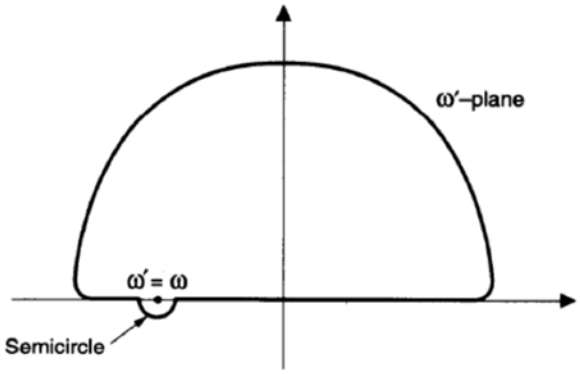
Homework 13.

1. (10 points) Derive the expression for the longitudinal loss factor of a bunch in terms of the impedance, $Z(\omega)$, and the bunch spectrum, $\tilde{\lambda}(\omega)=\frac{1}{c}\int_{-\infty}^{\infty}\lambda(s)e^{i\omega s/c}ds$.

2. (10 points) Perform a contour integral of $\frac{Z_{//}(\omega')}{\omega'-\omega}$ in the complex ω' plane over the upper half plane along the contour shown in the figure.



Show that if $Z_{//}(\omega')$ converges sufficiently fast as $|\omega'|\to\infty$,

$$Z_{//}(\omega)=-\frac{i}{\pi}P.V.\int_{-\infty}^{\infty}\frac{Z_{//}(\omega')}{\omega'-\omega}d\omega', \qquad (1)$$

and eq. (1) leads to Kramers-Kronig relations.

$$\mathrm{Re}\left[Z_{//}(\omega)\right]=\frac{1}{\pi}P.V.\int_{-\infty}^{\infty}\frac{\mathrm{Im}\left[Z_{//}(\omega')\right]}{\omega'-\omega}d\omega'$$

$$\mathrm{Im}\left[Z_{//}(\omega)\right]=-\frac{1}{\pi}P.V.\int_{-\infty}^{\infty}\frac{\mathrm{Re}\left[Z_{//}(\omega')\right]}{\omega'-\omega}d\omega'$$