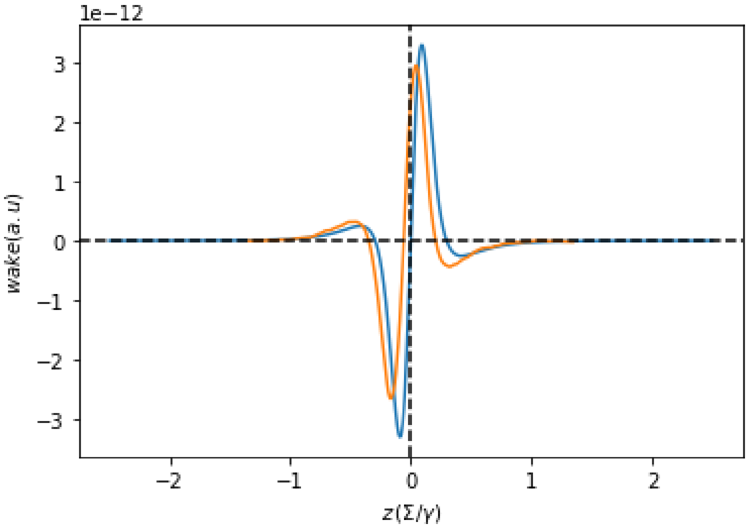
a) Without amplification



disk model wakes by W. Bergan

asymmetric 3D wake

b) With amplification

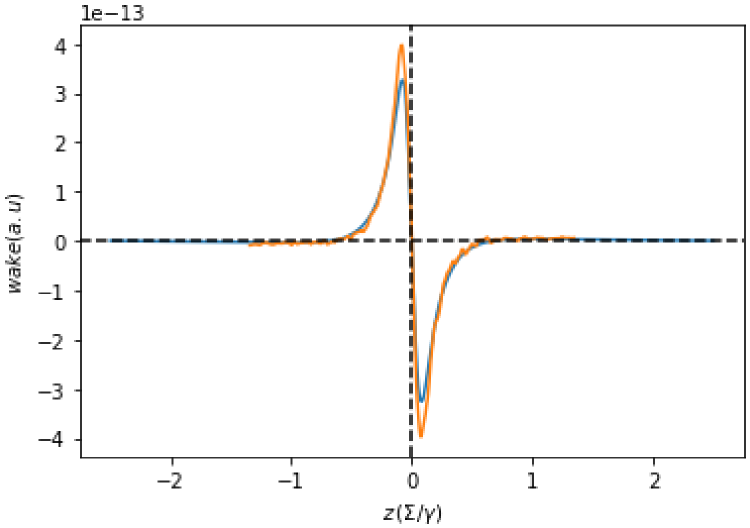


Compared to the disk model, the 3D calculation shows:

- *25% stronger wake without amplification*
- *33% smaller gain factor → **43% weaker 2-stage wake***

3D vs disk model – 275 GeV/Re1=-1.15 cm

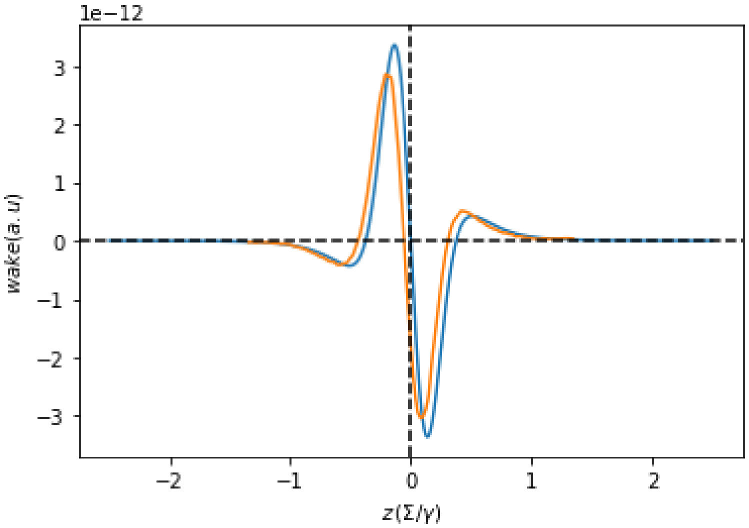
a) Without amplification



disk model wakes by W. Bergan

asymmetric 3D wake

b) With amplification



Compared to the disk model, the 3D calculation shows:

- *22% stronger wake without amplification*
- *28% smaller gain factor → **37% weaker 2-stage wake***

Summary

- We have developed theoretical and simulation tools for the study of microbunched electron cooling using a fully 3D model.
- This allows us to consider effects that were previously neglected, such as the e-beam angular spread and betatron oscillations.
- Good agreement has been observed between the simulation algorithm and the theoretical analysis.
- We have also compared the disk-to-disk wake calculated by the 3D analysis with its 1D counterpart.
- The 1D disk model appears to overestimate the 2-stage wake by a factor of up to 2 (for the specific EIC SHC parameter sets considered).
- This stresses the need to always benchmark with the 3D results.